Thermal Properties of the Glueball Gas

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Motivation

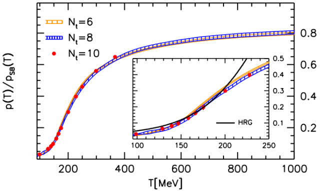
- Considering the Hadron Resonance Gas (HRG) model

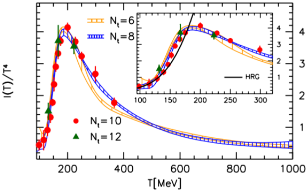
$$\frac{p^{HRG}}{T^4} = \frac{1}{VT^3}\left(\sum_{i\in\text{mes}} \log \mathcal{Z}^M(T,V,\mu_{X^a},m_i) + \sum_{i\in\text{bar}} \log \mathcal{Z}^B(T,V,\mu_{X^a},m_i)\right)$$

- Partition function for mesons and baryons $z_i := \exp\left\{\frac{1}{T}\sum_a X_i^a \mu_{X^a}\right\}$

$$\log \mathcal{Z}^{M,B}(T,V,\mu_{X^a},m_i) := \mp V d_i \int_0^\infty \frac{dk k^2}{2\pi^2} \log\left(1 \mp z_i e^{-\frac{\sqrt{k^2+m_i^2}}{T}}\right)$$

- All known baryons and mesons up to 2.5 GeV from PDG within HRG model [Borsnayi et al.JHEP11(2010)077]





- **How about pure Yang-Mills sector of QCD within Gluebal Resonance Gas=GRG?**

Lattice Mass Spectrum of Glueball

- [Chen et.al.arXiv:hep-lat/0510074]
- [Andreas Athenodorou and Michael Teper arXiv:hep-lat/2007.06422]
- [Harvey B. Meyer arXiv:hep-lat/0508002]

$n\,J^{PC}$	M[MeV]			$n\,J^{PC}$	M[MeV]		
	Chen et.al.	A & T	Meyer		Chen et.al.	A & T	Meyer
$1\,0^{++}$	1710(50)(80)	1651(23)	1475(30)(65)	$1\,1^{--}$	3830(40)(190)		3240(330)(150)
$2\,0^{++}$		2840(40)	2755(30)(120)	$1\,2^{--}$	4010(45)(200)	3920(90)	3660(130)(170)
$3\,0^{++}$	3650(60)		3370(100)(150)	$2\,2^{--}$			3740(200)(170)
$4\,0^{++}$			3990(210)(180)	$1\,3^{--}$	4200(45)(200)	4030(70)	4330(260)(200)
$1\,2^{++}$	2390(30)(120)	2376(32)	2150(30)(100)	$1\,0^{+-}$	4780(60)(230)		
$2\,2^{++}$		3300(50)	2880(100)(130)	$1\,1^{+-}$	2980(30)(140)	2944(42)	2670(65)(120)
$1\,3^{++}$	3670(50)(180)	3740(70)	3385(90)(150)	$2\,1^{+-}$		3800(60)	
$1\,4^{++}$		3690(80)	3640(90)(160)	$1\,2^{+-}$	4230(50)(200)	4240(80)	
$1\,6^{++}$			4360(260)(200)	$1\,3^{+-}$	3600(40)(170)	3530(80)	3270(90)(150)
$1\,0^{-+}$	2560(35)(120)	2561(40)	2250(60)(100)	$2\,3^{+-}$			3630(140)(160)
$2\,0^{-+}$		3540(80)	3370(150)(150)	$1\,4^{+-}$		4380(80)	
$1\,2^{-+}$	3040(40)(150)	3070(60)	2780(50)(130)	$1\,5^{+-}$			4110(170)(190)
$2\,2^{-+}$		3970(70)	3480(140)(160)				
$1\,5^{-+}$			3942(160)(180)				
$1\,1^{-+}$		4120(80)					
$2\,1^{-+}$		4160(80)					
$3\,1^{-+}$		4200(90)					

- Glueballs being predictions of QCD can be considered as hadronic d.o.f
- $T < T_c$ compare GRG to the Thermodynamic LQCD results for pure YM
- The role of the interaction of scalar glueball $\longrightarrow$ Validity of GRG?

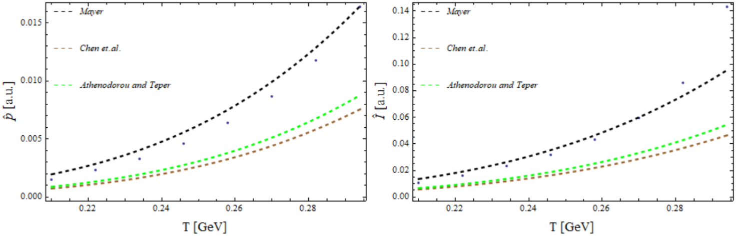
GRG vs LQCD in [Borsnayi et al.JHEP07(2012)056]

- Pressure and energy density

$$\hat{p} = -T \sum_J (2J+1) \int_0^\infty \frac{k^2}{2\pi^2} \ln\left(1 - e^{-\frac{\sqrt{k^2+m^2}}{T}}\right) dk$$

$$\hat{\rho} = (2J+1) \int_0^\infty \frac{k^2}{2\pi^2} \frac{\sqrt{k^2+m^2}}{\exp\left[\frac{\sqrt{k^2+m^2}}{T}\right] - 1} dk$$

- Trace anomaly $\hat{I} = \hat{p} - 3\hat{\rho}$.



- Consider $T_c \approx 0.3\text{GeV}$ [Braun et.al. Eur. Phys.J.C 70, 689 (2010).]

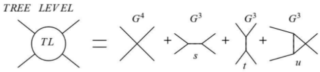
Glueball-glueball scattering

- Scalar field G that describes both the scalar glueball and the trace anomaly

$$\mathcal{L}_{\text{dil}} = \frac{1}{2}\left(\partial_\mu G\right)^2 - \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left(G^4 \ln\left|\frac{G}{\Lambda_G}\right| - \frac{G^4}{4}\right).$$

- Amplitude can be decomposed in terms of the Legendre polynomials

$$A(s,\cos\theta) = \sum_{\ell=0}^{\infty}(2\ell+1)A_\ell(s)P_\ell(\cos\theta)$$



- Partial waves in terms of phase shift

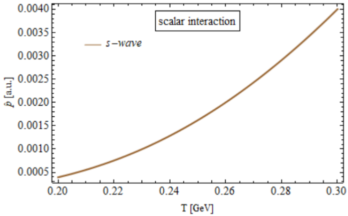
$$\frac{e^{2i\delta_\ell(s)}-1}{2i} = \frac{k}{16\pi\sqrt{s}}A_\ell(s) \longrightarrow \delta_\ell(s) = \frac{1}{2}\arg\left[1+2i\frac{k}{16\pi\sqrt{s}}A_\ell(s)\right]$$

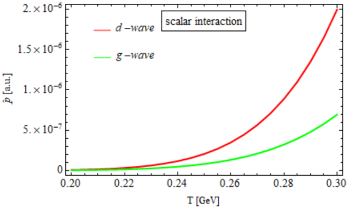
- Unitarized Amplitude $A_\ell^U(s) = \left(A_\ell^{-1}(s) - \Sigma(s)\right)^{-1}$ [Giacosa et.al. arXiv:2110.05582]

Scalar Glueball Interaction

- Phase shift approach to the interaction pressure [Dashen et.al Phys.Rev.187.345 (1969)],[Dashen and Rajaraman Phys.Rev.D10.694 (1974)], [Samanta and Giacosa arXiv:2110.14752]

$$P^{\text{int}} + P_B = -T \sum_{l=0}^{\infty} \int_0^{\infty} dx \frac{2l+1}{\pi} \frac{d\delta_l^U(s=x^2)}{dx} \int \frac{d^3k}{(2\pi)^3} \ln\left(1 - e^{-\beta \frac{\sqrt{k^2+x^2}}{T}}\right)$$





- Glueballonium $\Lambda_G \approx 0.4$GeV with mass 3.37GeV [Trotti et.al. arXiv:2112.15010]
- Contribution from interaction appears to be comparable less than free case

Thank you for the attention!