Exact equilibrium distributions for massive and massless fermions with rotation and acceleration

Large vorticity in the QGP. Phenomenological consequences: **spin polarization.**

Exact Wigner Function: quantum corrections in relativistic fluids at global equilibrium.

$$W(x,k) = \frac{1}{(2\pi)^3}\int \frac{\mathrm{d}^3p}{2\varepsilon}\sum_{n=1}^{\infty}(-1)^{n+1}e^{-n\widetilde{\beta}(in\phi)\cdot p}\times$$
$$\left[e^{-in\frac{\phi:\Sigma}{2}}(m+\not{p})\delta^4\left(k-\frac{\Lambda^n p+p}{2}\right)+(m-\not{p})e^{in\frac{\phi:\Sigma}{2}}\delta^4\left(k+\frac{\Lambda^n p+p}{2}\right)\right]$$

Exact **distribution functions** and **spin vector** at all orders in thermal vorticity.

Based on: A. P., M. Buzzegoli, F. Becattini 10.1007/JHEP10 077 and 10.1007/JHEP02 101 (2021)

Andrea Palermo

UNIVERSITÀ DEGLI STUDI FIRENZE

INFN Istituto Nazionale di Fisica Nucleare

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

CRC-TR 211 Strong-interaction matter under extreme conditions

Equilibrium density operataor

$$\widehat{\rho} = \frac{1}{Z} e^{-b_\mu \widehat{P}^\mu + \frac{1}{2}\varpi_{\mu\nu}\widehat{J}^{\mu\nu}}$$

The **generators of the Poincaré group** appear.

$$\varpi^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{\omega_\rho}{T}u_\sigma + \left(\frac{A^\mu}{T}u^\nu - \frac{A^\nu}{T}u^\mu\right)$$

ϖ is the thermal vorticity, a constant antisymmetric tensor.

Analytic continuation, **factorization** of the density operator.

$$\widehat{\rho} = \frac{1}{Z} e^{-b_\mu \widehat{P}^\mu - \frac{i}{2}\phi_{\mu\nu}\widehat{J}^{\mu\nu}} = \frac{1}{Z} e^{-\widetilde{b}_\mu(\phi)\widehat{P}^\mu} \underbrace{e^{-\frac{i}{2}\phi_{\mu\nu}\widehat{J}^{\mu\nu}}}_{\equiv\widehat{\Lambda}}$$

$$\widetilde{b}^\mu(\phi) = \sum_{k=0}^{\infty} \frac{1}{(k+1)!} \underbrace{\left(\phi^{\mu\nu_1}\phi_{\nu_1\nu_2}\dots\phi^{\nu_{k-1}\nu_k}\right)}_{k-\text{times}} b_{\nu_k}$$

Thermal expectation values

The Wigner function is obtained from the **mean number operator**

$$\langle \widehat{a}_s^\dagger(p)\widehat{a}_r(p')\rangle = 2\varepsilon' \sum_{n=1}^{\infty} (-1)^{2S(n+1)} \delta^3(\Lambda^n p - p') D^{(S)}(W_{rs}(\Lambda^n, p)) e^{-\widetilde{b}\cdot\sum_{k=1}^{n} \Lambda^k p}$$

$\mathrm{W}(\Lambda,\mathrm{p})$ is the Wigner rotation. Expectation values are series

$$\langle \widehat{T}^{00} \rangle = \frac{3T^4}{8\pi^2} \sum_{n=1}^{\infty} (-1)^{n+1} \phi^4 \frac{\sinh n\phi}{\sinh^5(n\phi/2)} \quad \rightarrow \quad \frac{7\pi^2}{60\beta^4} - \frac{\alpha^2}{24\beta^4} - \frac{17\alpha^4}{960\pi^2\beta^4}$$

Finite result for $\phi \rightarrow i\alpha$ is obtained by **analytic distillation.**

Distribution function

Exact Wigner function can be used to calculate expectation values.

$$W(x,k) = \frac{1}{(2\pi)^3}\int \frac{\mathrm{d}^3p}{2\varepsilon}\sum_{n=1}^{\infty}(-1)^{n+1}e^{-n\widetilde{\beta}(in\phi)\cdot p}\times$$
$$\left[e^{-in\frac{\phi:\Sigma}{2}}(m+\not{p})\delta^4\left(k-\frac{\Lambda^n p+p}{2}\right)+(m-\not{p})e^{in\frac{\phi:\Sigma}{2}}\delta^4\left(k+\frac{\Lambda^n p+p}{2}\right)\right]$$

We infer the distribution function from: $j_+^\mu(x) = \int \frac{\mathrm{d}^3p}{\varepsilon}p^\mu f(x,p) + N^\mu(x,p)$

$$f(x,p) = \frac{1}{2(2\pi)^3}\sum_{n=1}^{\infty}(-1)^{n+1}e^{-n\widetilde{\beta}(-n\varpi)\cdot p}\mathrm{tr}\left(e^{n\frac{\varpi}{2}:\Sigma}\right)$$

Differs form commonly used ansatz in kinetic theories:

$$f_A = \mathrm{tr}\left(\left(e^{\beta\cdot p}e^{-\frac{\varpi}{2}:\Sigma}+\mathbb{I}\right)^{-1}\right)$$

Spin-Polarization

Exact spin vector

$$S^\mu(p) = \frac{1}{2}\frac{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(\gamma^\mu\gamma_5 W_+(x,p)\right)}{\int \mathrm{d}\Sigma\cdot p\ \mathrm{tr}\left(W_+(x,p)\right)}$$

$$S^\mu(p) = \frac{1}{2m}\frac{\sum_{n=1}^\infty(-1)^{n+1}e^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(\gamma^\mu\gamma_5 e^{-in\frac{\phi:\Sigma}{2}}\not{p}\right)\delta^3(\Lambda^n p - p)}{\sum_{n=1}^\infty(-1)^{n+1}e^{-n\widetilde{b}(in\phi)\cdot p}\mathrm{tr}\left(e^{-in\frac{\phi:\Sigma}{2}}\right)\delta^3(\Lambda^n p - p)}$$

The leading order in vorticity agrees with the literature.

$$S^\mu(p) \simeq -\frac{1}{8m}\epsilon^{\mu\alpha\beta\rho}\varpi_{\alpha\beta}p_\rho\frac{n_F(1-n_F)}{n_F}$$

Full summation of the series under investigation...