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# New paradigm in anisotropic flow analyses with correlation techniques

Phys. Rev. C 105, 024912 (2022)  
arXiv:2106.05760

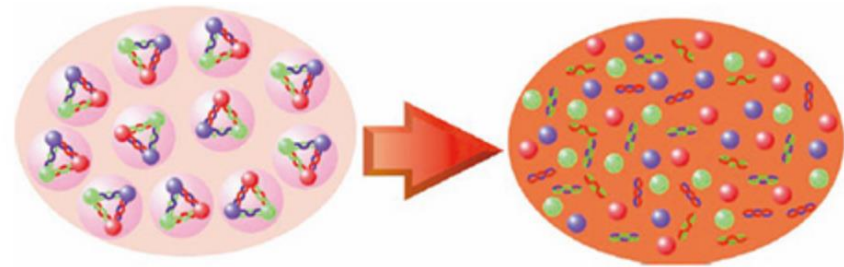
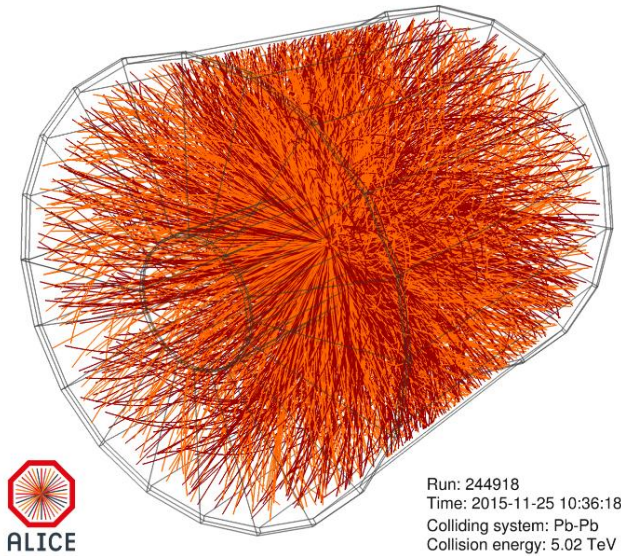
Ante Bilandzic  
Technical University of Munich  
“Quark Matter”, 08/04/2022



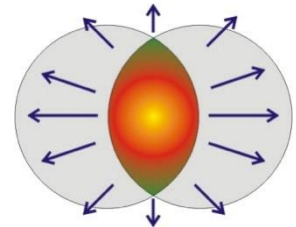
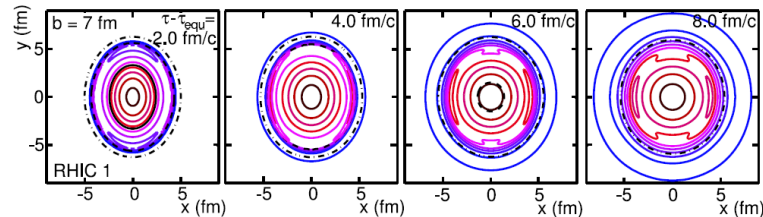
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# Introduction

- Properties of quark–gluon plasma (QGP) can be probed by analysing collective phenomena in heavy-ion collisions with multiparticle analysis techniques



P. F. Kolb, U. W. Heinz, nucl-th/0305084



## Open questions:

- What are the smallest collision systems and energies at which QGP can be formed?
- How to extract new and independent constraints from the available heavy-ion data?
- Is the observed universality of flow measurements in vastly different collision systems physical, or just a subtle artifact of using correlation techniques in the randomized data set?

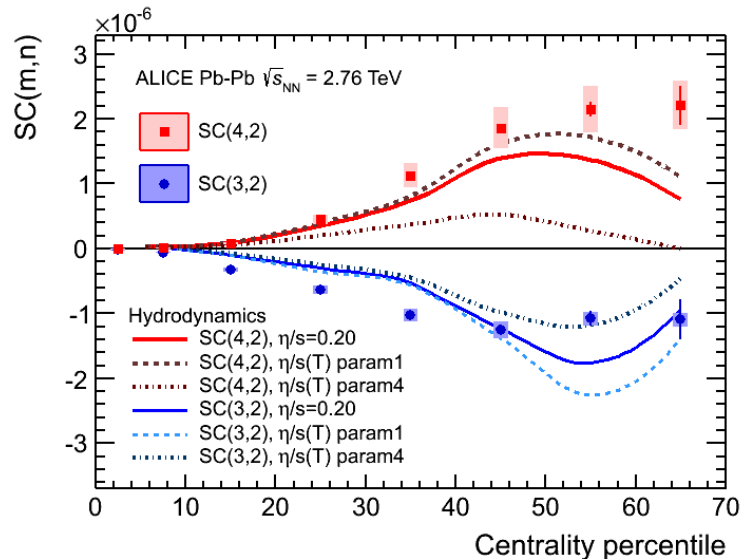
# Independent constraints

- By definition, higher-order multivariate cumulants extract information which is exactly free from lower-order contributions:

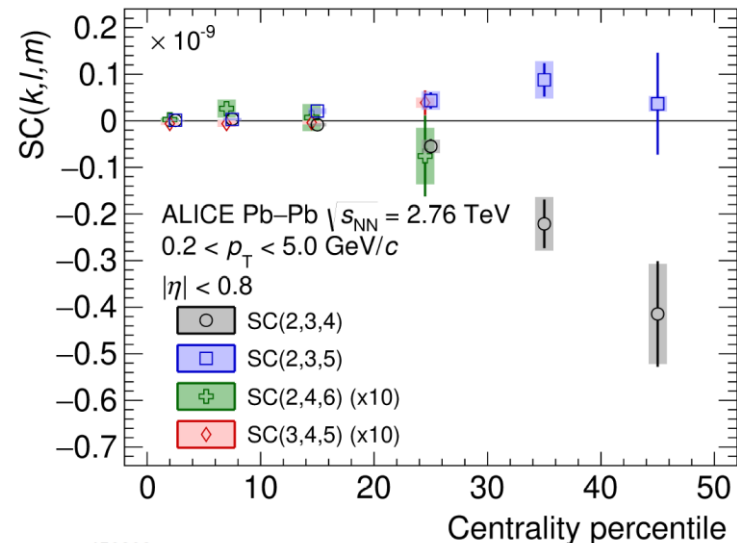
$$\begin{aligned} \langle X_1 X_2 X_3 \rangle_c &= \langle X_1 X_2 X_3 \rangle \\ &- \langle X_1 X_2 \rangle \langle X_3 \rangle - \langle X_1 X_3 \rangle \langle X_2 \rangle - \langle X_2 X_3 \rangle \langle X_1 \rangle \\ &+ 2 \langle X_1 \rangle \langle X_2 \rangle \langle X_3 \rangle \end{aligned}$$

R. Kubo, JPSJ, Vol. 17, No. 7 (1962)  
Borghini, Dinh, Ollitrault, PRC **64**, 054901 (2001)  
AB, Snellings, Voloshin, PRC **83**, 044913 (2011)  
'Generic Framework', PRC **89**, 064904 (2014)

- Old paradigm:** Cumulant expansion in flow analyses is performed on azimuthal angles – very difficult to reconcile with the mathematical properties of cumulants
- New paradigm:** Cumulant expansion is performed directly on flow harmonics



ALICE, PRL 117, 182301 (2016)



ALI-DER-479298

ALICE, PRL 127 (2021) 092302

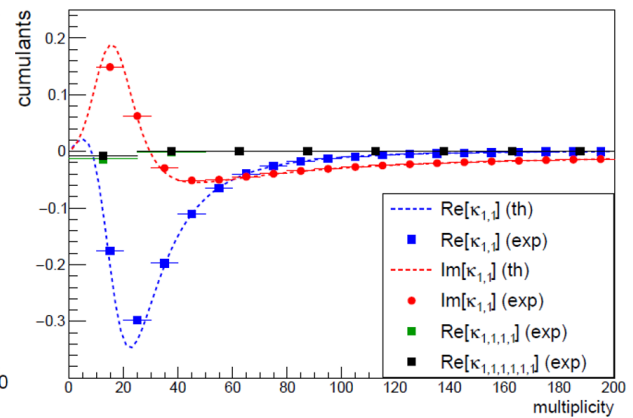
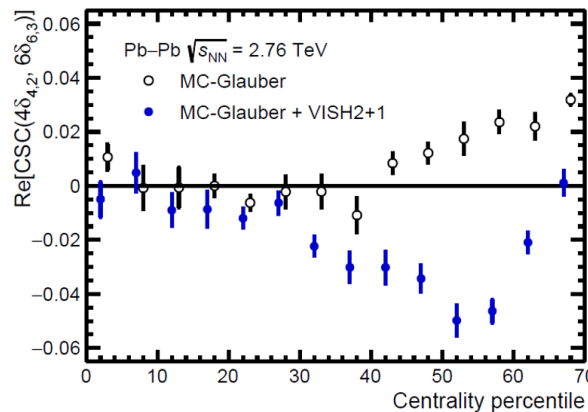
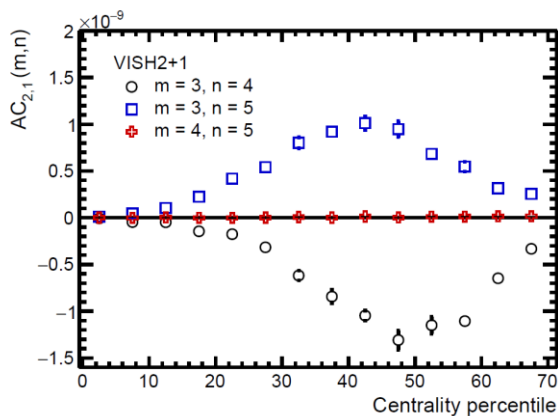
# New paradigm: Main conclusions

- **The main conclusion #1:** One cannot perform cumulant expansion in one set of stochastic observables, then in the resulting expression perform the transformation to some new set of observables, and then claim that the cumulant properties are preserved in the new set of observables
  - After such transformation, the fundamental properties of cumulants are lost in general
- **The main conclusion #2:** The formal properties of cumulants are valid only if there are no underlying symmetries due to which some terms in the cumulant expansion would vanish identically
  - Due to symmetries,  $\langle e^{in\varphi_i} \rangle = 0$ ,  $\langle e^{in(\varphi_i+\varphi_j)} \rangle = 0$ , etc., all vanish
  - There are no obvious symmetries for  $\langle v_k^2 \rangle$ ,  $\langle v_k^2 v_l^2 \rangle$ , etc., to vanish
- Example simple necessary condition for multivariate cumulants:

$$\lambda(aX + b) = a^N \lambda(X)$$

# Reconciliation

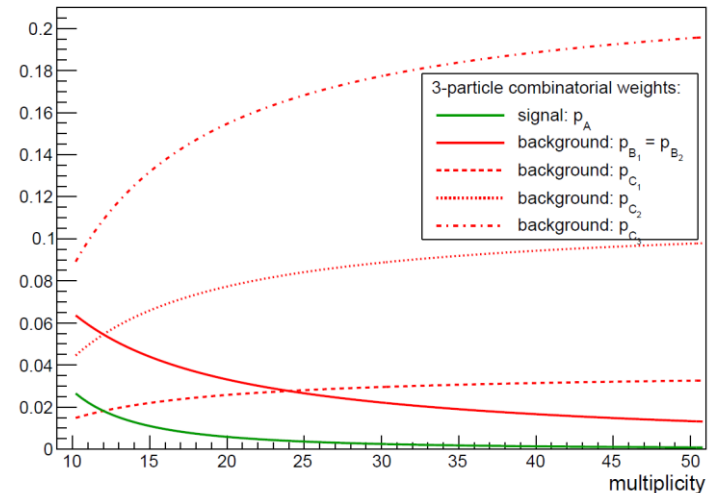
- New flow observables ('The Next Generation') which do satisfy all formal mathematical properties of cumulants:
  - **'Symmetric and Asymmetric Cumulants'** (genuine multiharmonic correlations of flow amplitudes)  
arXiv:1901.06968 and Sec. V in arXiv:2101.05619
  - **'Cumulants of symmetry plane correlations'**  
Sec. VI in arXiv:2101.05619
  - **'Event-by-event cumulants of azimuthal angles'**  
Sec. IV in arXiv:2101.05619 and arXiv: 2106.05760



# Explanation for universality

- Why flow measurements with multiparticle correlations exhibit the same universal behaviour in pp, p–Pb, peripheral Pb–Pb, and even in  $e^+e^-$  collisions?
- Example: 3-particle correlations
  - If particles are emitted from p.d.f.  $f(x,y,z)$ , and if the resulting sample is randomized, what is the p.d.f.  $w(x,y,z)$  which describes the final randomized sample?

$$\begin{aligned}
 w(x,y,z) = & p_A f_{xyz}(x,y,z) \\
 & + p_{B_1} [f_{xy}(x,y)f_x(z) + f_{xy}(x,y)f_y(z) + f_{xz}(x,z)f_x(y) \\
 & \quad + f_{xz}(x,z)f_z(y) + f_{yz}(y,z)f_y(x) + f_{yz}(y,z)f_z(x)] \\
 & + p_{B_2} [f_{xy}(x,y)f_z(z) + f_{xz}(x,z)f_y(y) + f_{yz}(y,z)f_x(x)] \\
 & + p_{C_1} [f_x(x)f_x(y)f_x(z) + f_y(y)f_y(x)f_y(z) + f_z(z)f_z(y)f_z(x)] \\
 & + p_{C_2} [f_x(x)f_x(z)f_y(y) + f_x(x)f_x(y)f_z(z) + f_y(y)f_y(z)f_x(x) \\
 & \quad + f_y(y)f_y(x)f_z(z) + f_z(z)f_z(y)f_x(x) + f_z(z)f_z(x)f_y(y)] \\
 & + p_{C_3} f_x(x)f_y(y)f_z(z).
 \end{aligned}$$



- The relation between original  $f(x,y,z)$  and its counterpart  $w(x,y,z)$  in the randomized sample is unique and generic:

Observed universality  $\Leftrightarrow$  for small multiplicities,  $w(x,y,z)$  is dominated by the universal combinatorial weights  $p$ , and not by the details of functional form of  $f(x,y,z)$