Probing the Weizsäcker-Williams gluon distribution with electron-dijet correlations at EIC

Andreas van Hameren

Institute of Nuclear Physics
Polish Academy of Sciences
Kraków

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ITMD* factorization in a nutshell

ITMD* factorization is identical to ITMD factorization for two jets, and an approximation for more complicated final states. The ITMD* formula of forward particle production, where a dilute projectile p (probed at large x) collides with a dense target A (probed at small x) is given by:

$$d\sigma_{pA\to n} = \int \frac{dx_1}{x_1} \sum_a x_1 f_{a/p}(x_1,\mu) \frac{dx_2}{x_2} \int \frac{d^2k_T}{\pi} d\Phi_{n_J} \times \sum_{b_1,\dots,b_{n_J}} \frac{\left|\mathcal{M}_{ag\to b_1\dots b_{n_J}}\right|^2_{\text{ITMD}^*}}{\text{flux}_{ag\to b_1\dots b_{n_J}}} \quad (1)$$

where $f_{a/p}$ is the collinear PDF for parton a, where $d\Phi_{n_J}$ is the n_J-particle differential phase space, and where $b_1,\dots,b_{n_J}$ are the various final state partons contributing to the partonic sub-process $ag \to b_1\dots b_{n_J}$. The factor $\text{flux}_{ag\to b_1\dots b_{n_J}}$ is assumed to contain the flux factor, the factors to turn the summations implied by the matrix element into averages regarding the initial-state partons, and the necessary factors in case there are identical final-state particles. The matrix element is given by

$$\left|\mathcal{M}_{ag\to b_1\dots b_{n_J}}\right|^2_{\text{ITMD}^*} = (N_c^2-1) \sum_{i_1,\dots,i_n} \sum_{j_1,\dots,j_n} \sum_{\bar\imath_1,\dots,\bar\imath_n} \sum_{\bar\jmath_1,\dots,\bar\jmath_n} \left(\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n}\right)^* \left(\tilde{\mathcal{M}}^{\bar\imath_1 \bar\imath_2\cdots \bar\imath_n}_{\bar\jmath_1 \bar\jmath_2\cdots \bar\jmath_n}\right) \times \left\langle\!\!\left\langle 2\left(\hat F^+(\xi)\right)^{j_1}_{i_1} \left(\hat F^+(0)\right)^{\bar\jmath_1}_{\bar\imath_1} \left(\mathcal{U}^{[\lambda_2]}\right)_{i_2\bar\imath_2} \left(\mathcal{U}^{[\lambda_2]\dagger}\right)^{j_2\bar\jmath_2} \cdots \cdots \left(\mathcal{U}^{[\lambda_n]}\right)_{i_n\bar\imath_n} \left(\mathcal{U}^{[\lambda_n]\dagger}\right)^{j_n\bar\jmath_n} \right\rangle\!\!\right\rangle . \quad (2)$$

Here, $\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n}$ is the parton-level scattering amplitude in the incarnation of the colorflow representation as given in [Kanaki:2000ms, Papadopoulos:2005ky]. Such a representation treats gluons on the same footing as quark-antiquark pairs in the color sum. The symbol n is the number of color-pairs in this representation, so the number of gluons plus the number of quarks, where the latter is equal to the number of antiquarks. The field strength operators $\hat F^+$ are separated in the light-cone 'minus' and transverse directions $\xi = \left(\xi^+=0,\xi^-,\vec\xi_T\right)$. The symbols $\mathcal{U}^{[\lambda]}$ denote two staple-like fundamental representation Wilson lines connecting the fields

$$\mathcal{U}^{[\pm]} = \left[\left(0^+,0^-,\vec 0_T\right),\left(0^+,\pm\infty^-,\vec 0_T\right)\right] \left[\left(0^+,\pm\infty^-,\vec 0_T\right),\left(0^+,\pm\infty^-,\vec\xi_T\right)\right] \times \left[\left(0^+,\pm\infty^-,\vec\xi_T\right),\left(0^+,\xi^-,\vec\xi_T\right)\right],$$

where the square brackets are the straight segments of the Wilson link. The value $\lambda = \pm$ depends on whether the parton whose color the Wilson link connects is incoming or outgoing. The Wilson loop obtained by two staples glued together is denoted

$$\mathcal{U}^{[\square]} = \mathcal{U}^{[-]\dagger}\mathcal{U}^{[+]} .$$

The angular double brackets denote the Fourier transform of the hadronic matrix element:

$$\left\langle\!\!\left\langle \cdots \right\rangle\!\!\right\rangle = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} \exp\left(ix_2P^+\xi^- - i\vec k_T\cdot\vec\xi_T\right) \langle P|\cdots|P\rangle$$

where P^+ is the longtudinal 'plus' momentum component of the hadron. In this Fourier transform, the variables x_2 and $\vec k_T$ from Eq. (1) appear. One feature of the chosen color representation is that the amplitude decomposes as

$$\tilde{\mathcal{M}}^{i_1 i_2\cdots i_n}_{j_1 j_2\cdots j_n} = \sum_{\sigma\in S_n} \delta^{i_1}_{j_{\sigma(1)}} \delta^{i_2}_{j_{\sigma(2)}} \cdots \delta^{i_n}_{j_{\sigma(n)}} \mathcal{A}_\sigma$$

where the *partial amplitudes* $\mathcal{A}_\sigma$ only depend on momenta, helicity, and the permutation σ, but not on color. They can be calculated using color-ordered Feynman rules. Inserting this decompostion, Eq. (2) collapses to

$$\left|\mathcal{M}_{ag\to b_1\dots b_{n_J}}\right|^2_{\text{ITMD}^*} = (N_c^2-1)\sum_{\sigma\in S_n}\sum_{\tau\in S_n} \mathcal{A}^*_\sigma\, \mathcal{C}_{\sigma\tau}(x_2,k_T)\, \mathcal{A}_\tau ,$$

where the entries of the "TMD-valued color matrix" $\mathcal{C}_{\sigma\tau}(x_2,k_T)$ consist of exactly a single power of N_c times one of the following 10 TMDs

$$\mathcal{F}^{(1)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(2)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(3)}_{qg}(x,k_T) = \left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(1)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(2)}_{gg}(x,k_T) = \frac{1}{N_c}\left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\text{Tr}\left[\hat F^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(3)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(4)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle\!\!\right\rangle,$$

$$\mathcal{F}^{(5)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle,$$

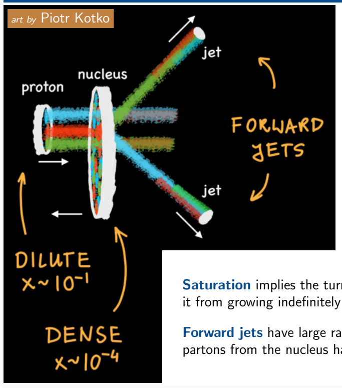
$$\mathcal{F}^{(7)}_{gg}(x,k_T) = \left\langle\!\!\left\langle \frac{\text{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\text{Tr}\left[\hat F^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat F^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle\!\!\right\rangle.$$

Below is the explicit example of the matrix $\mathcal{C}_{\sigma\tau}(x_2,k_T)$ and the column vector of partial amplitudes for both the processes $g_1^*\, q_2 \to q_4\, \bar q_3'\, q_5'$ and $g_1^*\, q_2 \to q_4\, \bar q_3\, q_5$:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_c\mathcal{F}^{(1)}_{qg} & \mathcal{F}^{(1)}_{qg} & 0 & \mathcal{F}^{(1)}_{qg} & 0 \\ 0 & \mathcal{F}^{(1)}_{qg} & N_c\mathcal{F}^{(2)}_{qg} & \mathcal{F}^{(3)}_{qg} & 0 & 0 \\ 0 & 0 & \mathcal{F}^{(3)}_{qg} & N_c\mathcal{F}^{(2)}_{qg} & \mathcal{F}^{(1)}_{qg} & 0 \\ 0 & \mathcal{F}^{(1)}_{qg} & 0 & \mathcal{F}^{(1)}_{qg} & N_c\mathcal{F}^{(1)}_{qg} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathcal{A}_{12345} \\ \mathcal{A}_{21345} \\ \mathcal{A}_{23145} \\ \mathcal{A}_{32145} \\ \mathcal{A}_{31245} \\ \mathcal{A}_{13245} \end{pmatrix}$$

The processes have 3 color pairs, so there are (at most) 3! partial amplitudes. They are explicitly labeled with their associated permutation, and the logic in the enumation of the partons is that gluons come first, then anti-quarks, and then quarks. Initial-state quarks count as negative-energy antiquarks. As can be seen, the first and last partial amplitudes do not contribute at all, but are included here for clearity. For processes with only gluons, the number of contributing partial amplitudes is only $(n-1)!$ rather than $n!$.

QCD evolution, dilute vs. dense, forward jets



A dilute system carries a few high-x partons contributing to the hard scattering.

A dense system carries many low-x partons.

At high density, gluons are imagined to undergo recombination, and to saturate.

This is modeled with non-linear evolution equations, involving explicit non-vanishing k_T.

Saturation implies the turnover of the gluon density, stopping it from growing indefinitely for small x.

Forward jets have large rapidities, and trigger events in which partons from the nucleus have small x.

ITMD Factorization

For forward dijet production in dilute-dense hadronic collisions

Generalized TMD factorization (Dominguez, Marquet, Xiao, Yuan 2011)

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_b \Phi^{(i)}_{gb}(x_A,k_T,\mu)\, f_{b/B}(x_B,\mu)\, d\hat\sigma^{(i)}_{gb\to X}(x_A,x_B,\mu)$$

For $x_A \ll 1$ and $P_T \gg k_T \sim Q_s$ (jets almost back-to-back).
TMD gluon distributions $\Phi^{(i)}_{gb}(x_A,k_T,\mu)$ satisfy non-linear evolution equations.
Partonic cross section $d\hat\sigma^{(i)}_{gb}$ is on-shell, but depends on color-structure i.

Improved TMD factorization (Kotko, Kutak, Marquet, Petreska, Sapeta, AvH 2015)

$$d\sigma_{AB\to X} = \int dk_T^2 \int dx_A \sum_i \int dx_B \sum_b \Phi^{(i)}_{gb}(x_A,k_T,\mu)\, f_{b/B}(x_B,\mu)\, d\hat\sigma^{(i)}_{gb\to X}(x_A,x_B,k_T,\mu)$$

Originally a model interpolating between High Energy Factorization and Generalized TMD factorization: $P_T \gtrsim k_T \gtrsim Q_s$.
Partonic cross section $d\hat\sigma^{(i)}_{gb}$ is off-shell and depends on color-structure i.

ITMD formalism is obtained from the CGC formalism, by including so-called kinematic twist corrections (Antinoluk, Boussarie, Kotko 2019).

Augmented TMD evolution

Kwieciński, Martin, Staśto 1997
Kwieciński, Kutak 2003

$$\phi(x,k^2) = \phi^{(0)}(x,k^2) + \frac{\alpha_s(k^2)N_c}{\pi}\int_x^1 \frac{dz}{z}\int_{k_0^2}^\infty \frac{dl^2}{l^2}\left\{\frac{l^2\phi(\frac{x}{z},l^2)\theta(\frac{k^2}{z}-l^2) - k^2\phi(\frac{x}{z},k^2)}{|l^2-k^2|} + \frac{k^2\phi(\frac{x}{z},k^2))}{\sqrt{|4l^4+k^4|}}\right\}$$

linear BFKL with kinematic constraint

$$+\frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz\left(P_{gg}(z)-\frac{2N_c}{z}\right)\int_{k_0^2}^{k^2} dl^2\,\phi\left(\frac{x}{z},l^2\right) + \frac{\alpha_s(k^2)}{2\pi}\int_x^1 dz\,P_{gq}(z)\Sigma\left(\frac{x}{z},k^2\right)$$

DGLAP corrections

$$-\frac{2\alpha_s^2(k^2)}{R^2}\left[\left(\int_{k^2}^\infty \frac{dl^2}{l^2}\phi(x,l^2)\right)^2 + \phi(x,k^2)\int_{k^2}^\infty\frac{dl^2}{l^2}\ln\left(\frac{l^2}{k^2}\right)\phi(x,l^2)\right]$$

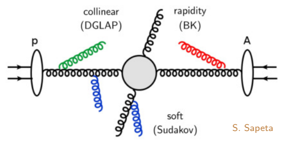
non-linear term from triple-pomeron vertex, with $R_A = R\,A^{1/3}$

Kutak, Sapeta 2012:

Starting distribution $\phi^{(0)}(x,k^2) = \frac{\alpha_s(k^2)}{2\pi k^2}\int_x^1 dz P_{gg}(z)\frac{x}{z}g\left(\frac{x}{z}\right)$, $xg(x) = N(1-x)^\beta(1-Dx)$
fitted to combined HERA F_2 data, and with $\phi(x,k^2<1) = k^2\phi(x,1)$.

Sudakov resummation for dijets

Having hard jets in the final state, large logarithms associated with the hard scale have to be resummed. This resummation can be accounted for by inclusion of the Sudakov factor.



Within the small-x saturation formalism, Sudakov effects are most conveniently included in b-space (Mueller, Xiao, Yuan 2013; Staśto, Wei, Xiao, Yuan 2018)

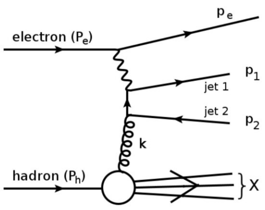
$$\mathcal{F}^{ag\to cd}_{g^*/B}(x,q_T,\mu) = \frac{-N_cS_\perp}{2\pi\alpha_s}\int\frac{b_Tdb_T}{2\pi}J_0(b_Tq_T)\,e^{-S^{ag\to cd}_{\text{Sud}}(\mu,b_T)}\,\nabla^2_{b_T}S(x,b_T)$$

where $S_\perp$ is the transverse area of the target, and $S(x,b_T)$ the dipole scattering amplitude. This can be translated into a relation for momentum dependent distributions as

$$\mathcal{F}^{ag\to cd}_{g^*/B}(x,k_T,\mu) = \int db_T\,b_T\,J_0(b_Tk_T)\,e^{-S^{ag\to cd}_{\text{Sud}}(\mu,b_T)}\int dk'_T\,k'_T\,J_0(b_Tk'_T)\,\mathcal{F}_{g^*/B}(x,k'_T)$$

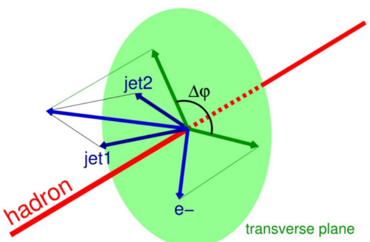
Dijets in DIS

AvH, Kotko, Kutak, Sapeta, Żarów 2021

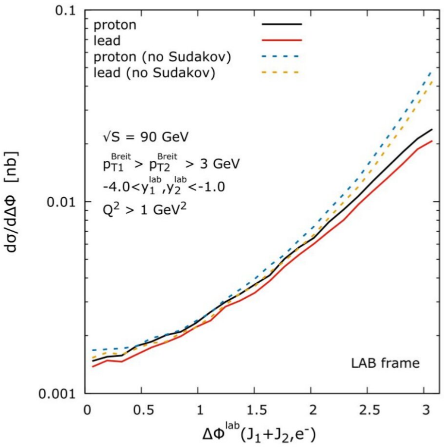


$$d\sigma_{eh\to e'+2j+X} = \int\frac{dx}{x}\frac{d^2k_T}{\pi}\mathcal{F}^{(3)}_{gg}(x,k_T,\mu) \times \frac{1}{4xP_e\cdot P_h}d\Phi(P_e,k;p_e,p_1,p_2)\,|\overline{M}_{eg^*\to e'+2j}|^2$$

ITMD for DIS only requires $\mathcal{F}^{(3)}_{gg}$, aka the Weizsäcker-Williams density



Saturation effects are typically visible in observables sensitive to the final-state momentum inbalance.



The Sudakov supression is even stronger than the saturation effect, but appears to cancel in ratio plots, that is in the nuclear modification factor.

Still, this ratio plot for the processes with, initial-state Pb over initial-state p, decreases towards large angle, an indication of saturation.