Jet quenching in anisotropic media

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based on 2202.08847 and work in progress

Jet Tomography

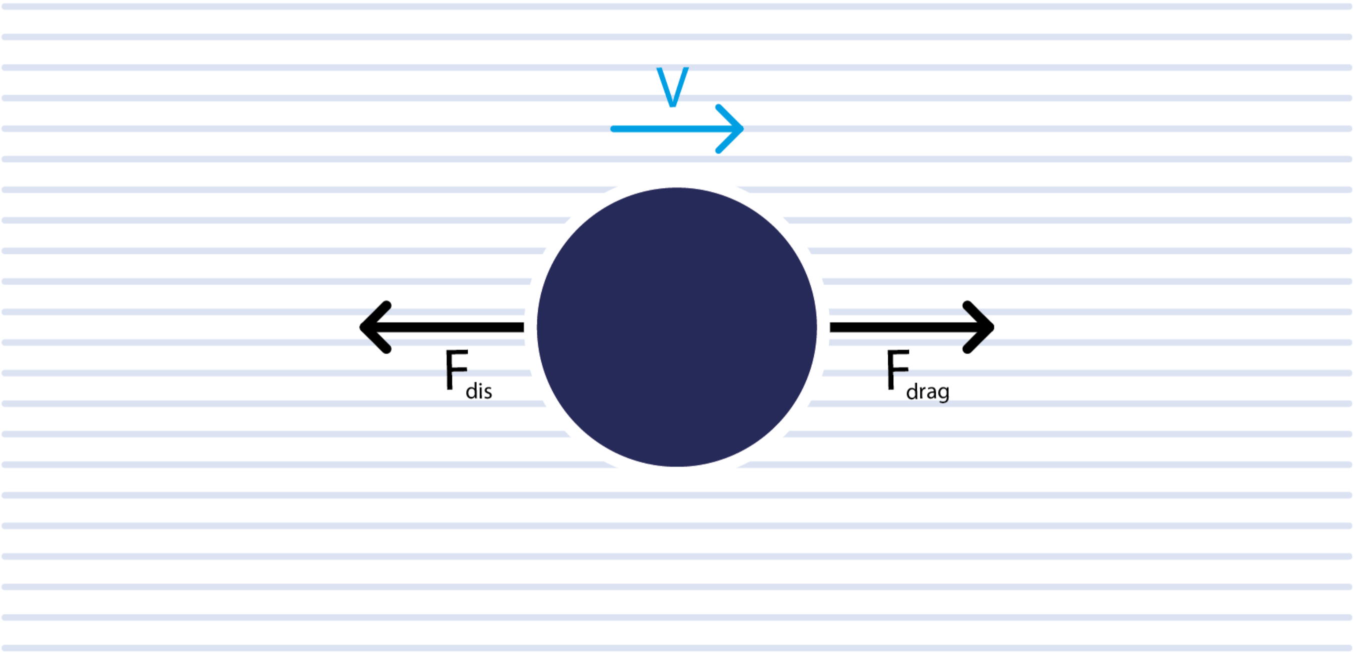
- The hot nuclear matter in HIC undergoes multi-phase evolution and its details are hard to access through the soft sector. In turn, jets see the matter at multiple scales, and essentially X-ray it;

- However, most approaches to the jet-medium interaction are either empirical or based on multiple simplifying assumptions – static matter, no fluctuations, etc;

- In what follows I will highlight our recent progress on the medium motion and structure effects in the QCD calculations for jet broadening and gluon emission;

- The developed formalism can be also applied to include orbital motion of nucleons and some of in-medium fluctuations (e.g. spatial inhomogeneities) in the DIS context;

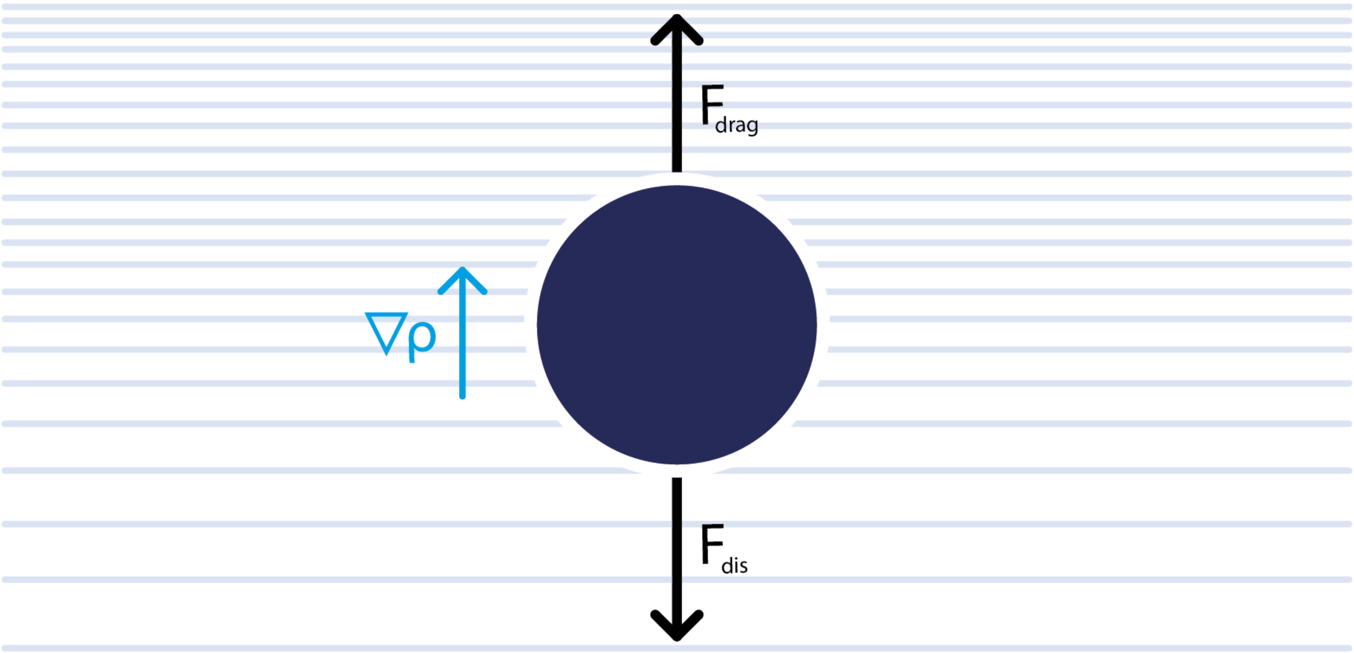
Drag Force



$$\vec{f} \sim T^2 \vec{v}$$

M. Lekaveckas, K. Rajagopal, JHEP, 2014
K. Rajagopal, AS, JHEP, 2017
J. Reiten, AS, JHEP, 2020

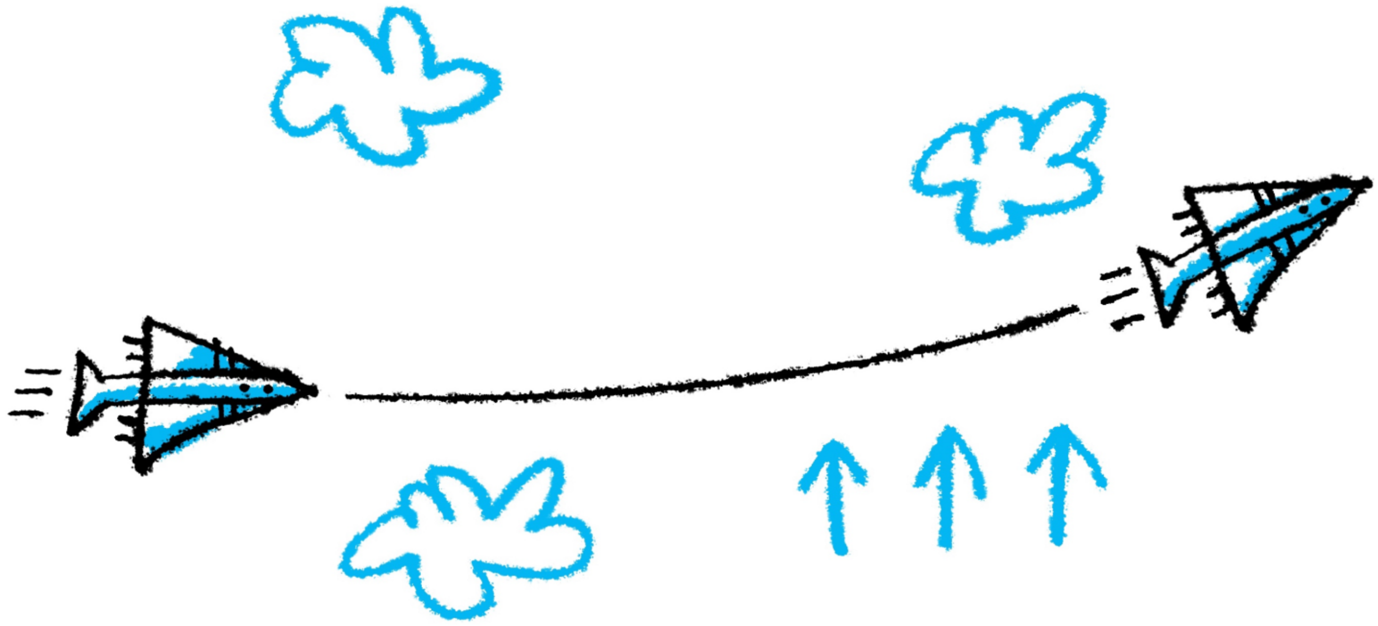
Drag Force



$$\vec{f} \sim \vec{\nabla}\rho$$

Jets

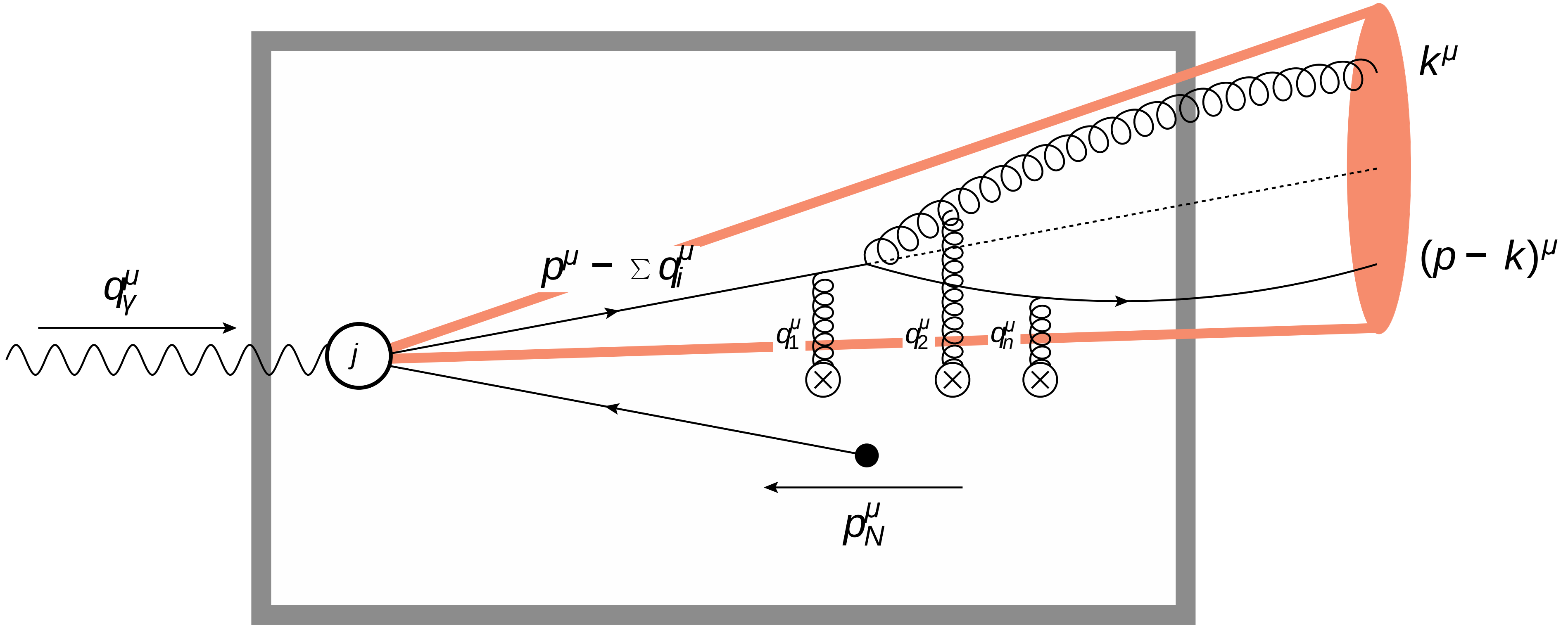
Does a jet feel the flow?



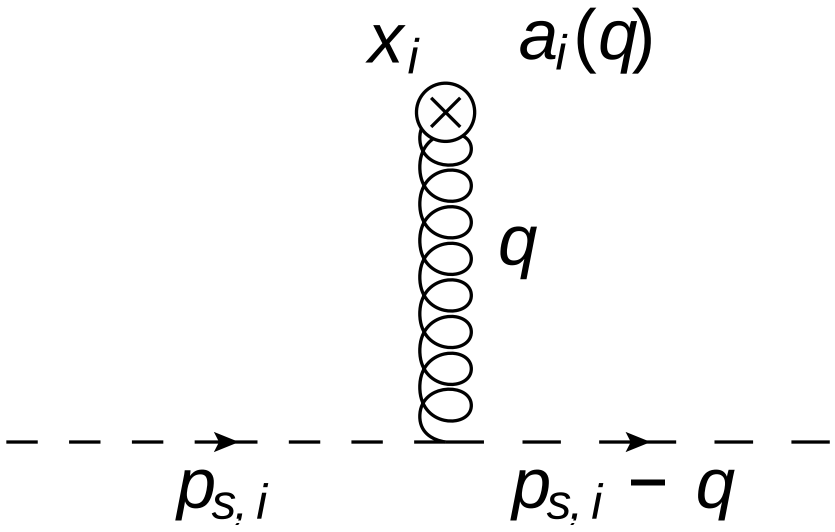
R. Baier et al, NPB, 1997
B. G. Zakharov, JETP, 1997
R. Baier et al, NPB, 1998
M. Gyulassy et al, NPB, 2000
M. Gyulassy et al, NPB, 2001
…

Jets

QCD broadening and gluon emission
(GLV/BDMPS-Z) with flow



Color Potential

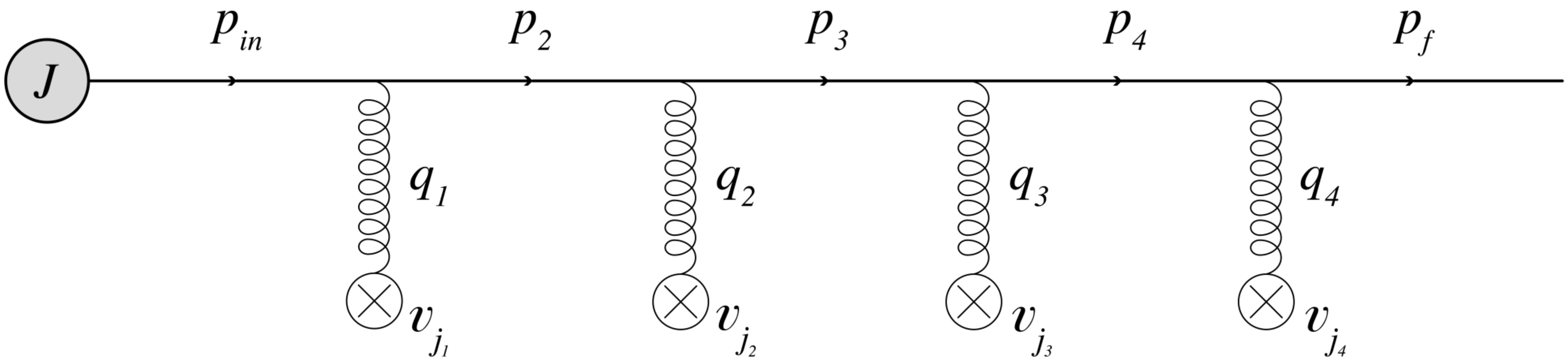


large

$$A^{\mu a}(q) = \sum_i (ig\, t_i^a)\, e^{iq\cdot x_i}\, (2p_{s\,i} - q)_\nu \frac{-ig^{\mu\nu}}{q^2 - \mu_i^2 + i\epsilon} (2\pi)\, \delta\Big((p_{s\,i} - q)^2 - M^2\Big)\,.$$

$v(q^2)$ -- **the Gyulassy-Wang potential**

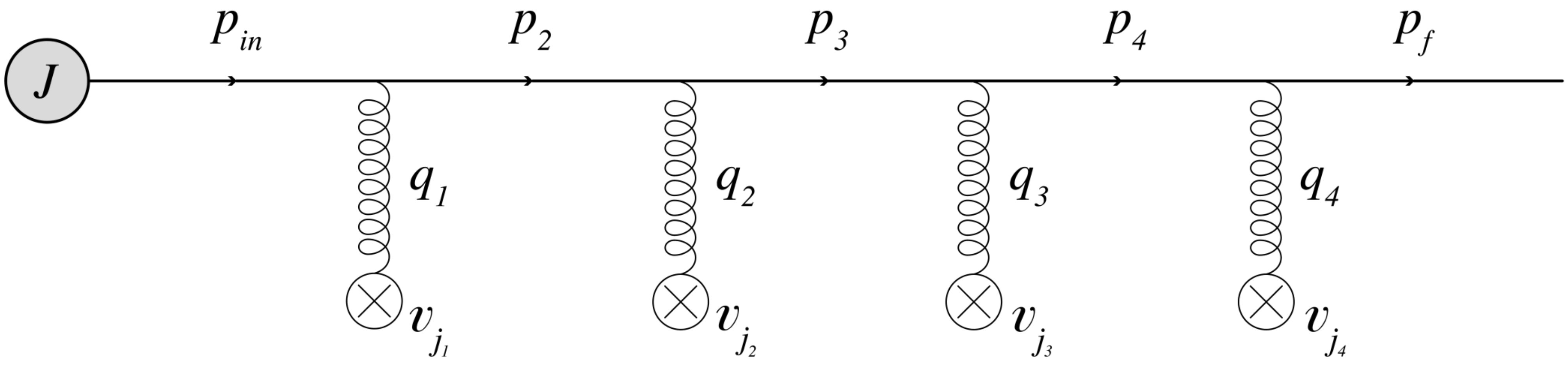
The Propagator



$$iM(p) = \int \frac{d^2\mathbf{p}_{in}}{(2\pi)^2} e^{i\frac{\mathbf{p}_f^2}{2E}L} \, G(\mathbf{p}_f, L; \mathbf{p}_{in}, 0) \, J\,(E, \mathbf{p}_{in})$$

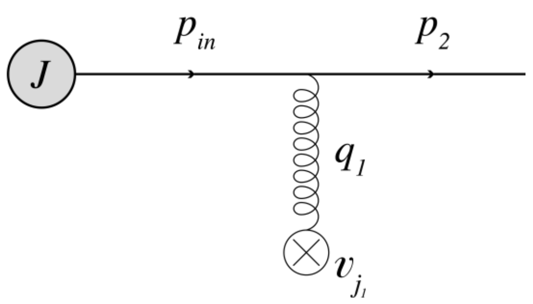
single particle propagator

The Propagator



$$G(\mathbf{x}_L, L; \mathbf{x}_0, 0) = \int_{\mathbf{x}_0}^{\mathbf{x}_L} \mathcal{D}\mathbf{r} \exp\left(\frac{iE}{2} \int_0^L d\tau\, \dot{\mathbf{r}}^2 \right) \mathcal{P} \exp\left(-i \int_0^L d\tau\, t^a_{\mathrm{proj}} v^a(\mathbf{r}(\tau), \tau) \right)$$

Medium Averaging

$\Bigg\langle \Bigg|$  $\Bigg|^2 \Bigg\rangle$

$$\langle t_i^a t_j^b \rangle = \mathcal{C}\delta_{ij}\delta^{ab}$$

color neutrality

$$\sum_i = \int d^3x \, \rho(\vec{x})$$

source averaging

Medium Averaging

$$\sum_i f_i = \int d^2\mathbf{x}\, dz\, \rho(\mathbf{x}, z)\, f(\mathbf{x}, z)$$

$$\int d^2\mathbf{x}_n\, e^{-i(\mathbf{q}_n \pm \overline{\mathbf{q}}_n)\cdot\mathbf{x}_n} = (2\pi)^2\, \delta^{(2)}(\mathbf{q}_n \pm \overline{\mathbf{q}}_n)$$

$$\int d^2\mathbf{x}_n\, x_n^\alpha\, e^{-i(\mathbf{q}_n \pm \overline{\mathbf{q}}_n)\cdot\mathbf{x}_n} = i\,(2\pi)^2\, \frac{\partial}{\partial(q_n \pm \overline{q}_n)_\alpha}\, \delta^{(2)}(\mathbf{q}_n \pm \overline{\mathbf{q}}_n)$$

Medium Averaging

$$\left\langle \mathcal{P}\exp\left(-i\int_0^L d\tau\, t^a_{\text{proj}} v^a(\boldsymbol{r}(\tau),\tau)\right)\mathcal{P}\exp\left(i\int_0^L d\overline{\tau}\, t^b_{\text{proj}} v^b(\overline{\boldsymbol{r}}(\overline{\tau}),\overline{\tau})\right)\right\rangle$$

$$= \exp\left\{-\int_0^L d\tau\,\left[1+\frac{\boldsymbol{r}(\tau)+\overline{\boldsymbol{r}}(\tau)}{2}\cdot\hat{\boldsymbol{g}}\right]\mathcal{V}\left(\boldsymbol{r}(\tau)-\overline{\boldsymbol{r}}(\tau)\right)\right\}$$

$$\hat{\boldsymbol{g}} \equiv \left(\boldsymbol{\nabla}\rho\frac{\delta}{\delta\rho}+\boldsymbol{\nabla}\mu^2\frac{\delta}{\delta\mu^2}\right)$$

$$\left\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0)G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0)\right\rangle$$

$$= \int^{\boldsymbol{u}_L}\mathcal{D}\boldsymbol{u}\int^{\boldsymbol{w}_L}\mathcal{D}\boldsymbol{w}\,\exp\left\{\int_0^L d\tau\,\left[iE\,\dot{\boldsymbol{u}}\cdot\dot{\boldsymbol{w}}-(1+\boldsymbol{w}\cdot\hat{\boldsymbol{g}})\,\mathcal{V}\left(\boldsymbol{u}(\tau)\right)\right]\right\}$$

Effective Dipole Potential

$$\mathcal{V}(\boldsymbol{q}, z) \equiv -\mathcal{C}\,\rho(z)\left(\left|v(q_\perp^2)\right|^2 - \delta^{(2)}(\boldsymbol{q})\int d^2\boldsymbol{l}\,\left|v(l_\perp^2)\right|^2\right)$$

$$\mathcal{V}(x_\perp) = \frac{\mathcal{C}g^4\rho}{4\pi\mu^2}\left(1 - \mu\,x_\perp K_1(\mu\,x_\perp)\right)$$

Jet Broadening

inhomogeneous matter

$$\left\langle G(\boldsymbol{x}_L, L; \boldsymbol{x}_0, 0) G^\dagger(\overline{\boldsymbol{x}}_L, L; \overline{\boldsymbol{x}}_0, 0) \right\rangle$$
$$= \int_{\boldsymbol{u}_0}^{\boldsymbol{u}_L} \mathcal{D}\boldsymbol{u} \int_{\boldsymbol{w}_0}^{\boldsymbol{w}_L} \mathcal{D}\boldsymbol{w} \, \exp\left\{ \int_0^L d\tau \, \left[iE \, \dot{\boldsymbol{u}} \cdot \dot{\boldsymbol{w}} - (1 + \boldsymbol{w} \cdot \hat{\boldsymbol{g}}) \mathcal{V}\left(\boldsymbol{u}(\tau) \right) \right] \right\}$$

- In the homogeneous case, the classical action is pretty simple, and the path integral can be evaluated;
- The leading gradient correction doesn't change that, and the only complication is a non-zero "force" in the EOM for u;
- The EOM can be solved perturbatively, giving

$$\boldsymbol{u}_c(\tau) = \boldsymbol{u}_L + \frac{i}{E} \hat{\boldsymbol{g}} \, \mathcal{V}\left(\boldsymbol{u}_L \right) \left\{ \frac{(\tau - L)^2}{2} \right\}$$

Jet Broadening

inhomogeneous matter

After some straightforward algebra, the re-summed broadening distribution reads

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\left\{-\mathcal{V}\left(\boldsymbol{x}\right)L\right\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}\left(\boldsymbol{x}\right)\cdot\hat{\boldsymbol{g}}\,\mathcal{V}\left(\boldsymbol{x}\right)\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\hat{\boldsymbol{g}}\,\mathcal{V}\left(\boldsymbol{x}\right)\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

and one can easily check that it satisfies the unitarity condition

$$\int d^2\boldsymbol{p}\,\frac{d\mathcal{N}}{d^2\boldsymbol{p}dE} = \left.\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \left.\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \int d^2\boldsymbol{p}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}dE}$$

One can also notice that the initial and final state distributions are not factorized anymore

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \mathcal{P}\left(\boldsymbol{x}\right)\hat{\mathsf{S}}\left(\boldsymbol{x}\right)\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

Jet Broadening

inhomogeneous matter

$$\langle F(\boldsymbol{p})\rangle = \frac{\int d^2\boldsymbol{p}\, F(\boldsymbol{p})\, \frac{d\mathcal{N}}{d^2\boldsymbol{p} dE}}{\int d^2\boldsymbol{p}\, \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p} dE}}$$

some distribution in energies

$$E\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p_\perp^2}{2w^2}}$$

a model source

$$\boxed{\langle p^\alpha\, p_\perp^2\rangle = \frac{w^2 L^2 \mu^2}{E\,\lambda}\frac{\nabla^\alpha \rho}{\rho} \ln\frac{E}{\mu}} + \frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2$$

N=1 contribution

$1/\rho\sigma_0$, the mean free path

Jet Broadening

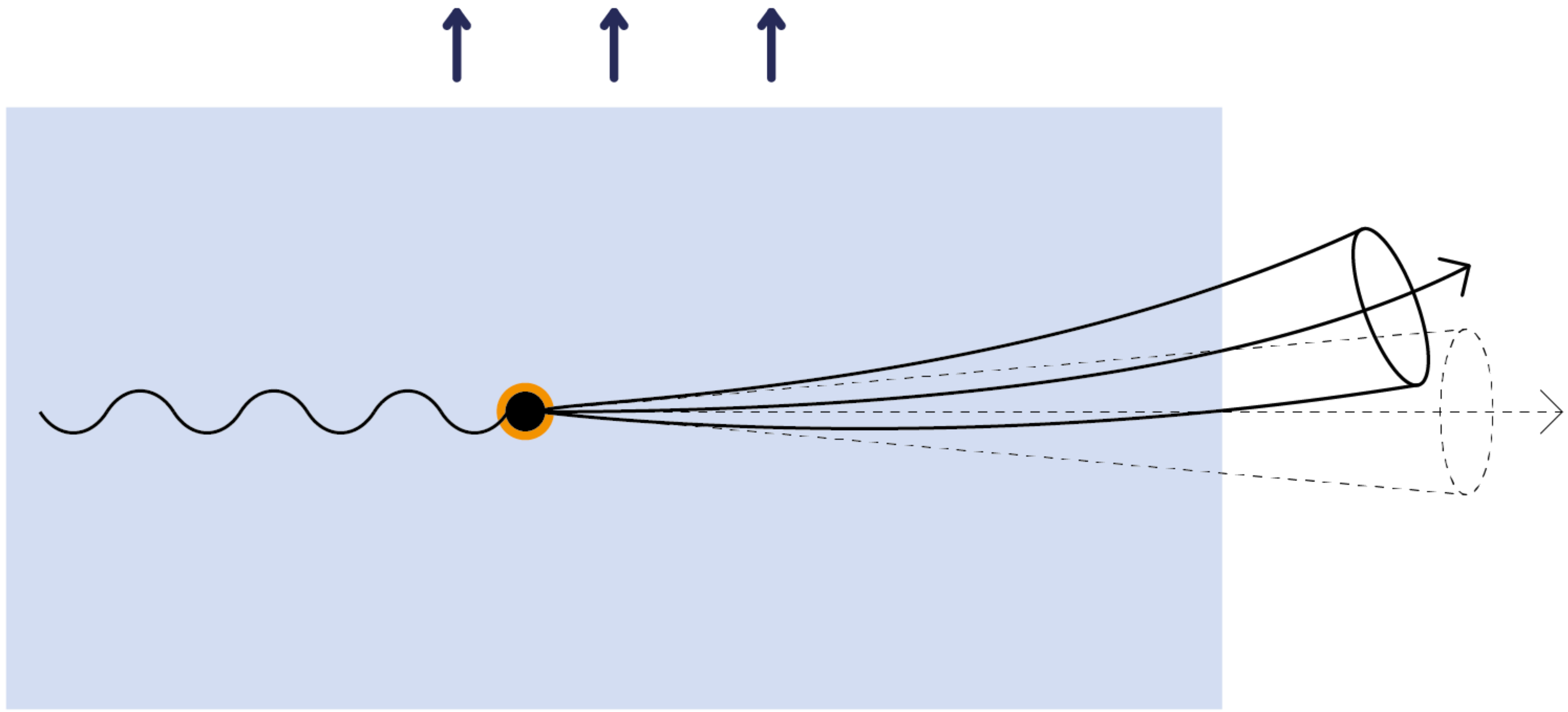
inhomogeneous matter

$$\langle F(\boldsymbol{p})\rangle = \frac{\int d^2\boldsymbol{p}\, F(\boldsymbol{p})\, \frac{d\mathcal{N}}{d^2\boldsymbol{p} dE}}{\int d^2\boldsymbol{p}\, \frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p} dE}}$$

$$E\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}\, dE} = \frac{f(E)}{2\pi w^2} e^{-\frac{p_\perp^2}{2w^2}}$$

$$\langle p^\alpha\, p_\perp^2\rangle = \frac{w^2 L^2 \mu^2}{E\,\lambda}\frac{\nabla^\alpha \rho}{\rho}\ln\frac{E}{\mu} + \frac{L^3\mu^4}{6E\,\lambda^2}\frac{\nabla^\alpha\rho}{\rho}\left(\ln\frac{E}{\mu}\right)^2$$

All order re-summed



Jet Broadening

inhomogeneous matter

After some straightforward algebra, the re-summed broadening distribution reads

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} \simeq \exp\{-\mathcal{V}(\boldsymbol{x})\,L\}\left\{\left[1-\frac{iL^3}{6E}\boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x})\cdot\hat{\boldsymbol{g}}\,\mathcal{V}(\boldsymbol{x})\right]\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}+\frac{iL^2}{2E}\,\hat{\boldsymbol{g}}\,\mathcal{V}(\boldsymbol{x})\cdot\boldsymbol{\nabla}\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right\}$$

and one can easily check that it satisfies the unitarity condition

$$\int d^2\boldsymbol{p}\,\frac{d\mathcal{N}}{d^2\boldsymbol{p}dE} = \left.\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \left.\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}\right|_{\boldsymbol{x}=0} = \int d^2\boldsymbol{p}\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{p}dE}$$

One can also notice that the initial and final state distributions are not (fully) factorized anymore

$$\frac{d\mathcal{N}}{d^2\boldsymbol{x}dE} = \mathcal{P}(\boldsymbol{x})\,\hat{\mathsf{S}}(\boldsymbol{x})\,\frac{d\mathcal{N}^{(0)}}{d^2\boldsymbol{x}dE}$$

Broadening Probability

inhomogeneous matter

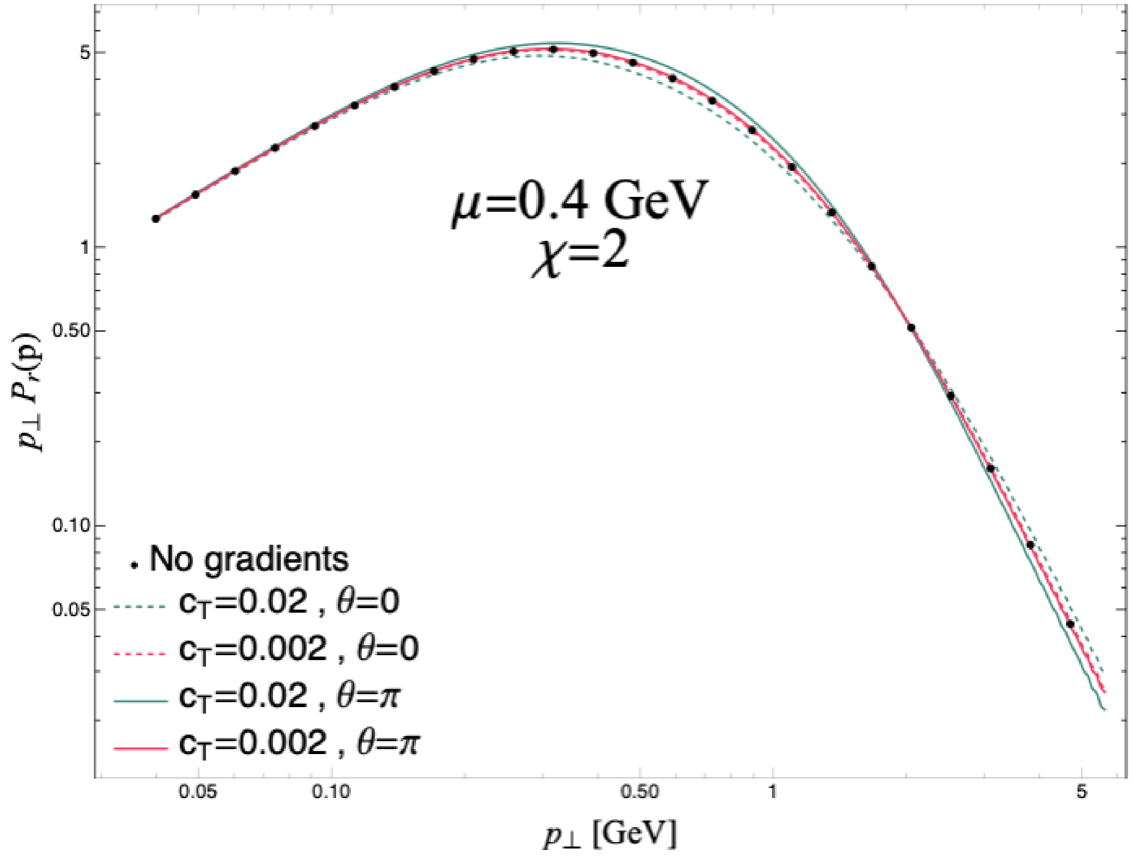
$$\mathcal{P}(\boldsymbol{p}) = \int d^2\boldsymbol{x}\, e^{-i\boldsymbol{p}\cdot\boldsymbol{x}} e^{-\mathcal{V}(\boldsymbol{x})L} \left[1 - \frac{iL^3}{6E} \boldsymbol{\nabla}\mathcal{V}(\boldsymbol{x}) \cdot \hat{\boldsymbol{g}}\mathcal{V}(\boldsymbol{x})\right]$$

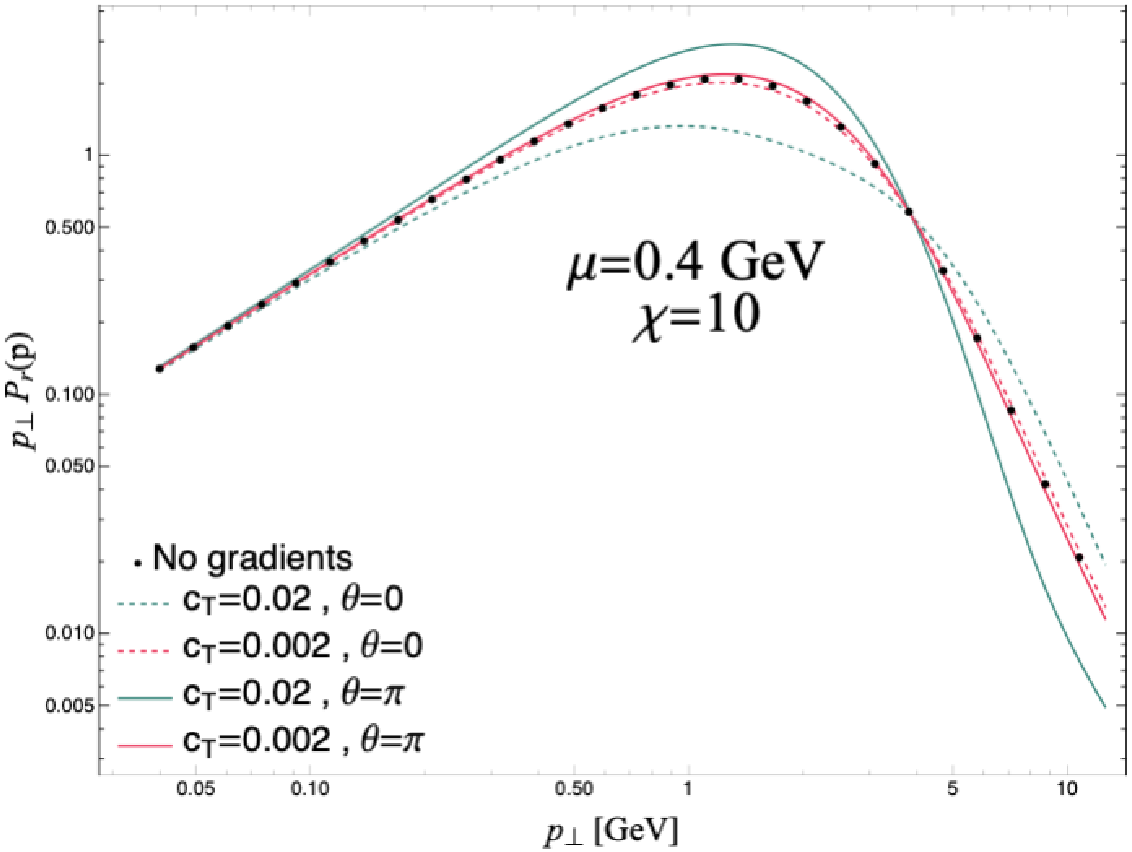
opacity, $\chi \equiv \frac{\mathcal{C} g^4 \rho}{4\pi\mu^2} L$

$$\mathcal{P}(\boldsymbol{p}) = 2\pi \int_0^\infty dx_\perp\, x_\perp\, e^{-\mathcal{V}^{\mathrm{GW}}(x_\perp)L} \Bigg\{ J_0(p_\perp x_\perp) - \frac{\chi^2\mu^2 L}{6} c_T\, x_\perp\, K_0(\mu\, x_\perp)\, J_1(p_\perp x_\perp)$$
$$\times \left[1 - 3\mu\, x_\perp K_1(\mu\, x_\perp) + \mu^2 x_\perp^2 K_2(\mu\, x_\perp)\right] \cos\theta \Bigg\}$$

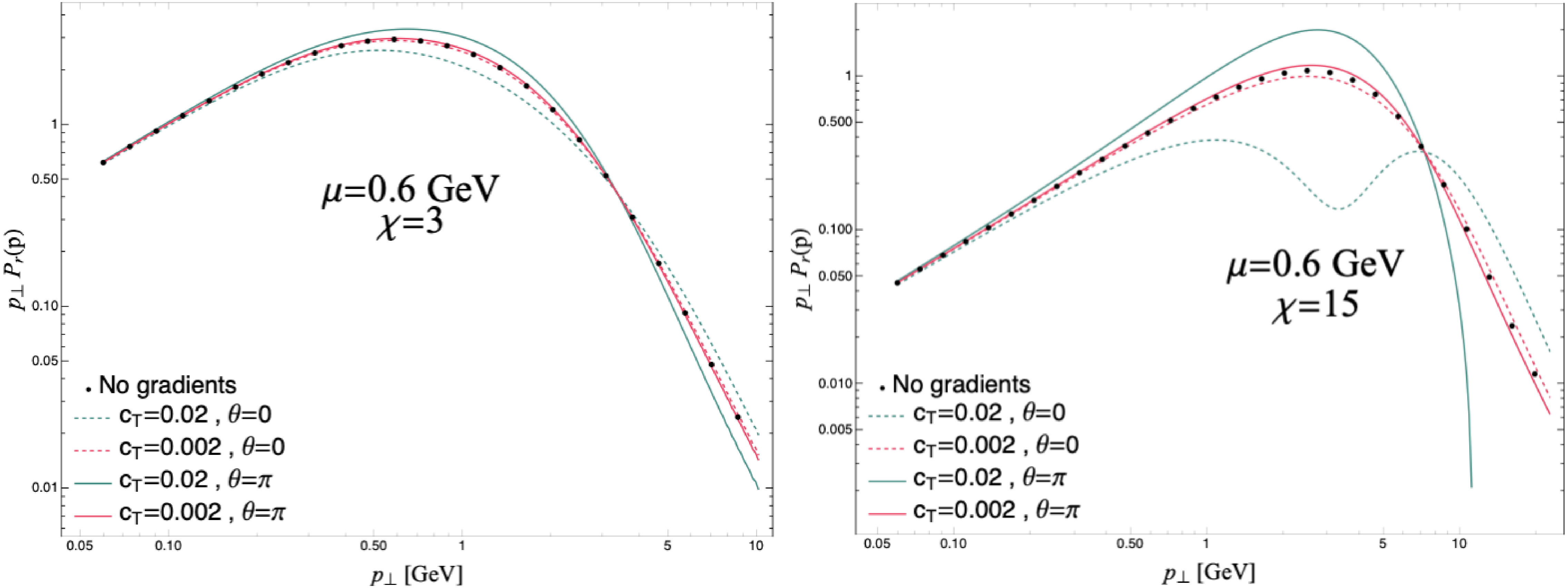
a parameter, $c_T \equiv \left|\frac{\boldsymbol{\nabla}T}{ET}\right|$

the angle between p and $\boldsymbol{\nabla}\boldsymbol{T}$





$$\mathcal{P}_r(\boldsymbol{p}) = \mathcal{P}(\boldsymbol{p}) - 2\pi \int_0^\infty dx_\perp \, x_\perp \, e^{-\mathcal{V}^{\mathrm{GW}}(\infty)L} \, J_0(p_\perp x_\perp)$$



$$\mathcal{P}_r(\boldsymbol{p}) = \mathcal{P}(\boldsymbol{p}) - 2\pi \int_0^\infty dx_\perp\, x_\perp\, e^{-\mathcal{V}^{\mathrm{GW}}(\infty)L}\, J_0(p_\perp x_\perp)$$

Summary

- We have re-summed the jet broadening distribution to all orders in opacity in inhomogeneous matter and constructed the corresponding single particle propagator in the presence of finite hydrodynamic gradients;

- The leading gradients bend the parton trajectory, contributing to odd moments of the momentum distribution (anisotropic broadening);

- The initial and final state effects are not (fully) factorized anymore, but the combination is still simple. We have studied the probability distribution, constraining the form of the initial distribution;

- The gradient effects are not large for reasonable opacities, but the anisotropic broadening should have lower background;

- These results open multiple opportunities to include the medium motion and in-medium fluctuation effects into studies of other probes of nuclear matter;

Outlook

- We have constructed a generalization of the GLV approach which includes the medium motion and structure effects. With this tool one can study general flow, temperature, and source density profiles in the HIC context;
- We have extended the BDMPS-Z formalism, including the same effects to all orders in opacity (partially work in progress: radiation, velocities, etc.);
- In the context of DIS our formalism can be also used to study nucleon orbital motion and spatial inhomogeneities in the system (a relation to GPDs and TMDs?);
- The same formalism can be used to study the initial dynamics, and one may attempt to couple jets to non-equilibrium nuclear matter;
- The medium response naturally appears in our considerations, and we will return to that in the near future;