Tensor Polarization of the Vector Meson Induced by the Flow Gradients

from a Linear Response Approach

Feng Li[1], **Shuai Liu**[2] **Yin Yi**[3]

[1]School of Physics, Lanzhou University, CN
[2]School of Physics and Electronics, Hunan University, CN
[3] Insititute of Modern Physics, Chinese Academy of Sciences

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Tensor Polarization in the Heavy-ion Collisions

- Tensor polarized $\rightarrow$ $\langle s|\frac{1}{2}(\hat{J}_i\hat{J}_j+\hat{J}_j\hat{J}_i)-\frac{\delta_{ij}}{3}\hat{\mathbf{J}}^2|s'\rangle \neq 0$
- Carried by massive particles with spin larger than or equal to 1
- Measured via the decay processes of ϕ, K^* and J/Ψ

$$\frac{dN}{d\cos\theta^*} \propto 1-\rho_{00}+\cos^2\theta^*(3\rho_{00}-1)$$

Angle between the spin of the mother and the momentum of the daughter particle in the P.R.F.

Angle-dependence vanishes if $\rho_{00}=1/3$(no tensor polarization)

- Measured results: significant deviation of ρ_{00} from $1/3$ (expectation value in the un-polarized case) observed in both the STAR & ALICE experiments
- Should be induced by the gradients of the flow fields

Spin-Density Matrix

Definition (written in terms of the fields operators $A_{\mp}$)

$$\rho_{ss'}(X,\mathbf{k}) = 2\omega_{\mathbf{k}} \int d^3\mathbf{r}\epsilon_s^*(\mathbf{k}) \cdot A_-(T,\mathbf{X}+\mathbf{r}/2)\epsilon_{s'}(\mathbf{k}) \cdot A_+(T,\mathbf{X}-\mathbf{r}/2)e^{-i\mathbf{k}\cdot\mathbf{r}}$$

$$A_-^\mu(x) = (2\pi)^{-3/2}\sum_s \int \frac{d^3\mathbf{k}}{\sqrt{2\omega_{\mathbf{k}}}}\epsilon_s^\mu(\mathbf{k})a(s,\mathbf{k})e^{-ik\cdot x}; \qquad A_+^\mu = A_-^{\mu\dagger}$$

ϵ_s: polarization vector of the fields

Normalization Condition

$$\sum_s \frac{1}{N}\int \frac{d^3\mathbf{X}d^3\mathbf{k}}{(2\pi)^3}\langle\rho_{ss}(X,\mathbf{k})\rangle \equiv \sum_s \frac{1}{N}\int \frac{d^3\mathbf{X}d^3\mathbf{k}}{(2\pi)^3}\mathrm{Tr}[e^{-\beta H_0}\rho_{ss}(X,\mathbf{k})] = 1$$

$\xrightarrow{M.R.F \to P.R.F.}$ Measured in the experiments with N being particle number

Linear Response to the Gradients of the Flows

Deviation (of spin-density matrix from 1/3) $\propto$ perturbation:

$$\overline{\delta\rho_{ss'}(x,\mathbf{k})} \approx -i\int d\bar{t}\theta(t-\bar{t})\langle[\rho_{ss'}(x,\mathbf{k}), H'_I(\bar{t})]\rangle_0$$

Perturbation (neglecting ∂T) aroused by the gradients of the flow:

$$H'(t) = \int^t dt' \int d^3\mathbf{x} T^{ji}(t')\partial_i u_j(t) \ll H_0.$$

Stress-energy tensor: $T^{\mu\nu} = \frac{1}{2}(\tau^{\mu\nu}_{\alpha\beta\gamma\delta}\partial^\alpha A^\gamma \partial^\beta A^\delta + m^2\chi^{\mu\nu}_{\gamma\delta}A^\gamma A^\delta)$

$\Downarrow$ Fourier transform on $x \to q$

$$\overline{\widetilde{\delta\rho}_{ss'}(q,\mathbf{k})} \approx 2\omega_{\mathbf{k}}\epsilon^*_{s\mu}(\mathbf{k})\epsilon_{s'\nu}(\mathbf{k})\widetilde{\Xi}^{\mu\nu ji}(q,\mathbf{k})\mathbf{q}^i\widetilde{u_j}(q)$$

$$\widetilde{\Xi}^{\mu\nu ji}(q,\mathbf{k}) = \int \frac{dz}{\pi}\frac{\mathrm{Im}\widetilde{\Pi}^{\mu\nu ji}(\omega_n,\mathbf{q},\mathbf{k})|_{i\omega_n\to z+i0^+}}{(z-i0^+)(q^0-z+i0^+)}$$

$$\Pi^{\mu\nu ji}(\tau,\mathbf{x},\mathbf{k}) = \int d^3\mathbf{r}\mathcal{T}_\tau\langle A^\mu_-(-i\tau,\mathbf{x}+\mathbf{r}/2)A^\nu_+(-i\tau,\mathbf{x}-\mathbf{r}/2)T^{ji}(0)\rangle_0 e^{-i\mathbf{k}\cdot\mathbf{r}}$$

Polarization of Ideal Meson Gas

After neglecting the interactions among the Spin-1 particles

$$\overline{\widetilde{\delta\rho}_{ss'}(q,\mathbf{k})} \approx \frac{-\omega_{\mathbf{k}}}{2\omega_{\mathbf{k}+\frac{\mathbf{q}}{2}}\omega_{\mathbf{k}-\frac{\mathbf{q}}{2}}}\mathcal{O}^{ji}_{\xi\zeta}(k_+;k_-)\left(-g^{\rho\xi}+\frac{k_+^{\rho}k_+^{\xi}}{m^2}\right)\left(-g^{\zeta\sigma}+\frac{k_-^{\zeta}k_-^{\sigma}}{m^2}\right)$$
$$\times\frac{f(\omega_{\mathbf{k}-\frac{\mathbf{q}}{2}})-f(\omega_{\mathbf{k}+\frac{\mathbf{q}}{2}})}{q^0-\omega_{\mathbf{k}+\frac{\mathbf{q}}{2}}+\omega_{\mathbf{k}-\frac{\mathbf{q}}{2}}+i0^+}\frac{\mathbf{q}^i\epsilon^*_{s\rho}(\mathbf{k})\epsilon_{s'\sigma}(\mathbf{k})\widetilde{u_j}(q)}{\omega_{\mathbf{k}+\frac{\mathbf{q}}{2}}-\omega_{\mathbf{k}-\frac{\mathbf{q}}{2}}-i0^+}$$

$\Downarrow$ Keep leading and next leading terms in q

$$\overline{\widetilde{\delta\rho}_{ss'}(q,\mathbf{k})} \approx \frac{f'(\omega_{\mathbf{k}})\mathbf{v}_{\mathbf{k}}\cdot\mathbf{q}}{q^0-\mathbf{v}_{\mathbf{k}}\cdot\mathbf{q}+i0^+}\left[k^j\delta_{ss'}+\frac{\mathbf{q}^i}{2}\frac{\left(\epsilon^*_{s\rho}(\mathbf{k})\epsilon_{s'\sigma}(\mathbf{k})-\epsilon_{s'\rho}(\mathbf{k})\epsilon^*_{s\sigma}(\mathbf{k})\right)}{\mathbf{k}\cdot\mathbf{q}-i0^+}\right.$$
$$\left.\times\left(\frac{k^{\zeta}}{4m^2}\mathcal{O}^{ji}_{\xi\zeta}(k;k)\left(\hat{q}^{\rho}g^{\xi\sigma}-\hat{q}^{\sigma}g^{\rho\xi}\right)-\frac{1}{2}\hat{q}^{\alpha}k^{\beta}\left(\tau^{ji;\rho\sigma}_{\alpha\beta}-\tau^{\mu i;\sigma\rho}_{\alpha\beta}\right)\right)\right]\widetilde{u_j}(q)$$

$\delta_{ss'}\rightarrow$ scalar polarization, which reflects the change of phase space distribution in the presence of u^{μ}, i.e., $f(\omega_{\mathbf{k}})\rightarrow f(\omega_{\mathbf{k}}-\mathbf{k}\cdot\mathbf{u})$, at a vanishing q^0

$\epsilon^*_{s\rho}(\mathbf{k})\epsilon_{s'\sigma}(\mathbf{k})-\epsilon_{s\sigma}(\mathbf{k})\epsilon^*_{s'\rho}(\mathbf{k})\propto\mathbf{J}_{ss'}\rightarrow$ vector polarization

Tensor-polarization is absent in ideal meson gas

Conclusions & Discussions

- We develop a universal formalism for calculating the polarization matrix of the vector mesons induced by the flow gradients near thermal equilibrium.
- See only the vector polarization being induced by the first order gradients of the flow field in the ideal gas composed of vector mesons
- The tensor polarization of the ϕ or K^* mesons, observed in the heavy ion collisions, should be caused by the interactions among the particles