Numerical solutions of the JIMWLK equation with the kinematical constraint

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in collaboration with L. Motyka based on: PK, SoftwareX (2021) arXiv:2009.02045, S. Cali *et al.*, Eur.Phys.J.C 81 (2021) 663 arXiv:2104.14254, PK, arXiv:2111.07427

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JIMWLK evolution equation with collinear improvement

Langevin equation formulation from Hatta, Iancu (2016)

At each point of the discretized transverse plane a Wilson line exists with an additional index: the scale at which the final correlator is evaluated.

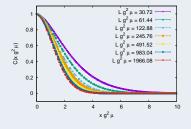
Computationally demanding!

For each Wilson line and scale R we have a separate equation:

$$\begin{split} U(\mathbf{x},R,s+\delta s) &= \exp\Big(\\ -\sqrt{\delta\varepsilon} \sum_{\mathbf{y}} \sqrt{\alpha_s} \frac{\theta(s-\rho_{\mathbf{xy}}^R)}{\theta(s-\rho_{\mathbf{xy}}^R)} U(\mathbf{y},\hat{R},s-\Delta_{\mathbf{xy}}^R) \big[\mathbf{K}(\mathbf{x}-\mathbf{y}) \cdot \boldsymbol{\xi}(\mathbf{y}) \big] \, U^\dagger(\mathbf{y},\hat{R},s-\Delta_{\mathbf{xy}}^R) \Big) \\ &\times U(\mathbf{x},R,s) \times \exp\Big(\sqrt{\delta\varepsilon} \sum_{\mathbf{y}} \sqrt{\alpha_s} \frac{\theta(s-\rho_{\mathbf{xy}}^R)}{\theta(s-\rho_{\mathbf{xy}}^R)} \mathbf{K}(\mathbf{x}-\mathbf{y}) \cdot \boldsymbol{\xi}(\mathbf{y}) \Big), \\ \rho_{\mathbf{xy}}^R &= \ln \frac{(\mathbf{x}-\mathbf{y})^2}{R^2}, \; \Delta_{\mathbf{xy}}^R = \theta \big(|\mathbf{x}-\mathbf{y}| - R \big) \rho_{\mathbf{xy}}^R, \; \hat{R} = \max(|\mathbf{x}-\mathbf{y}|,R), \; s = \varepsilon \alpha_s. \end{split}$$

Initial condition from the McLerran-Venugopalan model





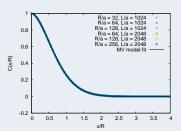


Figure: Volume dependence of the dipole amplitude in the MV model on the torus. Increasing torus size pushes the distribution to the left. Consecutive volumes differ by a factor 4 (factor 2 in linear extend). Convergence to some limiting distribution can be seen for very large volumes. Dipole amplitude generated by the new method shows negligible finite size and lattice spacing effects.

Saturation scale evolution speed

JIMWLK with running coupling without collinear improvement

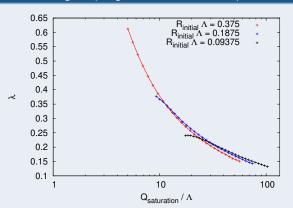


Figure: $R_{\rm initial}\Lambda$ is the only parameter of the initial condition and of the evolution. Coinciding data from evolution for different values of $R_{\rm initial}\Lambda$ corresponds to geometrical scaling.

Saturation scale evolution speed

JIMWLK with collinear improvement

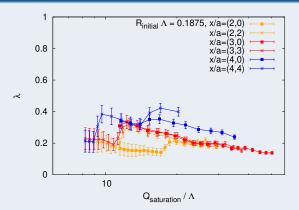


Figure: Saturation scale evolution speed at $R_{\rm initial}\Lambda=0.1875$ for different discretizations. Discontinuities signal large discretization effects which have to be address by an improved numerical approach where part of the evolution at scales shorter than a should be performed exactly using the BK equation.