Forward quark jet-nucleus scattering in a light-front Hamiltonian approach

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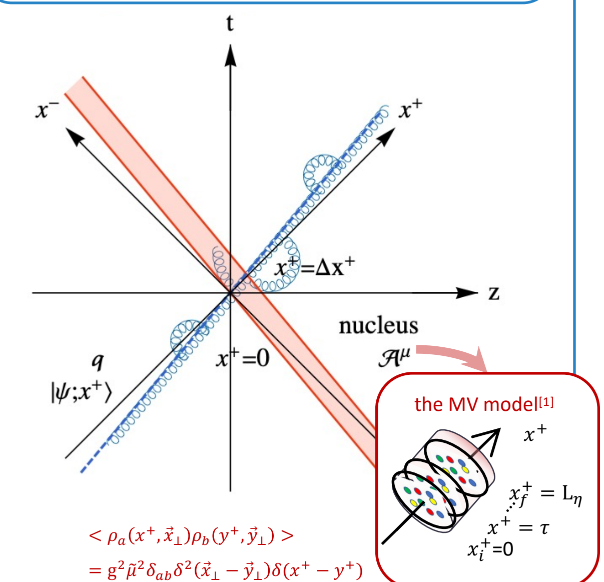
I. Introduction

We aim to understand the interplay of medium interaction and gluon emission in the quark-nucleus scattering, beyond the eikonal limit, and nonperturbatively.



A high-energy quark moving in the positive z direction, scattering on a high-energy nucleus moving in the negative z direction.

II. Methodology

❖ The light-front Hamiltonian

We derive the light-front Hamiltonian from the QCD Lagrangian with a background field of a Color Glass Condensate $\mathcal{A}_\mu$,

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}{}_a F^a_{\mu\nu} + \overline{\Psi}(i\gamma^\mu D_\mu - m)\Psi$$

where $D_\mu = \partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$. In the $|q\rangle + |qg\rangle$ space, it includes the kinetic energy, the interaction with the background field, and gluon emission/absorption:

$$\mathrm{P}^-(x^+) = \mathrm{P}^-_{\mathrm{KE}} + V(x^+), \qquad V(x^+) = V_{qg} + V_{\mathcal{A}}(x^+)$$

❖ A nonperturbative time evolution

We solve the time-evolution equation in the interaction picture

$$\frac{1}{2}V_I(x^+)|\psi; x^+\rangle_{\mathrm{I}} = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_{\mathrm{I}}$$

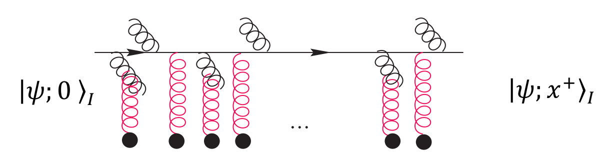
P^-_{KE} as a phase factor: $|\psi; x^+\rangle_{\mathrm{I}} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}|\psi; x^+\rangle, \mathrm{V_I}(\mathrm{x}^+) = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}V(x^+)e^{-\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}x^+}$

Evolution as **a product of many small timesteps**[2,3], each containing two operations,

$$|\psi; x^+\rangle_{\mathrm{I}} = \lim_{n\to\infty}\prod_{k=1}^{n}\underbrace{\mathcal{T}_+ \exp\{-\tfrac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_I(z^+)\}}|\psi; 0\rangle_{\mathrm{I}}$$

$\mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_{\mathcal{A},I}(z^+)\}$
matrix exponential in coordinate space

$\times\ \mathcal{T}_+ \exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k} dz^+ V_{qg,I}(z^+)\}$
4th-order Runge-Kutta



❖ The basis representation

The quark state is expanded in the basis space, and the information of the state is encoded in the wavefunction as **basis coefficients**:

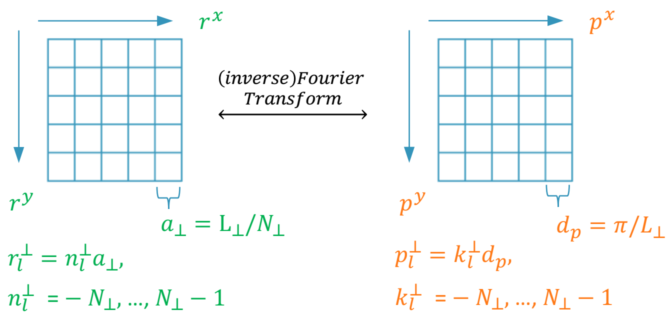
$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle$$

We choose the momentum states as the basis states[3]:

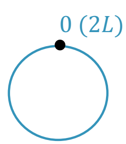
$$P^-_{\rm KE}|\beta\rangle = P^-_\beta|\beta\rangle$$

i. Each single particle state carries five quantum numbers: $\beta_l = \{k^x_l, k^y_l, k^+_l, \lambda_l, c_l\}, (l = q, g)$, the transverse momenta, the longitudinal momentum, helicity, and color; and the basis states in the transverse coordinate space $\bar{\beta}_l = \{n^x_l, n^y_l, k^+_l, \lambda_l, c_l\}$

- The transverse coordinate space is discretized on a lattice $[-L_\perp, L_\perp]$ with periodic boundary conditions:



- The longitudinal space is circle of $2L$ with (anti)periodic boundary condition for the gluon (quark):



$x^- = [0, 2L]$

$$p^+_l = \frac{2\pi}{L} k^+_l, \quad \begin{cases} k^+_q = \frac{1}{2}, \frac{3}{2}, \ldots, K + \frac{1}{2} \\ k^+_g = 1, 2, \ldots, K \end{cases}$$

ii. In each Fock sector, the many-particle basis states are direct products of single particle states:

$$|q\rangle: |\beta_q\rangle; \quad |qg\rangle: |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$$

III. Time evolution [PRD 104, 056014 (2021)]
(a) gluon emission

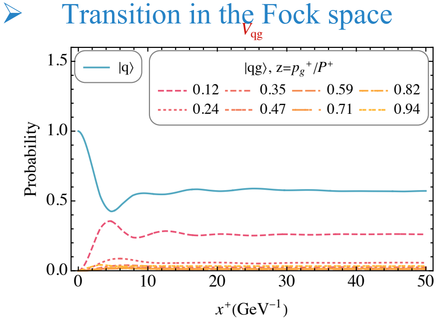
The interaction contains the gluon emission/absorption term:

$$V_I(x^+) = e^{\frac{i}{2}P^-_{KE} x^+} V_{qg} e^{-\frac{i}{2}P^-_{KE} x^+}$$

The initial state is a single quark state. It enables us to test the physical effects of different parts of the Hamiltonian in a clean and tractable setup.

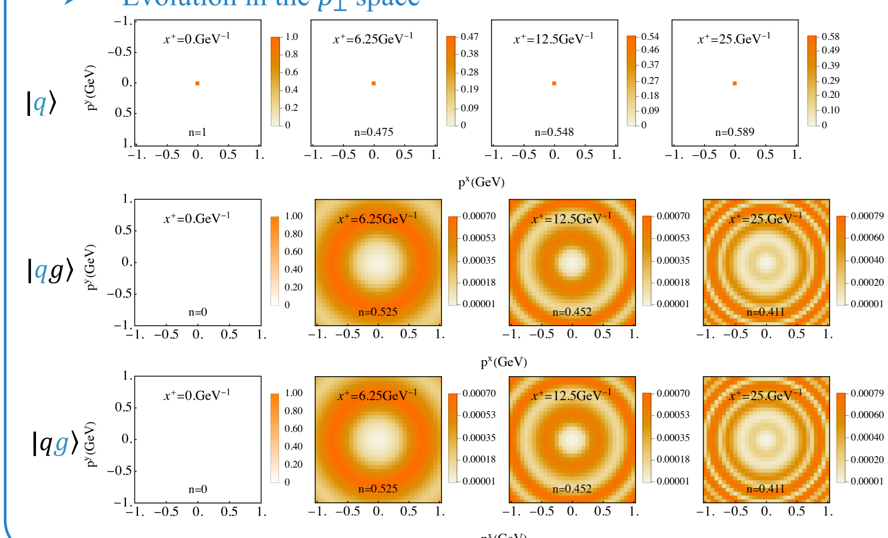
$$|q\rangle \text{ with } p^+ = 8.5\,GeV,\ \vec{p}_\perp = \vec{0}_\perp,\ \lambda_q = \uparrow,\ c_q = 1$$

➢ Transition in the Fock space



The probability of the initial $|q\rangle$ state decreases and that of the $|qg\rangle$ sector increases, indicating gluon radiations.

➢ Evolution in the $\vec{p}_\perp$ space



III. Time evolution [PRD 104, 056014 (2021)]
(b) medium interaction

The interaction contains the background field term :

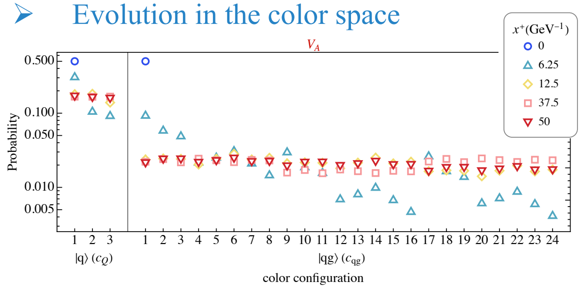
$$V_I(x^+) = e^{\frac{i}{2}P^-_{KE}x^+} V_{\mathcal{A}}(x^+) e^{-\frac{i}{2}P^-_{KE}x^+}$$

The initial state is a mixture of a single quark and a one-gluon-dressed quark states,

$|q\rangle(with\ \vec{p}_\perp = \vec{0}_\perp, p^+ = 8.5\ \text{GeV}) + |qg\rangle\ (with\ \vec{p}_{q,\perp} = \vec{p}_{g,\perp} = \vec{0}_\perp, p^+_q = 0.5\ \text{GeV}, p^+_g = 8\ \text{GeV}, \lambda_q = \uparrow,\ c_q = 1, \lambda_g = \uparrow,\ c_g = 1)$

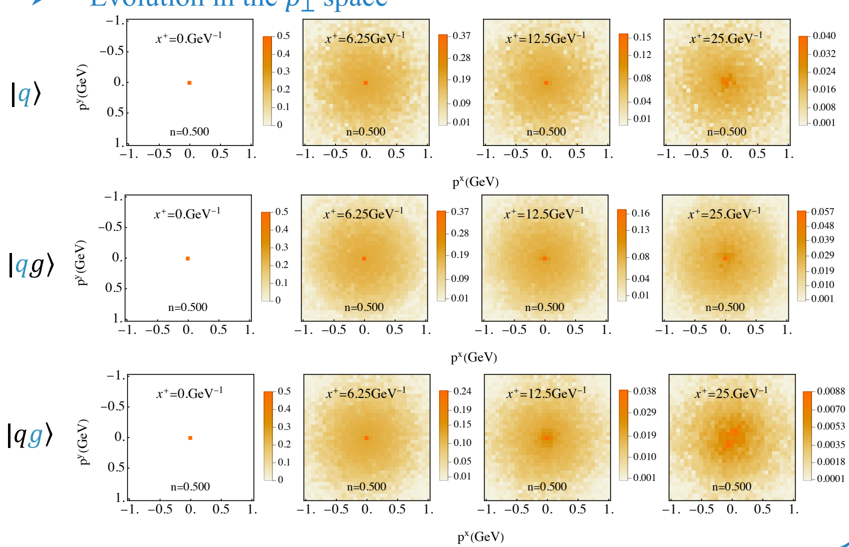
The charge density of the background field, $g^2\tilde{\mu} = 0.108\ \text{GeV}^{3/2}$

➢ Evolution in the color space



The background field leads to a color rotation.

➢ Evolution in the $\vec{p}_\perp$ space



(c) full interaction

The interaction contains the gluon emission/absorption, and the background field term

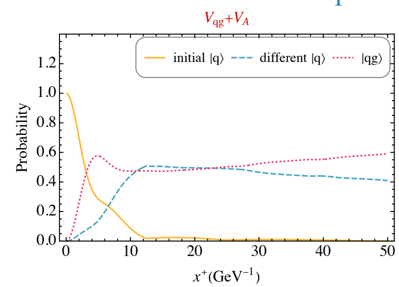
$$V_I(x^+) = e^{\frac{i}{2}P^-_{KE}x^+}[V_{qg} + V_{\mathcal{A}}(x^+)] e^{-\frac{i}{2}P^-_{KE}x^+}$$

The initial state is a single quark state,

$|q\rangle$ *with* $p^+ = 8.5\ \text{GeV},\ \vec{p}_\perp = \vec{0}_\perp,\ \lambda_q = \uparrow,\ c_q = 1$

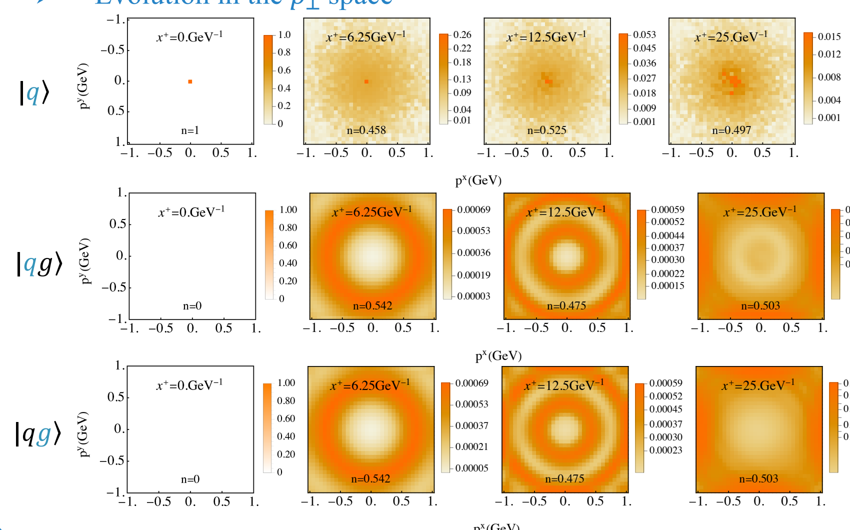
The charge density of the background field, $g^2\tilde{\mu} = 0.144\ \text{GeV}^{3/2}$

➢ Transition in the Fock space



The probability of the initial $|q\rangle$ state decreases and that of the $|qg\rangle$ sector increases, and different momentum state appear.

➢ Evolution in the $\vec{p}_\perp$ space



IV. Momentum broadening (a) bare quark $|q\rangle$

We have seen that the quark would transfer to different momentum modes through the interaction with the background field. The momentum broadening can be characterized by the quenching parameter,

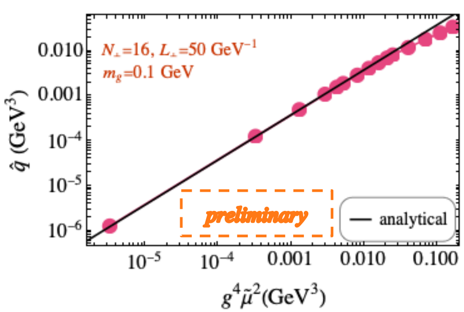
$$\hat{q} \equiv \frac{d\langle p_\perp^2(x^+)\rangle}{dx^+}$$

In the eikonal limit, the quenching parameter for a bare quark can be calculated analytically

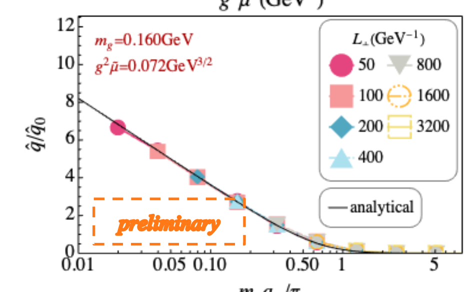
$$\hat{q} = \hat{q}_0 \left[\log\frac{\lambda_{UV}^2 + m_g^2}{m_g^2} - \frac{\lambda_{UV}^2}{\lambda_{UV}^2 + m_g^2}\right], \qquad \hat{q}_0 = C_F \frac{g^4\tilde{\mu}^2}{4\pi}$$

In the discrete basis space, the UV cutoff is determined by the lattice spacing, $\lambda_{UV} = \frac{\pi}{a_\perp}$.

- Broadening of a bare quark $|q\rangle$



- $\hat{q}$ is proportional to the charge density square of the background field.



- $\hat{q}$ is **logarithmically divergent** on the UV cutoff of the system.

(b) dressed quark $|q\rangle+|qg\rangle$

In the $|q\rangle+|qg\rangle$ Fock space, two mechanisms contribute to momentum broadening, medium interaction, and gluon radiation.

- Broadening of the dressed state $|q\rangle+|qg\rangle$

The bare quark,

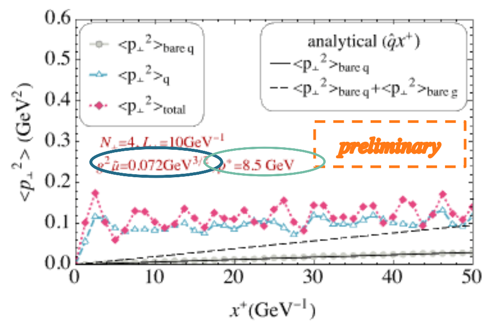
$\langle p_\perp^2(x^+)\rangle_{\text{bare q}} = \langle q|p_\perp^2(x^+)|q\rangle$

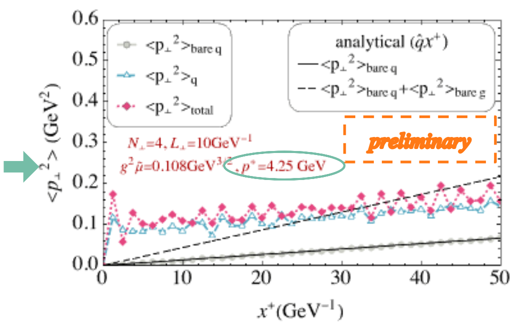
The quark component of the system,

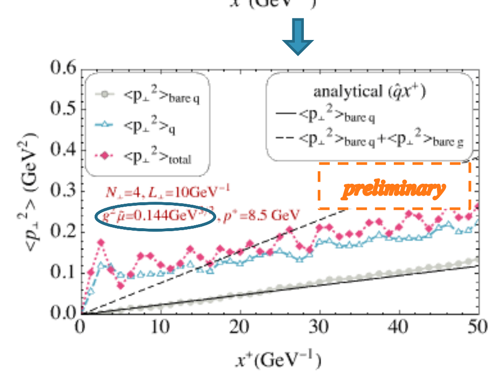
$\langle p_\perp^2(x^+)\rangle_{\text{q}} = \langle q|p_\perp^2(x^+)|q\rangle + \langle qg|p_{\perp,q}^2(x^+)|qg\rangle$

The total momentum of the system,

$\langle p_\perp^2(x^+)\rangle_{total} = \langle q|p_\perp^2(x^+)|q\rangle + \langle qg|p_{\perp,total}^2(x^+)|qg\rangle$







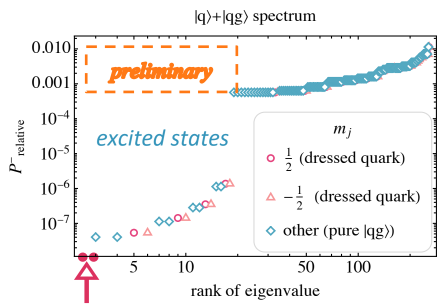
- The momentum broadening of the bare q agrees with the analytical.
- Going from a weaker field to a stronger field, the broadening becomes more profound.
- Going from a larger to a smaller p^+, the oscillation from gluon radiation becomes denser.

IV. Momentum broadening [preliminary]
(c) physical quark

In a physical high-energy scattering process, the initial quark state has already developed a gluon cloud before the interaction. The physical quark state is solved from the eigenvalue equation in the $|q\rangle + |qg\rangle$ space by matrix diagonalization with mass renormalization scheme[4] :

$$\hat{P}^-_{QCD}|\psi\rangle = P^-|\psi\rangle, \qquad \hat{P}^-_{QCD} = P^-_{KE} + V_{qg}$$

➢ Spectrum in the q-g relative light-front energy space



ground states ($P^+P^- - P_\perp^2 = m_q^2$)

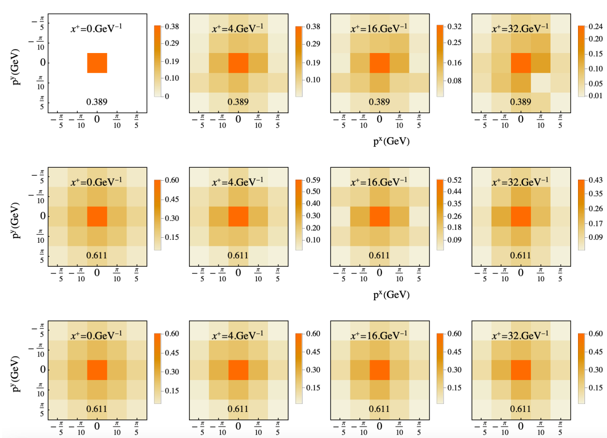
- Boost-invariant *(same at different center-of-mass momentum)*
- SU(3) gauge invariant *(degenerate in color space*
- Rotational symmetry *(eigenstates of J_z, m_j)*

➢ Evolution in the $\vec{p}_\perp$ space preliminary

$|q\rangle$

$|qg\rangle$

$|qg\rangle$



V. Summary and outlooks

- We demonstrated a nonperturbative method to investigate time-evolution problems with light-front Hamiltonian formalism We studied the quark-nucleus scattering in the$|q\rangle + |qg\rangle$ space, and analyzed the effects from
  - *the light-front energy*
  - *gluon emission/absorption*
  - *interaction with a background field*

  A main advantage of this method: we can smoothly change the magnitudes of the above three effects separately to match different physics regimes
- Ongoing and future works
  - momentum broadening from gluon radiation and medium induction
  - quark-nucleus scattering with the initial quark state as an eigenstate of the QCD Hamiltonian in the $|q\rangle + |qg\rangle$ space
  - jet quenching in the quark-gluon plasma

References

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