Spheroid expansion and freeze-out geometry heavy-ion collisions in the few-GeV energy regime

Jędrzej Kołaś,[1] Wojciech Florkowski,[2] Tetyana Galatyuk,[3,4] Małgorzata Gumberidze,[3] Szymon Harabasz,[4] Radosław Ryblewski,[5] Piotr Salabura,[2] Joachim Stroth,[3,6] and Hanna Paulina Zbroszczyk [1]

[1]*Warsaw University of Technology, 00-662 Warsaw, Poland*

[2]*Jagiellonian University, 30-348 Cracow, Poland*

[3]*GSI Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany*

[4]*Technische Universität Darmstadt, 64289 Darmstadt, Germany*

[5]*Institute of Nuclear Physics Polish Academy of Sciences, 31-342 Cracow, Poland*

[6]*Institut für Kernphysik, Goethe-Universität, 60438 Frankfurt, Germany*

Faculty of Physics
WARSAW UNIVERSITY OF TECHNOLOGY

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Spheroidal Siemens-Rassmusen Model

The basis for this model remains the Cooper-Frye formula [1] :

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x) p^\mu f(x,p)$$

Where:

- $d^3\Sigma_\mu(x)$ - the element of the 3D freeze-out hypersurface
- $f(x,p)$ - phase-space distribution function of particles

The Siemens - Rassmussen model is modified by the eccentricity parameters δ (for the momentum space) and ε (for the position space):

- The freeze-out hypersurface is then given by:

$$x^\mu = (t, r\sqrt{1-\varepsilon}\cos\phi\sin\theta\,, r\sqrt{1-\varepsilon}\sin\phi\sin\theta\,, r\sqrt{1+\varepsilon}\cos\theta)$$

- And the hydrodynamic flow by:

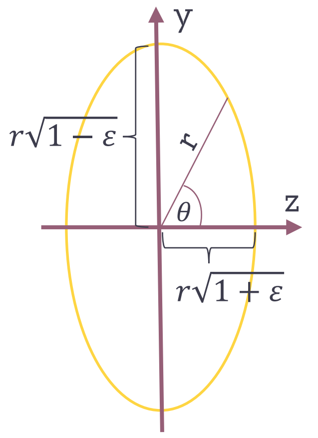
$$u^\mu = \gamma(\xi)(t, v(\xi)\sqrt{1-\delta}\cos\phi\sin\theta\,, v(\xi)\sqrt{1-\delta}\sin\phi\sin\theta\,, v(\xi)\sqrt{1+\delta}\cos\theta)$$

We assume Hubble-like radial profile of the flow velocity:

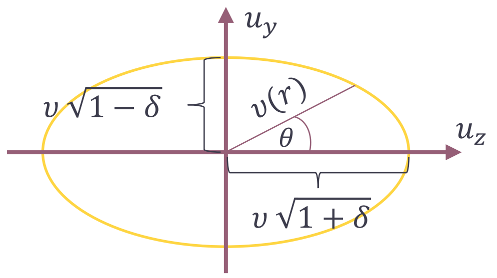
$$v(r) = \tanh(Hr)$$

[1] F. Cooper and G. Frye, Phys. Rev. D 10, 186 (1974)

Hypersurface parametrisation



Flow parametrisation



The δ and ε must be anticorrelated to align with the hydrodynamical description of heavy-ion collisions, hence the difference in orientation of the ellipses (and spheroids in 3D).

Used notation:
ϕ – azimuthal angle
θ – polar angle
r – radius
ξ – certain mapping of space-time ($\xi \to (r(\xi), t(\xi))$)
$v(\xi)$ – flow profile (for set value of time $v(\xi) = v(r)$)
$\gamma(\xi)$ – Lorenz factor
H – some constant

Our model predicts transverse momentum and rapidity spectra of particles.

A reproduction of the HADES data would mean that the same model applies to Au-Au collisions at HADES.

Thermal Parameters

As an input to our calculation, we consider three sets of thermodynamic parameters:

parameter	Case A	Case B	Case C
T (MeV)	49.6 ± 1.1	70.3 ± 2.0	63.5 ± 1.6
R (fm)	16.0	6.8 ± 0.9	10.4 ± 0.3
μ_B (MeV)	776 ± 3	872.1 ± 24.3	781.1 ± 3.3
γ_S	0.16 ± 0.02	0.05 ± 0.01	0.04 ± 0.01
χ^2/N_{df}	$N_{df} = 0$	1.13/2	62.30/5
H (GeV)	0.01	0.0225	0.016
δ	0.2	0.4	0.4
$\sqrt{Q^2}$	0.238	0.256	0.323

Where:

- T – temperature,
- R – radius of the freeze-out hypersurface,
- μ_B - baryon potential,
- γ_S - strangeness supression factor

Results for case A were obtained by the analytical solution of the Cooper-Frye formula.

Cases B and C used a numerical fit procedure; see [8].

[2] M. Szala (HADES), ECT* Workshop, Trento, Italy (2019)
[3] M. Szala (HADES), SQM2019 Proceedings, Springer (2020)
[4] J. Adamczewski-Musch et al. (HADES), Eur. Phys. J. A 56, 259 (2020)
[5] J. Adamczewski-Musch et al. (HADES), Phys. Lett. B778, 403 (2018)
[6] J. Adamczewski-Musch et al. (HADES), Phys. Lett. B793, 457 (2019)
[7] S. Harabasz et al., Phys. Rev. C 102, 054903 (2020)
[8] A. Motornenko et al. ,Phys. Lett. B 822, 136703 (2021)
[9] E. Shuryak and J. M. Torres-Rincon, Phys. Rev. C 101, 034914 (2020)
[10] V. Vovchenko et al., Phys. Lett. B, 135746 (2020),

Parameters were obtained by different fitting strategies of the particle multiplicities measured by the HADES collaboration in Au-Au collisions at $\sqrt{s_{NN}} = 2.4$ GeV for the most central collisions (0-10%) are showcased on the right.

particle	mulitplicity	uncertainity	Ref.
$p\ (free)$	77.6	± 2.4	[2,3]
$p\ (bound)$	46.5	± 1.5	[2,3]
π^+	9.3	± 0.6	[4]
π^+	17.1	± 1.1	[4]
K^+	$5.98 \cdot 10^{-2}$	$\pm 6.79 \cdot 10^{-3}$	[5]
K^+	$5.6 \cdot 10^{-4}$	$\pm 5.96 \cdot 10^{-5}$	[5]
Λ	$8.22 \cdot 10^{-2}$	$^{+5.2}_{-9.2} \cdot 10^{-2}$	[6]

The cases differentiate accordingly:

A. The total number of protons is equal to the sum of thermal and bound in detected light nuclei [7].

B. The same approach to proton counting is used as in the previous case but obtained in [8].

C. The light nuclei are counted separately from protons [8], and the feed-down from excited nuclear states has been included, following [9,10].

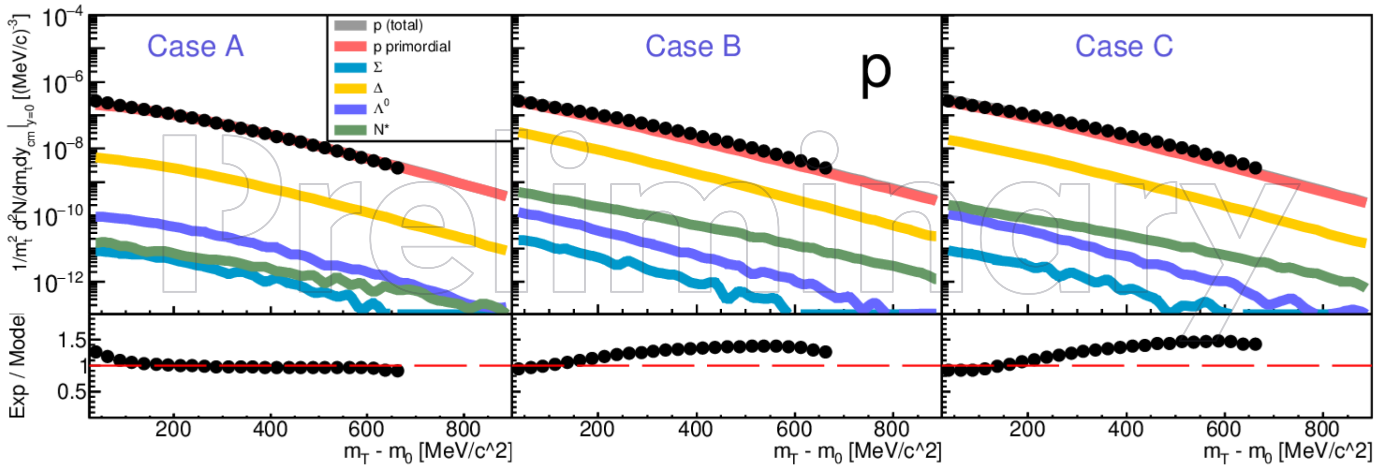
Best H, ε and δ are chosen by establishing the mean relative error between the experiment and the model for transverse momentum spectra of protons and pions:

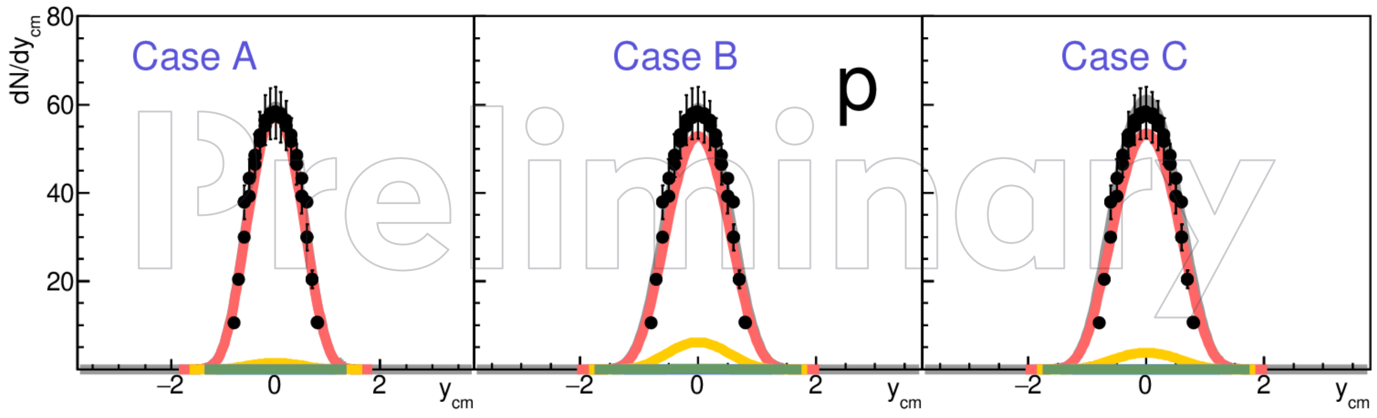
$$Q^2 = \sqrt{\frac{1}{N}\sum_{i=0}^{N} \frac{\left(Y_{i,model} - Y_{i,exp}\right)^2}{{Y_{i,exp}}^2}}$$

Fitting Procedure

Preliminary model comparison of mid-rapidity transverse mass and rapidity distribution of protons with HADES data (Au-Au collisions at $\sqrt{s_{NN}} = 2.4$ GeV).

- Primordial protons dominate in all cases.
- Reasonable reproduction of proton spectra.
- Comparable results in all three cases.
- For cases B and C Δ(1232) becomes non-negligible.
- The higher the temperature, the more significant the contribution from heavier particles becomes.
- No fitting procedure was performed in the case of rapidity distributions, yet good agreement with data was obtained.

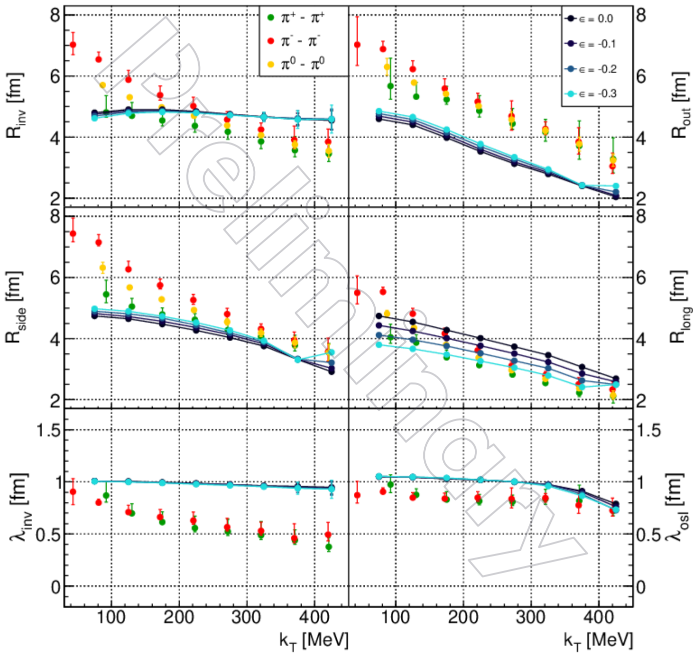


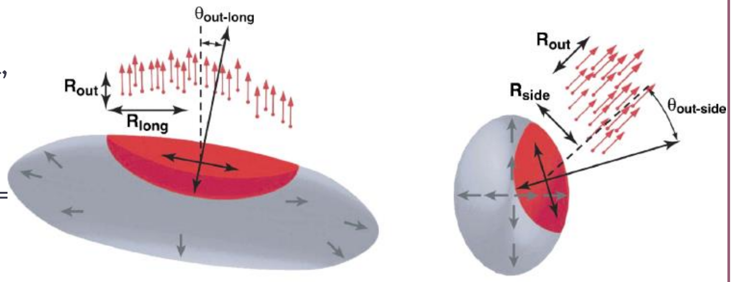


Particle Femtoscopy

Preliminary model results for the comparison of the source size at HADES.

- For the HBT analysis, ε parameter plays a significant role in the longitudinal direction.
- In the C case, the model has the size of the emission source (the fireball) similar to the one present in HADES.
- The plot on the right side compares the HBT radii of identical pions:
  - HADES data (red, green, and yellow) [11].
  - Case C of our model (dark to light blue) for a set of ε values.
- Notation:
  - p_T - momentum transverse to the beam
  - $k_T = \frac{p_{T_1}+p_{T_2}}{2}$ - pair momentum, transverse to the beam
  - $R_{out}, R_{side}, R_{long}$ - as shown on the right:
  - R_{inv} - same as above, but assumes $R_{out} = R_{side} = R_{long} = R_{inv}$





Lisa MA, et al. 2005.
Annu. Rev. Nucl. Part. Sci. 55:357–402

[11] Adamczewski-Musch, J. et al., *Eur. Phys. J. A* 56.5 (2020): 1-20.

Conclusions

We have studied transverse momentum, HBT radii, and rapidity distributions at HADES Au-Au collisions at $\sqrt{s_{NN}} = 2.4$ GeV.

- Our model allows for reproducing abundant particles' transverse momentum and rapidity spectra.
- Spheroidal symmetry of the fireball along the beam axis allows for improved rapidity reproduction.
- All three studied cases give similar results, despite their initial differences.
- Transverse momentum distributions are invariant of the ε parameter.
- HBT analysis favours case C.