Affects of criticality on the light nuclei yields

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Main Logic

Enhancement of correlation length (of order parameter) around CEP

Larger correlation length leads to larger light nuclei yield ratio

Assumption: the large correlation length survives the hadronic evolution

Appearance of CEP -> Enhancement of light nuclei yield ratio

At chemical freeze-out:

\[ \frac{\xi}{\text{fm}} \]

\[ \mu_c \]

\[ \Delta = 0.2 \text{ GeV} \]
\[ \Delta = 0.1 \text{ GeV} \]
\[ \Delta = 0.05 \text{ GeV} \]

Initial State

Pre-equilibrium Dynamics

Relativistic Fluid

Hadronization

Transport/Freeze out

Approach CEP

Light Nuclei Formation

Coalescence Model

- **Deuteron abundance:**
  \[ N_d = g_d \int dx_1 dx_2 dp_1 dp_2 f_{np}(x_1, p_1; x_2, p_2) W_d \left( \frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}} ; \sigma_d \right) \]

  \[ N_d = g_d \int dx_1 dx_2 dp_1 dp_2 f_{np}(x_1, p_1; x_2, p_2) W_d \left( \frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}} ; \sigma_d \right) \]

  - Joint probability:
    \[ f_{np}(x_1, p_1; x_2, p_2) \sim \rho_{np}(x_1, x_2) e^{-\frac{p_1^2 + p_2^2 + p_3^2}{2mT}} \]
    \[ \rho_{np}(x_1, x_2) = \rho_n(x_1)\rho_p(x_2) + C_2(|x_1 - x_2|) \]

  - Long-range Cor.:
    \[ C_2^*(r) = \langle \delta \rho_B(r)\delta \rho_B(0) \rangle \approx \left( \frac{\partial \rho_B}{\partial \phi} \right)^2 \langle \delta \phi(r)\delta \phi(0) \rangle \]
    \[ \zeta_2 = \frac{T}{\pi \rho_B^2} \left( \frac{\partial \rho_B}{\partial \phi} \right)^2 > 0 \]

- **Triton abundance:**
  \[ N_t = g_t \int dx_1 dx_2 dx_3 dp_1 dp_2 dp_3 f_{nnp}(x_1, p_1; x_2, p_2; x_3, p_3) W_t(\sigma_t) \]

  - Joint probability:
    \[ f_{nnp}(x_1, p_1; x_2, p_2; x_3, p_3) \sim \rho_{nnp}(x_1, x_2, x_3) e^{-\frac{p_1^2 + p_2^2 + p_3^2}{2mT}} \]
    \[ \rho_{nnp}(x_1, x_2, x_3) = \rho_n(x_1)\rho_n(x_2)\rho_p(x_3) + \rho_n(x_1)C_2(x_2, x_3) \]
    \[ + \rho_n(x_2)C_2(x_1, x_3) + \rho_p(x_3)C_2(x_1, x_2) + C_3(x_1, x_2, x_3) \]

  - Long-range Cor.:
    \[ \zeta_3 = \frac{-3g_3 T^2}{\pi^4 \rho_B^3} \left( \frac{\partial \rho_B}{\partial \phi} \right)^3 > 0 \]

\[ 1 \quad \text{Nucleons are emitted from a thermalized source} \]
\[ 2 \quad \text{Neglect the correlation among momenta} \]
\[ 3 \quad \text{Dominated by the singular term around CEP} \]
Yields & Correlation Length

\[ N_d \approx N_d^{(0)} \left[ 1 + C_{np} + \frac{\lambda_2}{\sigma_d} G_2 \left( \frac{\xi}{\sigma_d} \right) \right] \]
\[ N_t \approx N_t^{(0)} \left[ 1 + \Delta \rho_B + 2 C_{np} + \frac{3\lambda_2}{\sigma_t} G_2 \left( \frac{\xi}{\sigma_t} \right) + \lambda_3 G_3 \left( \frac{\xi}{\sigma_t} \right) \right] \]
\[ \frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[ 1 + \Delta \rho_B + \frac{\lambda_2}{\sigma_t} G_2 \left( \frac{\xi}{\sigma_t} \right) + \lambda_3 G_3 \left( \frac{\xi}{\sigma_t} \right) \right] \]

- Light nuclei yields (and their ratios) increase with the correlation length can thus be employed as a 'ruler' measuring the correlation length

- The resolution of such a 'ruler' is the size of the light nuclei (2-3 fm) close to \( \xi_{\text{max}} \)

- G-functions saturates at large \( \xi \) a broad peak in BES
Signal from critical point?

A dynamical simulation (maybe via hydro+) is necessary to see whether the long-range correlation survives during the hadronic evolution.
Appendix: Sign of $\zeta_3$

Quark-Meson Model:  

$$H = H_{\text{kin}} + H_{q-m} + \frac{\lambda}{4} \left( \sigma^2 + \pi^2 - v^2 \right)^2 - c\sigma$$

Order parameter for chiral phase transition:  

$$\sigma = \sigma^* + \sigma'$$

Expectation value of q-condensate:  

$$\sigma^* = \frac{M_Q}{g_{Q\sigma}} > 0$$

$$H = \ldots + \lambda \sigma^* \sigma'^3 + \ldots$$

$$g_3 \approx \lambda \sigma^* + O(\hbar) > 0$$

$$\left( \frac{\partial \rho_B}{\partial \sigma^*} \right)_T < 0$$

$$\zeta_3 = \frac{-3g_3T^2}{\pi^4 \rho_B^3} \left( \frac{\partial \rho_B}{\partial \phi} \right)^3 > 0!$$