Causal second-order magnetohydrodynamics from kinetic theory using RTA approximation

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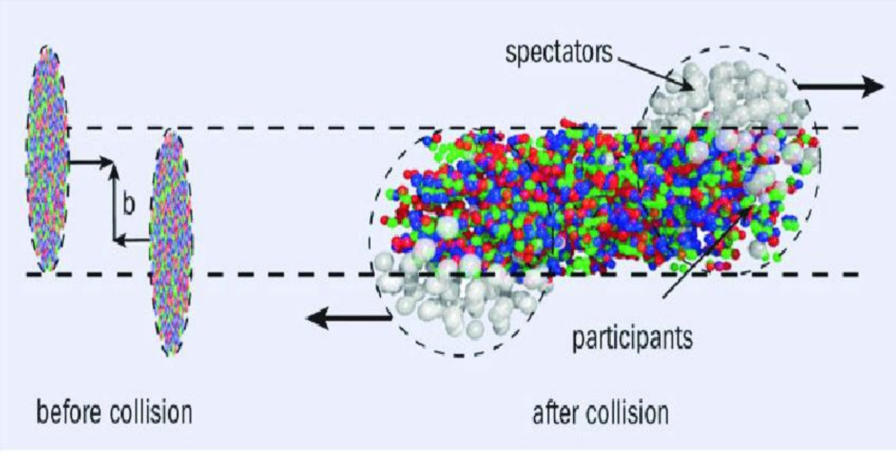
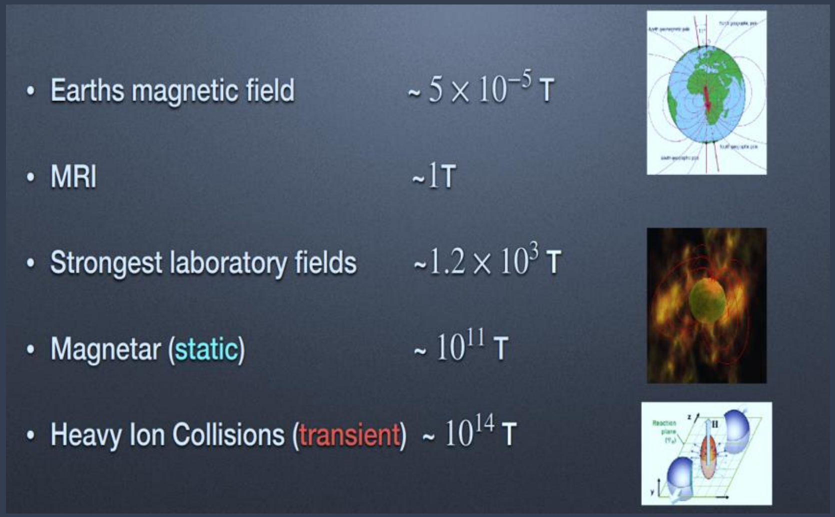
QGP AND MAGNETIC FIELD PRODUCTION

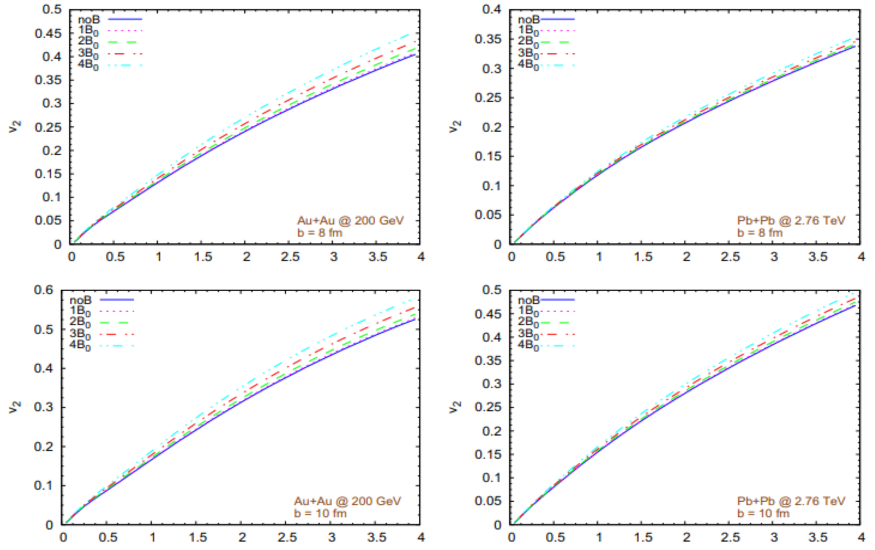


MAGNETIC FIELD MAGNITUDE COMPARISION

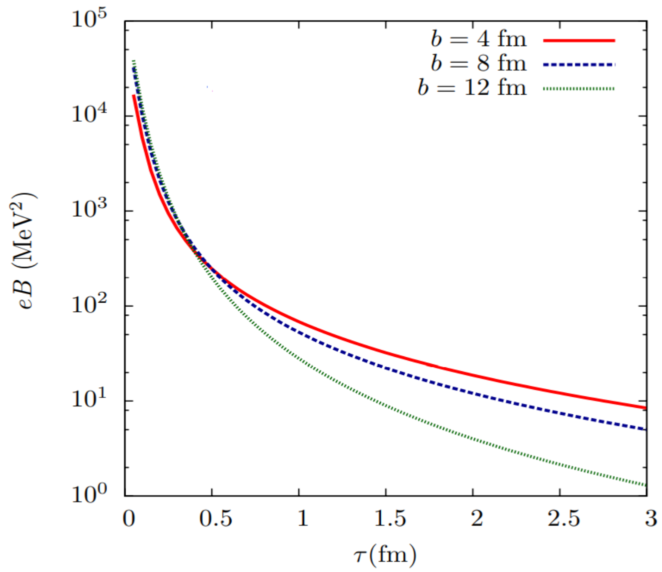


MOTIVATION

v_2 VARIATION WITH MAGNETIC-FIELD AND b



MAGNETIC-FIELD VARIATION WITH b



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D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl. Phys. A803, 227 (2008)

MICROSCOPIC THEORY

KINETIC THEORY

BOLTZMANN EQUATION

$$p^\mu \partial_\mu f + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} f = C[f]$$

RTA

$$p^\mu \partial_\mu f + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu} f = -\frac{u \cdot p}{\tau_c} \delta f$$

$$f = \sum_{n=0}^{\infty} (-1)^n \left(\frac{\tau_c}{u.p}\right)^n \left(p^\mu \partial_\mu + qF^{\mu\nu} p_\nu \frac{\partial}{\partial p^\mu}\right)^n f_0.$$

Find $(\delta f)^1$ and $(\delta f)^2$

$$V_f^\mu \equiv \Delta^\mu_\nu \delta N^\nu = \Delta^\mu_\nu \int dp p^\nu \left(\delta f - \delta \tilde{f}\right)$$

$$\Pi \equiv -\frac{\Delta_{\mu\nu}}{3} \delta T^{\mu\nu} = -\frac{\Delta_{\mu\nu}}{3} \int dp p^\mu p^\nu \left(\delta f + \delta \tilde{f}\right)$$

$$\pi^{\mu\nu} \equiv \Delta^{\mu\nu}_{\alpha\beta} \delta T^{\alpha\beta} = \Delta^{\mu\nu}_{\alpha\beta} \int dp p^\mu p^\nu \left(\delta f + \delta \tilde{f}\right)$$

FORMALISM

MACROSCOPIC THEORY

RELATIVISTIC MAGNETOHYDRODYNAMICS EQUATIONS

$$\partial_\mu T_f^{\mu\nu} = F^{\nu\lambda} J_{\lambda\, f} \qquad \partial_\mu J_f^\mu = 0.$$

$$\partial_\mu F^{\mu\nu} = J^\nu \qquad F^{\mu\nu} = E^\mu u^\nu - E^\nu u^\mu + \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta,$$

- NO POLARISATION
- NO MAGNETISATION

CALCULATE : $\dot{\alpha}$, $\dot{\beta}$, $\dot{u}^\mu$

EQUATION OF MOTION

FINDING EVOLUTION EQUATION FOR DISSIPATIVE STRESSES ($\pi^{\mu\nu}$, V^μ, Π)

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ANISOTROPIC CO-EFFICIENT

DIFFUSION COEFFICIENTS

$$\kappa_{\|} = \beta_V \tau_c,$$

$$\kappa_{\perp} = \frac{\beta_V \tau_c}{1+(qB\tau_c\delta_{VB})^2},$$

$$\kappa_{\times} = \frac{\beta_V qB\tau_c^2\delta_{VB}}{1+(qB\tau_c\delta_{VB})^2} = \kappa_{\perp} qB\tau_c\delta_{VB}.$$

CONDUCTIVITY COEFFICIENTS

$$\sigma_E^{\|} = q^2\tau_c\beta\beta_V,$$

$$\sigma_E^{\perp} = \frac{q^2\tau_c\beta\beta_V}{1+(qB\tau_c\delta_{VB})^2},$$

$$\sigma_E^{\times} = \frac{q^3B\tau_c^2\beta\beta_V\delta_{VB}}{1+(qB\tau_c\delta_{VB})^2}.$$

SHEAR COEFFICIENTS

$$\eta_0 = 2\beta_\pi\tau_c,$$

$$\eta_1 = \frac{2\beta_\pi\tau_c}{1+(2qB\tau_c\delta_{\pi B})^2},$$

$$\eta_2 = \frac{4\beta_\pi qB\tau_c^2\delta_{\pi B}}{1+(2qB\tau_c\delta_{\pi B})^2} = 2\eta_1 qB\tau_c\delta_{\pi B},$$

$$\eta_3 = \frac{2\beta_\pi\tau_c}{1+(qB\tau_c\delta_{\pi B})^2},$$

$$\eta_4 = \frac{2\beta_\pi qB\tau_c^2\delta_{\pi B}}{1+(qB\tau_c\delta_{\pi B})^2} = \eta_3 qB\tau_c\delta_{\pi B}.$$

IDEAL-MHD EVOLUTION EQUATION

$$\frac{\Pi}{\tau_c} = -\dot{\Pi} - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} - \tau_{\Pi V}V\cdot\dot{u} - \lambda_{\Pi V}V\cdot\nabla\alpha - l_{\Pi V}\partial\cdot V - \beta_\Pi\theta$$
$$+\tau_c\tau_{\Pi VB}\dot{u}_\alpha qBb^{\alpha\beta}V_\beta - \tau_c q\delta_{\Pi VB}\nabla_\mu\left(Bb^{\mu\beta}V_\beta\right) - \tau_c qB\lambda_{\Pi VB}b^{\mu\beta}V_\beta\nabla_\mu\alpha$$

$$\frac{V^\mu}{\tau_c} = -\dot{V}^{\langle\mu\rangle} - V_\nu\omega^{\nu\mu} - \lambda_{VV}V^\nu\sigma_\nu^\mu - \delta_{VV}V^\mu\theta + \lambda_{V\Pi}\Pi\nabla^\mu\alpha - \lambda_{V\pi}\pi^{\mu\nu}\nabla_\nu\alpha - \tau_{V\pi}\pi_\nu^\mu\dot{u}^\nu$$
$$+\tau_{V\Pi}\Pi\dot{u}^\mu + l_{V\pi}\Delta^{\mu\nu}\partial_\gamma\pi_\nu^\gamma - l_{V\Pi}\nabla^\mu\Pi + \beta_V\nabla^\mu\alpha - qB\delta_{VB}b^{\mu\gamma}V_\gamma + \tau_c qBl_{V\pi B}b^{\sigma\mu}\partial^\kappa\pi_{\kappa\sigma}$$
$$+\tau_c qB\tau_{V\Pi B}b^{\gamma\mu}\Pi\dot{u}_\gamma - \tau_c qBl_{V\Pi B}b^{\gamma\mu}\nabla_\gamma\Pi - q\tau_c\delta_{VVB}Bb^{\mu\nu}V_\nu\theta - q\tau_c\lambda_{VVB}Bb^{\gamma\nu}V_\nu\sigma_\gamma^\mu$$
$$-q\tau_c\rho_{VVB}Bb^{\gamma\nu}V_\nu\omega_\gamma^\mu - \tau_c q\tau_{VVB}\Delta_\gamma^\mu D\left(Bb^{\gamma\nu}V_\nu\right)$$

$$\frac{\pi^{\mu\nu}}{\tau_c} = -\dot{\pi}^{\mu\nu} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi V}V^{\langle\mu}\dot{u}^{\nu\rangle}$$
$$+\lambda_{\pi V}V^{\langle\mu}\nabla^{\nu\rangle}\alpha + l_{\pi V}\nabla^{\langle\mu}V^{\nu\rangle} + \delta_{\pi B}\Delta_{\eta\beta}^{\mu\nu}qBb^{\gamma\eta}g^{\beta\rho}\pi_{\gamma\rho} - \tau_c qB\tau_{\pi VB}\dot{u}^{\langle\mu}b^{\nu\rangle\sigma}V_\sigma$$
$$-\tau_c qB\lambda_{\pi VB}V_\gamma b^{\gamma\langle\mu}\nabla^{\nu\rangle}\alpha - q\tau_c\delta_{\pi VB}\nabla^{\langle\mu}\left(B^{\nu\rangle\gamma}V_\gamma\right)$$

RESULTS

$$\delta f^{(1)} = \frac{f_0\tilde{f}_0\tau_c}{u\cdot p}\left(A\Pi + B^\beta V_\beta + C^{\gamma\rho}\pi_{\gamma\rho}\right),$$

Cooper-Frye Formalism

NUMERICAL CODES

RESISTIVE-MHD EVOLUTION EQUATIONS

$$\frac{\Pi}{\tau_c} = -\dot{\Pi} - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} - \tau_{\Pi V}V\cdot\dot{u} - \lambda_{\Pi V}V\cdot\nabla\alpha - l_{\Pi V}\partial\cdot V - \beta_\Pi\theta - qB\lambda_{\Pi VB}b^{\mu\beta}V_\beta V_\mu$$
$$+\tau_c\tau_{\Pi VB}\dot{u}_\alpha qBb^{\alpha\beta}V_\beta - q\delta_{\Pi VB}\nabla_\mu\left(\tau_c Bb^{\mu\beta}V_\beta\right) - q^2\tau_c\chi_{\Pi EE}E^\mu E_\mu.$$

$$\frac{V^\mu}{\tau_c} = -\dot{V}^{\langle\mu\rangle} - V_\nu\omega^{\nu\mu} + \lambda_{VV}V^\nu\sigma_\nu^\mu - \delta_{VV}V^\mu\theta + \lambda_{V\Pi}\Pi\nabla^\mu\alpha - \lambda_{V\pi}\pi^{\mu\nu}\nabla_\nu\alpha - \tau_{V\pi}\pi_\nu^\mu\dot{u}^\nu - qB\delta_{VB}b^{\mu\gamma}V_\gamma$$
$$+\tau_{V\Pi}\Pi\dot{u}^\mu + l_{V\pi}\Delta^{\mu\nu}\partial_\gamma\pi_\nu^\gamma - l_{V\Pi}\nabla^\mu\Pi + \beta_V\nabla^\mu\alpha + \tau_c qBl_{V\pi B}b^{\sigma\mu}\partial^\kappa\pi_{\kappa\sigma} - q\tau_c\lambda_{VVB}Bb^{\gamma\nu}V_\nu\sigma_\gamma^\mu$$
$$+\tau_c qB\tau_{V\Pi B}b^{\gamma\mu}\Pi\dot{u}_\gamma - \tau_c qBl_{V\Pi B}b^{\gamma\mu}\nabla_\gamma\Pi - q\tau_c\delta_{VVB}Bb^{\mu\nu}V_\nu\theta - q\tau_c\rho_{VVB}Bb^{\gamma\nu}V_\nu\omega_\gamma^\mu$$
$$+\chi_{VE}qE^\mu + q\Delta_\alpha^\mu\chi_{VE}D\left(\tau_c E^\alpha\right) - q\tau_c\rho_{VE}E^\mu\theta - q\tau_{VVB}\Delta_\gamma^\mu D\left(\tau_c Bb^{\gamma\nu}V_\nu\right).$$

$$\frac{\pi^{\mu\nu}}{\tau_c} = -\dot{\pi}^{\langle\mu\nu\rangle} + 2\beta_\pi\sigma^{\mu\nu} + 2\pi_\gamma^{\langle\mu}\omega^{\nu\rangle\gamma} - \tau_{\pi\pi}\pi_\gamma^{\langle\mu}\sigma^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} - \tau_{\pi V}V^{\langle\mu}\dot{u}^{\nu\rangle} - \tau_c qB\tau_{\pi VB}\dot{u}^{\langle\mu}b^{\nu\rangle\sigma}V_\sigma$$
$$+\lambda_{\pi V}V^{\langle\mu}\nabla^{\nu\rangle}\alpha - l_{\pi V}\nabla^{\langle\mu}V^{\nu\rangle} + \delta_{\pi B}\Delta_{\eta\beta}^{\mu\nu}qBb^{\gamma\eta}g^{\beta\rho}\pi_{\gamma\rho} - qB\lambda_{\pi VB}V_\gamma b^{\gamma\langle\mu}V^{\nu\rangle} - q\delta_{\pi VB}\nabla^{\langle\mu}\left(\tau_c B^{\nu\rangle\gamma}V_\gamma\right)$$
$$+q^2\tau_c\chi_{\pi EE}\Delta_{\sigma\rho}^{\mu\nu}E^\sigma E^\rho.$$

SUMMARY

- ❑ We Found out the 2^{nd}-ORDER evolution equations of viscous stresses .
- ❑ All the Transport coefficients pertaining to this study have been evaluated .
- ❑ The anisotropic transport coefficients of shear , diffusion stresses and for the electrical conductivities have also been evaluated in the presence of external electromagnetic field.
- ❑ From kinetic theory perspective we have found out the small corrections to equilibrium distribution function which can be readily used in the cooper-frey formula to find out different flow harmonics.

THANK YOU