Evaluating in-medium Λ isospin impurity from charge symmetry breaking in four-body hypernuclei

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האוניברסיטה העברית בירושלים
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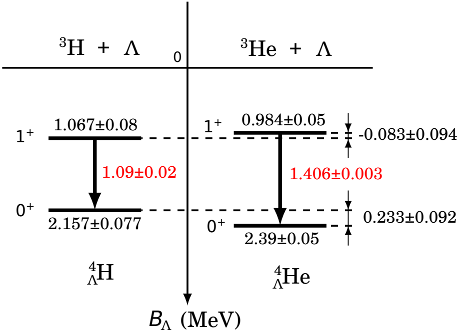
Charge symmetry breaking in ${}^4_\Lambda\mathrm{H}/{}^4_\Lambda\mathrm{He}$

- $B_\Lambda({}^4_\Lambda\mathrm{H};\, 0^+)$ measurement at MAMI
  (Nucl. Phys. A, 954 (2016) 149)
- $B_\Lambda({}^4_\Lambda\mathrm{He};\, 0^+)$ measurement (emulsion)
  (Nucl. Phys. A 754 (2005) 3c)
- $E_\gamma({}^4_\Lambda\mathrm{H};\, 1^+ \to 0^+)$, $E_\gamma({}^4_\Lambda\mathrm{He};\, 1^+ \to 0^+)$
  γ-ray energies (J-PARC)
  (Phys. Rev. Lett., 115 (2015) 222501)



Sizable CSB splitting in 0^+ ground states, while small in 1^+ excited states.

Theoretical works

- **R. H. Dalitz and F. von Hippel** (Phys. Lett. 10 (1964) 153)
  $\rightarrow$ CSB OPE contribution by allowing $\Lambda - \Sigma^0$ mixing in $SU(3)_f$

$$g_{\Lambda\Lambda\pi} = 2\mathcal{A}^{(0)}_{I=1} g_{\Lambda\Sigma\pi}; \qquad \mathcal{A}^{(0)}_{I=1} = -\frac{\langle \Sigma^0 | \, \delta M \, | \Lambda \rangle}{M_{\Sigma^0} - M_\Lambda} = -0.0148(6)$$

- **A. Gal** (Phys. Let. B 744 (2015) 352)
  $\rightarrow$ generalization of DvH

$$\langle N\Lambda | \, V^{CSB}_{\Lambda N} \, | N\Lambda \rangle = -\frac{2}{\sqrt{3}} \mathcal{A}^{(0)}_{I=1} \tau_{N_z} \langle N\Lambda | \, V \, | N\Sigma \rangle$$

$\rightarrow \Delta B_\Lambda(0^+_{\mathrm{g.s.}}) \approx 240$ keV $\qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) \approx 35$ keV

- **D. Gazda and A. Gal**
  (Phys. Rev. Lett. 116 (2016) 122501; Nucl. Phys. A 954 (2016) 161)
  $\rightarrow$ generalized DvH; LO χEFT *YN* interaction; NSCM

  $\rightarrow \Delta B_\Lambda(0^+_{\mathrm{g.s.}}) \approx 180 \pm 130$ keV $\qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) \approx -200 \pm 30$ keV

Theoretical works - J. Haidenbauer et al., Few-Body Syst. 62 (2021) 105

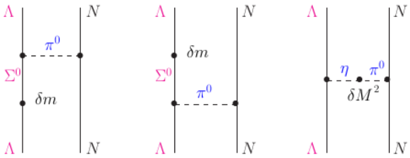


Fig. 1 CSB contributions involving pion exchange, according to Dalitz and von Hippel [1], due to $\Lambda - \Sigma^0$ mixing (left two diagrams) and $\pi^0 - \eta$ mixing (right diagram).

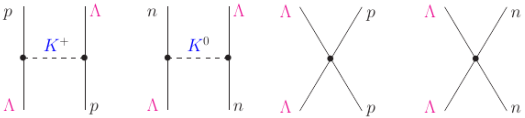


Fig. 2 CSB contributions from $K^{\pm}/K^0$ exchange (left) and from contact terms (right).

Λ	NLO13		NLO19	
	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]	C_s^{CSB}[MeV^{-2}]	C_t^{CSB}[MeV^{-2}]
500	4.691×10^{-3}	-9.294×10^{-4}	5.590×10^{-3}	-9.505×10^{-4}
550	6.724×10^{-3}	-8.625×10^{-4}	6.863×10^{-3}	-1.260×10^{-3}
600	9.960×10^{-3}	-9.870×10^{-4}	9.217×10^{-3}	-1.305×10^{-3}
650	1.500×10^{-2}	-1.142×10^{-3}	1.240×10^{-2}	-1.395×10^{-3}

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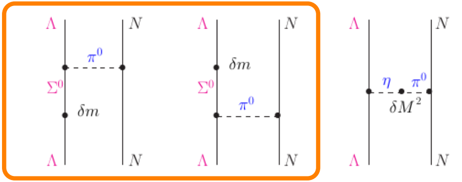

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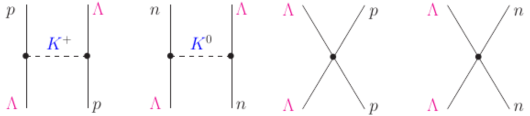

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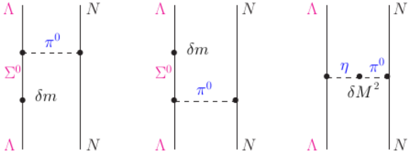

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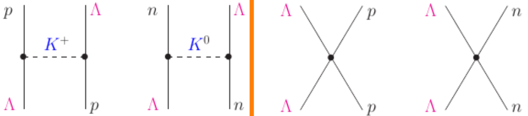

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Hypernuclear CSB within $\not\pi$EFT

Charge Symmetric (CS) LO $\not\pi$EFT

(introduced extensively in Nir Barnea's talk)

Nuclear :

$$V_{NN} = \sum_S C^S_{NN}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}r_{12}^2}$$

$$V_{NNN} = D_\lambda^{1/2\ 1/2}\mathcal{Q}^{1/2\ 1/2}\sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(r_{12}^2+r_{23}^2)}$$

Hypernuclear :

$$V_{\Lambda N} = \sum_S C^S_{\Lambda N}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}r_{12}^2}$$

$$V_{\Lambda NN} = \sum_{IS} D^{IS}_{\Lambda NN}(\lambda)\mathcal{Q}^{IS}\sum_{\mathrm{cyc}} e^{-\frac{\lambda^2}{4}(r_{12}^2+r_{23}^2)}$$

$\rightarrow$ fitted to explicit CS input : $B(^2\mathrm{H})$, $a_0^{nn/pp} = -18.13$ fm, several sets of $(a_0^{\Lambda N};\ a_1^{\Lambda N})$, $B_\Lambda(^3_\Lambda\mathrm{H})$, and CS averages $B(^3\mathrm{H}/^3\mathrm{He})$ and $B(^4_\Lambda\mathrm{H}/^4_\Lambda\mathrm{He})$

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CSB in ΛN interaction

$$C^S_{\Lambda N}\mathcal{P}^S \rightarrow \left(C^S_{\Lambda p}\frac{1+\tau_{Nz}}{2} + C^S_{\Lambda n}\frac{1-\tau_{Nz}}{2}\right)\mathcal{P}^S$$

$$C^S_{\Lambda N} = \frac{1}{2}(C^S_{\Lambda p} + C^S_{\Lambda n}), \qquad \delta C^S_{\Lambda N} = \frac{1}{2}(C^S_{\Lambda p} - C^S_{\Lambda n})$$

$$V_{\Lambda N} = \overbrace{\sum_S C^S_{\Lambda N}(\lambda)\mathcal{P}^S e^{-\frac{\lambda^2}{4}r_{12}^2}}^{\text{part of LO CS } \not\pi\text{EFT}} + \overbrace{\sum_S \delta C^S_{\Lambda N}(\lambda)\mathcal{P}^S \tau_{N_z} e^{-\frac{\lambda^2}{4}r_{12}^2}}^{\text{perturbative CSB}}$$

Fitting CSB LECs

→ perturbatively
→ two experimental constraints

$$\Delta B_\Lambda(0^+_{\mathrm{g.s.}}) = 233 \pm 92 \text{ keV} \qquad \Delta B_\Lambda(1^+_{\mathrm{exc.}}) = -83 \pm 94 \text{ keV}$$

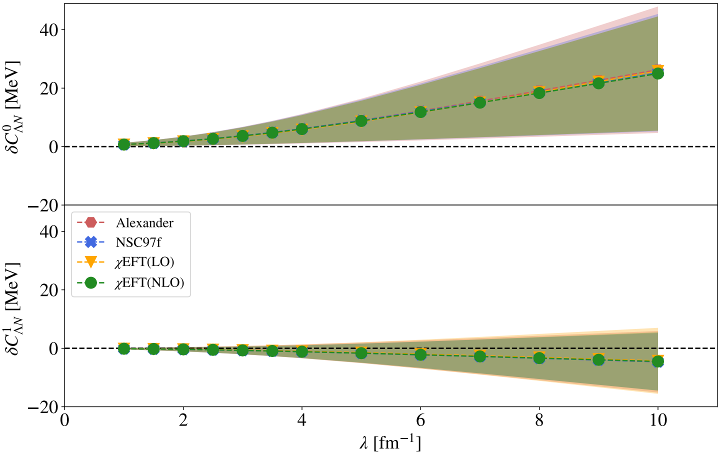
System of two linear equation for $\delta C^0_{\Lambda N}$ and $\delta C^1_{\Lambda N}$:

$$2\,\delta C^0_{\Lambda N}\,\Delta V^0_{\Lambda N;\,0^+} + 2\,\delta C^1_{\Lambda N}\,\Delta V^1_{\Lambda N;\,0^+} = \Delta B_\Lambda(0^+_{\mathrm{g.s.}})$$
$$2\,\delta C^0_{\Lambda N}\,\Delta V^0_{\Lambda N;\,1^+} + 2\,\delta C^1_{\Lambda N}\,\Delta V^1_{\Lambda N;\,1^+} = \Delta B_\Lambda(1^+_{\mathrm{exc.}})$$

where

$$\Delta V^S_{\Lambda N;\,J^\pi} = \underbrace{\langle {}^4_\Lambda\mathrm{H};J^\pi|\tau_{Nz}\mathcal{P}_S\delta_\lambda(r_{\Lambda N})|{}^4_\Lambda\mathrm{H};J^\pi\rangle}_{\text{CS LO } \not\pi\text{EFT wave function}}$$

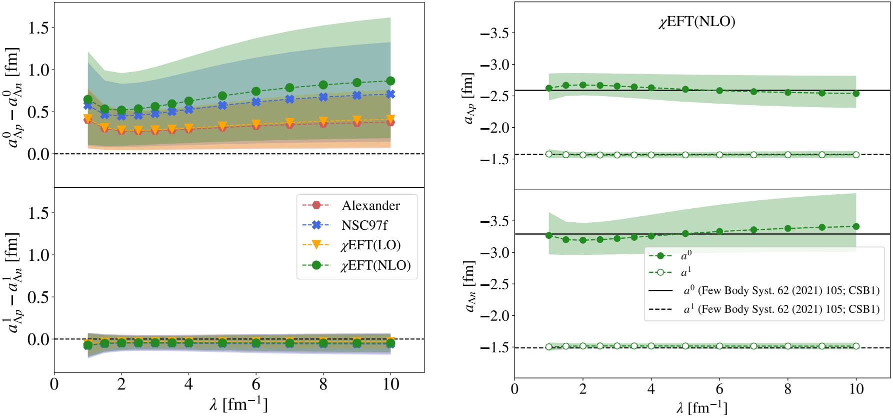
Fitting CSB LECs



$|\delta C^1_{\Lambda N}| < |\delta C^0_{\Lambda N}|$; predominantly opposite sign

Λp and Λn scattering lengths

$\rightarrow$ CSB propagated into ΛN scattering length (perturbatively; DWBA)



$S = 0$: Stronger Λn and weaker Λp interaction
$S = 1$: Hardly affected; mostly stronger Λp and weaker Λn interaction

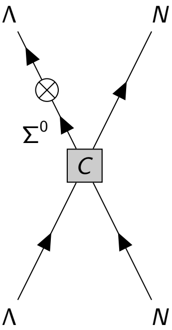
In-medium Λ isospin impurity

DvH ansatz :
(A. Gal, Phys. Lett. B 744 (2015) 352)

$$\langle \Lambda N | V_{\mathrm{CSB}} | \Lambda N \rangle = -\frac{2}{\sqrt{3}}\, \mathcal{A}^{(0)}_{I=1}\, \langle \Sigma N | V_{\mathrm{CS}} | \Lambda N \rangle\, \tau_{Nz}$$

$$\downarrow$$

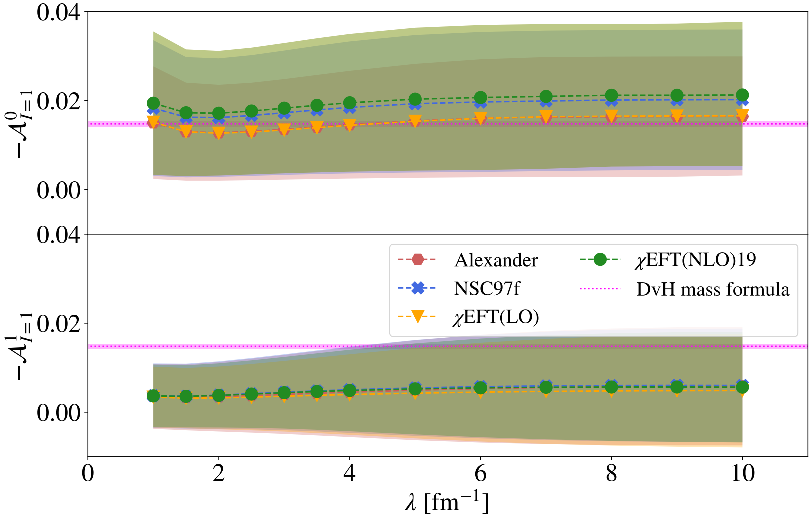
$$\delta C^{S}_{\Lambda N} = -\frac{2}{\sqrt{3}}\, \mathcal{A}^{S}_{I=1}\, C^{S}_{\Lambda N;\Sigma N}$$



SU(3)$_f$ symmetry:
(C.B. Dover, H. Feshbach, Ann. Phys. (NY) 198 (1990) 321)

$$\left.\begin{array}{ll} C^{0}_{\Lambda N,\Sigma N} = & -3(C^{0}_{NN} - C^{0}_{\Lambda N}) \\ C^{1}_{\Lambda N,\Sigma N} = & (C^{1}_{NN} - C^{1}_{\Lambda N}) \end{array}\right\} \longrightarrow \begin{array}{l} -\mathcal{A}^{0}_{I=1} = (\sqrt{3}/2)\delta C^{0}_{\Lambda N}/[-3(C^{0}_{NN} - C^{0}_{\Lambda N})] \\ -\mathcal{A}^{1}_{I=1} = (\sqrt{3}/2)\delta C^{1}_{\Lambda N}/[(C^{1}_{NN} - C^{1}_{\Lambda N})] \end{array}$$

In-medium Λ isospin impurity



In-medium Λ isospin impurity

→ considering more precise $\Delta E_\gamma = 316 \pm 20$ keV

Relation between CSB LECs and ΔE_γ :

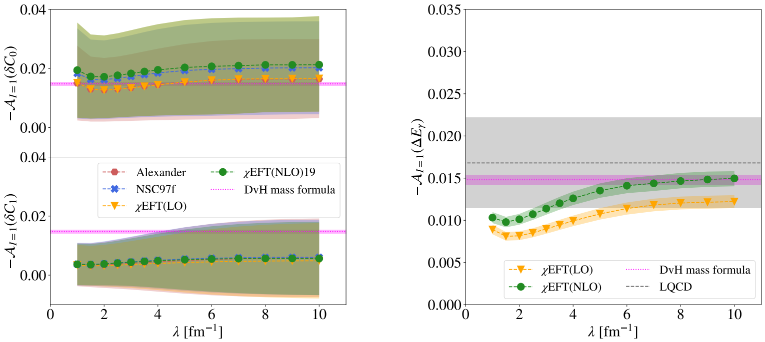
$$2\ \delta C^0_{\Lambda N}\left[\Delta V^0_{\Lambda N;\ 0^+} - \Delta V^0_{\Lambda N;\ 1^+}\right] + 2\ \delta C^1_{\Lambda N}\left[\Delta V^1_{\Lambda N;\ 0^+} - \Delta V^1_{\Lambda N;\ 1^+}\right] = \Delta E_\gamma$$

→ assuming DvH ansatz, $SU(3)_f$ symmetry, and $\mathcal{A}^0_{I=1} = \mathcal{A}^1_{I=1}$

Relation between $I = 1$ admixture amplitude and ΔE_γ :

$$-\mathcal{A}_{I=1} = \frac{\sqrt{3}}{2}\Delta E_\gamma \Big(-6(C^0_{NN} - C^0_{\Lambda N})[\Delta V^0_{\Lambda N;\ 0^+} - \Delta V^0_{\Lambda N;\ 1^+}] + 2\ (C^1_{NN} - C^1_{\Lambda N})[\Delta V^1_{\Lambda N;\ 0^+} - \Delta V^1_{\Lambda N;\ 1^+}]\Big)^{-1}$$

In-medium Λ isospin impurity



Method/Input	B	$-\mathcal{A}_{I=1}$
$\mathrm{SU}(3)_\mathrm{f}$ (Phys. Lett 10 (1964) 153)	1	0.0148 ± 0.0006
LQCD (Phys. Rev. D 101 (2020) 034517)	1	0.0168 ± 0.0054
$\not\pi$EFT (LO)/[χEFT(LO); $\Lambda \to \infty$]	4	0.0139 ± 0.0013
$\not\pi$EFT (LO)/[χEFT(NLO); $\Lambda \to \infty$]	4	0.0168 ± 0.0014

Conclusions

- perturbative inclusion of CSB in LO $\not{\pi}$EFT (fitted to CSB in ${}^4_\Lambda\mathrm{H}/{}^4_\Lambda\mathrm{He}$)
- Spin-singlet : Stronger Λn and weaker Λp interaction
  Spin-triplet : Hardly affected; mostly stronger Λp and weaker Λn
- assumtion of DvH ansatz and SU(3)$_f$ symmetry
- extraction of in-medium Λ isospin impurity $\mathcal{A}_{I=1}$; all cases in agreement with free-space LQCD prediction and in most cases with free-space DvH value
- using $\mathcal{A}^{(0)}_{I=1}$ DvH value the procedure can be applied in reverse thus predicting experimental CSB in ${}^4_\Lambda\mathrm{H}/{}^4_\Lambda\mathrm{He}$

Nuclear CSB within $\not\pi$EFT