Heavy quarks in the evolving Glasma
Diffusion, 2-particle correlations and comparison with Langevin dynamics

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Colored particle-in-cell method adapted for Glasma

Solving Wong’s equations with Glasma background fields

- Boost-invariant Glasma fields in the forward light-cone, expressed in Milne coordinates $(\tau, \eta)$
- Classical Yang-Mills equations solved by employing real-time lattice gauge theory (code available at gitlab.com/openpixi/curraun)
- Fields discretized using plaquettes in the transverse plane
  \[ U_{x,\mu\nu} \equiv U_{x,\mu} U_{x+\nu,\mu} U_{x+\mu,\mu} U_{x+\nu,\nu} \], no lattice discretization along $\eta$

- Wong’s equations for classical particles in Yang-Mills fields
  \[ p^\tau \frac{dx^\mu}{d\tau} = p^\mu , \quad p^\tau \frac{dp^\mu}{d\tau} = 2g \text{Tr}[Q F^{\mu\nu}] p_\nu \]

- Rotation of color charge $Q(\tau) = U_{x,\mu} U_{x+\nu,\mu} U_{x+\mu,\mu} U_{x+\nu,\nu} Q(\tau_0) U(\tau_0, \tau)$ when the nearest grid point (NGP) on lattice changes

- Numerical Wilson line $U(\tau_{n-1}, \tau_n) = U_{x,\mu} U_{x+\nu,\mu} U_{x+\mu,\mu} U_{x+\nu,\nu}$ contains an artificially constructed gauge link along $\eta$

- Symplectic solver assures $Q \in SU(3)$ and perfectly conserves the quadratic and cubic Casimirs $Q^a Q^a$ and $d_{abc} Q^a Q^b Q^c$
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Momentum broadening of hard probes in the Glasma

Limiting cases: highly energetic jets & infinitely massive heavy quarks

- **Infinitely massive heavy quarks**: static quarks, accumulate momentum only due to electric fields
  \[
  \langle \delta p_i^2(\tau) \rangle = g^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' \langle \text{Tr} \left[ E_i(\tau')E_i(\tau'') \right] \rangle
  \]
  Good agreement with numerical simulations in the limit \( m \to \infty \)
  Heavy quark diffusion coefficient can be extracted from electric fields correlators, see [arXiv:2005.02418] with overoccupied YM fields

- **Highly energetic jets**: move along \( x \)-axis, studied in [arXiv:2001.10001]
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  \langle \delta p_i^2(\tau) \rangle = g^2 \int_0^\tau d\tau' \int_0^\tau d\tau'' \langle \text{Tr} \left[ f_i(\tau')f_i(\tau'') \right] \rangle
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  with
  \[
  f_y(\tau) = U(\tau) \left[ E_y(\tau) - B_z(\tau) \right] U^\dagger(\tau)
  \]
  \[
  f_z(\tau) = U(\tau) \left[ E_z(\tau) + B_y(\tau) \right] U^\dagger(\tau)
  \]
  parallel transporter \( U(\tau) = \mathcal{P} \exp \left\{ -ig \int d\tau' A_x(\tau') \right\} \)
  Numerical solver for particles with \( p^x \to \infty \) reproduces these results
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Heavy quarks probing the Glasma
The effect of full dynamics & anisotropic momentum broadening

(Top) Simulated trajectories of heavy quarks in Glasma flux tubes

- Energy density at formation time in background
- Lines represent (left) infinitely massive, (middle) beauty and (right) charm quarks with different initial $p_T$
- Quarks with larger initial $p_T$ escape the correlation domains faster

(Right) Longitudinal and transverse momentum broadenings for charm and beauty quarks

- Numerical simulations with various initial $p_T$ show deviations from static quark scenario (grey dashed line), more significant for beauty quark
- Anisotropy, defined as $\langle \delta p_L^2 \rangle / \langle \delta p_T^2 \rangle$, decreases as initial $p_T$ increases since faster particles spent less time in the Glasma correlation regions
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Two-particle correlations & comparison with Langevin dynamics
Angular correlations of $Q\bar{Q}$ pairs & comparison with the QPM of the Catania group

- **(Left top & bottom)** angular correlations of $Q\bar{Q}$ pairs produced back-to-back in the Glasma
  - Pairs produced at the boundary of the fireball, with $Q$ going outwards and $\bar{Q}$ propagating in the Glasma
  - Angle $\theta$ between $Q\bar{Q}$ measured as a function of time, for different initial $p_T$, with $p_T$ affected by the Glasma medium
  - Angular correlation immediately washed out by the Glasma for pairs with small $p_T$
  - Large $p_T$ pairs still carry information about the early correlation even after 0.3 fm/c, when the Glasma picture should be replaced by a hydrodynamic or kinetic approach

- **(Right bottom)** Transverse momentum spectra and momentum broadening for charm quarks in a PbPb@5.02 TeV Glasma
  - Different behaviour of momentum broadening compared to a Langevin approach from Catania’s QPM, mainly due to longitudinal expansion in Glasma
  - Spectra show that quarks with larger $p_T$ diffuse more in both cases

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Gauge invariant correlators of color Lorentz forces

Color Lorentz force correlators evaluated along the trajectory of the heavy quarks

\[ \langle \delta p_i^2(\tau) \rangle = g^2 \tau \int_0^\tau d\tau' \int_0^\tau d\tau'' \langle \text{Tr} \left[ \widetilde{F}_i(\tau') \widetilde{F}_i(\tau'') \right] \rangle \]

with \( \widetilde{F}_i \) parallel transported Lorentz force and \( F_{\mu \nu} \equiv F_{\mu \nu} p^\nu / p^\tau \) color Lorentz force in Milne frame

Concluding remarks & future prospects

- Study of the interplay between mass, formation time and initial momentum for the propagation of heavy quarks in Glasma
- Effect on longitudinal and transverse momentum broadenings, momentum anisotropy, color Lorentz force correlators of charm and beauty quarks, showing that full dynamics is relevant
- Extend the study to jets, include more realistic initial distribution of color charge in the Glasma and of heavy quarks positions and momenta, implement an improved method to evaluate angular correlations

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The correlator decays faster for heavy quarks with larger initial \( p_T \). They jump quicker between correlation domains and thus lose the memory of the early phase more rapidly, see [arXiv:2110.14610]
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\langle \delta p^2_i(\tau) \rangle = g^2 \int_0^\tau d\tau' \int_0^{\tau'} d\tau'' \langle \text{Tr} [\tilde{F}_i(\tau')\tilde{F}_i(\tau'')] \rangle
\]
with \(\tilde{F}_i\) parallel transported Lorentz force and \(F_{\mu\nu} = F_{\mu\nu}^k p^k / p^\tau\) color Lorentz force in Milne frame.

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