Light nuclei production with/without critical fluctuation

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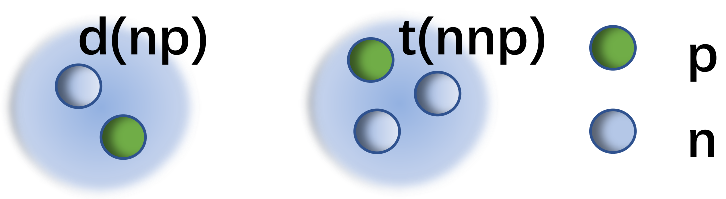
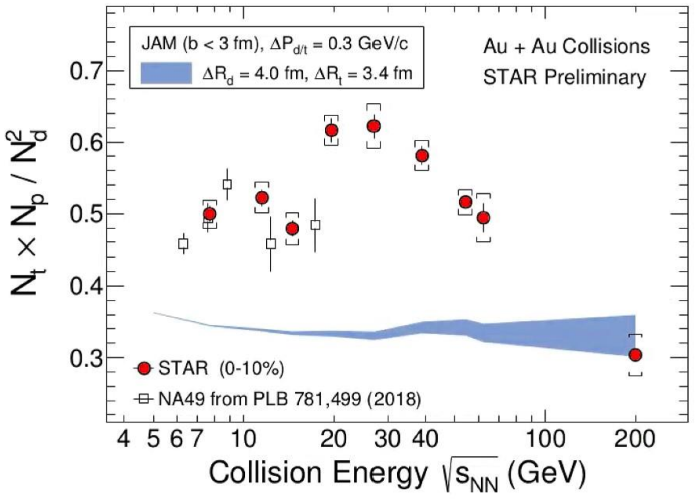
Light nuclei in heavy ion collisions



H. Liu et al., Phys. Lett. B805, 135452 (2020)



Experiment shows non-monotonicity as a function of $\sqrt{s_{NN}}$=> relates to critical point?

- Light nuclei are loosely bounded objects (~MeV), with binding energy << T_{kf}
  $\Rightarrow$ Form at late stages of collisions
  $\Rightarrow$ Detecting the phase-space distribution at freeze-out
- Widely used models: Thermal model and coalescence model

$$N_A = g_A \int \left[\prod_i^A d^3\boldsymbol{r}_i d^3\boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

$$= g_A \int \langle \prod_{i=1}^A \left[d^3 r_i d^3 p_i (\boxed{f_0(\boldsymbol{r}_i; \boldsymbol{p}_i)} + \boxed{\delta f(\boldsymbol{r}_i; \boldsymbol{p}_i)}) \right] \rangle_\sigma W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Background **Critical**

- Critical δf: nucleons interact with the order parameter field $\delta f = -f_0 \frac{g_\sigma \sigma}{\gamma T}$, which strongly fluctuates e-by-e
- but we have little knowledge on the coupling constants g_σ => proper treatment of background is important.

Decompose light-nuclei yield in terms of **phase-space cumulants**

SW, K.Murase, S.Tang, H.Song, in preparation

The production of light-nuclei with A-constituent nucleons N_A based on the Coalescence model:

$$N_A = g_A \int \left[\prod_i^A d^3 \boldsymbol{r}_i d^3 \boldsymbol{p}_i f(\boldsymbol{r}_i, \boldsymbol{p}_i) \right] W_A(\{\boldsymbol{r}_i, \boldsymbol{p}_i\}_{i=1}^A)$$

Use characteristic function to decompose the phase-space distribution in terms of phase-space cumulants: $\mathcal{C}_\alpha \coloneqq \langle z^\alpha \rangle_c$

$$f(\boldsymbol{r}_i, \boldsymbol{p}_i)/N_p := \rho(\boldsymbol{z}_i) = \int \frac{d^6 \boldsymbol{k}_i}{(2\pi)^6} e^{-\mathrm{i}\boldsymbol{k}_i \cdot \boldsymbol{z}_i} \exp\left\{ \sum_{|\alpha| \le n} \frac{\mathcal{C}_\alpha}{\alpha!} (\mathrm{i}\boldsymbol{k}_i)^\alpha \right\}$$

$$\langle z^\alpha \rangle \coloneqq 1/N_p \int d^6 z f(z)$$

N_A share a common structure $N_A \propto [\ldots]^{A-1}$ at low order $|\boldsymbol{\alpha}| < 3$

$$N_A = g_A N_p \left[\frac{8 N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}} \right]^{A-1} \cdot [1 + \mathcal{O}(\{\mathcal{C}_\alpha\}_{|\alpha| \ge 3})]$$

Combinations of N_A to eliminate the background effect of the lowest order

$$\frac{N_p^{B-A} N_B^{A-1}}{N_A^{B-1}} = \frac{g_B^{A-1}}{g_A^{B-1}} [1 + \mathcal{O}(\{\mathcal{C}_\alpha\}_{|\alpha| \ge 3})].$$

Light-nuclei yield with **critical fluctuations**

SW, K.Murase, S.Zhao, H.Song, in preparation

Introduce the critical fluctuation δf in the phase-space distribution of constituent nucleons f

$$N_A = g_A \int \langle \prod_{i=1}^{A} \left[d^3 r_i d^3 p_i (f_0(\boldsymbol{r}_i;\boldsymbol{p}_i) + \delta f(\boldsymbol{r}_i;\boldsymbol{p}_i)) \right] \rangle_\sigma W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^{A})$$

Use characteristic function to decompose the phase-space distribution in terms of phase-space cumulants: $C_\alpha := \langle z^\alpha \rangle_c$

$$f(\boldsymbol{r}_i,\boldsymbol{p}_i)/N_p := \rho(\boldsymbol{z}_i) = \int \frac{d^6 \boldsymbol{k}_i}{(2\pi)^6} e^{-i\boldsymbol{k}_i\cdot\boldsymbol{z}_i} \exp\left\{ \sum_{|\alpha|\le n} \frac{\mathcal{C}_\alpha}{\alpha!} (i\boldsymbol{k}_i)^\alpha \right\}$$

N_A share a common structure $N_A \propto [\ldots]^{A-1}[Bkg + Cri]$ at low order $|\boldsymbol{\alpha}| < 3$

$$N_A = g_A N_p \left[\frac{8N_p}{\sqrt{\det(\mathcal{C}_2 + \mathcal{I}_6)}} \right]^{A-1} \cdot [1 + \sum_{i=2}^{A} \Xi(A,i)]$$

$$R(A,B) \equiv \left(\frac{\langle N_B \rangle}{N_p}\right)^{A-1} \left(\frac{N_p}{\langle N_A \rangle}\right)^{B-1} - \frac{g_B^{A-1}}{g_A^{B-1}}$$

2pt. :$\langle \delta f_1 \delta f_2 \rangle_\sigma \sim \Xi(A,2)$, **3pt.**: $\langle \delta f_1 \delta f_2 \delta f_3 \rangle_\sigma \sim \Xi(A,3)$,
4pt.: $\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4 \rangle_\sigma \sim \Xi(A,4)$

At low order $|\boldsymbol{\alpha}| < 3$, combinations of N_A and the const g_A are directly proportional to critical contribution Ξ

$$\frac{N_p^{B-A} N_B^{A-1}}{N_A^{B-1}} - \frac{g_B^{A-1}}{g_A^{B-1}} \propto \left\{ \sum_{n=1}^{A-1} C_{(A-1)}^{n} \left[\sum_{i=2}^{B} (-1)^i C_B^i \Xi(B,i) \right]^n - \sum_{m=1}^{B-1} C_{(B-1)}^{m} \left[\sum_{i=2}^{A} (-1)^i C_A^i \Xi(A,i) \right]^m \right\}$$

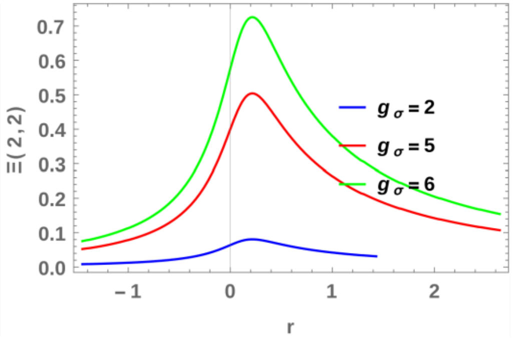
New light-nuclei yield ratio: example

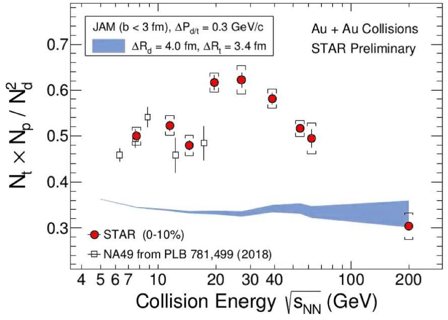
SW, K.Murase, S.Zhao, H.Song, in preparation

$$N_A = g_A \int \langle \prod_{i=1}^{A} \left[d^3 r_i d^3 p_i (f_0(\boldsymbol{r}_i;\boldsymbol{p}_i) + \delta f(\boldsymbol{r}_i;\boldsymbol{p}_i)) \right] \rangle_\sigma W_A(\{\boldsymbol{r}_i,\boldsymbol{p}_i\}_{i=1}^{A})$$

Phase-space density in non-relativistic form $f_{p,n}(\boldsymbol{r},\boldsymbol{p}) = \frac{N_{p,n}}{(2\pi)^3 (MTR_s^2)^{3/2}} e^{-\frac{\boldsymbol{p}^2}{2MT} - \frac{\boldsymbol{r}^2}{2R_s^2}}$

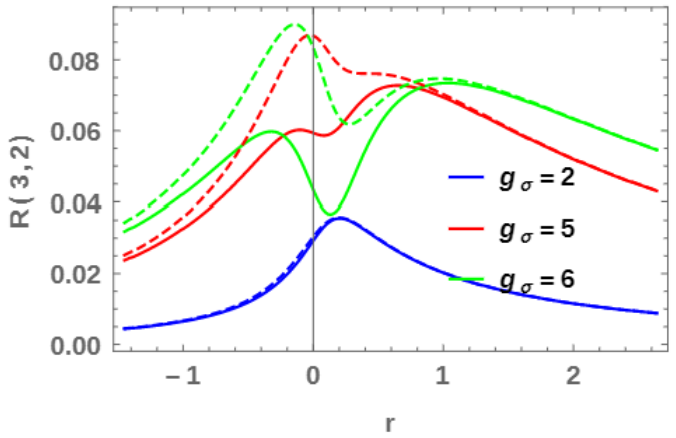
Critical correlator $\langle \Pi_i^B \, \delta f_i \rangle_\sigma \sim \Xi(A,B)$ from Ising model $\frac{\mu - \mu_c}{\Delta\mu} = -\frac{r}{\Delta r}$





R(3,2) ~2pt.-3pt.- 2pt.^{2}:

- 2pt.>3pt. because of $f_0 > \delta f$
- 2pt. ~ $(g_\sigma \xi)^2$
  =>larger g_σ larger 2pt.^{2} contribution
  ⇒ dip when g_σ is large



R(4,2)~ $3 \times$ 2pt. $-4 \times$ 3pt. $-$4pt. $-3 \times$ 2pt.^{2} $-$2pt.^{3}

- 2pt.>3pt.>4pt. because $f_0 > \delta f$
- R(4,2)~ $3 \times$ 2pt. $-3 \times$ 2pt.^{2} $-$2pt.^{3} cubic term introduces more obvious dip comparing R(3,2)
- Indicates the dip structure