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QM QUARK MATTER KRAKÓW 2022

Stony Brook University

Scaling approach to nuclear structure in high-energy heavy-ion collisions

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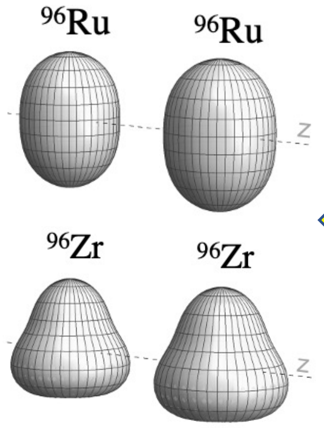
Poster Session 2 T14_2, April 6, 2022

Based on the: arXiv:2111.15559

Nuclear Structure

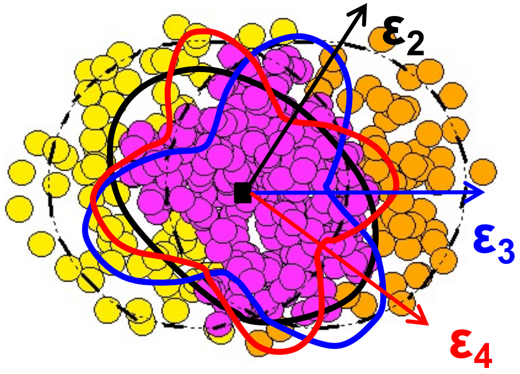


$$\rho(r,\theta,\phi) = \frac{\rho_0}{1+e^{(r-R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi)))/a_0}}$$

J. Jia, PRC105, 014905(2022)

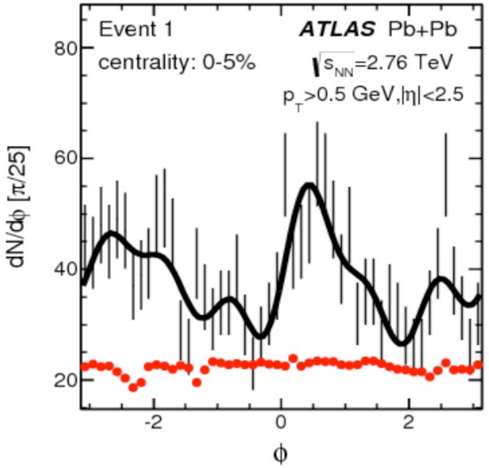
Imaging?

Initial state

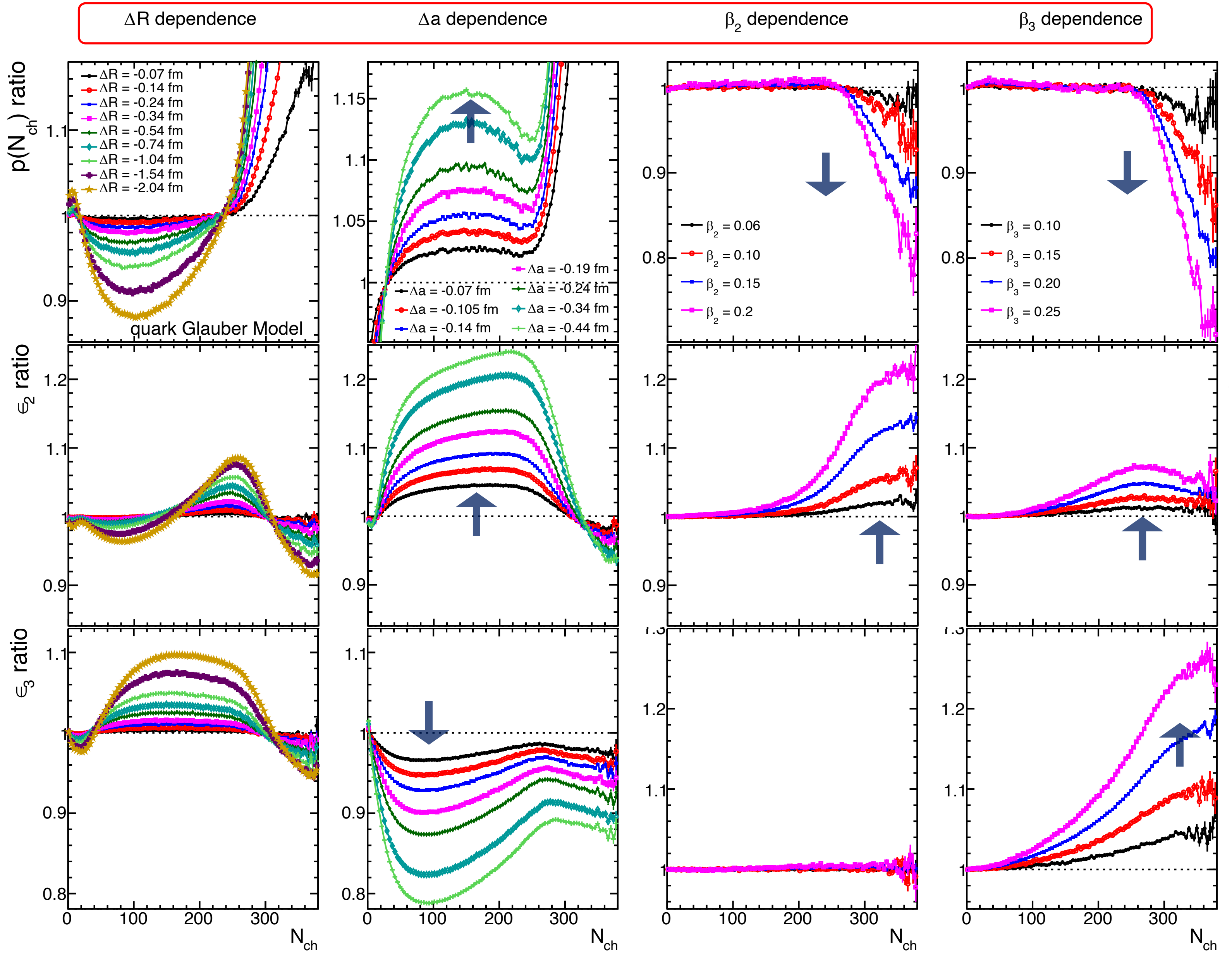


Hydrodynamic

Final state



Nuclear structure effect on the initial state



Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

R_0 has some effect with unexpected change

a_0 ↑ in mid-central

β_2 ↓ in central

β_3 ↓ in central

R_0 has some effect with unexpected change

a_0 ↑ ϵ_2 in mid-central

β_2 ↑ ϵ_2 in from mid-central to central

β_3 ↑ ϵ_2 in mid-central

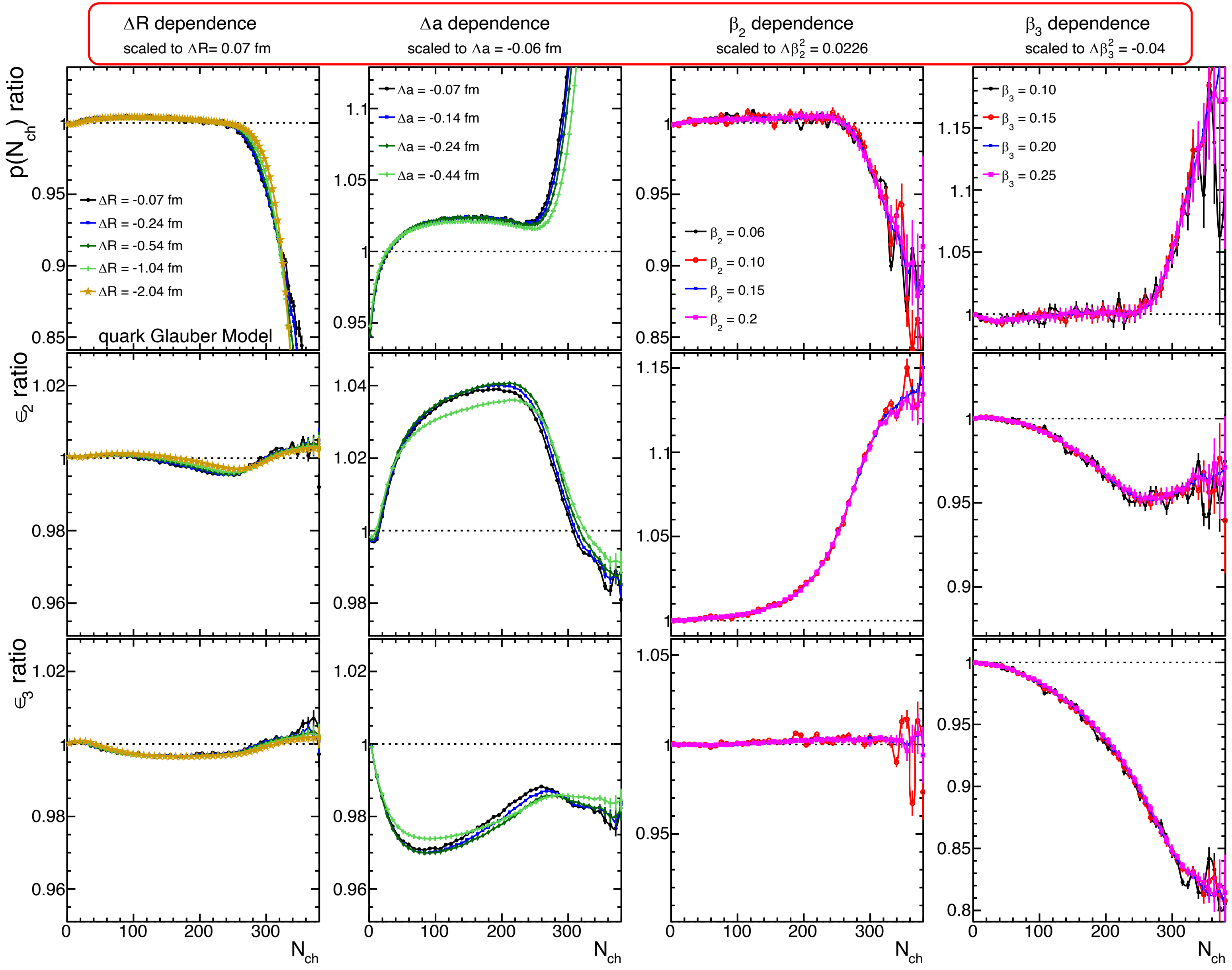
R_0 has some effect with unexpected change

a_0 ↓ ϵ_3 in mid-central

β_2 has no effect on ϵ_3

β_3 ↑ ϵ_3 in mid-central

Scaling approach to nuclear structure on the initial state



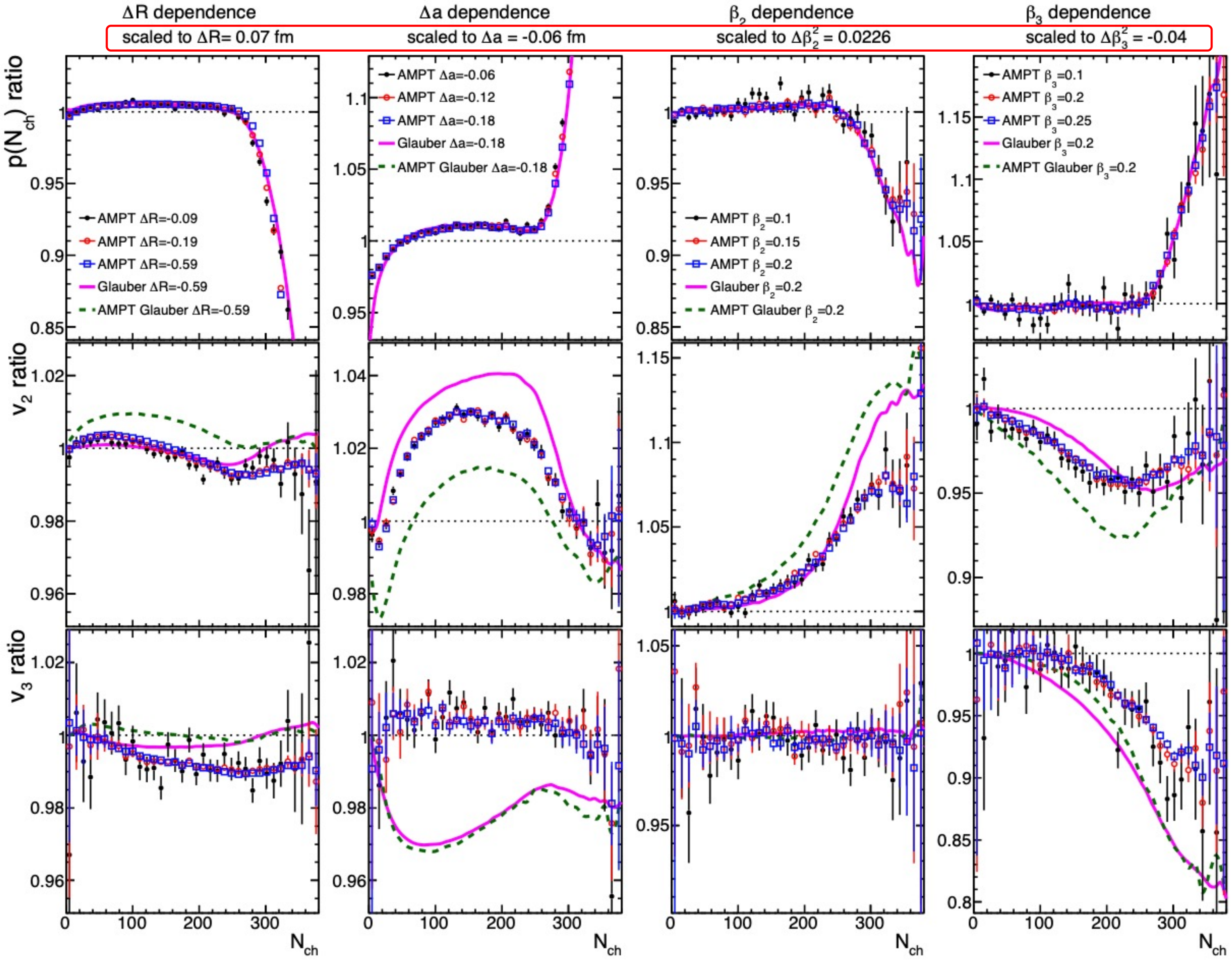
Is it linear?

Yes, only scale a constant value

nearly perfect scaling over the wide range of parameter values

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Scaling approach to nuclear structure on the final state



Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta\beta_2^2$	$\Delta\beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

nearly perfect scaling over the wide range of parameter values

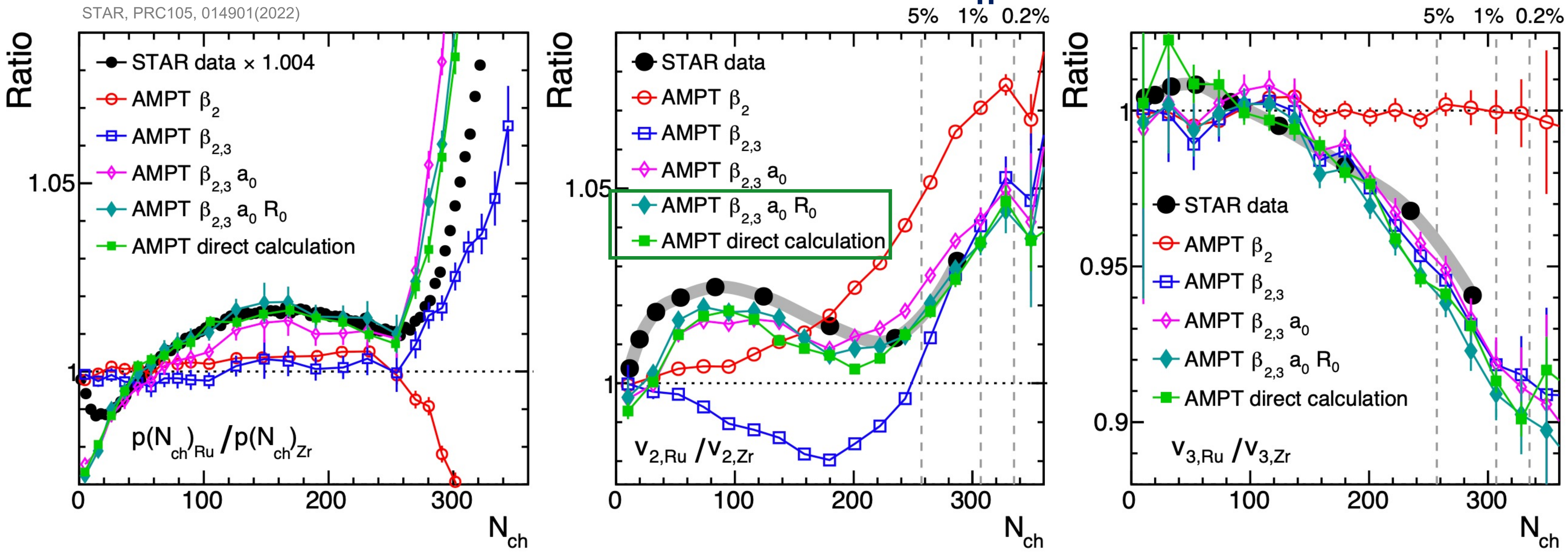
c_n can be determined more precisely by using a larger change of these parameters

Verifies the relation:

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

Nuclear structure via v_n ratio



Heavy-ion expectation:

$$v_2^2 = a_2 + b_2\beta_2^2 + b_{2,3}\beta_3^2, \quad v_3^2 = a_3 + b_3\beta_3^2$$

$$\frac{v_{2,\mathrm{Ru}}^2}{v_{2,\mathrm{Zr}}^2} \approx 1 + \frac{b_2}{a_2}\left(\beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2\right) - \frac{b_{2,3}}{a_2}\beta_{3,\mathrm{Zr}}^2$$

$$\frac{v_{3,\mathrm{Ru}}^2}{v_{3,\mathrm{Zr}}^2} \approx 1 - \frac{b_3}{a_3}\beta_{3,\mathrm{Zr}}^2 < 1$$

Cancelation expected in non-central collisions

1) v_2 ratio: large $\beta_{2,\mathrm{Ru}}$, negative contribution from $\beta_{3,\mathrm{Zr}}$ ⇒ Sharper increase in central

2) v_3 ratio: strong decrease from $\beta_{3,\mathrm{Zr}}$ with negligible $\beta_{2,\mathrm{Ru}}$ distortion

3) Residual effect due to radial structure, e.g., neutron skin in Zr

4) No significant effect due to nuclear size

✓ **Direct calculations are same as initial whole β_2, β_3, a_2, R_0 input.**

A direct algebra linked to neutron skin

Using relation for WS: $R^2 \equiv \langle r^2 \rangle \approx \left(\frac{3}{5}R_0^2 + \frac{7}{5}\pi^2 a^2\right) / \left(1 + \frac{5}{4\pi^2}\sum_n \beta_n^2\right)$

Neutron skin expressed by R and a parameters for nucleons and protons:

$$\Delta r_{np} \approx \frac{R^2 - R_p^2}{R(\delta+1)} \approx \frac{3\left(R_0^2 - R_{0,p}^2\right) + 7\pi^2\left(a^2 - a_p^2\right)}{\sqrt{15}R_0\sqrt{1 + \frac{7\pi^2}{3}\frac{a^2}{R_0^2}}\left(1 + \delta + \frac{5}{8\pi^2}\sum_n \beta_n^2\right)}$$

$$\delta = (N - Z)/A$$

The difference between two isobar systems can be expressed as:

$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{\bar{R}_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0\left(1 + \bar{\delta} + \frac{5}{8\pi^2}\sum_n \overline{\beta_n^2}\right)}$$

where $Y \equiv 3\left(R_0^2 - R_{0,p}^2\right) + 7\pi^2\left(a^2 - a_p^2\right)$ $\quad \Delta x = x_1 - x_2 \quad \bar{x} = (x_1 + x_2)/2$

Can obtain skin diff. from ΔR_0 Δa for nucleons and known ΔR_0 Δa for protons

Table from: H.J. Xu et. al., PLB819, 1136453(2021)

	^{96}Ru		^{96}Zr	
	R	*a*	*R*	*a*
p	5.060	0.493	4.915	0.521
n	5.075	0.505	5.015	0.574
p+n	5.067	0.500	4.965	0.556

Direct calc.: $\Delta(\Delta r_{np})$=0.0296 fm – 0.1606 fm = -0.1310 fm

Formula: $\Delta(\Delta r_{np})$= -0.1319 fm

<1% difference

Conclusions and Outlooks

1) Demonstration: the nuclear structure effect on bulk observables.

2) AMPT could describe the STAR published data quantitatively.

3) A new approach to constrain the collective nuclear structure parameters:

- ✓ The final state bulk observables v_2, v_3 and $p(N_{ch})$ follow a simple dependences on the variation of parameters:

$$\mathcal{O} \approx b_0 + b_1\beta_2^2 + b_2\beta_3^2 + b_3(R_0 - R_{0,\,\mathrm{ref}}) + b_4(a - a_{\mathrm{ref}})$$

$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\mathrm{Ru}}}{\mathcal{O}_{\mathrm{Zr}}} \approx 1 + c_1\Delta\beta_2^2 + c_2\Delta\beta_3^2 + c_3\Delta R_0 + c_4\Delta a$$

- ✓ The c_n can be determined precisely in a given model.
- ✓ The data-model comparison can precisely constrain the dependence of nuclear parameters:

$$\Delta\beta_2^2 = \beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2 \qquad \Delta\beta_3^2 = \beta_{3,\mathrm{Ru}}^2 - \beta_{3,\mathrm{Zr}}^2 \qquad \Delta R_0 = R_{0,\mathrm{Ru}} - R_{0,\mathrm{Zr}} \qquad \Delta a = a_{\mathrm{Ru}} - a_{\mathrm{Zr}}$$

- ✓ Achieve to obtain the difference of neutron skin between two isobar systems:

$$\Delta(\Delta r_{np}) = \Delta r_{np,1} - \Delta r_{np,2} \approx \frac{\Delta Y - \frac{7\pi^2}{3}\frac{\bar{a}^2}{\bar{R}_0^2}\left(\frac{\Delta Y}{2} + \bar{Y}\left(\frac{\Delta a}{\bar{a}} - \frac{\Delta R_0}{\bar{R}_0}\right)\right)}{\sqrt{15}\bar{R}_0\left(1 + \bar{\delta} + \frac{5}{8\pi^2}\sum_n \overline{\beta_n^2}\right)}$$

4) Unique opportunities by relativistic collisions of isobars as a tool to study nuclear structure.

Thank you @ QUARK MATTER KRAKÓW 2022