Data-driven analysis of light parton transport properties in a hard-soft factorized energy loss model

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Hard-soft factorization of parton energy loss

Weakly-coupled effective kinetic formalism
Leading-order realizations (e.g. MARTINI):

\[(\partial_t + \vec{v} \cdot \nabla_x) f^a (\vec{p}, \vec{x}, t) = -C^{2\leftrightarrow 2}_a[f] - C^{1\leftrightarrow 2}_a[f]\]

Hard-soft factorization:

\[C^{2\leftrightarrow 2}_a + C^{1\leftrightarrow 2}_a\]

\[= C^{\text{large-angle}}_a (\mu_{\tilde{q}_\perp}, \Lambda) + C^{\text{split}}_a (\Lambda) + C^{\text{large-\omega}}_a (\mu_{\omega}) + C^{\text{diff}}_a (\mu_{\tilde{q}_\perp}, \mu_{\omega})\]

Interactions with the medium:
- Large number of soft interactions
- Rare hard scatterings

Inelastic interactions

\[\tilde{q}_\perp \rightarrow \mu_{\tilde{q}_\perp}\]

\[C^{2\leftrightarrow 2}_a: \text{vacuum matrix elements}\]

\[C^{\text{split}}_a: \text{vacuum matrix elements}\]

\[C^{\text{soft}}_a: \text{Diffusion process (Langevin model)}\]

\[C^{1\leftrightarrow 2}_a: \text{resummed integral equations}\]
Hard-soft factorization of parton energy loss (Tequila)

Benefits of the hard-soft factorization

- Non-perturbative effects absorbed in transport coefficients $\hat{q}_{\text{soft}}, \hat{q}_{L,\text{soft}}$
- Stochastic description is numerically more efficient
- Soft transport coefficients can be constrained from measurements
- Can be extended to next-to-leading order

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<td>Interaction factorization</td>
<td>$C_\text{hard}^{2\leftrightarrow2} + C_\text{hard}^{1\leftrightarrow2} + C_{\text{diff}}$</td>
<td>$C_\text{hard}^{2\leftrightarrow2} + C_\text{hard}^{1\leftrightarrow2}$</td>
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Brick test: weak coupling and beyond

We compare: collision rate treatment v.s. stochastic treatment
We use: pure glue medium; screened matrix elements for collision rates
We plot: energy distribution of a hard gluon propagating in a static medium

\[ T = 300 \text{ MeV} \]
\[ E_0 = 100 \text{GeV} \]
Evolution time:
\[ t = \left( \frac{0.3}{\alpha_s} \right)^2 \text{fm/c} \]

We validate: the dependence of the single parton energy distribution on the hard-soft cutoff

Energy loss is weakly dependent on the cutoffs.
Data-driven analysis

**Hard-soft factorized model**
- Hard interactions:
  \[
  (\partial_t + \vec{v} \cdot \nabla_x) f^a(\vec{p}, \vec{x}, t) = -C_{a, \text{vacuum}}^{2 \leftrightarrow 2}[f] - C_{a, \text{AMY}}^{1 \leftrightarrow 2}[f]
  \]
- Soft interactions:
  \[
  C_{\text{diff}}[f] = -\frac{\partial}{\partial p^i} \left[ \eta_D(p) p^i f(p) \right] - \frac{1}{2} \frac{\partial^2}{\partial p^i \partial p^j} \left[ \left( p^i \hat{q}^j \hat{q}_L(p) + \frac{1}{2} (\delta^{ij} - \hat{p}^i \hat{p}^j) \hat{q}(p) \right) f(p) \right]
  \]

**Goal: model-to-data comparison**
- Simultaneously describe several set of experimental data
- Quantitatively estimate the model parameters:
  - Coupling constant of hard elastic interactions $\alpha_s^{\text{hard,elas}}$
  - Coupling constant of hard inelastic interactions $\alpha_s^{\text{hard,inel}}$
  - Parameterize soft $\hat{q}, \hat{q}_L$ (based on perturbative formula):
    \[
    \hat{q}_{\text{soft}}^{2 \leftrightarrow 2} = 4\pi (\alpha_s^{\text{soft}})^2 \left( \frac{N_c}{3} + \frac{N_f}{3} \right) C_R T^3 \ln \left[ 1 + \frac{1}{4\pi \alpha_s^{\text{soft}} \left( \frac{N_c}{3} + \frac{N_f}{3} \right)} \right]
    \]
    \[
    \hat{q}_{\text{soft}}^{1 \leftrightarrow 2} = \frac{4(2 - \ln 2)}{\pi} \left( \alpha_s^{\text{soft}} \right)^2 C_R C_T A T^3
    \]
- Quantify the non-perturbative effects of the soft interactions

**Gaussian process emulator**
Fast regression to predict model outputs

**Physics model**
Hard-soft factorized parton energy loss model

**Markov Chain Monte Carlo**
Random walk in parameter space

**Bayes’ theorem**
\[ p(\theta | y = y_{\text{exp}}) \propto p(\theta) \times L(y = y_{\text{exp}} | \theta) \]

**Posterior distribution of model parameters**

**Experimental data**
Charged pion $R_{AA}$ of Au+Au 200GeV at centrality 0-10%, 40-50%
Data-driven analysis

Posterior distribution of the model parameters

Model parameters: $k$, $\alpha_s^{\text{hard,elas}}$, $\alpha_s^{\text{hard,inel}}$

Observables:
- Charged pion $R_{p\pi}$ in Au + Au 200 GeV at centrality 0-10% and centrality 40-50% (PHENIX 2013*)

Validate the observables using Maximum a Posteriori (MAP) estimation of the model parameters