Finite Nc and non-eikonal corrections to induced emission

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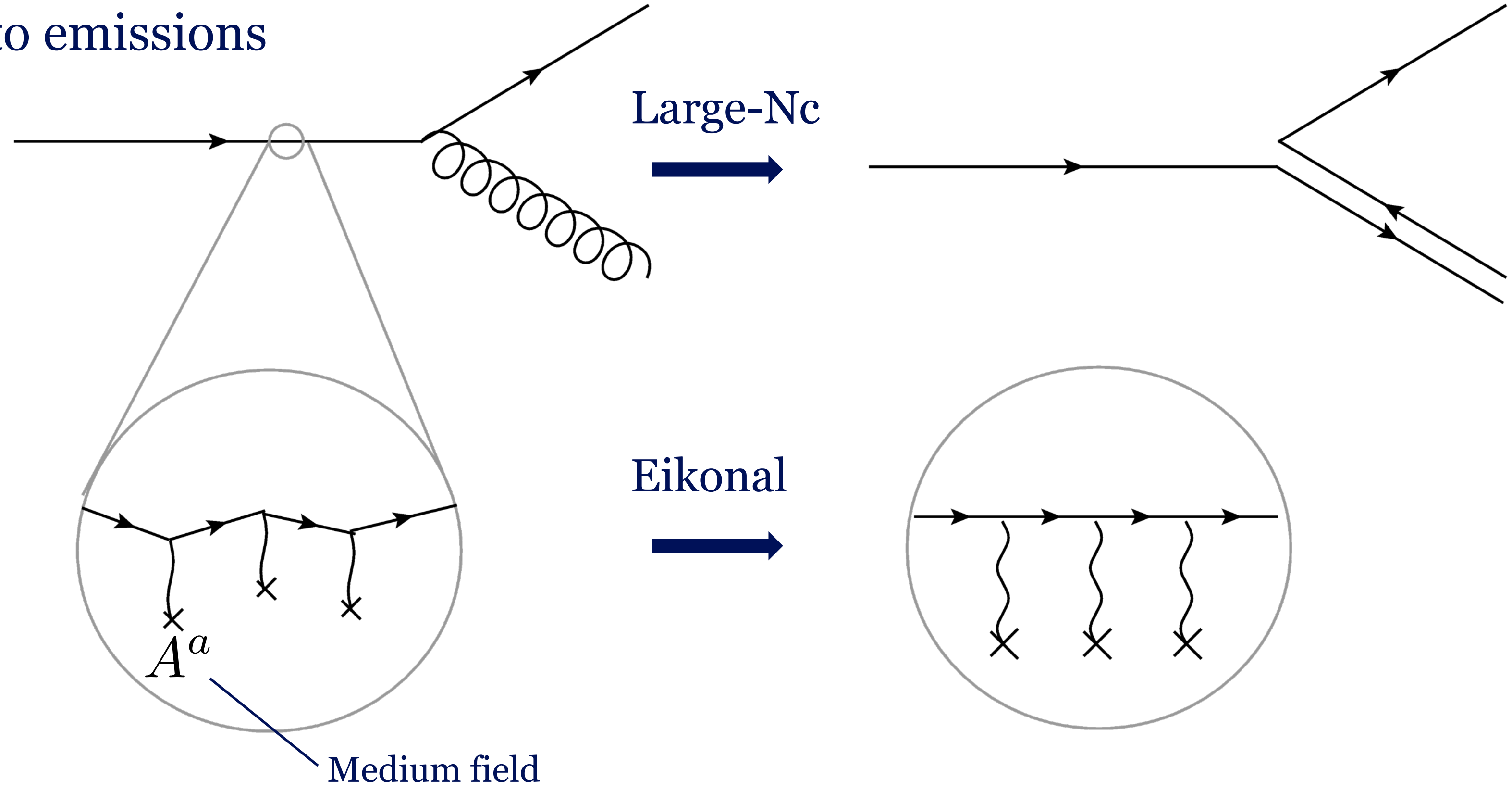
Based on https://arxiv.org/abs/2107.02542, and ongoing work

Medium induced emissions

- Interactions with the medium lead to emissions
- Two simplifying approximations:

1. Large-Nc approximation
   - Take number of colors (Nc) to infinity
2. Eikonal approximation
   - Let partons travel on straight lines
   - Good for high energy

- Important to understand the error of the approximations

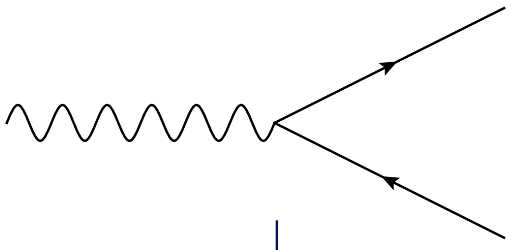


- Constant medium interactions means color continuously rotates
- Effect encapsulated in Wilson line: $V_R\left(t,t_0;\mathbf{r}(t)\right) = \mathcal{P}\exp\left[ig\int_{t_0}^{t}\mathrm{d}s\,A^a(s,\mathbf{r}(s))T_R^a\right]$

Splitting processes

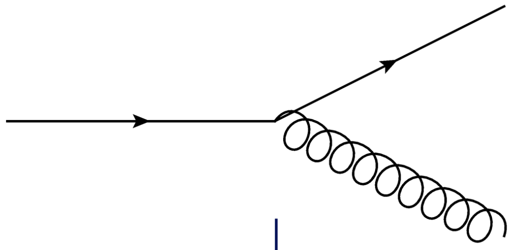
Calculate emission spectrum for three processes

Pair production



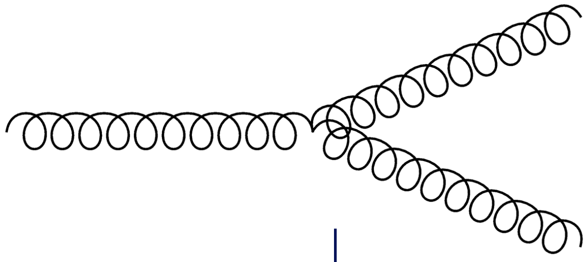
$$\frac{1}{N_c}\langle \mathrm{tr}[V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger]\rangle$$

Quark emitting gluon



$$\frac{1}{N_c^2-1}\left\langle \mathrm{tr}[V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}]\,\mathrm{tr}[V_{\bar{2}}^\dagger V_2] - \frac{1}{N_c}\,\mathrm{tr}[V_{\bar{1}}^\dagger V_1]\right\rangle$$

Gluon emitting gluon



$$\frac{1}{N_c(N_c^2-1)}\left\langle \mathrm{tr}[V_1 V_{\bar{1}}^\dagger]\,\mathrm{tr}[V_2 V_{\bar{2}}^\dagger V_{\bar{1}} V_1^\dagger]\,\mathrm{tr}[V_{\bar{2}} V_2^\dagger] - \mathrm{tr}[V_1 V_{\bar{1}}^\dagger V_2 V_{\bar{2}}^\dagger V_{\bar{1}} V_1^\dagger V_{\bar{2}} V_2^\dagger]\right\rangle$$

- Average out medium effects, represented by $\langle \dots \rangle$
- Leads to increasingly complex medium averaged traces of Wilson lines

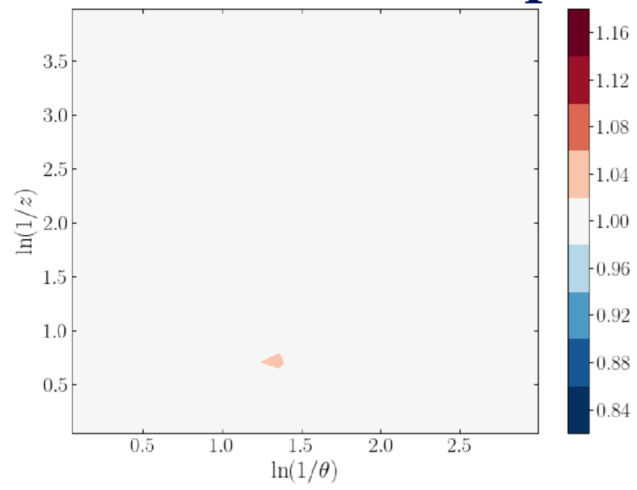
Finite Nc corrections

Wilson line correlators can be calculated through system of differential equations

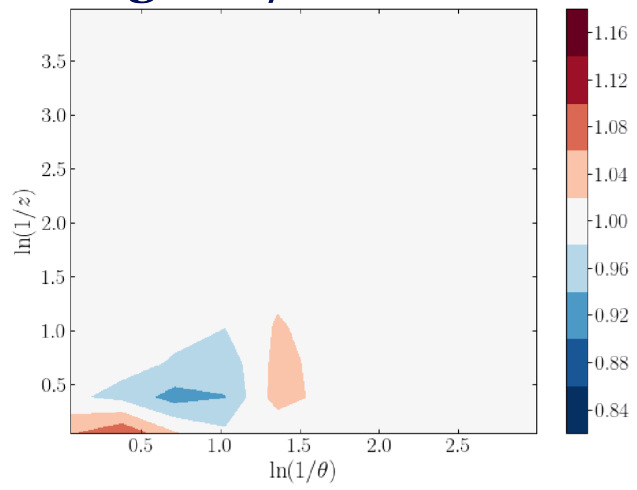
$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix}\langle \mathrm{tr}[VV]\,\mathrm{tr}[VV]\rangle \\ \langle \mathrm{tr}[VVVV]\rangle\end{bmatrix} = \mathbb{V}(t)\begin{bmatrix}\langle \mathrm{tr}[VV]\,\mathrm{tr}[VV]\rangle \\ \langle \mathrm{tr}[VVVV]\rangle\end{bmatrix}$$

- The potential matrix $\mathbb{V}(t)$ simplifies greatly in the large-Nc limit
  - Example: $\langle \mathrm{tr}[VV]\,\mathrm{tr}[VV]\rangle \overset{\text{large-}N_c}{\rightarrow} \langle \mathrm{tr}[VV]\rangle\langle \mathrm{tr}[VV]\rangle$
- Expect correction around $\sim 1/N_c^2 \simeq 10\%$

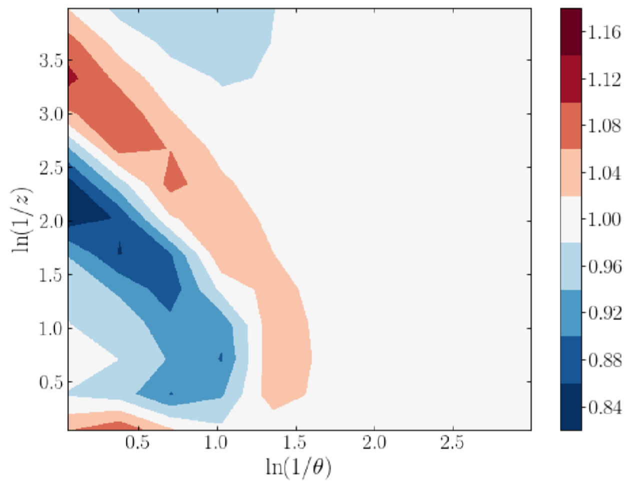
Ratio of emission spectrum for large-Nc/finite Nc



(a) Photon splitting.



(b) Quark-gluon splitting.



(c) Gluon-gluon splitting.

Up to 16%

More complex color structure leads to bigger correction!

Non-eikonal corrections

Partons get kicked around by medium

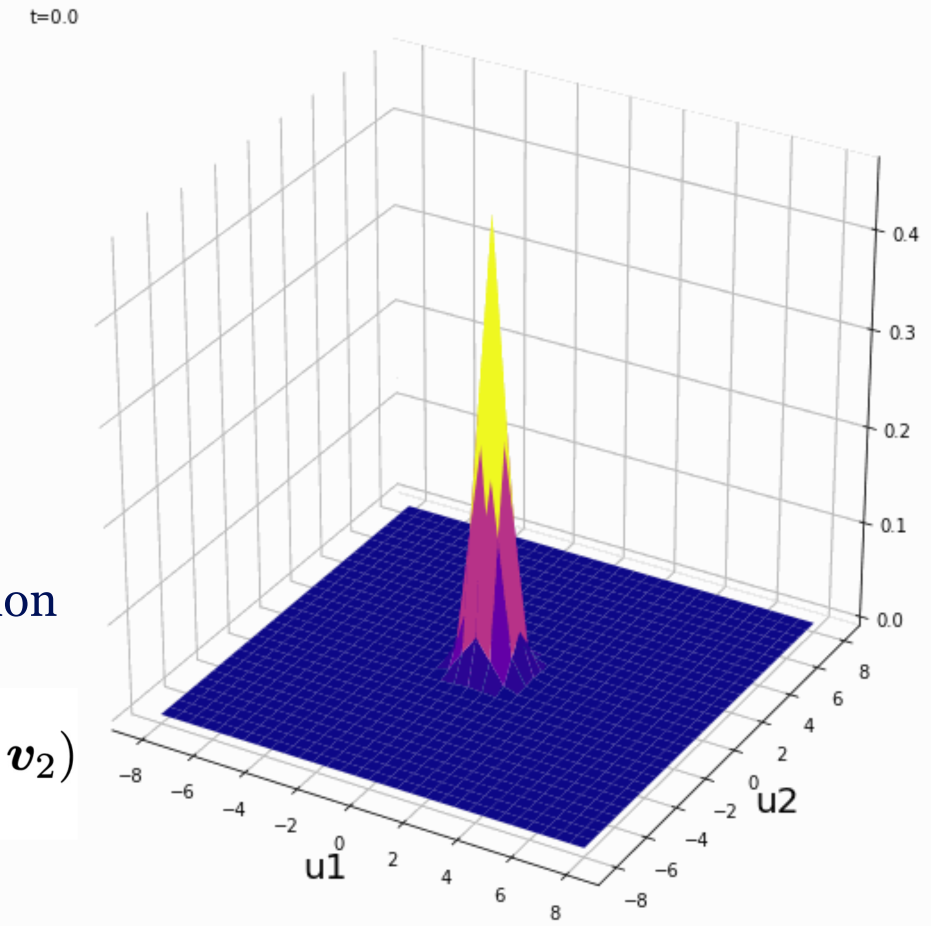
- Path through the medium is not straight
- Important for lower energy
- This leads to path integral over Wilson lines

$$\mathcal{K}_1 \equiv \int \mathcal{D}\boldsymbol{u} \int \mathcal{D}\boldsymbol{v}\, \mathrm{e}^{i\frac{\omega}{2}\int \mathrm{d}s\,(\dot{\boldsymbol{u}}^2-\dot{\boldsymbol{v}}^2)} \langle \mathrm{tr}[VV]\,\mathrm{tr}[VV]\rangle$$

$$\mathcal{K}_2 \equiv \int \mathcal{D}\boldsymbol{u} \int \mathcal{D}\boldsymbol{v}\, \mathrm{e}^{i\frac{\omega}{2}\int \mathrm{d}s\,(\dot{\boldsymbol{u}}^2-\dot{\boldsymbol{v}}^2)} \langle \mathrm{tr}[VVVV]\rangle\,,$$

- Differential equation from last slide becomes a Schrödinger equation

$$\left(i\frac{\partial}{\partial t} + \frac{\partial_{\boldsymbol{u}}^2 - \partial_{\boldsymbol{v}}^2}{2\omega} + i\mathbb{V}(t,\boldsymbol{u},\boldsymbol{v})\right)\begin{bmatrix}\mathcal{K}_1\\ \mathcal{K}_2\end{bmatrix} = i\delta(t-t_2)\delta^2(\boldsymbol{u}-\boldsymbol{u}_2)\delta^2(\boldsymbol{v}-\boldsymbol{v}_2)$$



- Ongoing work: Solving this numerically
- Expect small corrections for high energy, bigger corrections for low energy