Λ(1405) mediated triangle singularity in the $K^- d \rightarrow p \Sigma^-$ reaction

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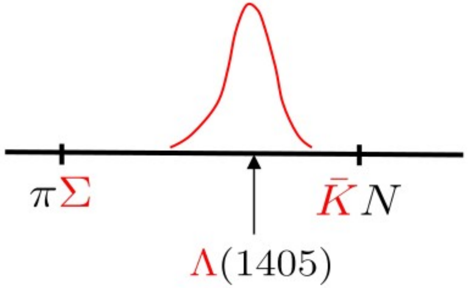
Kbar N update
Triangle singularities
$K^- d \rightarrow p \Sigma^-$ reaction

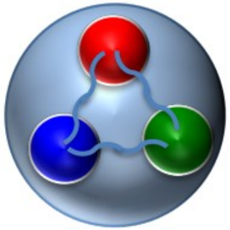
The $\Lambda(1405)$

- The $\bar{K}N$ interaction in the isospin I=0 channel is able develop a **quasi-bound state**, the $\Lambda(1405)$, located only 27 MeV below the $\bar{K}N$ threshold



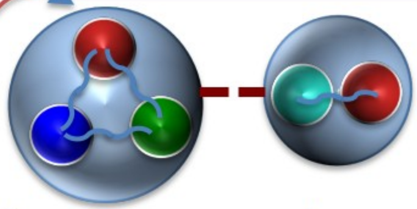
- It may be considered the first ”pentaquark” ever observed

in **conventional quark models:**
baryons are qqq states



exotic baryons:
pentaquarks (5q states)

compact



baryon-meson bound state
(or *molecule*)

Late fifties/sixties (first stellar period)

- Intense activity in bubble-chamber experiments (BNL, CERN, Rutherford) established the presence of a resonance with strangeness -1 around 1405 MeV. (The $\Lambda(1405)$ becomes a PDG baryon in 1963)

- The idea of the $\Lambda(1405)$ being a meson-baryon molecule was originally proposed by Dalitz and Tuan in the late 1950's (a quasibound state was found from solving a coupled-channel Schrödinger equation involving $\bar{K}N$ and $\pi\Sigma$)

R. H. Dalitz and S. F. Tuan, Annals of Phys. 10 (1960) 307

seventies/eighties

- Continuous experimental activity in bubble-chambers and in emulsions (cross section measurements, threshold branching ratios, …)

- The $\Lambda(1405)$ cannot be accommodated in quark models, which systematically predicted for it a too high mass.

1990 – 2005 (around the turn of the century)

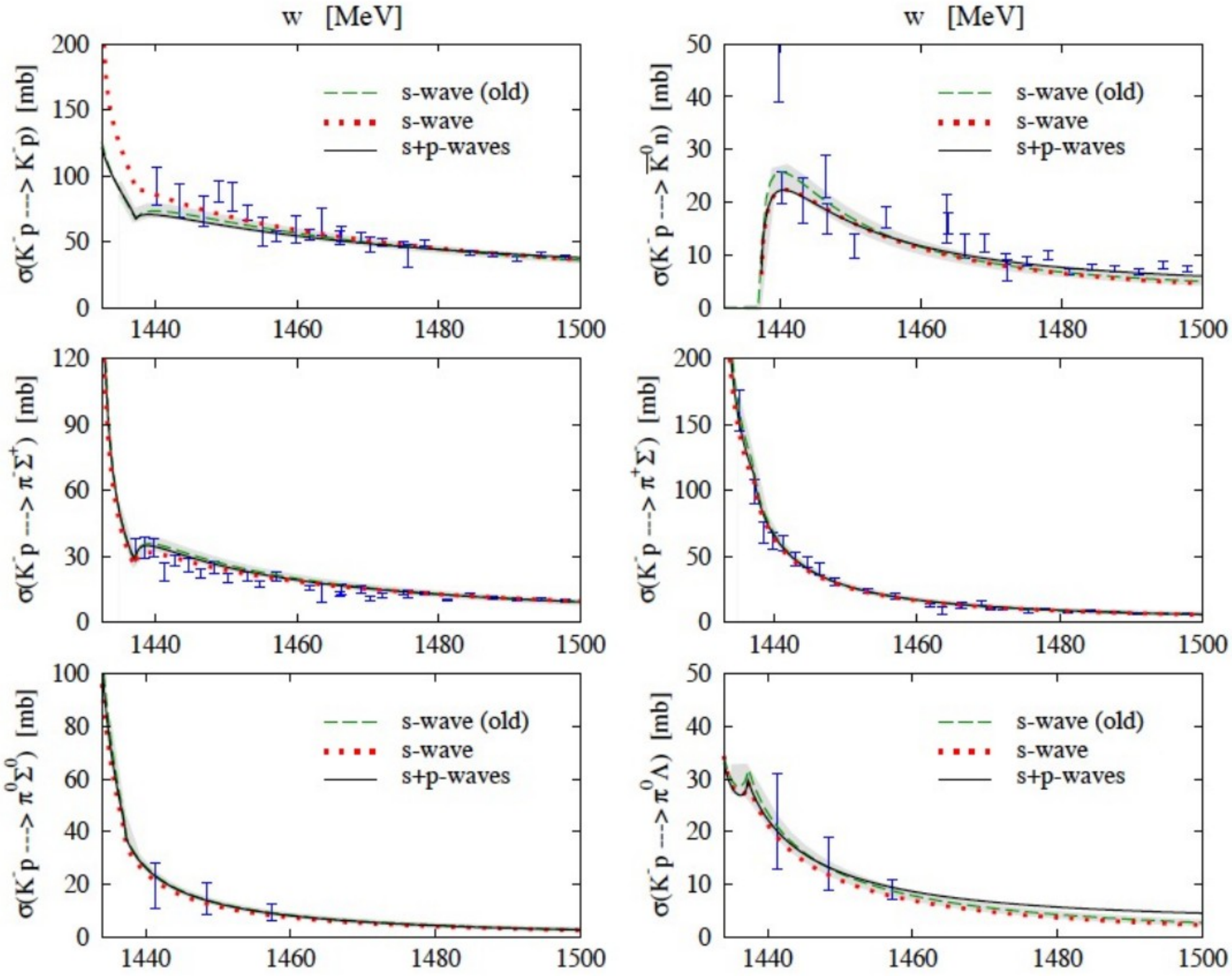
- Conflicting measurements of the kaonic hydrogen shift and width of the 1s state: KEK-PS E228 (1998) and DEAR (2005)

- The Dalitz/Tuan idea of a quasi-bound meson-baryon interpretation of the $\Lambda(1405)$ is reformulated in terms of an effective chiral unitary theory in coupled channels (in s-wave) N. Kaiser, P. B. Siegel, and W. Weise, Nucl. Phys. A594 (1995) 325
E. Oset and A. Ramos, Nucl. Phys. A635 (1998) 99

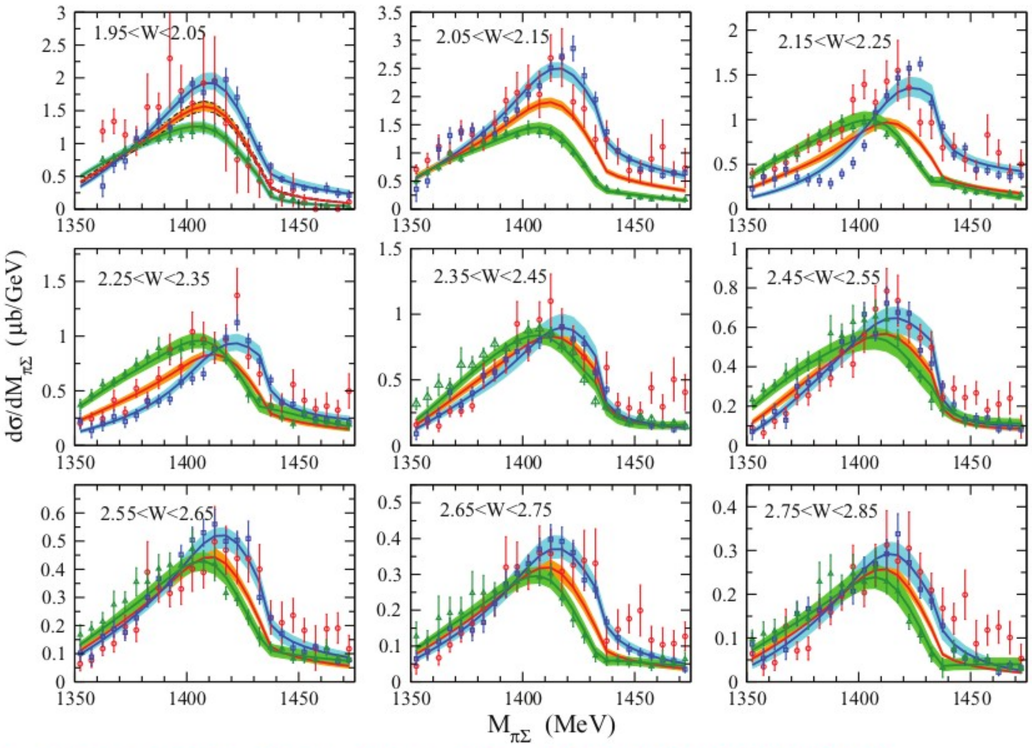
- For the next ten years, intense theoretical work (NLO Lagrangian, s-channel and u-channel Born terms…) finding similar features:
  - ✓ $\bar{K}N$ scattering data reproduced very satisfactorily
  - ✓ Two-pole structure of $\Lambda(1405)$

J. A. Oller, U. -G. Meissner, Phys. Lett. B 500, 263 (2001).
M. F. M. Lutz, E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002).
B. Borasoy, E. Marco, S. Wetzel, Phys. Rev. C 66, 055208 (2002).
C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003).
D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A 725, 181 (2003).
B. Borasoy, R. Nissler, W. Wiese, Eur. Phys. J. A 25, 79 (2005).
V.K. Magas, E. Oset, A. Ramos, Phys. Rev. Lett. 95, 052301 (2005).
B. Borasoy, U. -G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006)

Results: cross sections (classical processes)



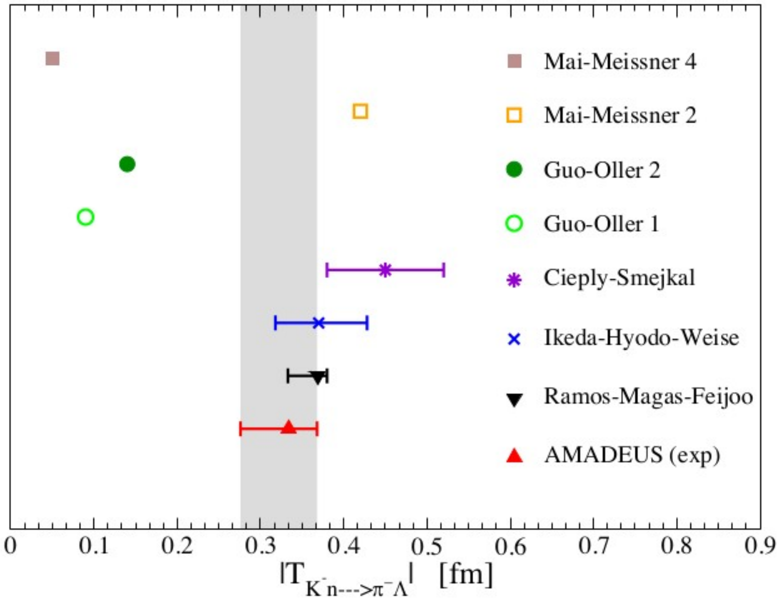
Many efforts have been made in order to extract information about subthreshold amplitudes…



L. Roca and E. Oset, Phys. Rev. C 88, 055206 (2013).

Fit to photoproduction data from CLAS

K. Moriya et al. (CLAS Collaboration), Phys. Rev. C 87, 035206 (2013).



$K^- n \longrightarrow \pi^- \Lambda$ amplitude (pure $I = 1$ process)

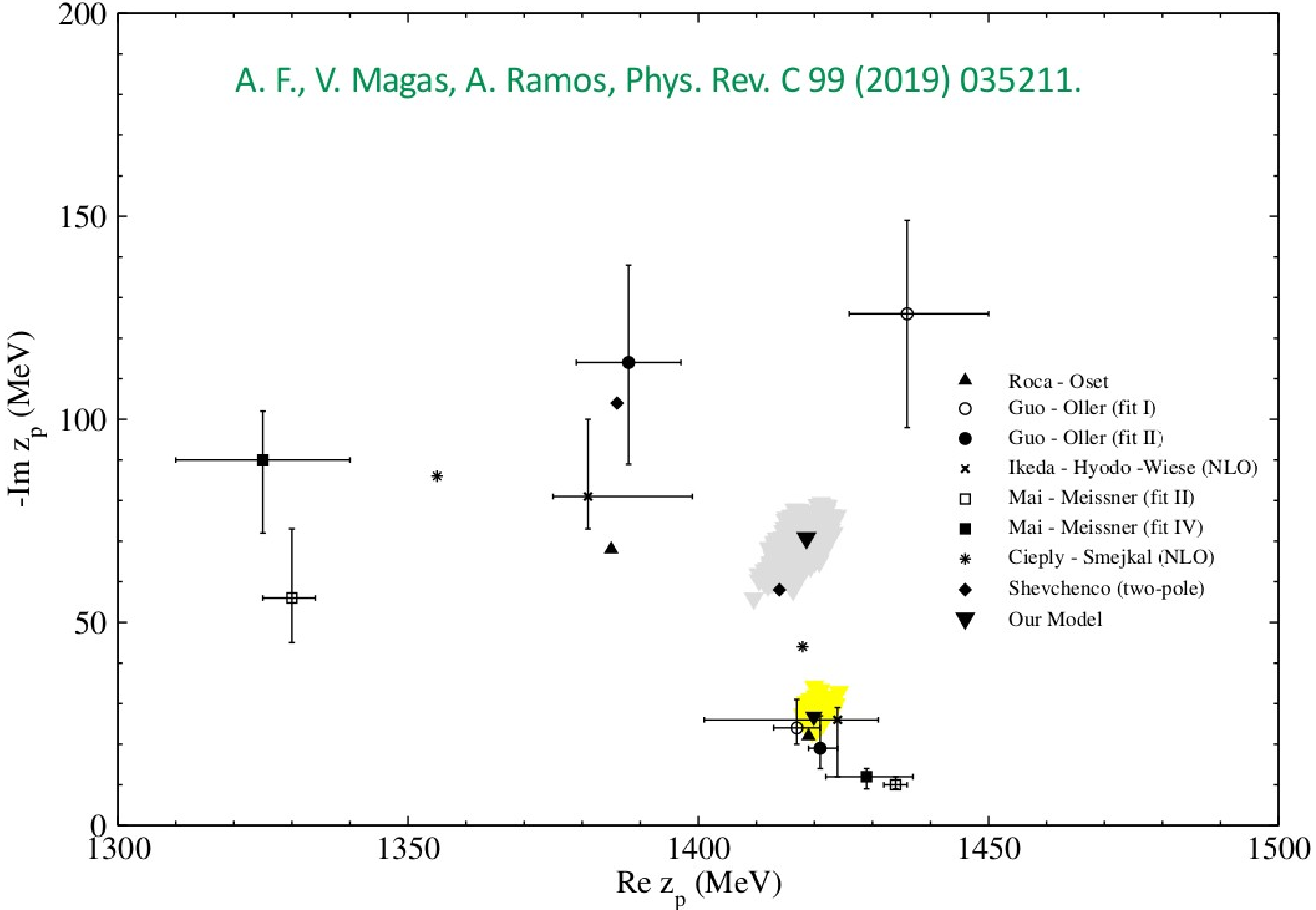
K. Piscicchia et al.., Phys.Lett. B782 (2018) 339-345.

AMADEUS collaboration, KLOE detector at DAFNE

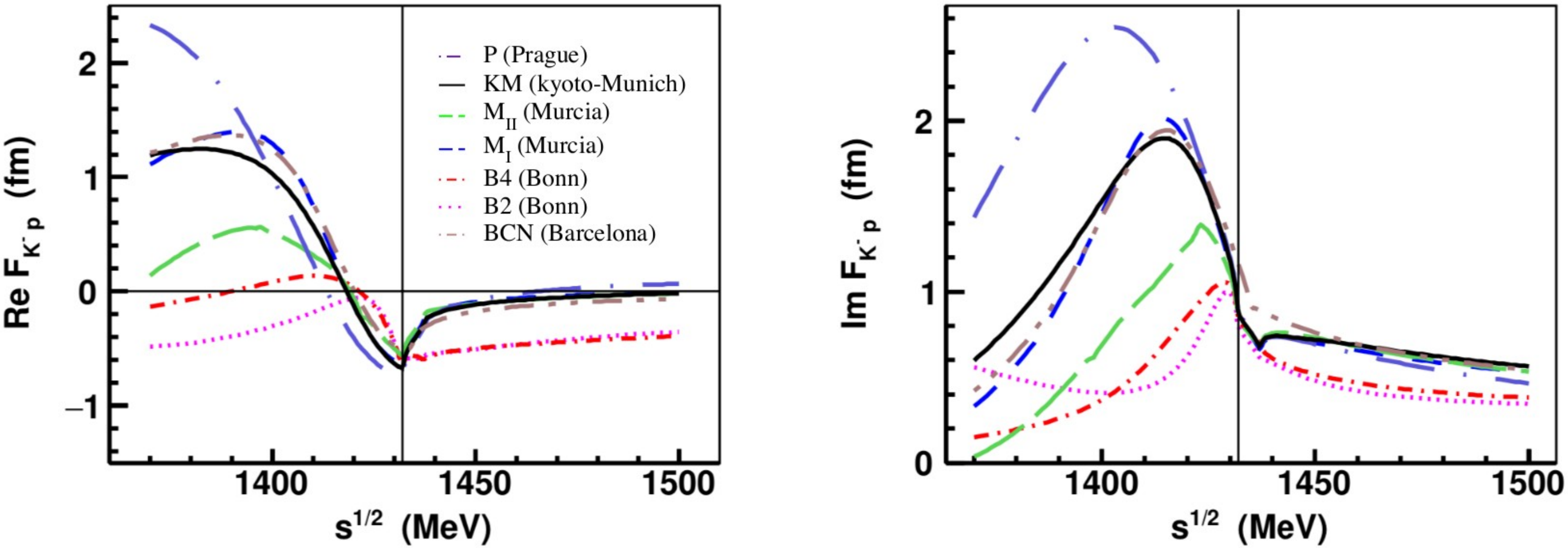
Coming work from SIDHARTA-II

M. Mai and U.G. Meissner Eur.Phys.J.A 51 (2015) 3, 30

Pole positions of the $\Lambda(1405)$ for some state-of-the-art models:



$K^- p \longrightarrow K^- p$ scattering amplitudes generated by recent chirally motivated approaches:



A. Cieply, J. Hrtánková, J. Mareš, E. Friedman, A. Gal and A. Ramos, AIP Conf. Proc. 2249, no.1, 030014 (2020).

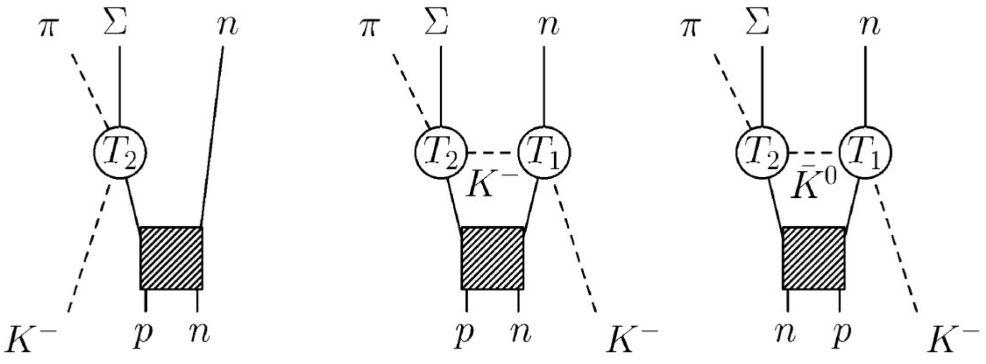
How to learn about the Kbar N amplitude below threshold and the Λ(1405)?

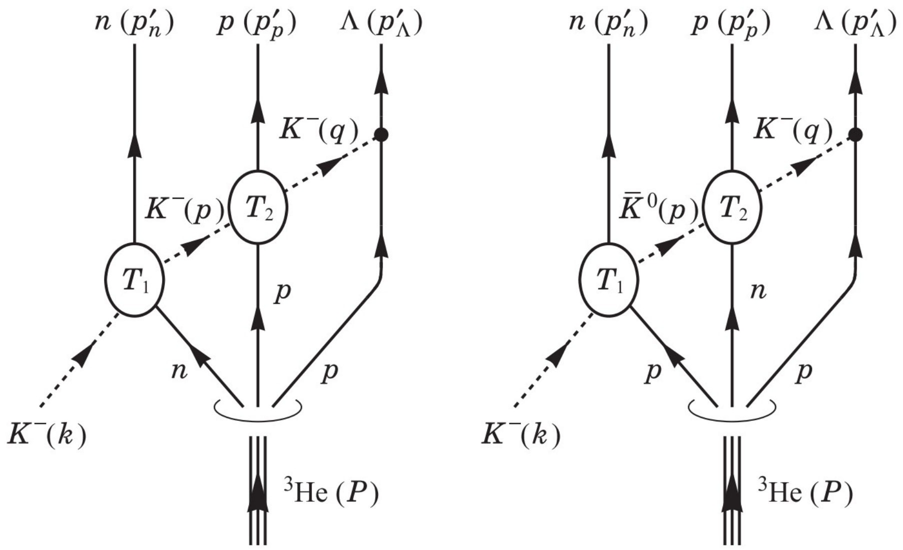
The photonuclear data provides information.

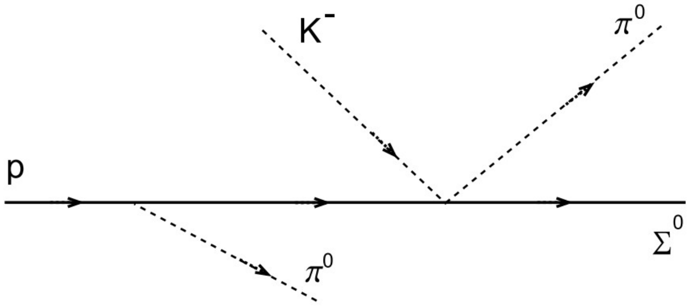
The K^- d → π n Σ reaction

K- 3He → Λ p n

K^- p → $\pi^0\pi^0\Sigma^0$

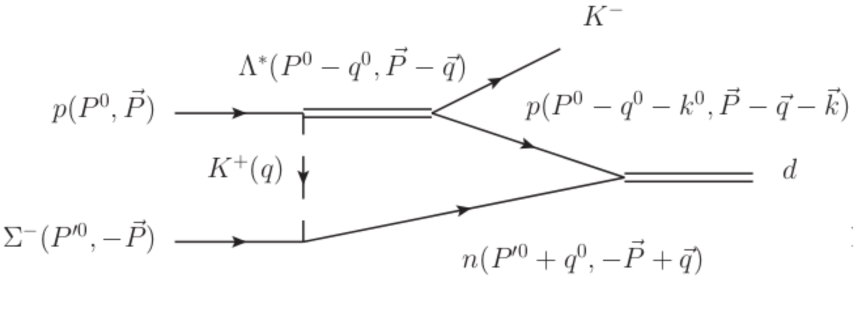




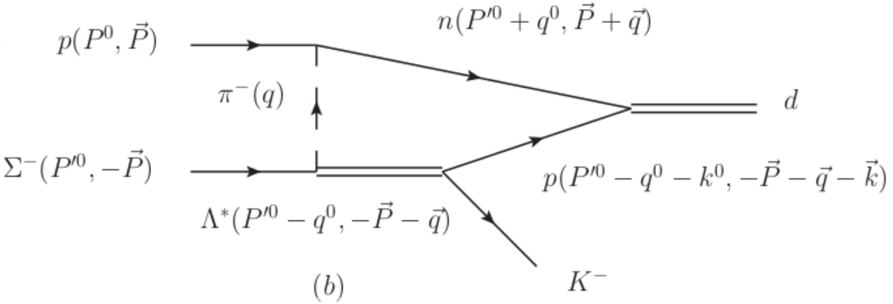


A novel reaction using real kaons providing information on the Kbar N interaction below threshold

$p\Sigma^- \longrightarrow K^- d$ reaction proceeds via these 2 mechanisms:



(a)

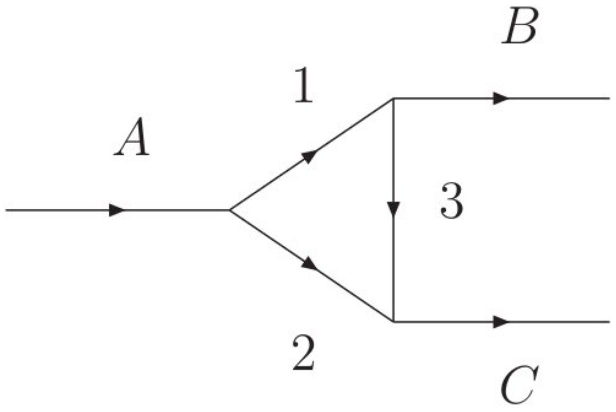


(b)

$$-it^{(a)} = (-i) g_{\Lambda^*,K^-p} (-i) g_{\Lambda^*,K^-p} (-i) g_d \frac{D-F}{2f} \int \frac{d^4q}{(2\pi)^4} \vec\sigma_2 \vec q \frac{i}{q^2-m_K^2+i\epsilon} \frac{M_{\Lambda^*}}{E^*_\Lambda} \frac{i}{P^0-q^0-E^*_\Lambda(\vec P-\vec q)+i\frac{\Gamma_{\Lambda^*}}{2}}$$
$$\times \frac{M_N}{E_N}\frac{i}{P^0-q^0-k^0-E_N(\vec P-\vec q-\vec k)+i\epsilon}\frac{M_N}{E'_N}\frac{i}{P'^0+q^0-E'_N(-\vec P+\vec q)+i\epsilon}\theta(q_{\rm max}-|\vec P-\vec q-\frac{\vec k}{2}|),$$

$$-it^{(b)} = -g_{\Lambda^*,K^-p}\, g_{\Lambda^*,\pi^+\Sigma^-}\, g_d \frac{f_{\pi NN}}{m_\pi} i\int\frac{d^4q}{(2\pi)^4}\frac{1}{q^2-m_\pi^2+i\epsilon}\frac{M_{\Lambda^*}}{E_{\Lambda^*}}\frac{1}{P'^0-q^0-E_{\Lambda^*}(-\vec P-\vec q)+i\frac{\Gamma_{\Lambda^*}}{2}}$$
$$\times \vec\sigma_1\cdot\vec q\, \frac{M_N}{E_N}\frac{1}{P'^0-q^0-k^0-E_N(-\vec P-\vec q-\vec k)+i\epsilon}\frac{M_N}{E'_N}\frac{1}{P^0+q^0-E'_N(\vec P+\vec q)+i\epsilon}\theta(q_{\rm max}-|-\vec P-\vec q-\frac{\vec k}{2}|),$$

TS can be developed when the 3 intermediate particles $\Lambda(1405)$ (1), n (2), p (3):



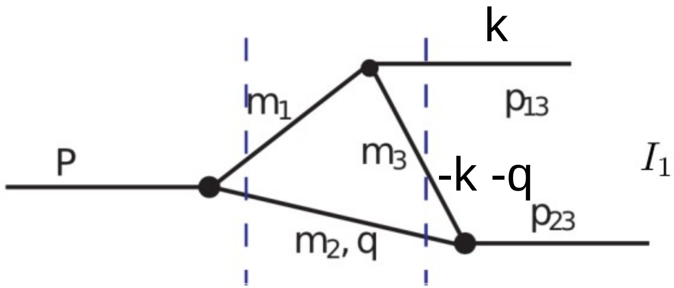
+ ***1, 2, 3 particles are simultaneously placed on Shell and they are colinear fulfilling Norton-Coleman theorem***

S. Coleman and R. E. Norton, Nuovo Cim. 38, 438 (1965).

This conditions are encoded in the following equation:

*Momentum of the **n** in the $p\Sigma^-$ rest frame* → $$q_{\rm on} = q_{a^-}$$ ← *Solution for the **n** momentum in the decay of the **d** for the moving **d** in the $p\Sigma^-$ rest frame*

M. Bayar, F. Aceti, F.-K. Guo, and E. Oset, Phys. Rev. D 94, 074039 (2016)



Explicit iintegral of the intermediate loop containing the 3 propagators:

$$I_1 = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{(q^2-m_2^2+i\epsilon)[(P-q)^2-m_1^2+i\epsilon][(P-q-p_{13})^2-m_3^2+i\epsilon]}$$

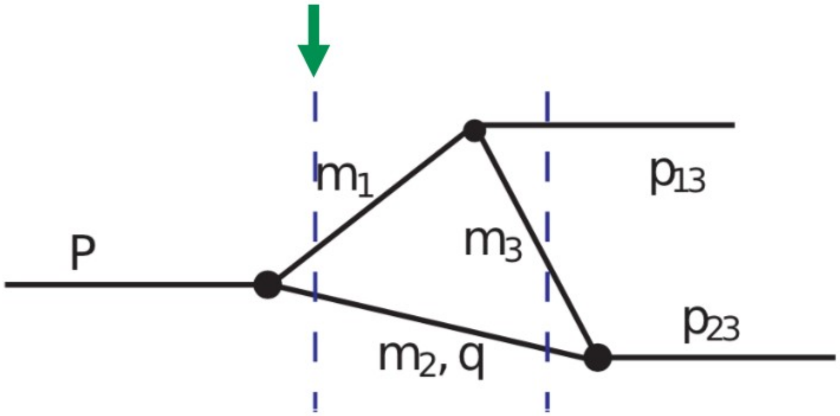
Integrating over q^0 … and taking only the part of the integral containing the singularity structure

$$I(m_{23}) = \int \frac{d^3q}{(P^0-\omega_1(\vec{q})-\omega_2(\vec{q})+i\,\epsilon)\left(E_{23}-\omega_2(\vec{q})-\omega_3(\vec{k}+\vec{q})+i\,\epsilon\right)}$$

$$= 2\,\pi\int_0^\infty dq\,\frac{q^2}{P^0-\omega_1(q)-\omega_2(q)+i\,\epsilon}f(q)\,,\quad f(q)=\int_{-1}^{1}dz\,\frac{1}{E_{23}-\omega_2(q)-\sqrt{m_3^2+q^2+k^2+2\,q\,k\,z}+i\,\epsilon}$$

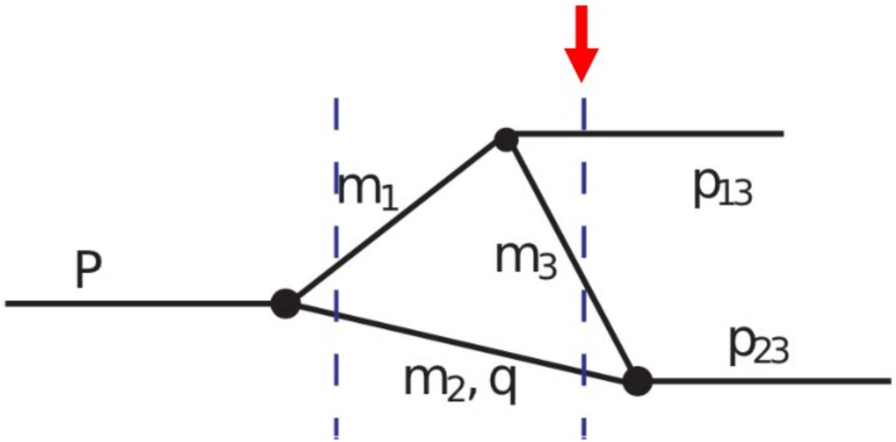
$$\omega_{1,2}(q)=\sqrt{m_{1,2}^2+q^2},\ \omega_3(\vec{q}+\vec{k})=\sqrt{m_3^2+(\vec{q}+\vec{k})^2},\ E_{23}=P^0-k^0$$

$$q=\vec{q},\ k=|\vec{k}|=\sqrt{\lambda(M^2,m_{13}^2,m_{23}^2)},\ M=\sqrt{P^2},\ m_{13,23}=\sqrt{p_{13,23}^2}$$



$$P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon = 0$$

$$q_{on+} = q_{on} + i\epsilon,\; q_{on} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_1^2)}$$

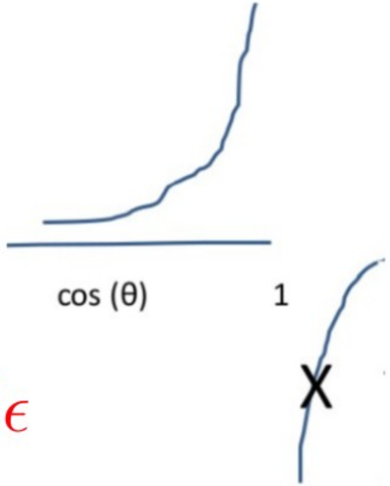


$$P^0 - \omega_1(\vec{q}) - \omega_2(\vec{q}) + i\epsilon = 0$$

$$q_{on+} = q_{on} + i\epsilon,\ q_{on} = \frac{1}{2M}\sqrt{\lambda(M^2, m_1^2, m_1^2)}$$

f(q) contains end-point singularities (logarithmic branch points) for $z = \pm 1$

$$E_{23} - \omega_2(q) - \sqrt{m_3^2 + q^2 + k^2 \pm 2qk} + i\epsilon = 0$$



$z = -1$

$$q_{a+} = \gamma(vE_2^* + p_2^*) + i\epsilon$$

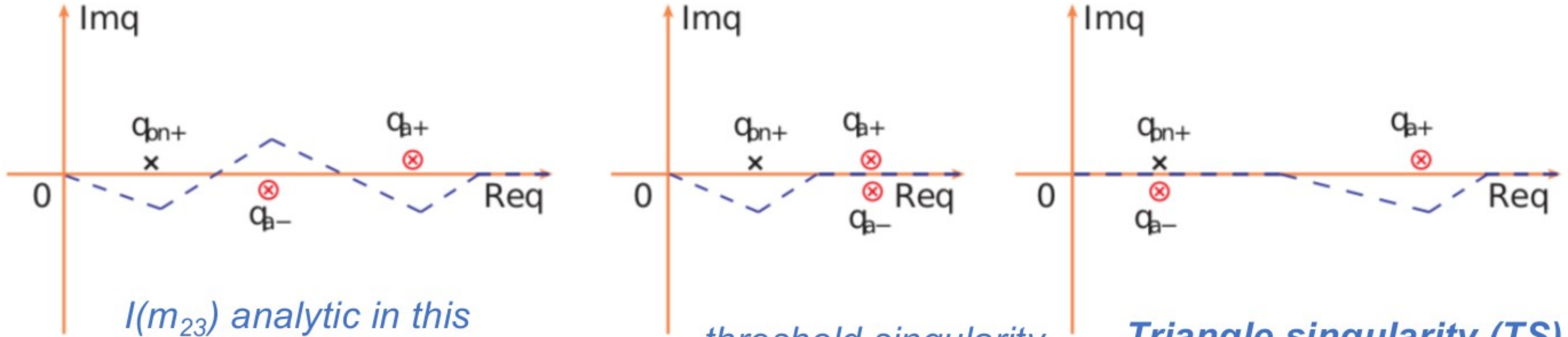
$$q_{a-} = \gamma(vE_2^* - p_2^*) - i\epsilon$$

$z = 1$

$$q_{b+} = \gamma(-vE_2^* + p_2^*) + i\epsilon$$

~~$q_{b-} = -\gamma(vE_2^* + p_2^*) - i\epsilon$~~

q_{b+} and q_{a-} are mutually exclusive as solutions that are simultaneously in the q (positive) integration range.
The interesting casuistry for TS is given by ***q_{a-}, q_{a+}, q_{on+}***:



$I(m_{23})$ analytic in this Kinematic region

threshold singularity

Triangle singularity (TS)

$$\lim_{\epsilon \to 0} (q_{on+} - q_{a-}) = 0$$

This is only fulfilled when all three intermediate particles are placed on shell and when:

$z = -1$ *Momentum of part. 2 is anti-parallel to that of (2,3) system from the decaying particle rest system*

$$\omega_1(q_{on}) - p_{13}^0 - \sqrt{m_3^2 + (q_{on} - k)^2} = 0$$

→ For this study, **TS** should appear at $\sqrt{s} \approx 2380 MeV$

Differential cross section for the $K^- d \longrightarrow p\Sigma^-$ reaction.

$$\frac{d\sigma}{d\cos\theta_p} = \frac{1}{4\pi}\frac{1}{s}M_p M_{\Sigma^-} M_d \frac{p}{k}\bar{\sum}\sum |t|^2 \qquad \bar{\sum}\sum |t|^2 = \frac{1}{3}\sum_{i,j}|t_{ij}^{(a)} + t_{ij}^{(b)}|^2$$

$p\Sigma^-$spin configurations

$d\ (S=1)$ polarizations

$$i = \uparrow\uparrow, \uparrow\downarrow, \downarrow\uparrow, \downarrow\downarrow \longrightarrow j = \uparrow\uparrow, \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \downarrow\downarrow$$

Pole couplings and coordinates needed to compute the cross section:

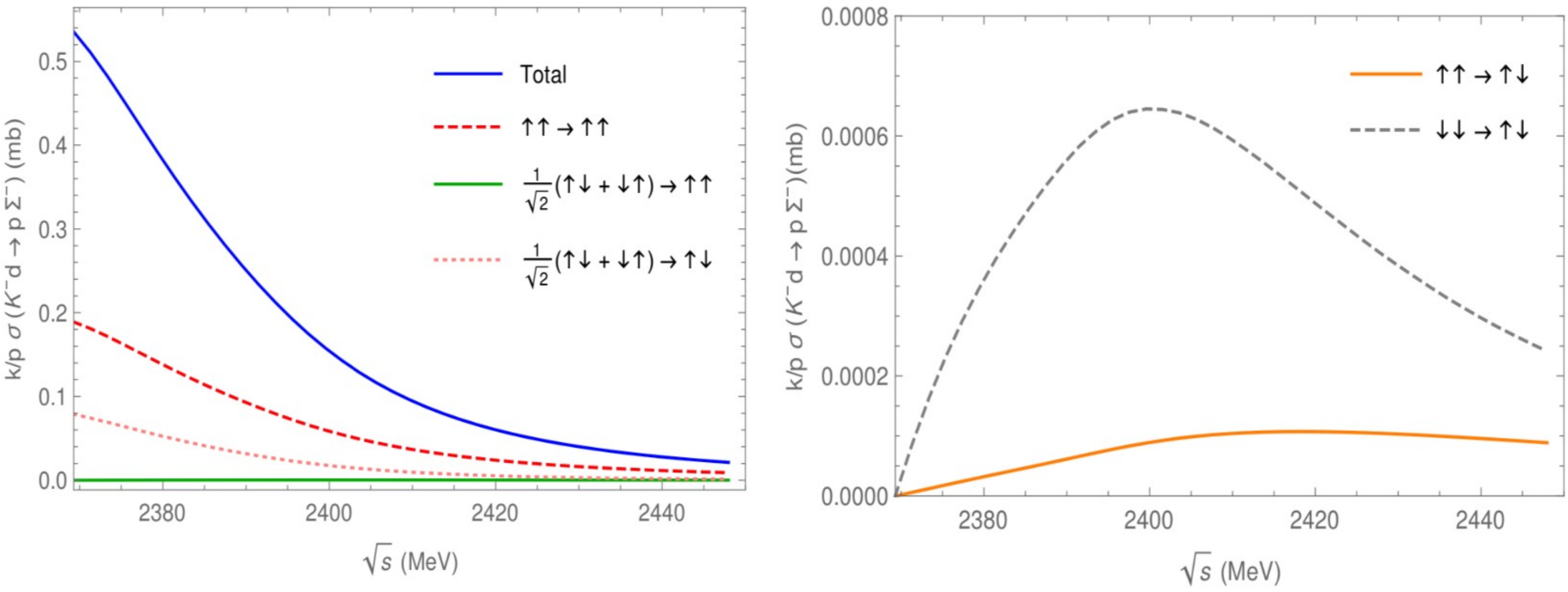
State	$g_{\Lambda^*,\bar{K}N}$	$g_{\Lambda^*,\pi\Sigma}$	(Mass, $\frac{\Gamma}{2}$)
$\Lambda(1390)$	$1.2 + i\,1.7$	$-2.5 - i\,1.5$	$(1390, 66)$
$\Lambda(1426)$	$-2.5 + i\,0.94$	$0.42 - i\,1.4$	$(1426, 16)$

$$g_{\Lambda^*,K^-p} = \frac{1}{\sqrt{2}}g_{\Lambda^*,\bar{K}N}$$

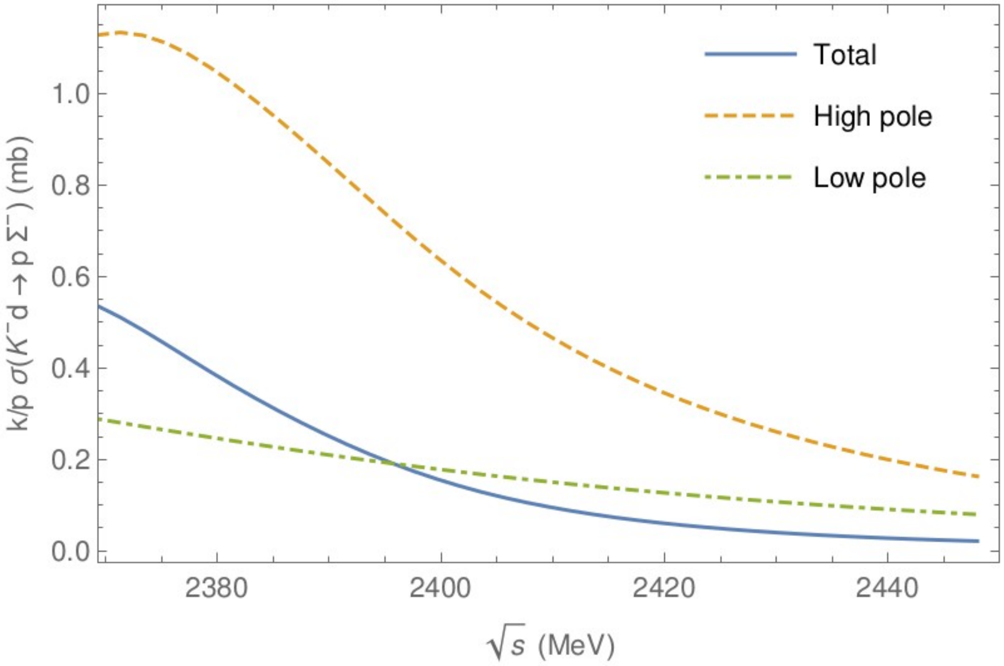
$$g_{\Lambda^*,\pi^+\Sigma^-} = -\frac{1}{\sqrt{3}}g_{\Lambda^*,\pi\Sigma}$$

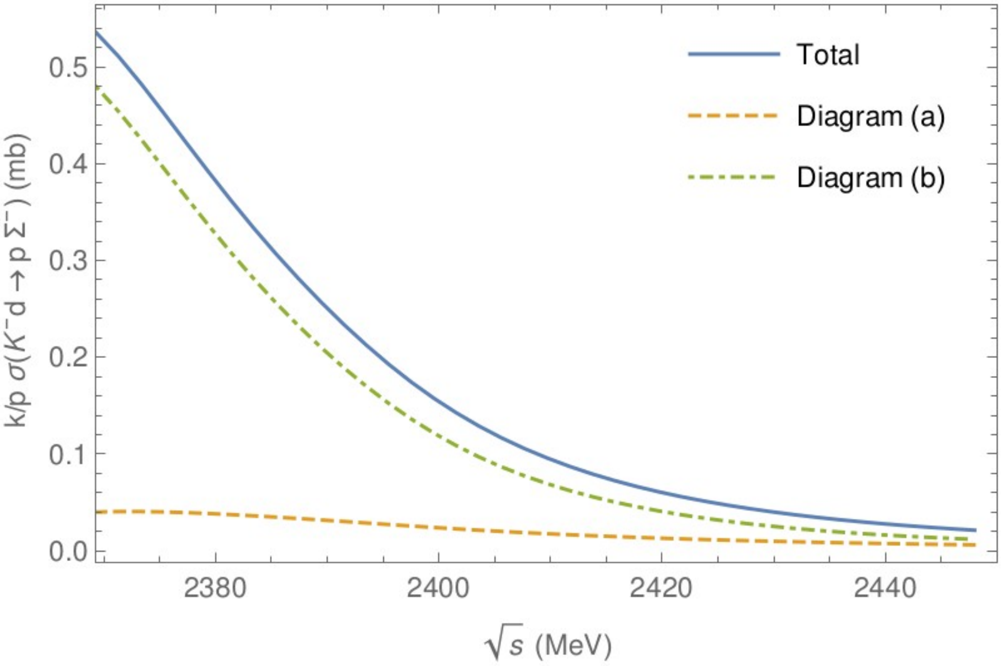
E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).

Contribution of several spin transitions to the $K^- d \longrightarrow p\Sigma^-$ cross section.



Contribution of the high and low mass poles and the mechanisms (a) and (b) to $\mathrm{K}/p \cdot \sigma(K^- d \longrightarrow p\Sigma^-)$.





Deuteron wave function replacement:

$$g_d \frac{M_N}{E(\vec{P}-\vec{q}-\vec{k})} \frac{M_N}{E_N(-\vec{P}+\vec{q})} \frac{\theta(q_{max}-|\vec{P}-\vec{q}-\frac{\vec{k}}{2}|)}{\sqrt{s}-k^0-E_N(-\vec{P}+\vec{q})-E_N(\vec{P}-\vec{q}-\vec{k})+i\epsilon} \longrightarrow -(2\pi)^{3/2}\psi(\vec{P}-\vec{q}-\frac{\vec{k}}{2})$$

R. Machleidt, Phys. Rev. C 63, 024001 (2001)

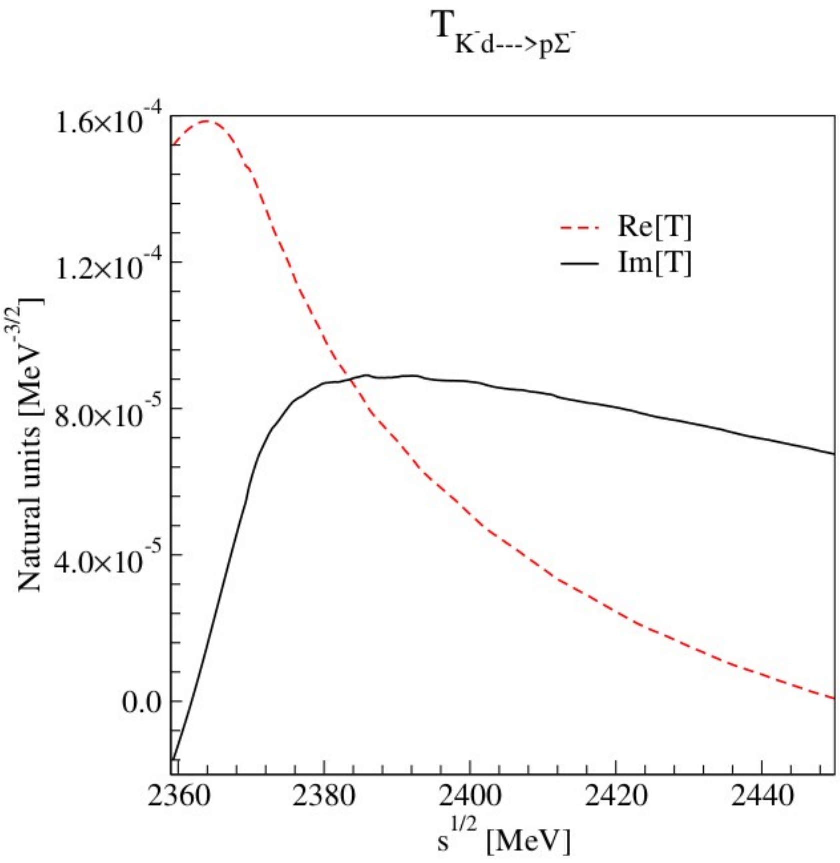
Formal equivalence between Breit-Wigner amplitudes and theoretical amplitudes:

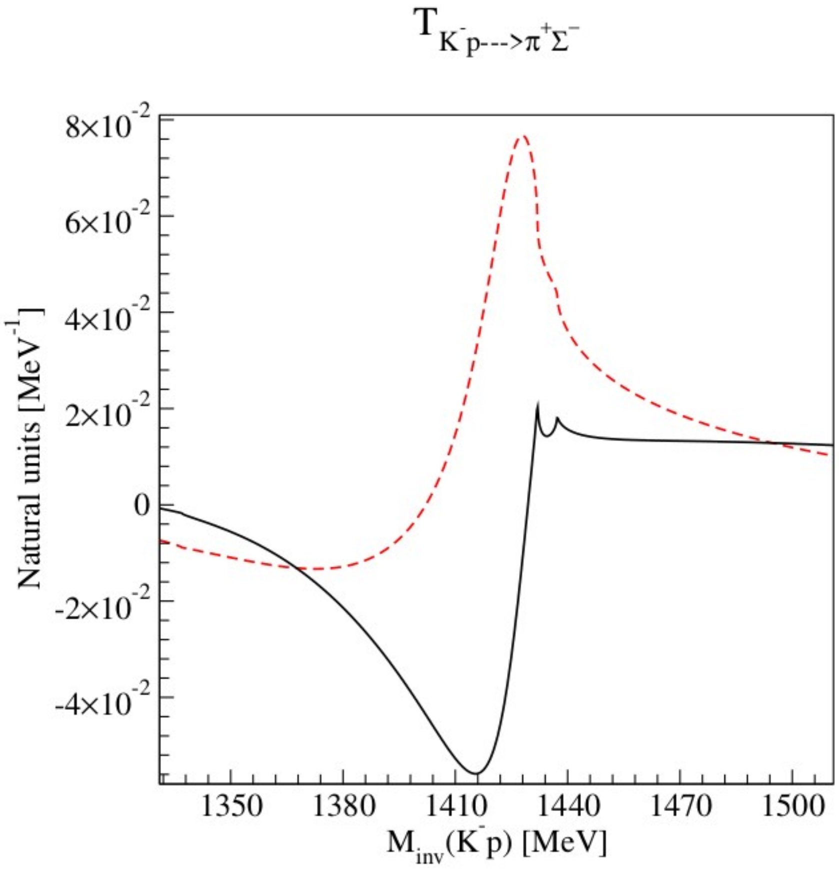
$$\sum_{i=1}^{2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,K^-p}^{(i)}}{\sqrt{s}-E_N(-\vec{p}+\vec{q})-E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \equiv t_{K^-p,K^-p}(M_{inv})$$

$$\sum_{i=1}^{2} \frac{M_{\Lambda^*}^{(i)}}{E_{\Lambda^*}^{(i)}(\vec{P}-\vec{q})} \frac{g_{\Lambda^*,K^-p}^{(i)} g_{\Lambda^*,\pi^+\Sigma^-}^{(i)}}{\sqrt{s}-E_N(\vec{p}+\vec{q})-E_{\Lambda^*}^{(i)}(-\vec{P}-\vec{q})+i\frac{\Gamma_{\Lambda^*}^{(i)}}{2}} \equiv t_{K^-p,\pi^+\Sigma^-}(M'_{inv})$$

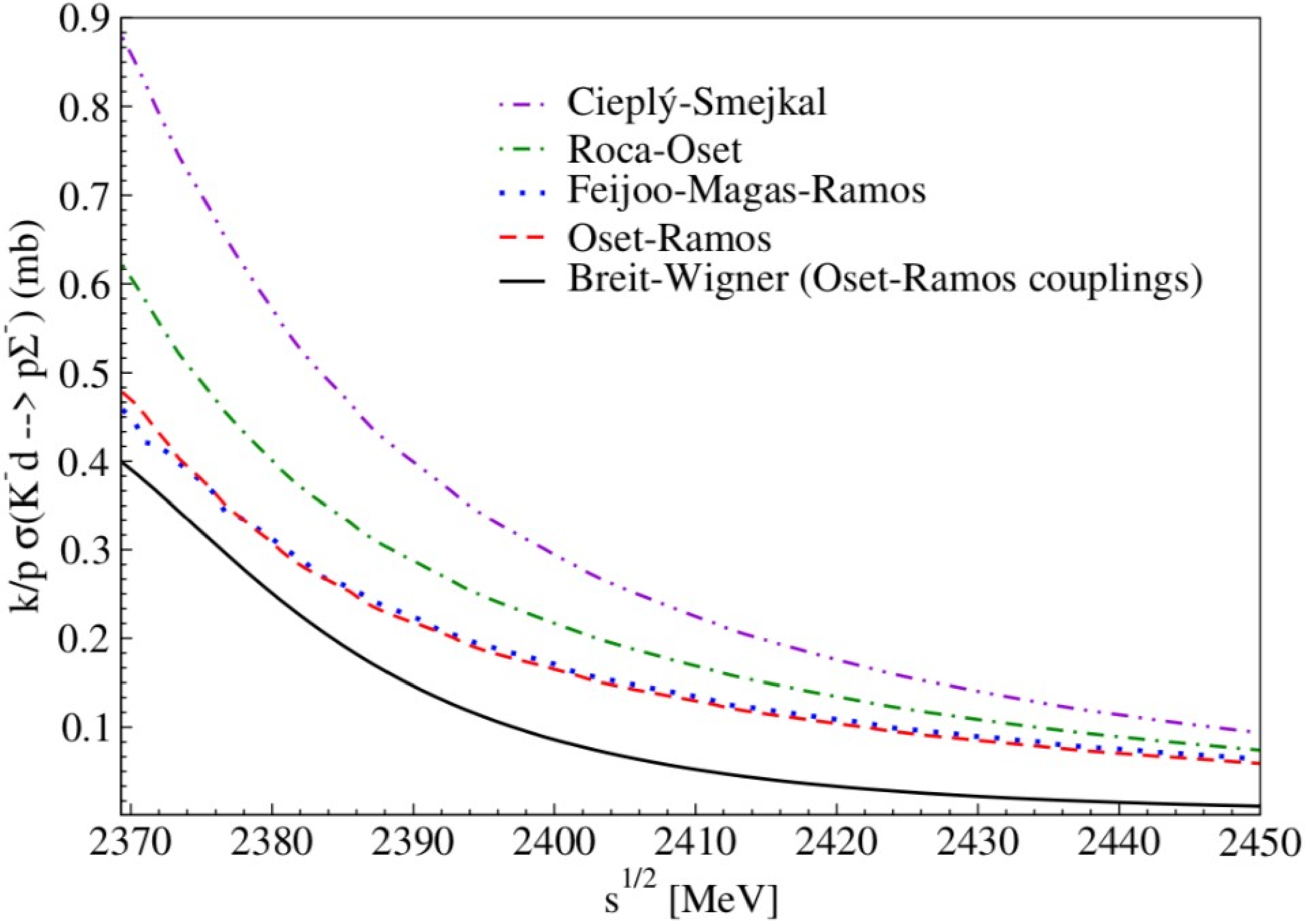
$$M_{\rm inv}^2 = s + M_N^2 - 2\sqrt{s}E_N(-\vec{P}+\vec{q}) \qquad M_{\rm inv}'^2 = s + M_N^2 - 2\sqrt{s}E_N(\vec{P}+\vec{q})$$

Energy dependence of the real and the imaginary parts of the $K^- d \longrightarrow p\Sigma^-$ and $K^- p \longrightarrow \pi^+\Sigma^-$ amplitudes.





$K^- d \longrightarrow p\Sigma^-$ cross sections for different considered models.



Conclusions:

Much progress has been done concerning the Kbar N interaction and the Λ(1405) both experimentally and theoretically.

In spite of it, we still have much ignorance concerning the information below threshold and the position of the lower pole.

Much work, theoretical and experimental is needed to advance in this topic

The fusion reaction proposed, $K^- d \to p \Sigma^-$ reaction, certainly will help in this direction