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Transient Relativistic Fluid Dynamics in a General Hydrodynamic Frame

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Hydrodynamic frames

Eckart, Phys. Rev. 58 919 (1940); Landau, Lifshitz, Fluid Mechanics (1987);
Israel, Stewart, Ann. Phys. 118, 341 (1979); Tsumura, Kunihiro, Ohnishi, Phys. Lett. B656, 274 (2007)

- Most general decomposition

$$T^{\mu\nu} = (\varepsilon + \mathcal{A})u^\mu u^\nu + (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu} + \mathcal{Q}^\mu u^\nu + \mathcal{Q}^\nu u^\mu$$
$$J^\mu = (n + \mathcal{N})u^\mu + \mathcal{J}^\mu$$

- Hydrodynamic variables out of equilibrium can be defined in many ways - In equilibrium the definition is unique
- Hydrodynamic frames are specific choices of hydrodynamic variables
- Examples:
  - **Landau** frame: $T^{\mu\nu}u_\nu = -\varepsilon u^\mu$
  - **Eckart** frame: $J^\mu = nu^\mu$
- In relativity causality must hold regardless how you define out of equilibrium variables

How does one formulate causal and stable relativistic hydrodynamics?

BDNK - Generalized first-order theory

Bemfica, Disconzi, Noronha, PRD **98**, 104064 (2018); PRD **100**, 104020 (2019); 2009.11388 (2020);
Kovtun JHEP **10**, 034 (2019); Hoult, Kovtun, JHEP **06**, 067 (2020)

- Write most general expansion using first order derivatives of u^μ, T, μ/T compatible with symmetries

$$\begin{aligned}
\mathcal{A} &= \varepsilon_1 \frac{u^\alpha \partial_\alpha T}{T} + \varepsilon_2 \partial_\alpha u^\alpha + \varepsilon_3 u^\alpha \partial_\alpha (\mu/T) \\
\Pi &= \pi_1 \frac{u^\alpha \partial_\alpha T}{T} + \pi_2 \partial_\alpha u^\alpha + \pi_3 u^\alpha \partial_\alpha (\mu/T) \\
\mathcal{N} &= \nu_1 \frac{u^\alpha \partial_\alpha T}{T} + \nu_2 \partial_\alpha u^\alpha + \nu_3 u^\alpha \partial_\alpha (\mu/T) \\
\mathcal{Q}^\mu &= \theta_1 \frac{\Delta^{\mu\nu} \partial_\nu T}{T} + \theta_2 u^\alpha \partial_\alpha u^\mu + \theta_3 \Delta^{\mu\nu} \partial_\nu (\mu/T) \\
\mathcal{J}^\mu &= \gamma_1 \frac{\Delta^{\mu\nu} \partial_\nu T}{T} + \gamma_2 u^\alpha \partial_\alpha u^\mu + \gamma_3 \Delta^{\mu\nu} \partial_\nu (\mu/T) \\
\pi^{\mu\nu} &= -2\eta\sigma^{\mu\nu}
\end{aligned}$$

- Causal and stable in region of parameter space in nonlinear regime
- **Note**: First order theory in Landau and Eckart frames are acausal and unstable - Not suitable for numerical calculations
  Hiscock, Lindblom, PRD 31, 725 (1985)

Transient theory in a general hydrodynamic frame

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- $\mathcal{A}$, Π, $\mathcal{Q}^\mu$, $\pi^{\mu\nu}$, $\mathcal{N}$, $\mathcal{J}^\mu$ promoted to be dynamical variables
- Second-order entropy current

$$S^\nu_{\text{neq}} = \left(s + \frac{\mathcal{A}}{T} - \frac{\mu}{T}\mathcal{N}\right)u^\nu + \frac{\mathcal{Q}^\nu}{T} - \frac{\mu}{T}\mathcal{J}^\nu - \frac{1}{2T}\Big(\beta_\pi \pi^{\lambda\rho}\pi_{\rho\sigma} + \beta_{\mathcal{Q}}\mathcal{Q}^\lambda \mathcal{Q}_\lambda + \beta_{\mathcal{A}}\mathcal{A}^2 + \beta_\Pi \Pi^2$$

$$+\beta_{\mathcal{N}}\mathcal{N}^2 + \beta_{\mathcal{J}}\mathcal{J}^\lambda\mathcal{J}_\lambda - 2\alpha_{\mathcal{N}\Pi}\mathcal{N}\Pi - 2\alpha_{\mathcal{N}\mathcal{A}}\mathcal{N}\mathcal{A} - 2\alpha_{\Pi\mathcal{A}}\Pi\mathcal{A} - 2\alpha_{\mathcal{Q}\mathcal{J}}\mathcal{Q}^\lambda\mathcal{J}_\lambda\Big)u^\nu$$

$\partial_\nu S^\nu_{\text{neq}} \geq 0$ →

$$\frac{\mathcal{A}}{\varphi T} = -\frac{\beta_{\mathcal{A}}}{T}D\mathcal{A} + D\left(\frac{1}{T}\right) - \frac{1}{2}\mathcal{A}\partial_\nu\left(\frac{\beta_{\mathcal{A}}}{T}u^\nu\right) + \frac{\alpha_{\mathcal{N}\mathcal{A}}}{T}D\mathcal{N} + \frac{\alpha_{\Pi\mathcal{A}}}{T}D\Pi,$$

$$\frac{\Pi}{\zeta T} = -\frac{\beta_\Pi}{T}D\Pi - \frac{1}{T}\theta - \frac{1}{2}\Pi\partial_\nu\left(\frac{\beta_\Pi}{T}u^\nu\right) + \frac{\alpha_{\mathcal{N}\Pi}}{T}D\mathcal{N} + \frac{\alpha_{\Pi\mathcal{A}}}{T}D\mathcal{A},$$

$$\frac{\mathcal{Q}^\nu}{\psi T} = -\frac{\beta_{\mathcal{Q}}}{T}\Delta^\nu_\lambda D\mathcal{Q}^\lambda + \Delta^\nu_\lambda\partial^\lambda\left(\frac{1}{T}\right) - \frac{1}{T}Du^\mu - \frac{1}{2}\mathcal{Q}^\nu\partial_\lambda\left(\frac{\beta_{\mathcal{Q}}}{T}u^\lambda\right) + \frac{\alpha_{\mathcal{Q}\mathcal{J}}}{T}\Delta^\nu_\lambda D\mathcal{J}^\lambda,$$

$$\frac{\pi^{\nu\lambda}}{2\eta T} = -\frac{\beta_\pi}{T}\Delta^{\nu\lambda\delta\rho}D\pi_{\delta\rho} - \frac{\sigma^{\nu\lambda}}{T} - \frac{1}{2}\pi^{\nu\lambda}\partial_\rho\left(\frac{\beta_\pi}{T}u^\rho\right),$$

$$\frac{\mathcal{N}}{\xi T} = -\frac{\beta_{\mathcal{N}}}{T}D\mathcal{N} - D\left(\frac{\mu}{T}\right) - \frac{1}{2}\mathcal{N}\partial_\nu\left(\frac{\beta_{\mathcal{N}}}{T}u^\nu\right) + \frac{\alpha_{\mathcal{N}\Pi}}{T}D\Pi + \frac{\alpha_{\mathcal{N}\mathcal{A}}}{T}D\mathcal{A} + \Pi\partial_\nu\left(\frac{\alpha_{\mathcal{N}\Pi}}{T}u^\nu\right) + \mathcal{A}\partial_\nu\left(\frac{\alpha_{\mathcal{N}\mathcal{A}}}{T}u^\nu\right),$$

$$\frac{\mathcal{J}^\nu}{\kappa T} = -\frac{\beta_{\mathcal{J}}}{T}\Delta^\nu_\lambda D\mathcal{J}^\lambda - \Delta^\nu_\lambda\partial^\lambda\left(\frac{\mu}{T}\right) - \frac{1}{2}\mathcal{J}^\nu\partial_\lambda\left(\frac{\beta_{\mathcal{J}}}{T}u^\lambda\right) + \frac{\alpha_{\mathcal{Q}\mathcal{J}}}{T}\Delta^\nu_\lambda D\mathcal{Q}^\lambda + \mathcal{Q}^\nu\partial_\lambda\left(\frac{\alpha_{\mathcal{Q}\mathcal{J}}}{T}u^\lambda\right),$$

- If one initiates system in Landau frame, it will dynamically deviate from it
- Gradient expansion originates BDNK!
- Kinetic theory approach Rocha, Denicol, PRD **104**, 096016 (2021)

Linearized theory - Conformal case

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- **Shear channel** - Dispersion relations

$$\Gamma^{\perp}_{\mathrm{h}} = -\frac{\eta}{sT}|\mathbf{k}|^2,$$
$$\Gamma^{\perp}_{\mathrm{nh},1} = -\frac{1}{\tau_\pi} + \frac{\eta}{sT}\frac{(\tau_Q - \tau_\pi)}{(\tau_Q - \tau_\pi - 3\tau_\psi)}|\mathbf{k}|^2,$$
$$\Gamma^{\perp}_{\mathrm{nh},2} = -\frac{1}{\tau_Q - 3\tau_\psi} - \frac{\eta}{sT}\frac{3\tau_\psi}{(\tau_Q - \tau_\pi - 3\tau_\psi)}|\mathbf{k}|^2$$

  One additional nonhydro mode relative to Israel Stewart theory in Landau frame

- **Sound channel** - Two additional nonhydro modes relative to Israel Stewart theory in Landau frame
- Linearized theory causal and stable in some region of parameter space
- Causality in nonlinear regime is still not known

Attractor behavior

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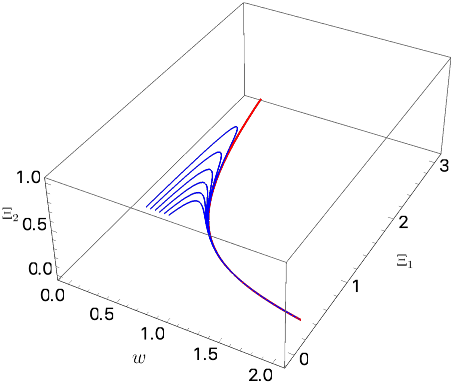
- Simple application: Bjorken solution in the conformal case
  $\mathcal{Q}^\mu = 0,\ \varepsilon(\tau),\ \mathcal{A}(\tau),\ \pi(\tau)$

$$\Xi_1 = 6\left(1 + \frac{3}{4}\tau\partial_\tau \ln\varepsilon\right) \qquad \Xi_2 = \frac{\mathcal{A}}{\varepsilon}$$

$$w = \tau T$$



Conclusions

- Generalized Israel-Stewart theory in arbitrary hydrodyanamic frame
- Conformal case is causal and stable in the linearized regime
- Attractor behavior illustrates deviations from Landau frame
- Useful for applications to neutron-star mergers and heavy-ion collisions