האוניברסיטה העברית בירושלים
THE HEBREW UNIVERSITY OF JERUSALEM

Baryonic EFT for Light Hypernuclei

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Collaboration

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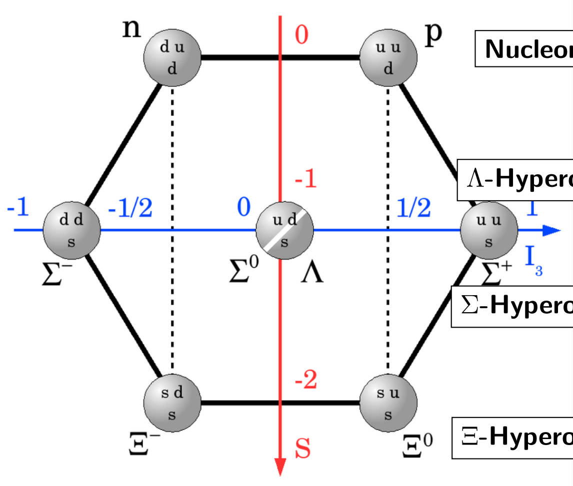


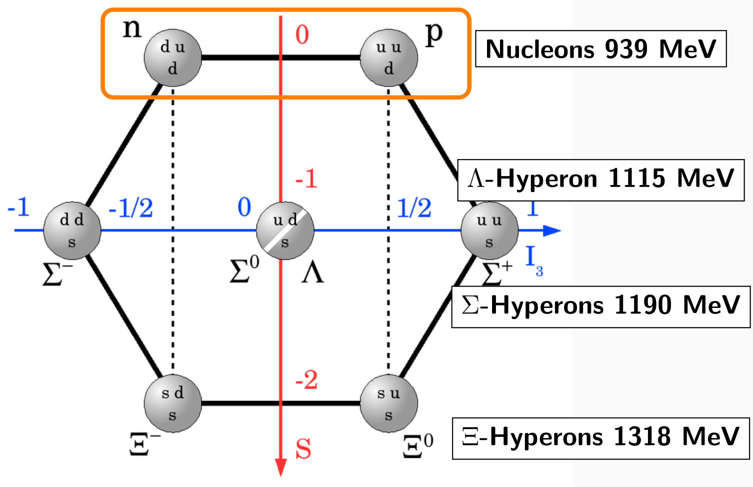
Nucleons 939 MeV

Λ-Hyperon 1115 MeV

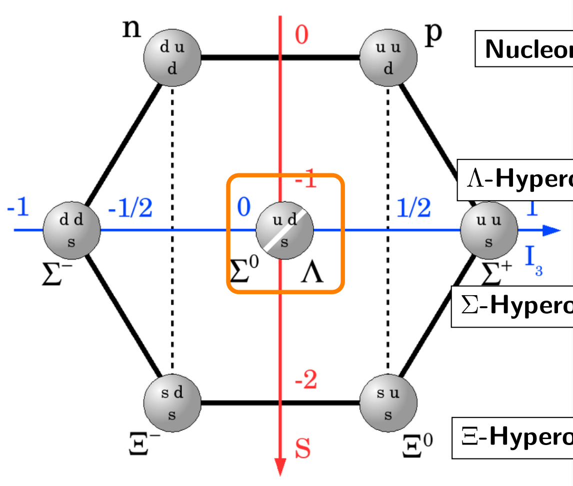
Σ-Hyperons 1190 MeV

Ξ-Hyperons 1318 MeV

Introduction - the baryon octet



Introduction - the baryon octet



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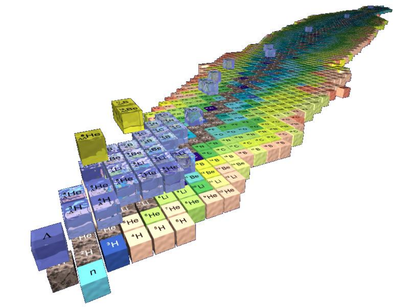
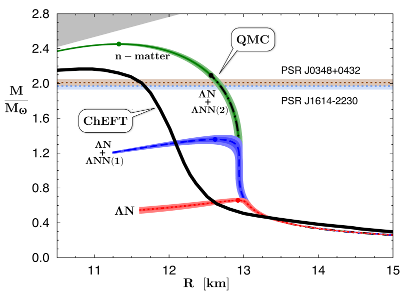




Nuclei & Hypernuclei

≈3300 nuclear isotopes

≈40 single Lambda hypernuclei

3 double Lambda hypernuclei

In neutron stars

Hyperons softens the EOS

Adding ΛNN-force stiffens the EOS

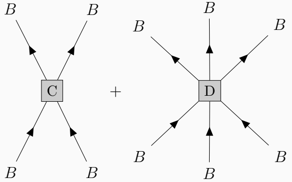
`Lonardoni et al.  PRL 114 (2015) 092301`

Baryonic EFT

Baryonic EFT aka $\not{\pi}$EFT

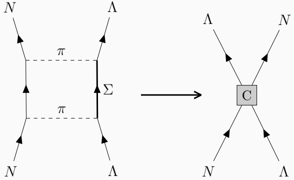
- $B = n, p, \Lambda$ are the only DOF.

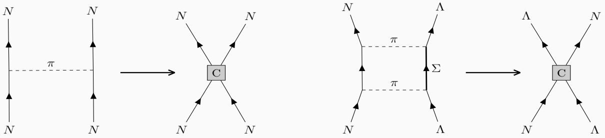
$$\mathcal{L}_{QCD}(q, G) \longrightarrow \mathcal{L}_{\chi EFT}(B, \pi, K) \longrightarrow \mathcal{L}(B)$$



- $\mathcal{L}$ is expanded in powers of Q/M_h.
- Include contact terms and derivatives.
- Not too many parameters

$$\mathcal{L} = N^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m}\right) N + \Lambda^\dagger \left(i\partial_0 + \frac{\nabla^2}{2m}\right) \Lambda + \mathcal{L}_{2B} + \mathcal{L}_{3B} + \ldots$$





Accuracy for light nuclei

Nuclei The pion mass is our breaking scale M_h

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_N M_N}}{m_\pi} \approx 0.5 - 0.8$$

Seems to work better in practice as $\Delta B(^4\text{He}) \approx 10\%$

Hypernuclei No OPE therefore breaking scale is $2m_\pi$

$$\left(\frac{Q}{M_h}\right) = \frac{\sqrt{2B_\Lambda M_\Lambda}}{2m_\pi} \approx 0.3$$

At LO accuracy goes as $(Q/M_h)^2$

Two-body interaction

- Leading order,

$$V_{LO} = C_{00}$$

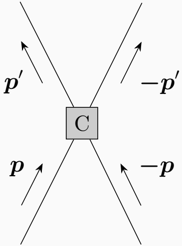
- NLO,

$$V_{NLO} = C_{20}p^2 + C_{02}p'^2$$

- N^{2}LO,

$$V_{N^2LO} = C_{40}p^4 + C_{04}p'^4 + C_{22}p^2p'^2$$

- Symmetry $C_{nm} = C_{mn}$
- Regularization: $V \to F(\boldsymbol{p}'/\lambda)\ V\ F(\boldsymbol{p}/\lambda)$
- Normalization: $C_{nm} \to C_{nm}(\lambda)$
- Equivalent to the effective range expansion (van Kolck, Beane & Savage,...).



1 The Wigner Bound   Phillips, Beane and Cohen (1997-1998)

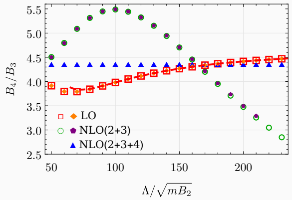
- The effective range is bounded by the cutoff $r_{\text{eff}} \leq W/\lambda$
- All orders but LO are perturbation (Kaplan, van Kolck, ...).

2 The Thomas collapse   Bedaque, Hammer, and van Kolck (1999)

- With LO 2-body interaction $B_3 \propto \hbar\lambda^2/m.$
- A 3-body counter term must be introduced at LO.

3 NLO - no end to surprises   Bazak et al. (2019)

- At NLO the 4-body system is unstable.
- **Conclusion:** the 4-body force is promoted to NLO.



What do we have?

- LO and NLO $\not\pi$EFT fitted to low-energy experimental constraints
- No Coulomb (not a principle problem)
- The Schrödinger equation

$$[T + V_{LO}]\,|\Psi_{LO}\rangle = E_{LO}|\Psi_{LO}\rangle \quad ; \quad \Delta E_{NLO} = \langle\Psi_{LO}|V_{NLO}|\Psi_{LO}\rangle$$

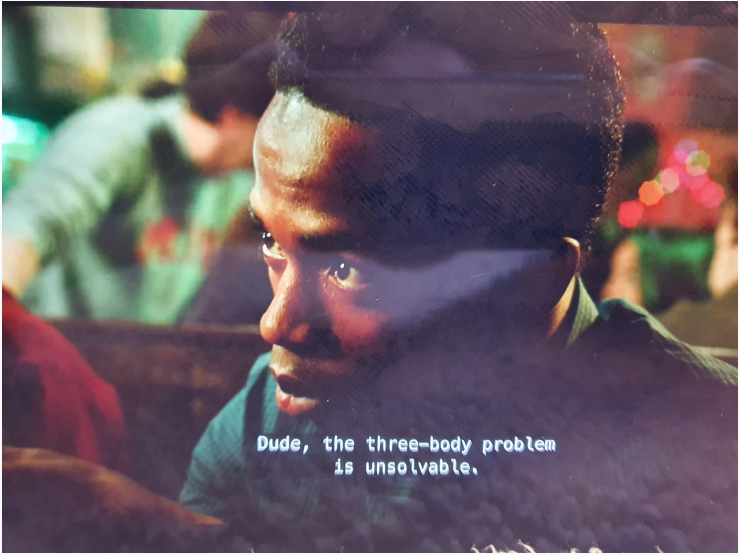
What do we want to know?

- Bound states, resonances, scattering

How do we get there?

- Gaussian basis functions
- Few-body bound states $\Rightarrow$ SVM
- Scattering $\Rightarrow$ Busch formula
- Complex rotation, analytic continuation $\Rightarrow$ Resonances

Some words of wisdom (Salvation 1st episode)



- **Nuclear scattering**

  Elastic s-wave scattering @NLO for $A \leq 5$

- **Λ hypernuclei ($^{A}_{\Lambda}Z$)**

  s-shell hypernuclei - overbinding of $^{5}_{\Lambda}$He

  Hypernuclear resonances

- **$\Lambda\Lambda$ hypernuclei ($^{A}_{\Lambda\Lambda}Z$)**

  Onset of binding, A=4 or 5?

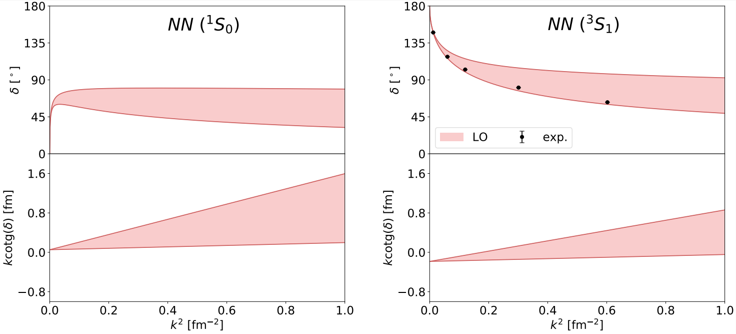
- **Charge symmetry breaking** (Martin's talk)

  The Dalitz von Hippel paramaters from SU(3) symmetry.

- **Nucleons in a box** (Betzalel's talk)

  EFT matching of LQCD calcs.

The nuclear sector



Leading order (LO):

(exp. constraints)

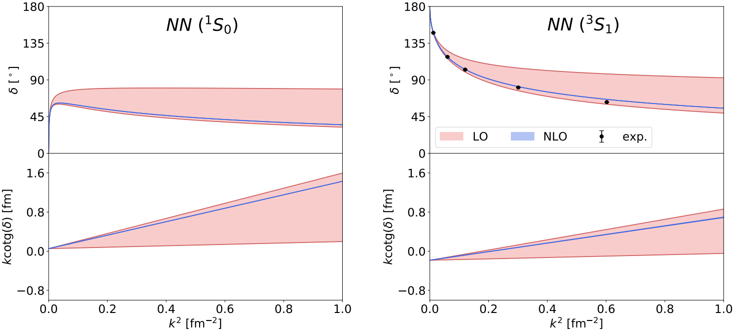
$a_0^{nn} = \quad -18.95(40)$ fm

$a_1^{np} = \qquad 5.419(7)$ fm

$B(^3\mathrm{H}) = 8.482$ MeV

Effective range expansion:

$$k\mathrm{cotg}(\delta) = -\frac{1}{a} + \frac{1}{2} r k^2 + \ldots$$



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(exp. constraints)

$a_0^{nn} = \; -18.95(40)$ fm

$a_1^{np} = \; 5.419(7)$ fm

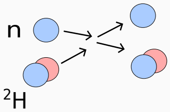
$B(^3\mathrm{H}) = 8.482$ MeV

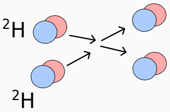
Next-to-leading order (NLO):
(exp. constraints)

$r_0^{nn} = \; 2.75(11)$ fm

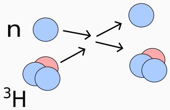
$r_1^{np} = \; 1.753(8)$ fm

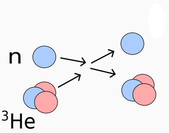
$B(^4\mathrm{He}) = 28.296$ MeV





Few-body s-wave scattering





1. **Near-threshold $^3\mathrm{H}^*$ virtual state**

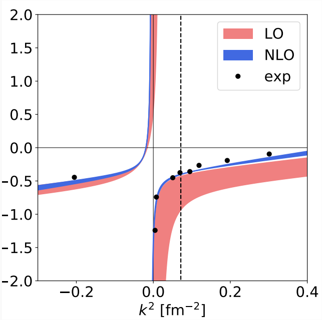
   $\Rightarrow$ pole of S-matrix

2. **Near-threshold zero in S-matrix**

$$\frac{1}{k\mathrm{cotg}(\delta) - ik} = 0$$

$$\lim_{k \to k_0} k\mathrm{cotg}(\delta) = \pm\infty$$

$\Rightarrow$ modified ERE

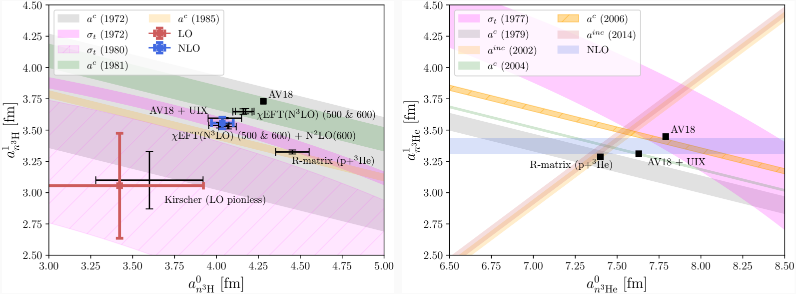


Oers and Seagrave, PLB 24, 11 (1967)

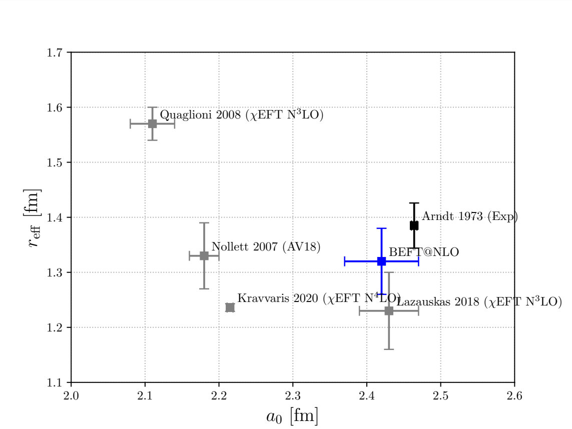
$$a^{1/2}_{n^2\mathrm{H}} = 0.29 \text{ fm}$$
$$r^{1/2}_{n^2\mathrm{H}} = 1.70 \text{ fm}$$

$$k\mathrm{cotg}(\delta) = A + B\ k^2 + \frac{C}{(1 + D\ k^2)} \quad ; \quad a = -\frac{1}{A+C} \quad \text{and} \quad r = 2B$$

Experiment & Theory : $n + {}^3\mathrm{H}$ and $n + {}^3\mathrm{He}$ scattering lengths



Phys. Rev. C 42 (1990) 438; Phys. Rev. C 102 (2020) 034007; Few-Body Syst. 34 (2004) 105; Phys. Lett B 721 (2013) 355; Phys. Rev. C 68(R) (2003) 021002



Exp	a_0 [fm]	r_{eff} [fm]
Arndt 1973	2.4641 ± 0.0037	1.385 ± 0.041
Haun 2020	$2.4746^{\pm 0.0017\ \text{[stat]}}_{\pm 0.0011\ \text{[syst]}}$	-

Light Hypernuclei @LO

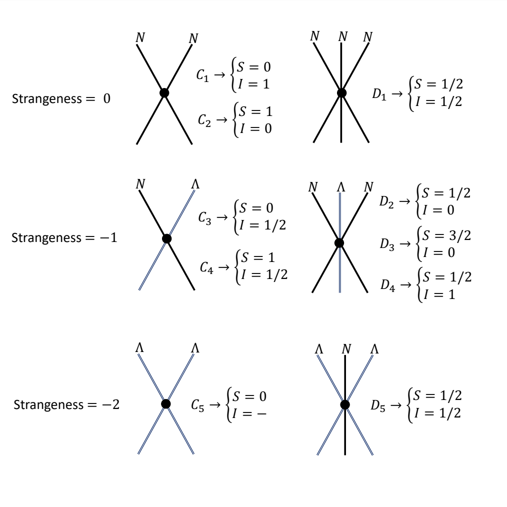
Data shrotage

- Limited scattering data
- No real **low** energy data
- Only 5 known bound states for $A \leq 6$ (+mirror nuclei)
- No resonance data
- ...

Issues

- Contradicting results for life time and binding energy of ${}^{3}_{\Lambda}$H
- Charge symmetry breaking
- What is the onset of double Lambda binding?
- Stability of the Λnn system
- Overbinding of ^{5}He
- ...

2-body & 3-body diagrams in $\not\pi$EFT



Nuclei:

C_1, C_2 - NN scat. lengths

D_1 - Triton B.E. B(^{3}H)

Hypernuclei:

One $\Lambda\Lambda$ term

Two $N\Lambda$ terms

Three $NN\Lambda$ terms

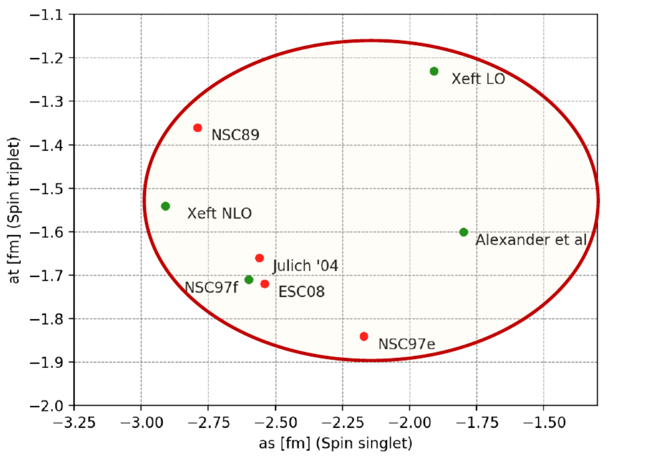
Single $N\Lambda\Lambda$ term

No $\Lambda\Lambda\Lambda$ force at LO

How to fit the Λ & $\Lambda\Lambda$ LECs?

L. Contessi, M. Schafer, N. Barnea, A. Gal, J. Mareš, PLB 797 (2019) 134893

$N\Lambda$ scattering lengths

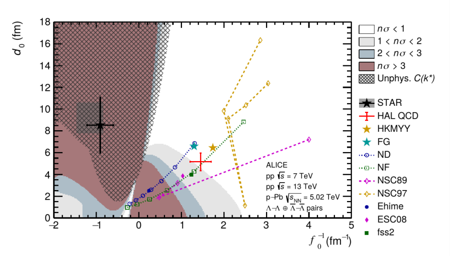


A. Gal et al., Rev. Mod. Phys. 88 (2016) 035004

$\Lambda\Lambda$ scattering length

Heavy Ion Collisions
ALICE/STAR

- Scattering length $a_0 = -f_0$ and effective range d_0
- For $r_{eff} \to 0$: a_0 is bounded $|a_0| < 0.5$
- There is a possible bound $\Lambda\Lambda$ state



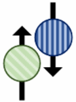
ALICE Collaboration, Phys. Lett. B 797 (2019) 134822

Analysis of ^{12}C$(K^-, K^+\Lambda\Lambda X)$ data

Scattering length $a_{\Lambda\Lambda} = -1.2 \pm 0.6$ fm

Gasparyan-Haidenbauer-Hanhart, PRC 85 (2012) 015204

Two body



$2\times NN$ — S = 0, S = 1 → a_{NN}

$2\times N\Lambda$ — S = 0, S = 1 → $a_{N\Lambda}$

$\Lambda\Lambda \longrightarrow a_{\Lambda\Lambda}$

No precise Experimental data

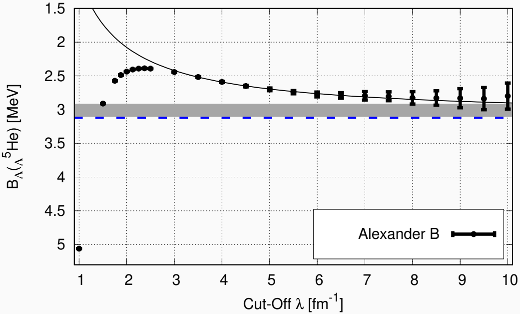
Three body

NNN — ${}^{3}\mathrm{H}$

$3\times NN\Lambda$:
- ${}^{3}_{\Lambda}\mathrm{H}$
- ~~${}^{3}_{\Lambda}\mathrm{H}$~~$(S=2/3)$ → ${}^{4}_{\Lambda}\mathrm{H}(S=0, I=1/2)$
- ~~${}^{3}_{\Lambda}\mathrm{He}$~~ → ${}^{4}_{\Lambda}\mathrm{H}(S=1, I=1/2)$

$N\Lambda\Lambda$ — ~~${}^{3}_{\Lambda\Lambda}\mathrm{H}$~~ → ${}^{6}_{\Lambda\Lambda}\mathrm{He}$

$B_\Lambda(^{5}_{\Lambda}$He) vs. cut-off λ in LO $\not\pi$EFT



L.Contessi N.Barnea A.Gal, PRL 121 (2018) 102502

With Alexander B scattering lengths a_s, a_t $\not\pi$EFT reproduces $B_\Lambda(^{5}_{\Lambda}$He)

Cut-off dependence
$$\frac{B_\Lambda(\lambda)}{B_\Lambda(\infty)} = 1 + \frac{\alpha}{\lambda} + \frac{\beta}{\lambda^2} + \ldots$$

Double-Lambda Hypernuclei

The elusive H dibaryon

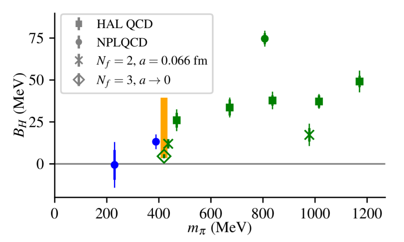
Jaffe predicted **stable** $\mathbf{H}(uuddss)$ PRL 38 (1977) 195

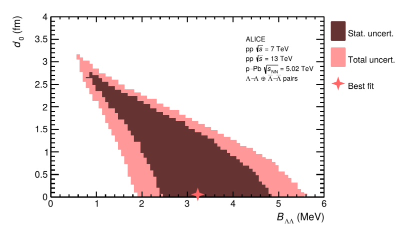
$$\mathbf{H} \sim \mathcal{A}[\sqrt{1/8}\;\Lambda\Lambda + \sqrt{1/2}\;N\Xi - \sqrt{3/8}\;\Sigma\Sigma,]_{I=S=0}$$

- To forbid ${}^{6}_{\Lambda\Lambda}$He→H+^{4}He, impose $B(H) \leq 7$ MeV.
- Weakly bound H in LQCD calculations at unphysical pion mass

  PRL 127 (2021) 24, 242003
- A bound H is not ruled out in $\Lambda\Lambda$ correlation femtoscopy; ALICE -

  PLB 797 (2019) 134822.

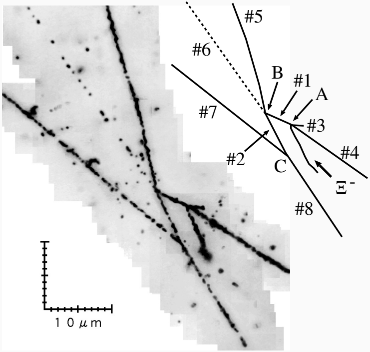




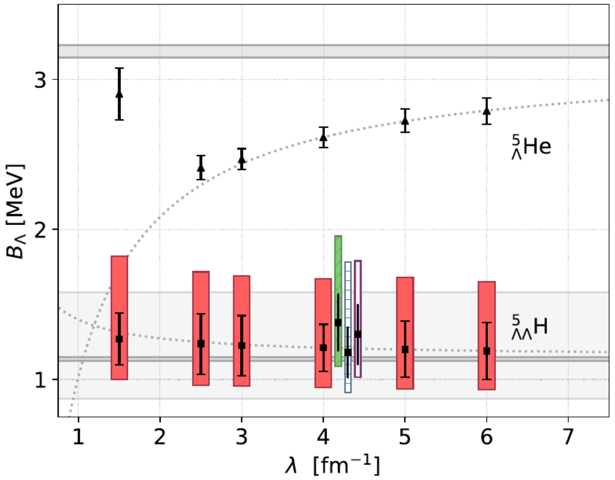
The Nagara event - ${}^{6}_{\Lambda\Lambda}\mathrm{He}$ (KEK-E373)

$B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}_{\mathrm{g.s.}}) = 6.91 \pm 0.16$ MeV

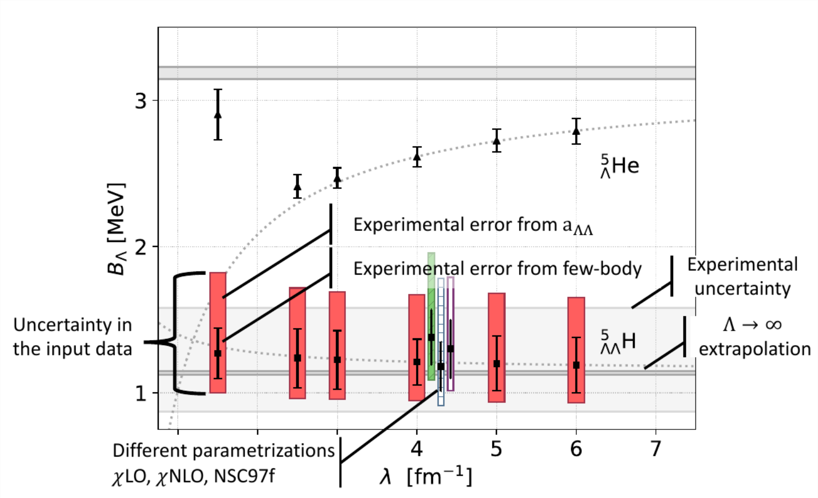
unambiguously determined.



- Ξ^- capture $\Xi^- + {}^{12}\mathrm{C} \rightarrow {}^{6}_{\Lambda\Lambda}\mathrm{He} + t + \alpha$
- Weak decay ${}^{6}_{\Lambda\Lambda}\mathrm{He} \rightarrow {}^{5}_{\Lambda}\mathrm{He} + p + \pi^-$ (no ${}^{6}_{\Lambda\Lambda}\mathrm{He} \rightarrow {}^{4}\mathrm{He} + H$)
- ${}^{5}_{\Lambda}\mathrm{He}$ nonmesic weak decay to 2 Z=1 recoils + n.



Binding ${}_{\Lambda\Lambda}{}^{5}H = 1.14(1)^{+(44)}_{-(25)}$MeV



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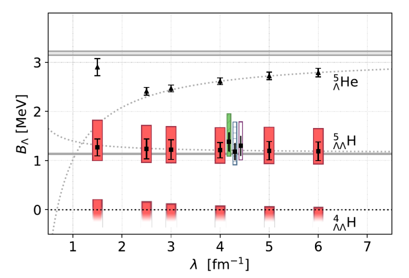
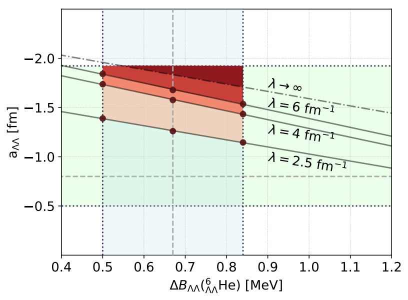


B$_\Lambda$ vs. $\not\pi$EFT cutoff λ



Binding of ${}_{\Lambda\Lambda}{}^{4}$H

Contessi-Schafer-Barnea-Gal-Mareš, PLB 797 (2019) 134893.

The question of ${}_{\Lambda\Lambda}{}^{4}H$ binding depends on the input data

$\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\text{He}) = B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\text{He}) - 2B_{\Lambda}({}_{\Lambda}^{5}\text{He})$

Conclusions: ${}_{\Lambda\Lambda}^{5}$H bound, ${}_{\Lambda\Lambda}^{4}$H unlikely

to bind $|a_{\Lambda\Lambda}| > 1.5$ fm, larger $\Delta B_{\Lambda\Lambda}({}_{\Lambda\Lambda}^{6}\text{He})$

What about the neutral systems?

Constraint (MeV)	${}^{3}_{\Lambda\Lambda}$n	${}^{4}_{\Lambda\Lambda}$n	${}^{4}_{\Lambda\Lambda}$H	${}^{5}_{\Lambda\Lambda}$H	${}^{6}_{\Lambda\Lambda}$He
$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\text{He}) = 0.67$	–	–	–	1.21	3.28
$B_{\Lambda}({}^{4}_{\Lambda\Lambda}\text{H}) = 0.05$	–	–	0.05	2.28	4.76
$B({}^{4}_{\Lambda\Lambda}\text{n}) = 0.10$	–	0.10	0.86	4.89	7.89
$B({}^{3}_{\Lambda\Lambda}\text{n}) = 0.10$	0.10	15.15	18.40	22.13	25.66

Λ separation energies $B_{\Lambda}({}^{A}_{\Lambda\Lambda}\text{Z})$
Calculated using

- $a_{\Lambda\Lambda} = -0.8$ fm
- Alexander[B] ΛN interaction model
- Cutoff $\lambda = 4$ fm^{-1}
- In each row a $\Lambda\Lambda N$ LEC was fitted to the underlined binding energy constraint.

Continuum states

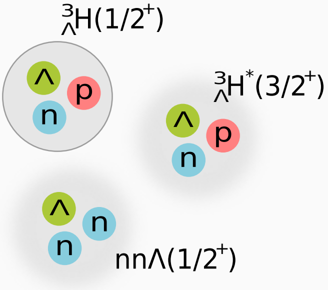
Search for continuum states

${}^{3}_{\Lambda}\mathrm{H}^{*}(3/2^{+})$

- no experimental evidence
- JLab C12-19-002 proposal

$\Lambda\mathrm{nn}(1/2^{+})$

- experiment (HypHI)
- JLab E12-17-003 experiment

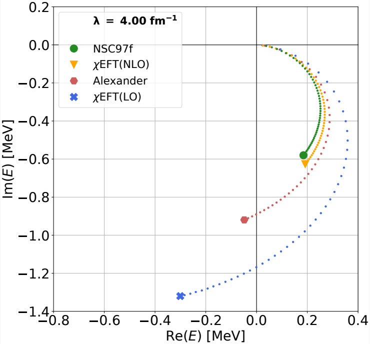


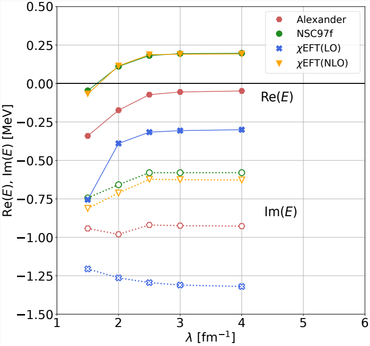
Calculating resonance states is a non-trivial task

We have used two techniques:

- Complex scaling method (CSM)
- Inverse analysis continuation in coupling constraint (IACCC)

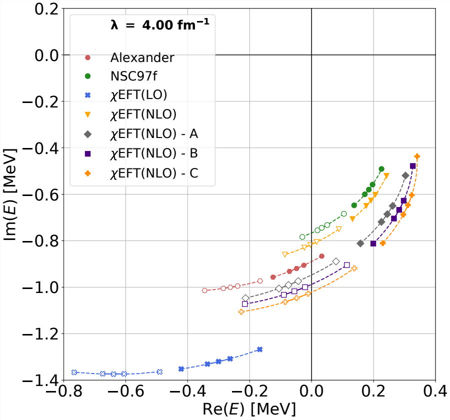
Resonance in Λnn system





- Λnn resonance pole moves with increasing cut-off towards physical Riemann sheet

$\Lambda nn\ (J^\pi = 1/2^+; I = 1)$ resonance



- Full symbols
  $B_\Lambda({}^3_\Lambda\mathrm{H}) = 0.13(5)$ MeV
- Empty symbols
  $B_\Lambda({}^3_\Lambda\mathrm{H}) = 0.41(12)$ MeV

$\Rightarrow$ increasing $B_\Lambda({}^3_\Lambda\mathrm{H})$ shifts Λnn resonance pole towards the third quadrant

$\Rightarrow$ $B_\Lambda({}^3_\Lambda\mathrm{H})$ experimental error yields considerable uncertainty in $E_{\Lambda nn}$ prediction

$\Rightarrow$

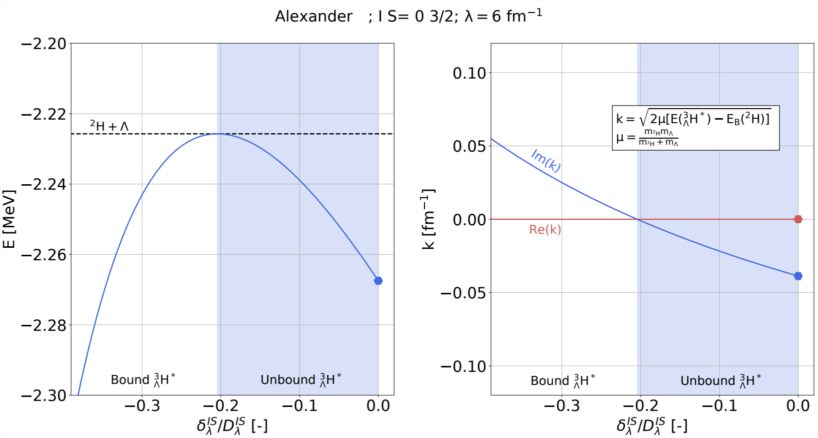
$\Gamma_{\Lambda nn} = -2\mathrm{Im}(E_{\Lambda nn}) \geq 0.8$ MeV

CSM

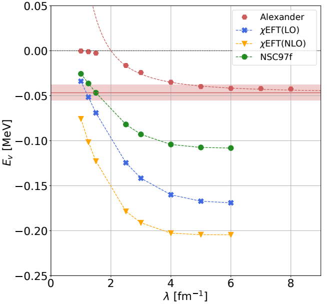
$\Rightarrow$ no sign of resonance

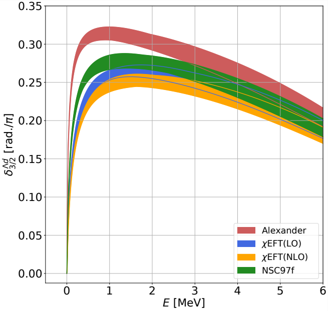
IACCC

$\Rightarrow \delta(\kappa),\ \kappa = -ik = -i\sqrt{E}$



${}^{3}_{\Lambda}\mathrm{H}^*$ as a virtual state





- ${}^{3}_{\Lambda}\mathrm{H}^*$ virtual state - for all considered cut-offs and scattering lengths
- Nice convergence with cut-off
- At LO $\not\pi$EFTthe virtual state is about 0.02- 0.25 MeV near the ${}^{2}\mathrm{H} + \Lambda$ threshold
- We see its trace in the $\Lambda - d$ cross-section

Summary

In a nut shell

$^{3}_{\Lambda}\mathrm{H}$, $^{4}_{\Lambda}\mathrm{H}$
$^{4}_{\Lambda}\mathrm{H}^{*}$, $^{5}_{\Lambda}\mathrm{He}$

All described together
(No overbinding problem!)

$^{5}_{\Lambda\Lambda}\mathrm{He}$

Solidaly bound

$^{3}_{\Lambda}\mathrm{H}^{*}$

Virtual state

$np\Lambda\Lambda$

Marginal case
needs better scattering data

$nn\Lambda\Lambda,\ n\Lambda\Lambda$

Unbound

Λnn

Physical resonance?