Pionless effective field theory in the flavor SU(3) symmetric limit

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Illustration: Harald Ritsch/IQOQI

Effective Field Theory (EFT)

- Typically in physics we have an "underlying" theory, valid at a mass scale M_{hi}, but we want to study processes at momenta $Q \approx M_{lo} \ll M_{hi}$.
- For example, nuclear structure involves energies that are much smaller than the typical QCD mass scale, $M_{QCD} \approx 1$ GeV.
- Effective Field Theory (EFT) is a framework to construct the interactions systematically. The high-energy degrees of freedom are integrated out, while the effective Lagrangian has the same symmetries as the underlying theory.
- The details of the underlying dynamics are contained in the interaction strengths.

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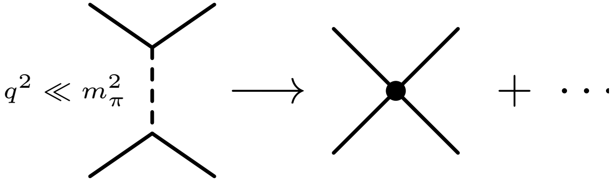

Pionless or Short-Range EFT

- For spinless particles, the two body-sector has a single term at LO,

$$V_{LO} = a_1.$$

- and another one at NLO,

$$V_{NLO} = b_1(p^2 + p'^2).$$

- The LO term is to be iterated.

- The NLO term is treated as perturbation.

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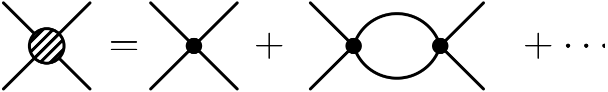

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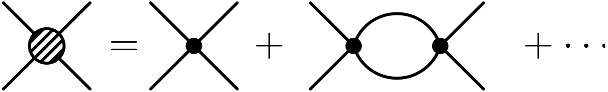

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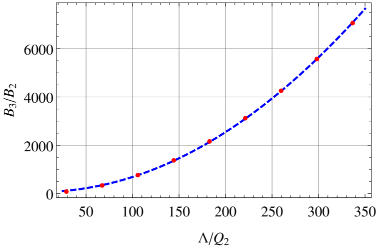
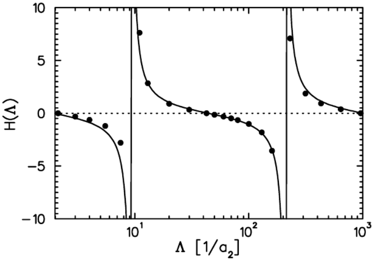
Three-boson system

Trying to calculate the trimer binding energy one gets the Thomas collapse:

$$B_3 \propto \frac{\hbar^2 \Lambda^2}{m}$$



To stabilize the system, a 3-body counter term must be introduced at LO



LO: Bedaque, Hammer, and van Kolck, PRL **82**, 463 (1999).

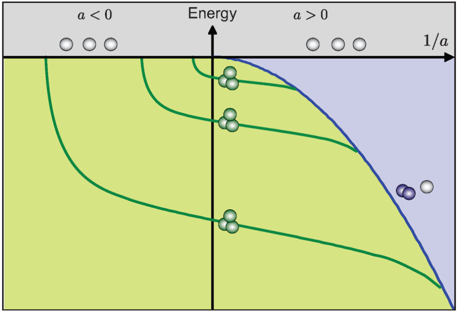
NLO: Ji, Phillips, and Platter, Ann. Phys. **327**, 1803 (2012).

Efimov Physics

- Actually we see here the Efimov effect.
- discrete scale invariance: $\lambda_n = e^{-\pi n/|s|}$
- infinite number of bound states $E_n = E_0 e^{-2\pi n/|s_0|}$ with $e^{2\pi/|s_0|} \approx 515$
- Borromean binding

Efimov, Phys. Lett. B **33**, 563 (1970)

Review: Naidon and Endo (2017)

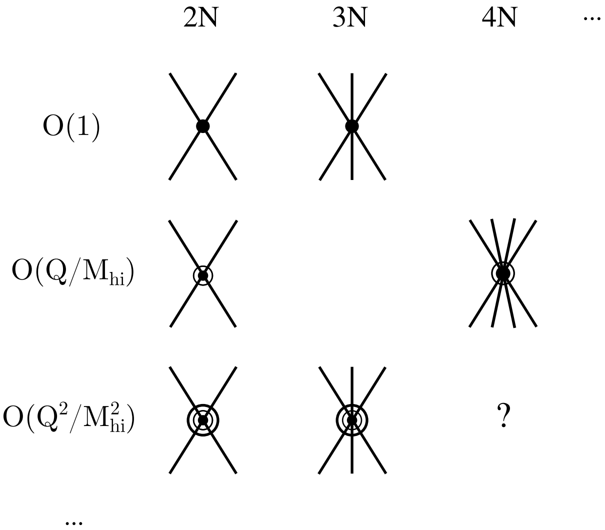


Ferlaino and Grimm, Physics **3**, 9 (2010)

$\not\pi$EFT potential

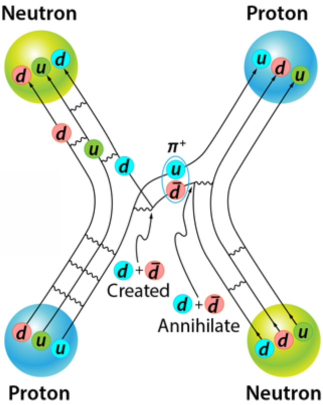
- At NLO, a four-body force is needed!

Bazak, Kirscher, König, Pavón Valderrama, Barnea, and van Kolck, PRL **122**, 143001 (2019)

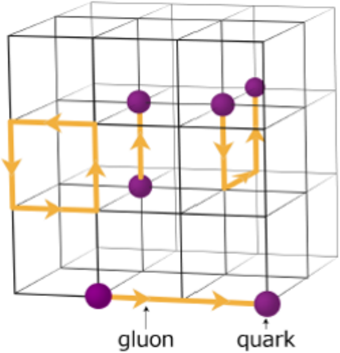


Hammer, König and van Kolck, Rev. Mod. Phys. **92**, 025004 (2020)

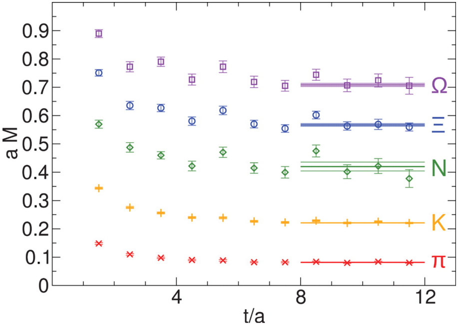
Lattice QCD

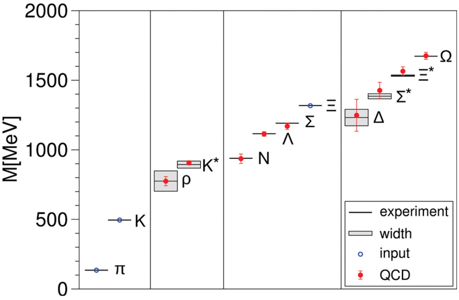


APS/Alan Stonebraker



The light hadron spectrum from Lattice QCD





Dürr et al., Science **322**, 1224 (2008)

How to let LQCD results out of the box?

- For the two-body case,

$$B_2(L) \approx B_2^{\text{free}} + \frac{6\kappa_2 |\mathcal{A}_2|^2}{\mu_2 L} e^{-\kappa_2 L}$$

M. Lüscher, Commun. Math. Phys. **104**, 177 (1986)

$\mathcal{A}_2$ is the dimensionless asymptotic normalization coefficient (ANC).

- Generalization for two-clusters breaking of N-body system,

$$B_N(L) = B_N^{\text{free}} + \frac{6\kappa_N |\mathcal{A}_N|^2}{\mu_N L} e^{-\kappa_N L}$$

S. König and D. Lee, Phys. Lett. B **779**, 9 (2018)

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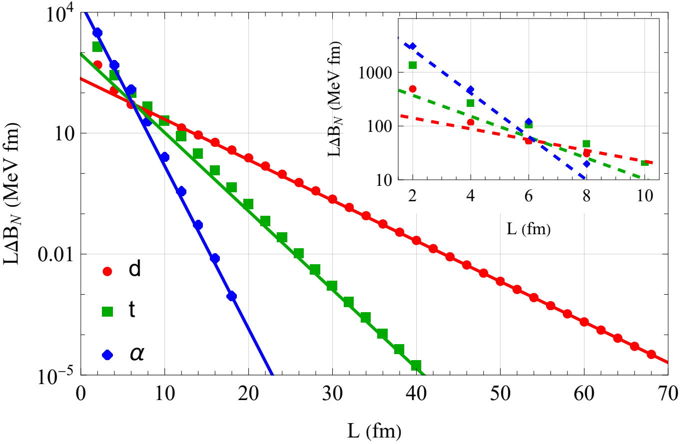
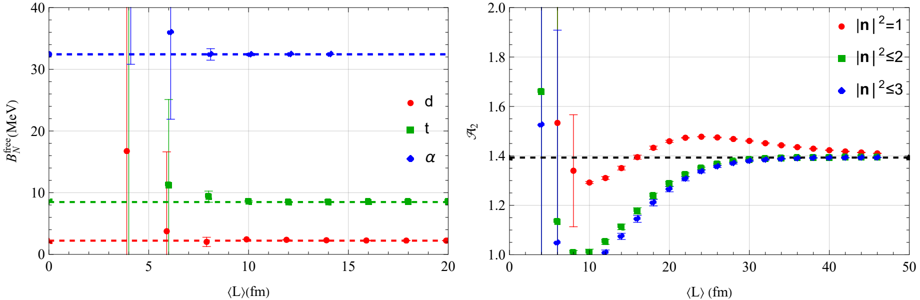
...but these are assymptotic formulas!



Yaron, Bazak, Schäfer, and Barnea, arXiv:2206.04497

How large should the box be?



Yaron, Bazak, Schäfer, and Barnea, arXiv:2206.04497

Values are extracted from three adjacent boxes.

$$\Delta B_2 = \frac{\kappa_2|\mathcal{A}_2|^2}{\mu_2 L}\left(\overbrace{6e^{-\kappa_2 L}}^{|\mathbf{n}|^2=1} + \overbrace{\frac{12}{\sqrt{2}}e^{-\sqrt{2}\kappa_2 L}}^{|\mathbf{n}|^2=2} + \overbrace{\frac{8}{\sqrt{3}}e^{-\sqrt{3}\kappa_2 L}}^{|\mathbf{n}|^2=3} + \dots\right)$$

Extrapolation of LQCD results

Free-space vs. Lüscher formula results using boxes with $L = \{6, 8, 10\}$ fm.

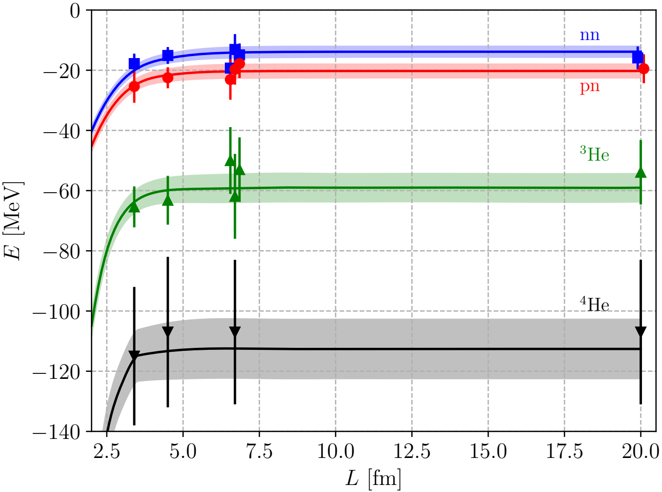
	Free-space		Lüscher	
N	B_N^{free} [MeV]	ANC	B_N^{free} [MeV]	ANC
2	2.2246	1.40	2.0(8)	1.3(2)
3	8.482	2.024	9.4(9)	2.2(1)
4	32.48	6.00	32.4(9)	4.9(1)

- Alternatively, one can fit effective field theory directly to the results in finite boxes.
- The EFT is then solved in free space to get the physical values.

Eliyahu, Bazak, and Barnea, Phys. Rev. C **102**, 044003 (2020).

W. Detmold and P. E. Shanahan, Phys. Rev. D **103**, 074503 (2021).

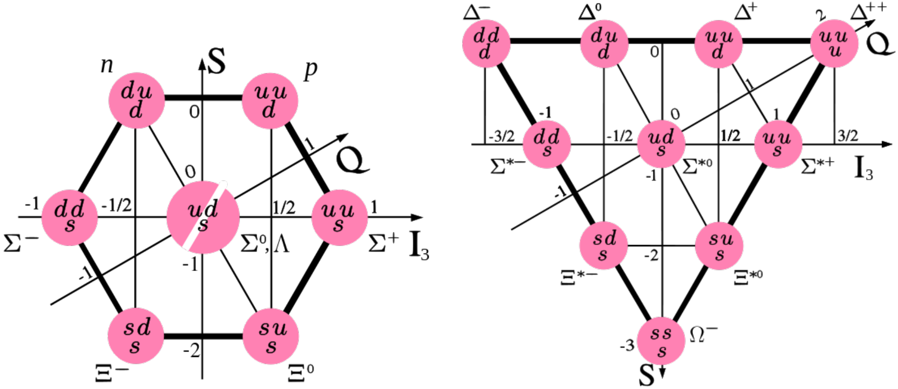
EFT for LQCD: extrapolation



Symbols: NPLQCD results for $m_\pi = 806$MeV, Beane *et al.*, Phys. Rev. D **87**, 034506 (2013).

Curves: $\not\pi$EFT results, Eliyahu, Bazak, and Barnea, Phys. Rev. C **102**, 044003 (2020).

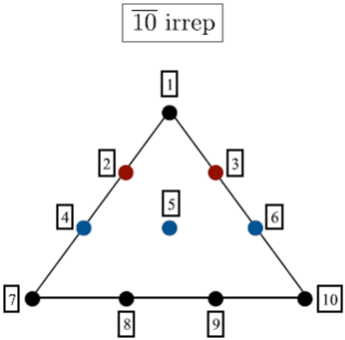
Flavor SU(3) symmerty of quarks



from wikipedia

Two baryon states

- flavor: $8 \otimes 8 = 1_S \oplus 8_S \oplus 27_S \oplus 8_A \oplus 10_A \oplus \overline{10}_A$
- spin: $2 \otimes 2 = 1_A \oplus 3_S$



	Flavor channel
1	$\frac{1}{\sqrt{2}}(pn - np)$
2	$-\sqrt{\frac{1}{3}}\Sigma^0 n + \sqrt{\frac{2}{3}}\Sigma^- p / \Lambda n$
3	$\sqrt{\frac{2}{3}}\Sigma^+ n + \sqrt{\frac{1}{3}}\Sigma^0 p / \Lambda p$
4	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^0 - \Sigma^0\Sigma^-) / \Xi^- n / \Lambda\Sigma^-$
5	$\frac{1}{\sqrt{2}}(\Sigma^-\Sigma^+ - \Sigma^+\Sigma^-) / \frac{1}{\sqrt{2}}(\Xi^- p - \Xi^0 n) / \Lambda\Sigma^0$
6	$\frac{1}{\sqrt{2}}(\Sigma^0\Sigma^+ - \Sigma^+\Sigma^0) / \Xi^0 p / \Lambda\Sigma^+$
7	$\Sigma^-\Xi^-$
8	$-\sqrt{\frac{2}{3}}\Sigma^0\Xi^- + \sqrt{\frac{1}{3}}\Sigma^-\Xi^0$
9	$\sqrt{\frac{1}{3}}\Sigma^+\Xi^- + \sqrt{\frac{2}{3}}\Sigma^0\Xi^0$
10	$\Sigma^+\Xi^0$

Wagman *et al.* (NPLQCD Collaboration) Phys. Rev. D 96, 114510 (2017)

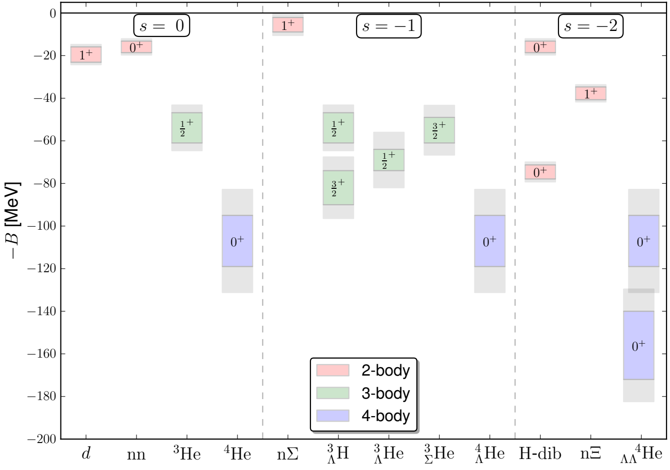
Pionless EFT for SU(3)⊗SU(2) case

- We need potential consistent with SU(3)⊗SU(2) symmetries.
- Use the identity + Casimir operators!

$$\{\hat{1}, \hat{C}_2, \hat{C}_3\} \otimes \{\hat{1}, \hat{S}^2\}$$

- $\hat{C}_3$ e.v. is 0 for all symmetric irreps $1, 8_S, 27$.
- $\{\hat{1}, \hat{S}^2\} \longrightarrow \{\hat{P}_S, \hat{P}_T\}$ therefore $\hat{C}_3\hat{P}_S = 0$; only 5 parameters are needed!
  Dover and Feshbach, Ann. Phys. 198, 321 (1990)
- Other approaches find 6 operators.
  Savage and Wise, Phys. Rev. D 53, 349 (1996); ...

NPLQCD calculations for SU(3) flavor symmetry

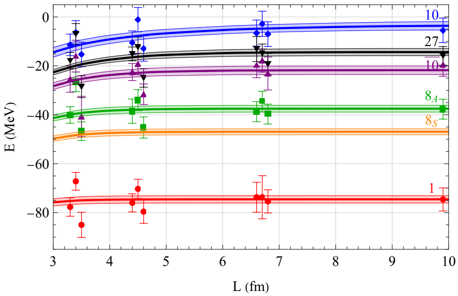


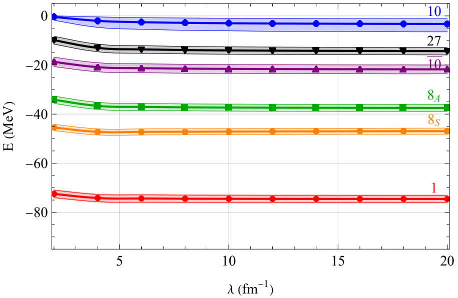
NPLQCD Collaboration, Phys. Rev. D **87**, 034506 (2013).

- In nature,
  $m_u \approx m_d \ll m_s$
  $m_\pi \approx 140$ MeV
  $m_N \approx 939$ MeV.
- SU(3) flavor symmetry:
  $m_u = m_d = m_s$
  $m_\pi \approx 806$ MeV
  $m_N \approx 1634$ MeV.

Results: two baryon systems

- flavor: $8 \otimes 8 = 1_S \oplus 8_S \oplus 27_S \oplus 8_A \oplus 10_A \oplus \overline{10}_A$
- spin: $2 \otimes 2 = 1_A \oplus 3_S$





$\lambda = 20\,\text{fm}^{-1}$; $L \longrightarrow \infty$ extrapolation

$\lambda \longrightarrow \infty$ extrapolation

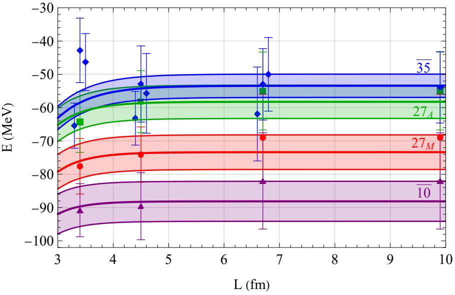
8_S results is a prediction!

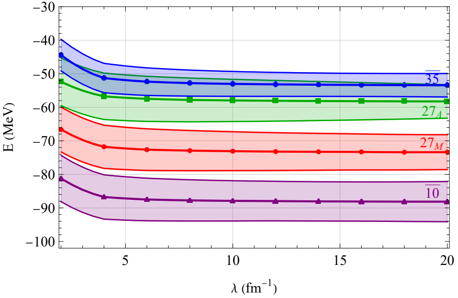
Results: three baryon systems

- flavor:

$$8 \otimes 8 \otimes 8 = 1_S \oplus 8_S \oplus \overline{10}_S \oplus 10_S \oplus 27_S \oplus 64_S \oplus$$
$$\oplus 27_A \oplus 10_A \oplus \overline{10}_A \oplus 8_A \oplus 1_A \oplus$$
$$\oplus 4 \cdot 27_M \oplus 2 \cdot 35_M \oplus 2 \cdot \overline{35}_M \oplus 2 \cdot 10_M \oplus 2 \cdot \overline{10}_M \oplus 6 \cdot 8_M$$

- spin: $2 \otimes 2 \otimes 2 = 2 \cdot 2_M \oplus 4_S$





$\lambda = 20\,\text{fm}^{-1}$; $L \longrightarrow \infty$ extrapolation $\quad$ $\lambda \longrightarrow \infty$ extrapolation

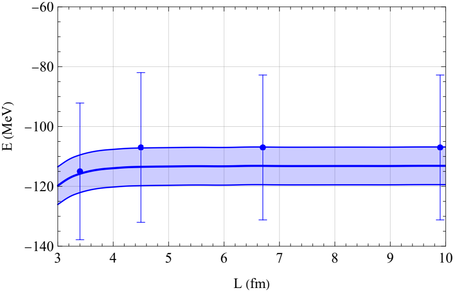
There are 14 physicaly allowed irreps; we don't have enough data to fit all needed LECs.

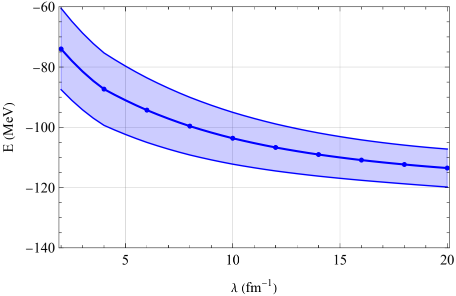
Results: four baryon systems

- flavor:

$$\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}\otimes\mathbf{8}=\mathbf{125}\oplus 3\cdot\mathbf{81}\oplus 3\cdot\overline{\mathbf{81}}\oplus 12\cdot\mathbf{64}\oplus 15\cdot\overline{\mathbf{35}}\oplus 15\cdot\mathbf{35}\oplus$$
$$\oplus 2\cdot\mathbf{28}\oplus 2\cdot\overline{\mathbf{28}}\oplus\oplus 33\cdot\mathbf{27}\oplus 20\cdot\mathbf{10}\oplus 20\cdot\overline{\mathbf{10}}\oplus 32\cdot\mathbf{8}\oplus 8\cdot\mathbf{1}$$

- spin: $2\otimes 2\otimes 2\otimes 2=2\cdot\mathbf{1}\oplus 3\cdot\mathbf{3}\oplus\mathbf{5}$





$\lambda=20\,\mathrm{fm}^{-1}$; $L\longrightarrow\infty$ extrapolation

$\lambda\longrightarrow\infty$ extrapolation

- The symmetry considerations becomes a bit cumbersome...
- We did only the simplest $\overline{28}$ irrep, i.e. ^{4}He.
- NLO calculations are important here!

Conclusion

- $\not\pi$EFT and its power counting were introduced.
- LQCD results should be done in quite large boxes to enable accurate extrapolation with Lüscher formula.
- $\not\pi$EFT can be used to extrapolate LQCD results to infinite volume and to heavier nuclei.
- NPLQCD results at the flavor symmetric point was used to fit the two-body $\not\pi$EFT potential, thus predicting the 8_S irrep.
- More data is needed to fit all 14 three-body channels.

1. BB, Johannes Kirscher, Sebastian König, Manuel Pavón Valderrama, Nir Barnea, and Bira van Kolck, Phys. Rev. Lett. **122**, 143001 (2019).
2. Moti Eliyahu, BB, and Nir Barnea, Phys. Rev. C **102**, 044003 (2020).
3. Roee Yaron, BB, Martin Schäfer, and Nir Barnea, arXiv:2206.04497.
4. BB and Martin Schäfer, *in preparation*.
5. Michael Leveson, Roee Yaron, Moti Eliyahu, Nir Barnea and BB, *in preparation*.