Dijet production in DIS at NLO in the CGC

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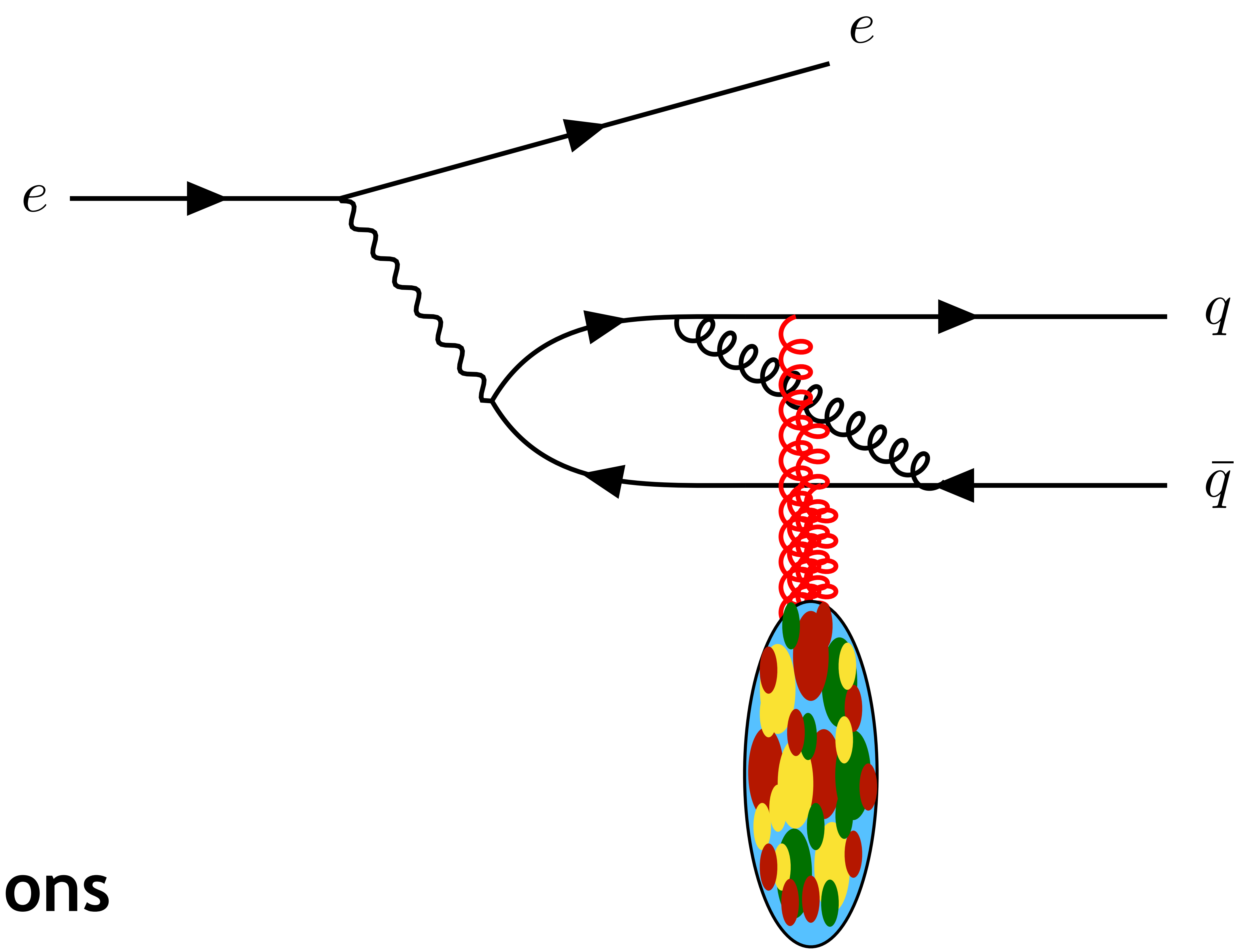
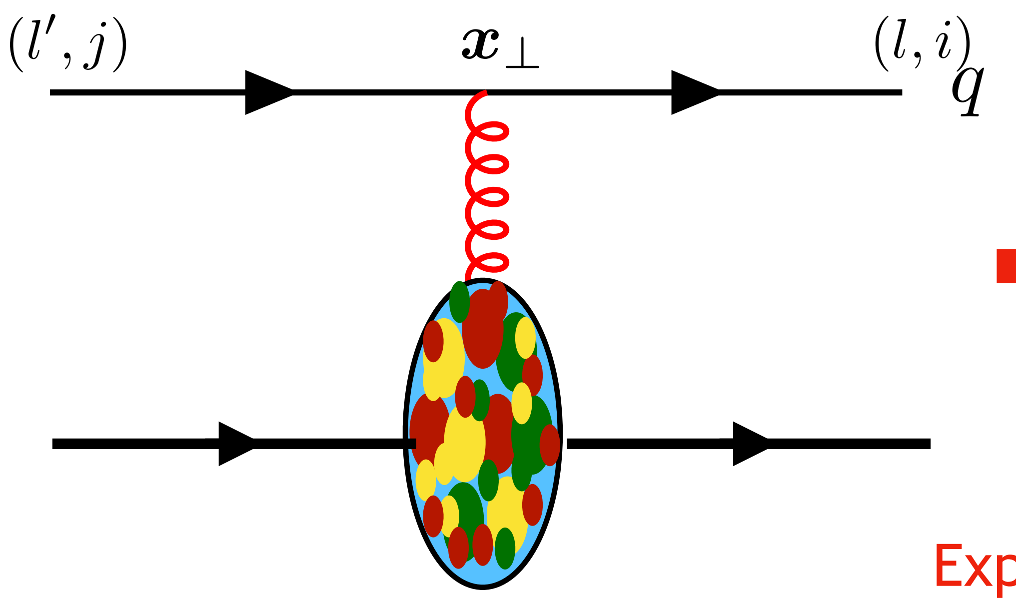


Based on P. Caucal, F. Salazar, R. Venugopalan. *JHEP* 11 (2021) 222. arXiv: 2108.06347

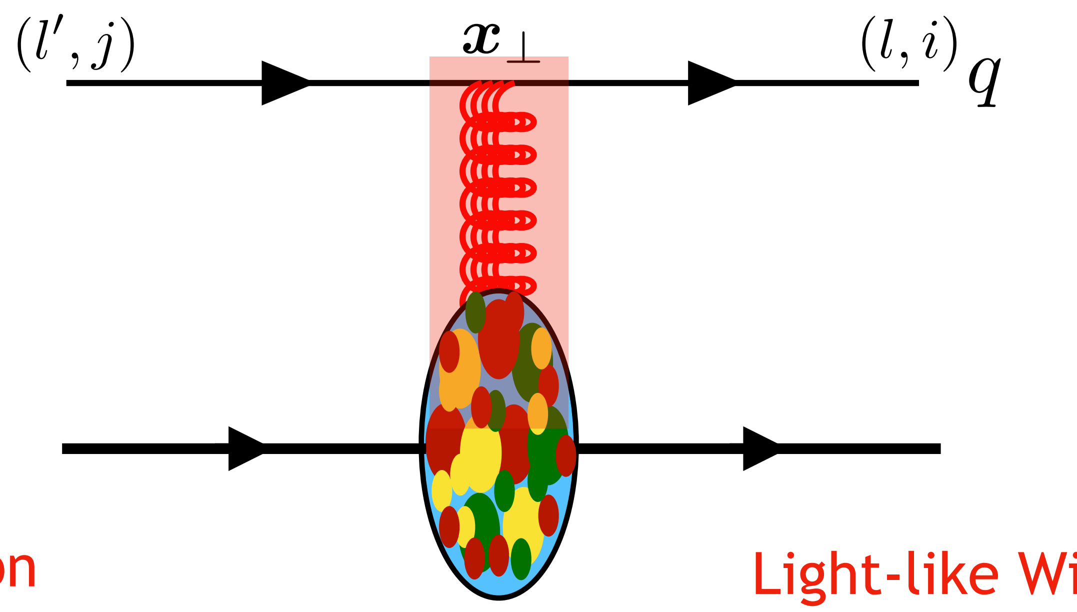
+ work in progress P. Caucal, B. Schenke, F. Salazar ,R. Venugopalan.

Formalism and NLO contributions

- Covariant perturbation theory (light-cone gauge) with effective CGC Feynman rules for multiple scattering



Exponentiation
(within eikonal approximation)



Effective vertex quark with CGC shock-wave

$$\mathcal{T}^q_{ij}(l,l') = (2\pi)\delta(l^- - l'^-)\gamma^- \mathrm{sgn}(l^-) \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} V_{ij}^{\mathrm{sgn}(l^-)}(\boldsymbol{z}_\perp)$$

Effective vertex gluon with CGC shock-wave

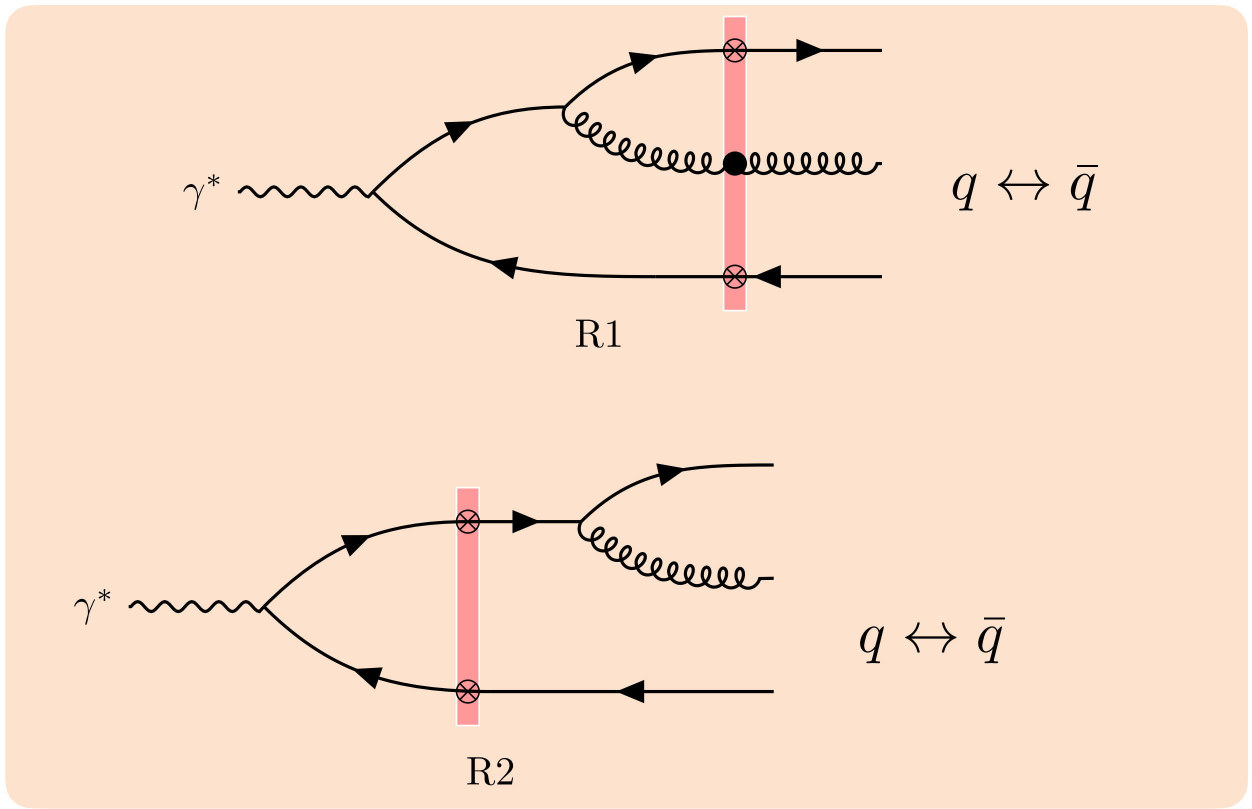
$$\mathcal{T}^g_{ab}(l,l') = -(2\pi)\delta(l^- - l'^-)(2l^-)g_{\mu\nu} \mathrm{sgn}(l^-) \int \mathrm{d}^2\boldsymbol{z}_\perp e^{-i(\boldsymbol{l}'_\perp - \boldsymbol{l}_\perp)\cdot \boldsymbol{z}_\perp} U_{ab}^{\mathrm{sgn}(l^-)}(\boldsymbol{z}_\perp)$$

Light-like Wilson line

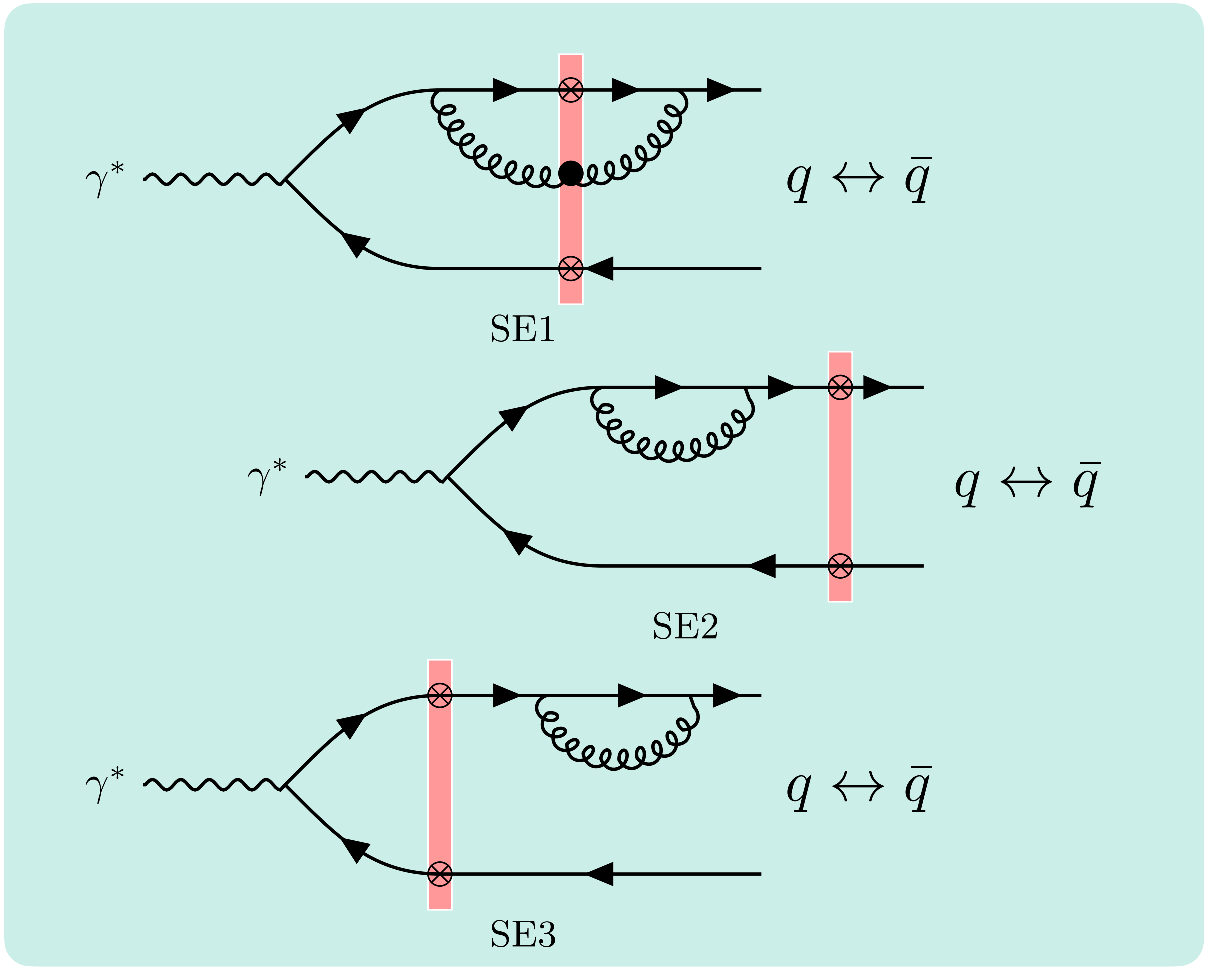
$$V_{ij}(\boldsymbol{z}_\perp) = \mathcal{P}\left[\exp\left(ig\int \mathrm{d}z^- A^{+,a}(z^-,\boldsymbol{z}_\perp)t^a_{ij}\right)\right] \quad U_{ab}(\boldsymbol{z}_\perp) = \mathcal{P}\left[\exp\left(ig\int \mathrm{d}z^- A^{+,c}(z^-,\boldsymbol{z}_\perp)t^c_{ab}\right)\right]$$

- Dimensional regularization integrals over transverse coordinates, and hard cut-off $\Lambda^- = z_0 q^-$ for $\mathrm{d}l^-$ loop integrals
- One-loop contributions

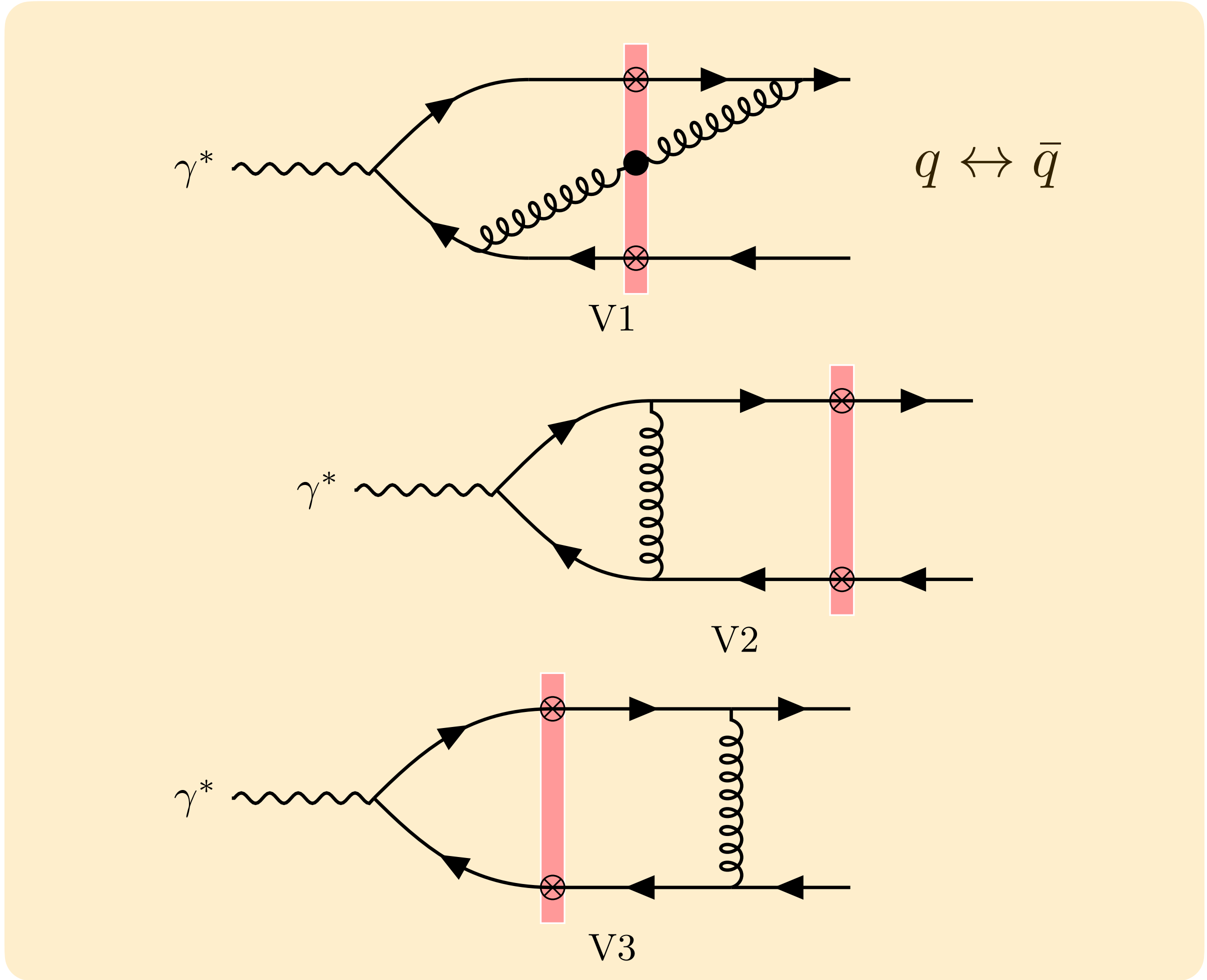
Real emission diagrams



Self-energy contributions

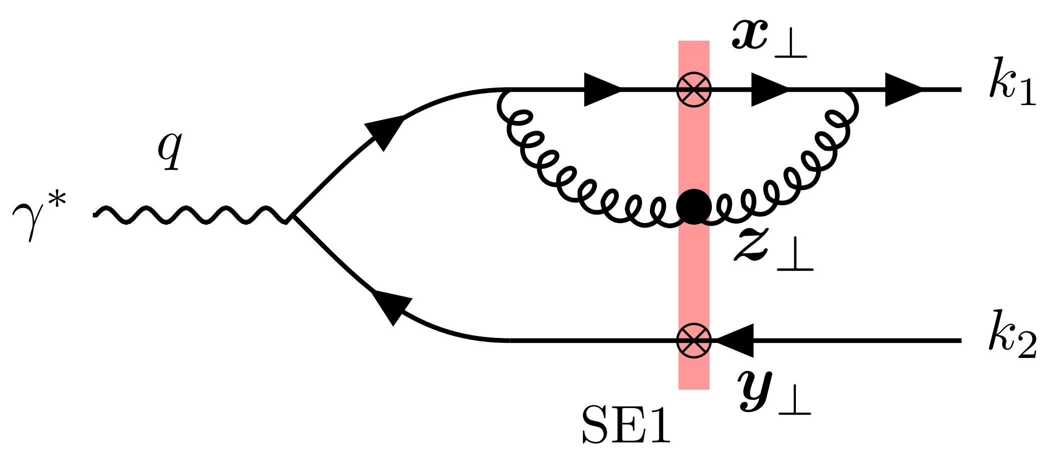


Vertex contributions



Anatomy of NLO amplitudes

Self energy with gluon crossing SW (only results for longitudinally polarized photon shown)



$$\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp) = \left[t^a V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{z}_\perp)t_a V(\boldsymbol{z}_\perp)V^\dagger(\boldsymbol{y}_\perp) - C_F\right]_{ij}$$

$$\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = C_F\left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij}$$

z_1, z_2, z_g	Quark, anti-quark and gluon longitudinal momentum fractions
$\boldsymbol{k_{1\perp}}, \boldsymbol{k_{2\perp}}, \boldsymbol{k_{g\perp}}$	Quark, anti-quark and gluon transverse momentum

UV finite piece

$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UVfinite},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\left[\mathcal{C}_{\mathrm{SE1},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp,\boldsymbol{z}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) - \mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})\right]$$

$$\mathcal{N}^{\lambda=0,\sigma\sigma'}_{\mathrm{SE1}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_1}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-i\frac{z_g}{z_1}\boldsymbol{k_{1\perp}}\cdot\boldsymbol{r}_{zx}}}{\boldsymbol{r}^2_{zx}}2(z_1z_2)^{3/2}QK_0\left(QX_{\mathrm{V}}\right)\delta_{\sigma_1,-\sigma_2}$$

$$\mathcal{N}^{\lambda=0}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx}) = -\frac{\alpha_s}{\pi^2}\int_{z_0}^{z_q}\frac{\mathrm{d}z_g}{z_g}\frac{1}{2}\left[1+\left(1-\frac{z_g}{z_1}\right)^2\right]\frac{e^{-\frac{\boldsymbol{r}^2_{zx}}{2\xi}}}{\boldsymbol{r}^2_{zx}}2(z_1z_2)^{3/2}QK_0\left(Q\sqrt{z_1z_2}r_{xy}\right)\delta_{\sigma_1,-\sigma_2}$$

UV divergent piece

$$\mathcal{M}^{\lambda}_{\mathrm{SE1,UV},ij,\sigma_1\sigma_2} = \frac{ee_f q^-}{\pi}\int_{\boldsymbol{x}_\perp,\boldsymbol{y}_\perp} e^{-i(\boldsymbol{k_{1\perp}}\cdot\boldsymbol{x}_\perp+\boldsymbol{k_{2\perp}}\cdot\boldsymbol{y}_\perp)}\,\mathcal{C}_{\mathrm{UV},ij}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp)\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy},\boldsymbol{r}_{zx})$$

$$\mathcal{N}^{\lambda}_{\mathrm{SE1,UV},\sigma_1\sigma_2}(\boldsymbol{r}_{xy}) = \frac{\alpha_s}{2\pi}\left\{\left(2\ln\left(\frac{z_1}{z_0}\right)-\frac{3}{2}\right)\left(\frac{2}{\varepsilon}+\ln(2\pi\mu^2\xi)\right)-\frac{1}{2}+\mathcal{O}(\varepsilon)\right\}\mathcal{N}^{\lambda}_{\mathrm{LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

UV pole

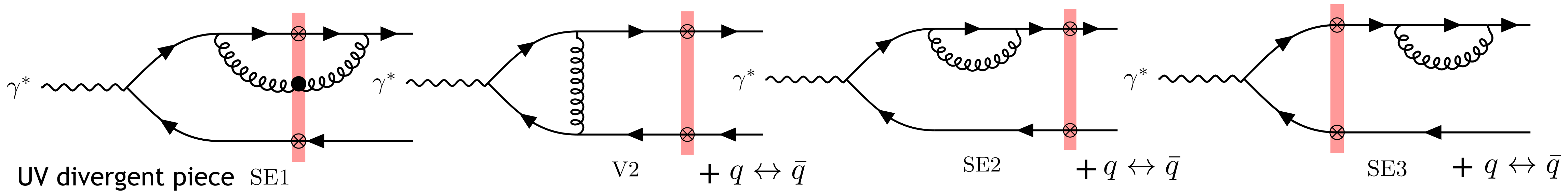
$$X_{\mathrm{V}}^2 = z_2(z_1-z_g)\boldsymbol{r}^2_{xy} + z_g(z_1-z_g)\boldsymbol{r}^2_{zx} + z_2z_g\boldsymbol{r}^2_{zy}$$

Explicit result for other amplitudes can be found in
2108.06347

Divergences in NLO contribution

- Divergences in virtual emissions

Diagrams containing poles



Sum of divergent contributions:

$$\mathcal{M}_{\rm IR} = \mathcal{M}_{\rm V2} + (\mathcal{M}_{\rm SE1,UV} + \mathcal{M}_{\rm SE2} + \mathcal{M}_{\rm SE3} + q \leftrightarrow \bar{q})$$

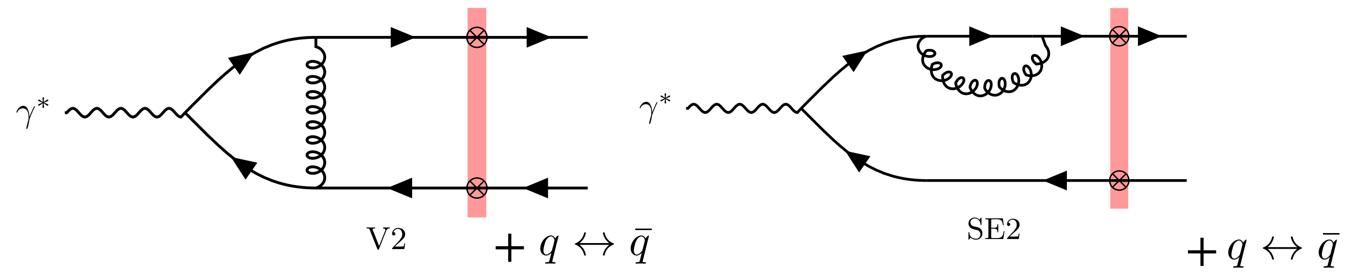
← Contributions proportional to LO color structure

$$= \frac{ee_f q^-}{\pi} \int_{\boldsymbol{x}_\perp, \boldsymbol{y}_\perp} e^{-i(\boldsymbol{k}_{1\perp}\cdot\boldsymbol{x}_\perp + \boldsymbol{k}_{2\perp}\cdot\boldsymbol{y}_\perp)} C_F \left[V(\boldsymbol{x}_\perp)V^\dagger(\boldsymbol{y}_\perp) - \mathbb{1}\right]_{ij} \mathcal{N}^{\lambda}_{{\rm LO},\varepsilon,\sigma_1\sigma_2}(\boldsymbol{r}_{xy})$$

$$\times \frac{\alpha_s}{2\pi}\left\{\left(\ln\left(\frac{z_q}{z_0}\right) + \ln\left(\frac{z_{\bar{q}}}{z_0}\right) - \frac{3}{2}\right)\left(\frac{2}{\varepsilon} - 2\gamma_E - \ln\left(\frac{\boldsymbol{r}_{xy}^2\tilde{\mu}^2}{4}\right) + 2\ln(2\pi\mu^2\xi)\right) + \frac{1}{2}\ln^2\left(\frac{z_{\bar{q}}}{z_q}\right) - \frac{\pi^2}{6} + \frac{5}{2} - \frac{1}{2}\right\}$$

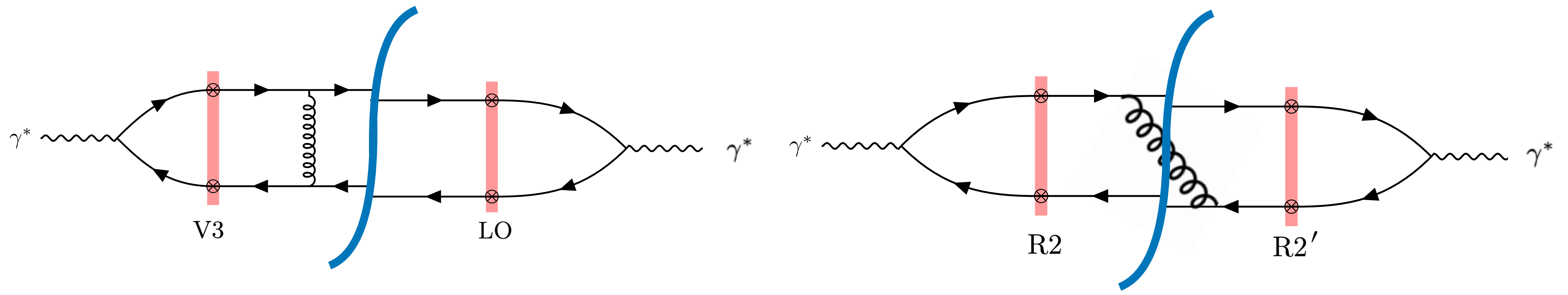
- Double logs (double poles)

IR divergences manifest as double logs $\log^2(z_0)$ in our calculation (our regularization scheme)



Double logs $\log^2(z_0)$ cancel
These are analogous to double poles in full dim reg.

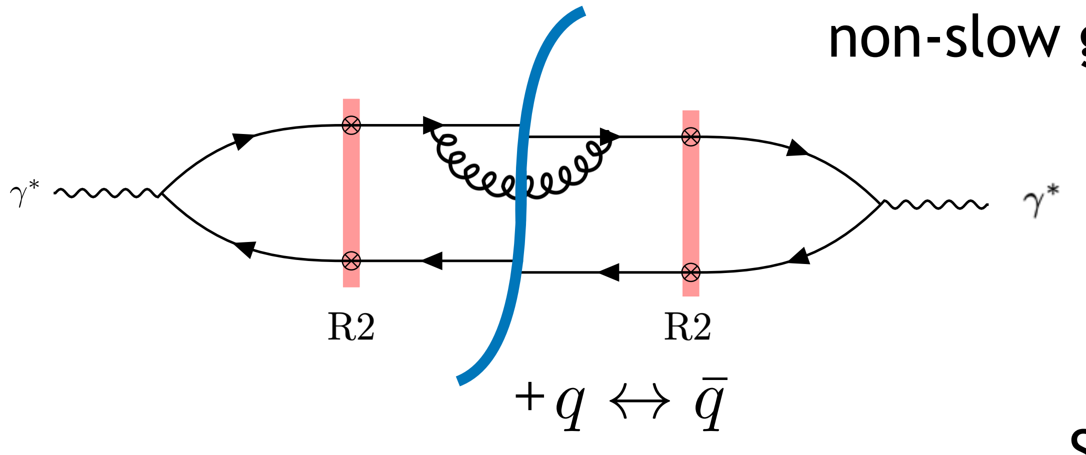
Double logs also occur in



Sum V3xLO + R2xR2' is free of double logs

- Collinear divergences

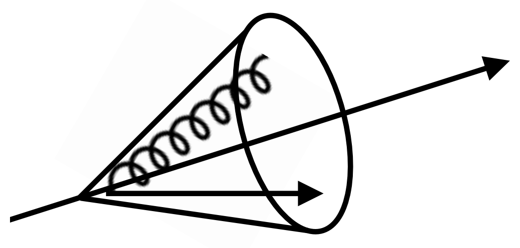
Implement a jet algorithm (small cone)



Phase space for collinear non-slow gluon $\displaystyle\int_{z_f}^{z_j} \frac{\mathrm{d}z_g}{z_g}\mu^\varepsilon \int \frac{\mathrm{d}^{2-\varepsilon}\mathcal{C}_{qg,\perp}}{(2\pi)^{2-\varepsilon}} \frac{1}{\mathcal{C}^2_{qg,\perp}}$

Collinearity variable:

$$\mathcal{C}_{qg,\perp} = \frac{z_q}{z_j}\left(\boldsymbol{k}_{g\perp} - \frac{z_g}{z_q}\boldsymbol{k}_\perp\right)$$



Small-cone condition:

$$\mathcal{C}^2_{qg,\perp} \leq \mathcal{C}^2_{qg,\perp}\big|_{\max} = R^2\boldsymbol{p}_j^2 \min\left(\frac{z_g^2}{z_j^2}, \frac{(z_j - z_g)^2}{z_j^2}\right)$$

$$\frac{\mathrm{d}\sigma^{\lambda}_{\rm R2\times R2,dijet,in-cone}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_2} = \frac{\alpha_s C_F}{\pi}\frac{\mathrm{d}\sigma^{\lambda}_{\rm LO,\varepsilon}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_2} \times \left\{\left(\frac{3}{4} - \ln\left(\frac{z_{J1}}{z_f}\right)\right)\frac{2}{\varepsilon}\right.$$

Collinear pole

$$\left. + \ln^2(z_{J1}) - \ln^2(z_f) - \frac{\pi^2}{6} + \left(\ln\left(\frac{z_{J1}}{z_f}\right) - \frac{3}{4}\right)\ln\left(\frac{R^2\boldsymbol{p}_{J1}^2}{\tilde{\mu}^2 z_{J1}^2}\right) + \frac{1}{4} + \frac{3}{2}\left(1 - \ln\left(\frac{z_{J1}}{2}\right)\right)\right\}$$

Collinear poles from R2 × R2 and R2' × R2' cancel against IR pole of virtual contributions

Final result is free of divergences without need of counter-terms! Note rapidity divergence is not physical (artifact of infinite energy limit, here regulated by z_0 cut-off)

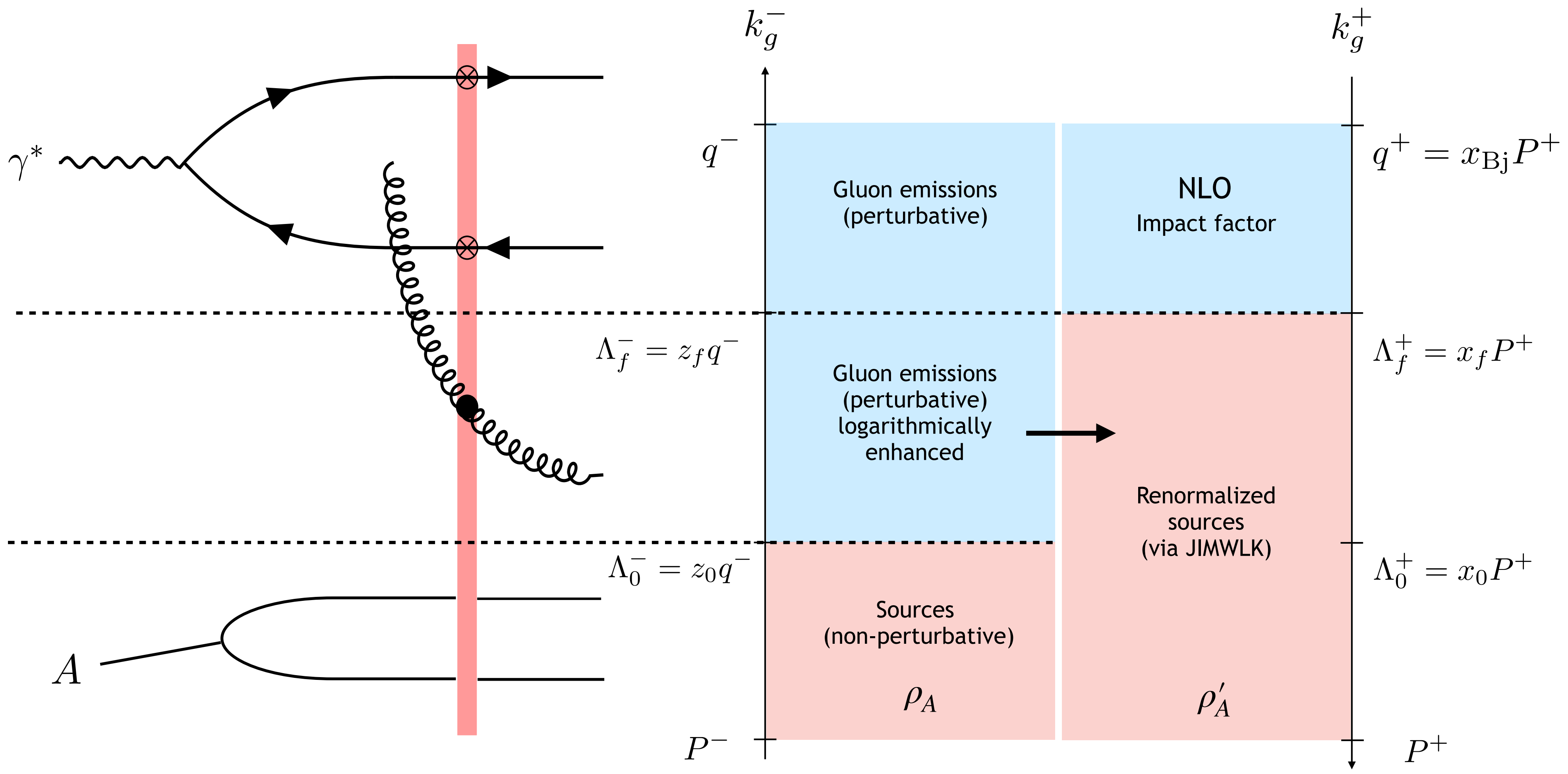
Rapidity factorization of NLO contributions

Slow gluon limit and JIMWLK factorization

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \int_{z_0}^{z} \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_0 + \sum_{n=1}^{\infty} \mathrm{d}\widetilde{\sigma}_n z_g^n\right]$$

$$\mathrm{d}\sigma_{\mathrm{NLO}} = \underbrace{\mathrm{d}\widetilde{\sigma}_0 \ln\left(\frac{z_f}{z_0}\right)}_{\text{Slow gluon piece}} + \underbrace{\int_0^{z} \frac{\mathrm{d}z_g}{z_g}\left[\mathrm{d}\widetilde{\sigma}_{\mathrm{NLO}} - \mathrm{d}\widetilde{\sigma}_0 \Theta(z_f - z_g)\right]}_{\text{Impact factor}} + \mathcal{O}(z_0)$$

$$\left.\frac{\mathrm{d}\sigma^{\lambda}_{\mathrm{NLO}}}{\mathrm{d}^2\boldsymbol{k}_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2\boldsymbol{k}_{2\perp}\mathrm{d}\eta_2}\right|_{\mathrm{slow}} = \frac{\alpha_{\mathrm{em}}e_f^2 N_c}{(2\pi)^6}\delta(1-z_q-z_{\bar{q}})\int \mathrm{d}\boldsymbol{X}_\perp \mathcal{R}^{\lambda}_{\mathrm{LO}}(\boldsymbol{r}_{xy},\boldsymbol{r}_{x'y'})\ln\left(\frac{z_f}{z_0}\right)$$
$$\times\frac{\alpha_s N_c}{4\pi^2}\left\langle\int \mathrm{d}^2\boldsymbol{z}_\perp\left\{\frac{\boldsymbol{r}_{xy}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy}^2}(2D_{xy}-2D_{xz}D_{zy}+D_{zy}Q_{y'x',xz}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xy}D_{y'x'})+\frac{\boldsymbol{r}_{xy'}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zy'}^2}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{zx'}Q_{xy,y'z}-D_{zy}Q_{y'x',xz})\right.\right.$$
$$+\frac{\boldsymbol{r}_{x'y'}^2}{\boldsymbol{r}_{zx'}^2\boldsymbol{r}_{zy'}^2}(2D_{y'x'}-2D_{y'z}D_{zx'}+D_{zx'}Q_{xy,y'z}+D_{y'z}Q_{xy,zx'}-Q_{xy,y'x'}-D_{xy}D_{y'x'})+\frac{\boldsymbol{r}_{xx'}^2}{\boldsymbol{r}_{zx}^2\boldsymbol{r}_{zx'}^2}(D_{zx'}Q_{xy,y'z}+D_{xz}Q_{y'x',zy}-Q_{xy,y'x'}-D_{xx'}D_{y'y})$$
$$\left.\left.+\frac{\boldsymbol{r}_{yy'}^2}{\boldsymbol{r}_{zy}^2\boldsymbol{r}_{zy'}^2}(D_{y'z}Q_{xy,zx'}+D_{zy}Q_{y'x',xz}-Q_{xy,y'x'}-D_{xx'}D_{y'y})+\frac{\boldsymbol{r}_{x'y}^2}{\boldsymbol{r}_{zx'}^2\boldsymbol{r}_{zy}^2}(D_{xx'}D_{y'y}+D_{xy}D_{y'x'}-D_{y'z}Q_{xy,zx'}-D_{xz}Q_{y'x',zy})\right\}\right\rangle_Y$$



Slow gluon pieces equals JIMWLK LL Hamiltonian acting on LO cross-section

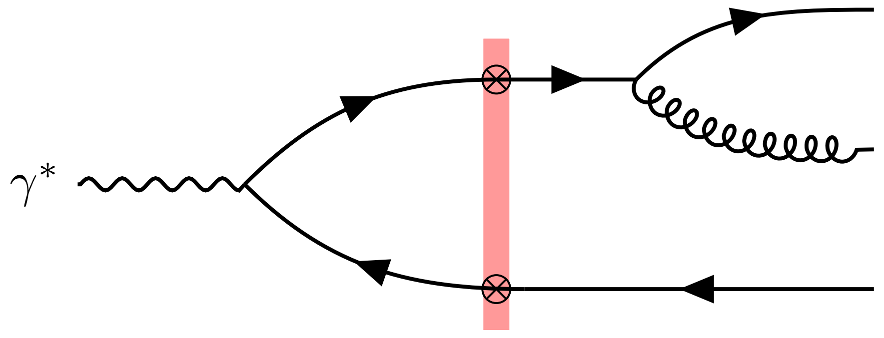
$$\mathrm{d}\sigma_{\mathrm{NLO,slow}} = \underbrace{\ln\left(\frac{z_f}{z_0}\right)\mathcal{H}_{\mathrm{JIMWLK}}}_{\alpha_s \ln(s)}\ \mathrm{d}\sigma_{\mathrm{LO}}$$

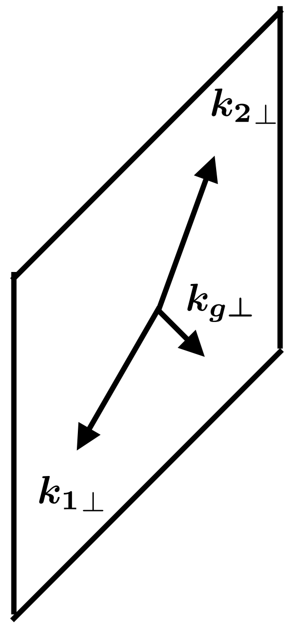
resummation via renormalization of target sources

$$W_{x_0}[\rho_A] \to W_{x_f}[\rho'_A]$$

Back-to-back limit: emergence of Sudakov

In the back-to-back limit we expect the appearance of double and single Sudakov logarithm





$$\boldsymbol{k}_\perp = \boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}$$

$$\boldsymbol{P}_\perp = z_2 \boldsymbol{k}_{1\perp} - z_1 \boldsymbol{k}_{2\perp}$$

Soft gluon emissions reduce the probability that dijets are back-to-back

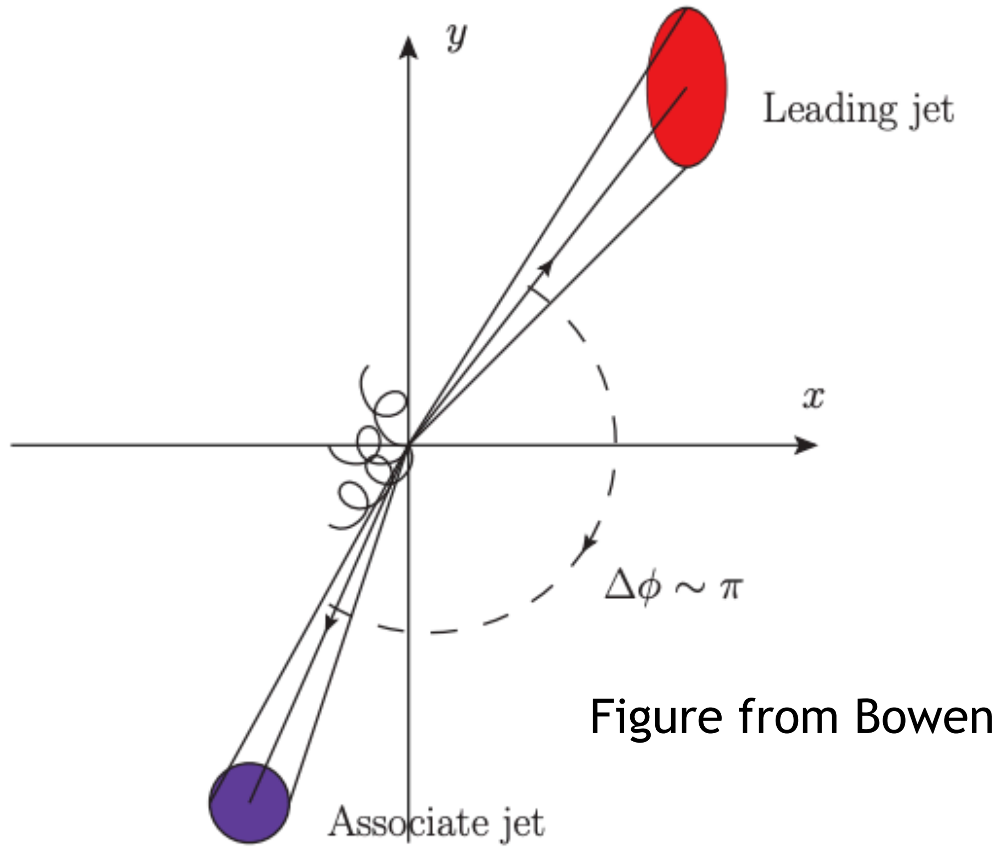


Figure from Bowen Xiao, lecture notes

resummation

$$\mathrm{d}\sigma \sim \mathcal{H}(\boldsymbol{P}_\perp, Q, z_1) \int \mathrm{d}^2\boldsymbol{b}_\perp \mathrm{d}^2\boldsymbol{b}'_\perp e^{-i\boldsymbol{k}_\perp\cdot(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)} e^{-S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp)} xG(\boldsymbol{b}_\perp, \boldsymbol{b}'_\perp; x)$$

$$= 1 - S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) + \dots$$

one-loop contribution

$$\alpha_s \ln^2(\boldsymbol{P}_\perp / \boldsymbol{k}_\perp)$$

Sudakov factor

Mueller, Xiao, Yuan (2013)

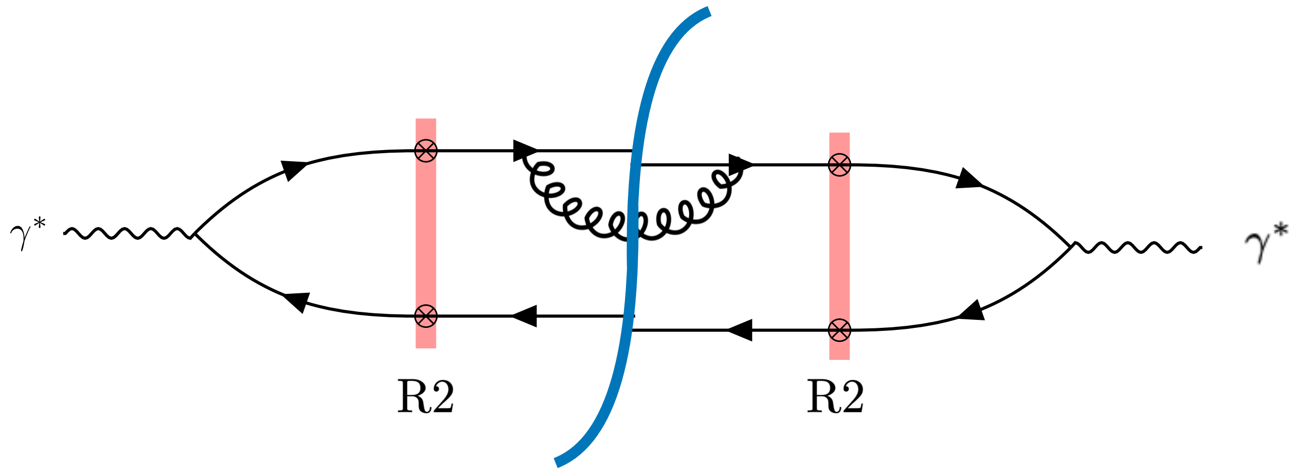
$$S_{\mathrm{Sud}}(\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp, \boldsymbol{P}_\perp) = \frac{\alpha_s N_c}{4\pi} \ln^2\left(\frac{\boldsymbol{P}_\perp^2 (\boldsymbol{b}_\perp - \boldsymbol{b}'_\perp)^2}{c_0^2}\right) + \dots$$

Double logs occur due to incomplete cancellation between real and virtual emission, real emission is highly constrained.

Sudakov double logs, where are they in our computation? Look inside the impact factor!

Diagrams that reduce to the WW in the back-to-back (other diagrams contribute to the evolution of WW)

Examine out-cone contribution



Back-to-back dijets seem to be sensitive to kinematic constraint too!
Correct Sudakov double log obtained in real emission when imposing **kinematic constraint!**

$$\mathrm{R}_2 \times \mathrm{R}_2 \ (\text{out-cone}) \propto \frac{\alpha_s C_F}{\pi} \left\{ \left(\ln\left(\frac{x_0}{x_{\mathrm{Bj}}}\right) + \ln\left(\frac{1}{R}\right) \right) \left[-\frac{2}{\varepsilon} - \ln(e^{\gamma_E}\mu^2\pi\Delta\boldsymbol{b}_\perp^2) \right] + \frac{2}{\varepsilon^2} - \frac{1}{\varepsilon}\ln\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) + \frac{1}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2}{\mu^2}\right) - \frac{1}{4}\ln^2\left(\frac{\boldsymbol{P}_\perp^2 \Delta\boldsymbol{b}_\perp^2}{c_0^2}\right) - \frac{\pi^2}{24} \right\}$$