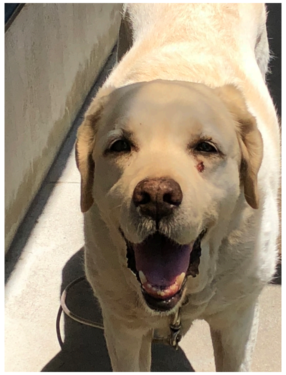
Structure of exotic hadrons by weak-binding relation with finite range correction



T. Kinugawa, T. Hyodo, arXiv:2205.08470[hep-ph]
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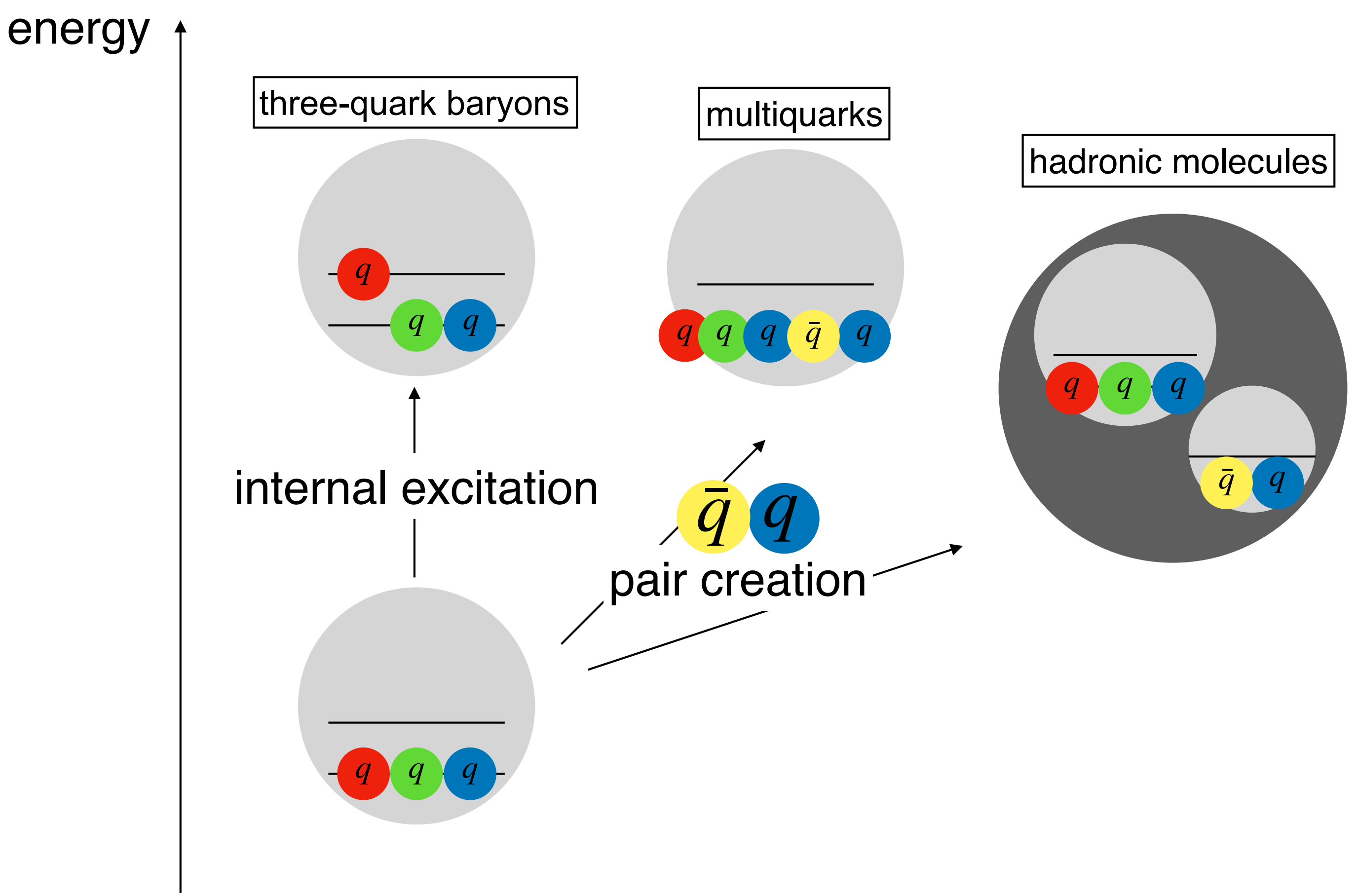
Background

candidates for exotic hadrons

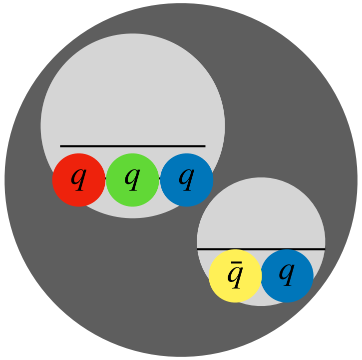
$\Lambda(1405)$, XYZ meson etc… → multiquarks, hadronic molecules

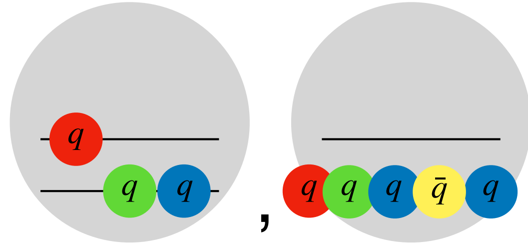


structure of hadrons ← model independent observable

Previous work

Hadron wave function



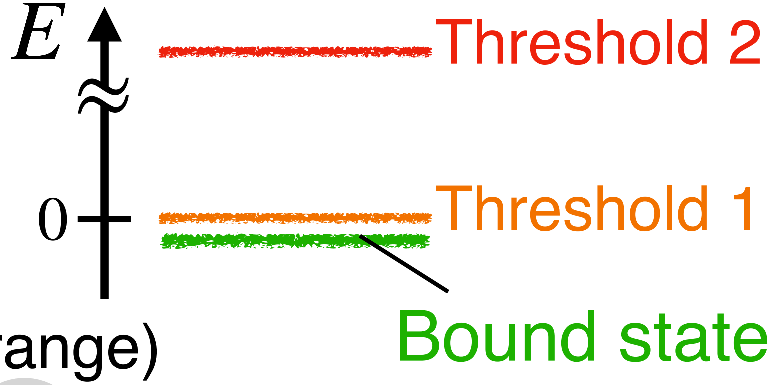


$$|\Psi\rangle = \sqrt{X}\,|\text{hadronic molecule}\rangle + \sqrt{1-X}\,|\text{others}\rangle$$

Compositeness (weight of hadronic molecule)

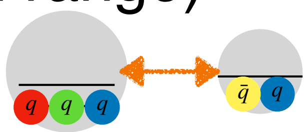
Weak-binding relation for bound state

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$



a_0 (scattering length) $R_{\rm typ}$(interaction range)

$R \equiv (2\mu B)^{-1/2}$, B (binding energy)



When $R \gg R_{\rm typ}$: observable(a_0, B) → compositeness(X)

S. Weinberg, Phys. Rev. 137, B672 (1965); Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

Motivation

Low-energy universality → $R = a_0$

→ The range correction by introducing the effective range r_e

Range correction

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

Properties of the effective range model:

-Single channel: $|$hadronic molecule$\rangle$ only $\Leftrightarrow X = 1$

-Zero range limit: $R_{\rm typ} \to 0$

$$\Rightarrow a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\} \Leftrightarrow a_0 = R \,?$$

Renormalized scattering amplitude ($R_{\rm typ} \to 0$):

E. Braaten, M. Kusunoki, and D. Zhang, Annals Phys. 323, 1770 (2008), 0709.0499.

$$a_0 = R\frac{2r_e/R}{1-(r_e/R-1)^2} = R\left[1 + \mathcal{O}\left(\left|\frac{r_e}{R}\right|\right)\right] \Rightarrow \quad a_0 \neq R\,!$$

→ range correction in the weak-binding relation form r_e

Improved weak-binding relation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\mathrm{typ}}}{R}\right)\right\}$$

interaction range: R_{typ} → R_{int}

Redefinition of R_{typ}: $R_{\mathrm{typ}} = \max\left\{R_{\mathrm{int}}, R_{\mathrm{eff}}\right\}$,

$$R_{\mathrm{eff}} = \max\left\{ |r_e|, \frac{|P_s|}{R^2}, \cdots \right\}.$$

It reduces to previous weak-binding relation for $R_{\mathrm{typ}} = R_{\mathrm{int}}$.

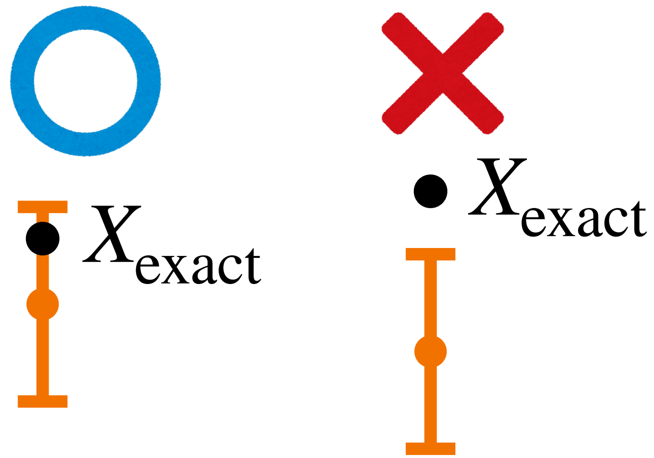
Numerical calculation

Effective range model ($R_{\mathrm{int}} \neq 0$)

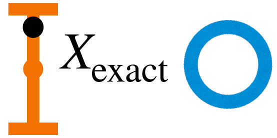
Weak-binding relation works when…

$$X_l < X_{\mathrm{exact}} < X_u$$

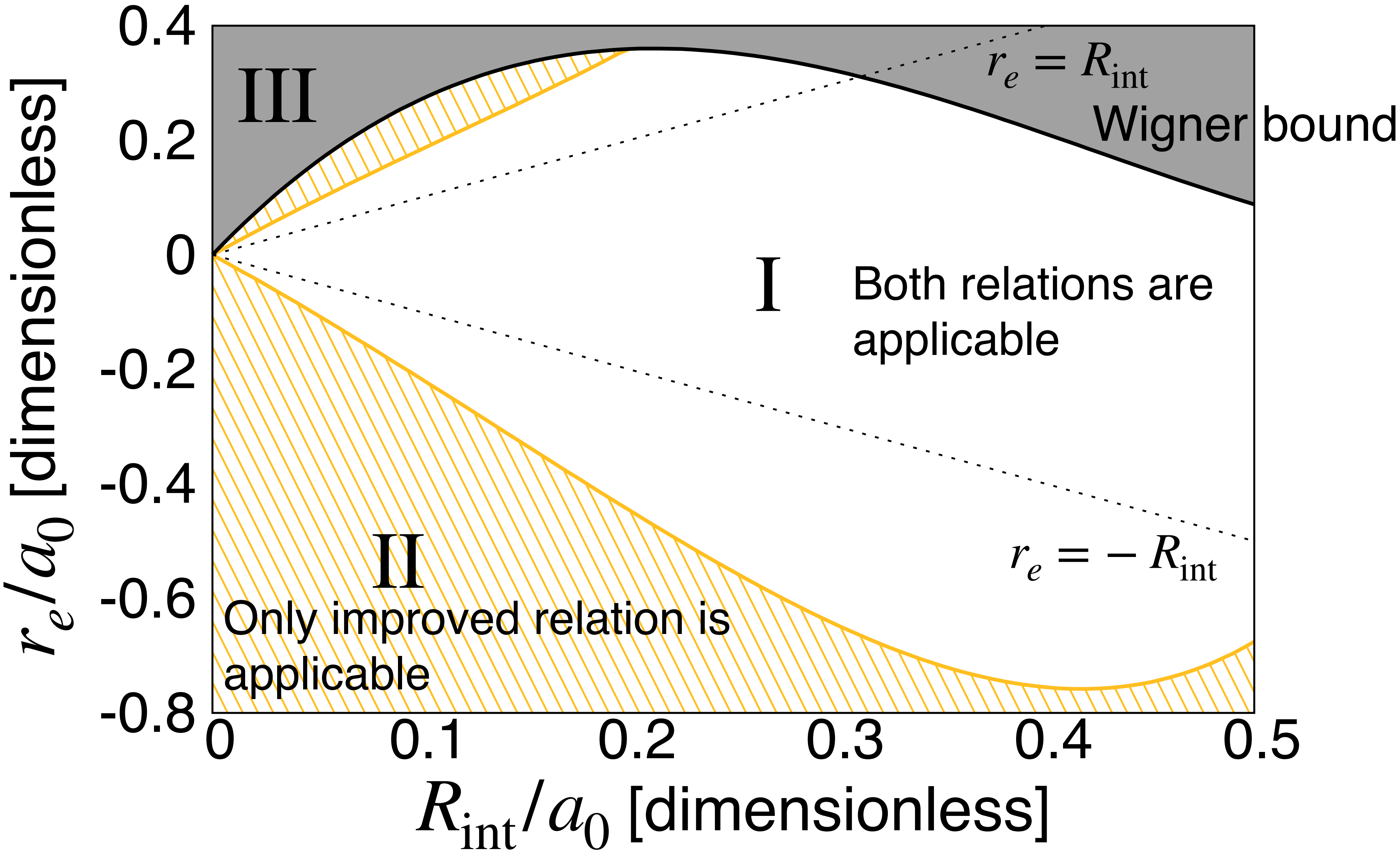
→ Validity condition



Numerical calculation



Applicable region of weak-binding relations in R_{int}/a_0 - r_e/a_0 plane



Range correction enlarge the applicable region (I→I+II).

Application

u: atomic mass unit, mK: millikelvin B.R.: Bohr Radius

bound state	particle 1	particle 2	m_1	m_2	B	a_0	r_e	R	$R_{\rm int}$
d	p	n	938.3 MeV	939.6 MeV	2.22 MeV	5.42 fm	1.75 fm	4.32 fm	1.43 fm
$X(3872)$	D^0	$\bar{D}^{*0}$	1865 MeV	2010 MeV	0.018 MeV	28.5 fm	−5.34 fm	33.4 fm	1.43 fm
$D^*_{s0}(2317)$	D	K	1867 MeV	495.6 MeV	44.8 MeV	1.3 fm	−0.1 fm	1.05 fm	0.359 fm
$D_{s1}(2460)$	D^*	K	2009 MeV	495.6 MeV	45.1 MeV	1.1 fm	−0.2 fm	1.03 fm	0.359 fm
$N\Omega$ dibaryon	N	Ω	955 MeV	1712 MeV	1.54 MeV	5.30 fm	1.26 fm	4.54 fm	0.676 fm
$\Omega\Omega$ dibaryon	Ω	Ω	1712 MeV	1712 MeV	1.6 MeV	4.6 fm	1.27 fm	3.77 fm	0.949 fm
${}^3_\Lambda$H	d	Λ	1876 MeV	1116 MeV	0.13 MeV	16.8 fm	2.3 fm	14.6 fm	4.32 fm
^{4}He dimer	^{4}He	^{4}He	4.003 u	4.003 u	1.30 mK	189 B.R.	13.8 B.R.	182.2 B.R.	10.2 B.R.

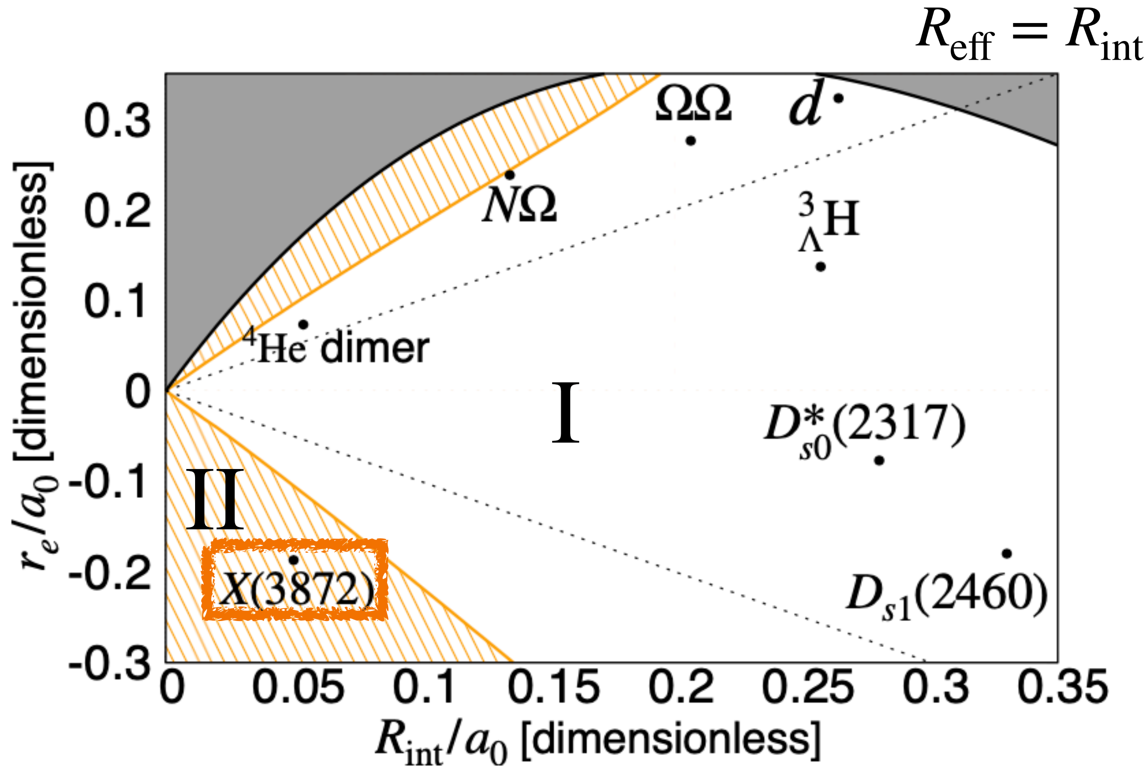
Low-energy universality holds $(a_0 > R_{\rm int})$

Bound states other than D_s and ${}^3_\Lambda$H have larger $|r_e| > R_{\rm int}$

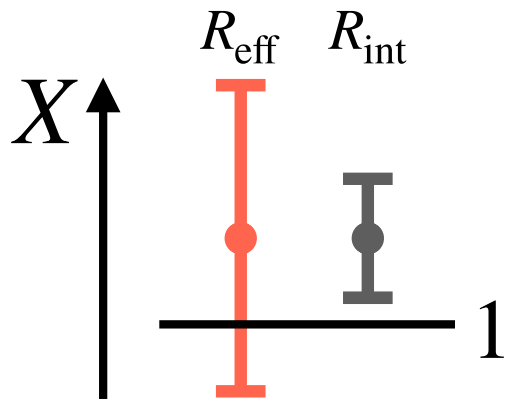
→ Range correction $R_{\rm typ} = R_{\rm eff}$

Determination of $R_{\rm int}$

- d, $X(3872)$: Compton wavelength of π
- D_s: Compton wavelength of σ ($m_\sigma \sim 550$ MeV)
- $N\Omega$: lattice QCD [HAL QCD, Phys. Lett. B 792, 284 (2019)]
- $\Omega\Omega$: lattice QCD [S. Gongyo *et al.*, Phys. Rev. Lett. 120 212001 (2018)]
- ${}^3_\Lambda$H: Radius of d ($d - \Lambda$ interaction range)
- ^{4}He dimer: van der Waals length [Z.-C. Yan *et al.*, Phys. Rev. A 54, 2824 (1996)]



$X(3872)$ is contained in applicable region for improved weak-binding relation (region II)



bound state	$\xi_{\rm eff}$	$\xi_{\rm int}$	$X(\xi_{\rm eff})$	$X(\xi_{\rm int})$	$R_{\rm typ}$
d	0.405	0.331	$1.68^{+3.18}_{-0.943}$	$1.68^{+2.14}_{-0.824}$	$R_{\rm eff}$
$X(3872)$	0.160	0.0428	$0.743^{+0.282}_{-0.213}$	$0.743^{+0.0675}_{-0.0626}$	$R_{\rm eff}$
$D^*_{s0}(2317)$	0.0949	0.341	$1.61^{+0.369}_{-0.288}$	$1.61^{+2.09}_{-0.804}$	$R_{\rm int}$
$D_{s1}(2460)$	0.192	0.345	$1.12^{+0.540}_{-0.358}$	$1.12^{+1.22}_{-0.566}$	$R_{\rm int}$
$N\Omega$ dibaryon	0.277	0.149	$1.40^{+1.20}_{-0.600}$	$1.40^{+0.523}_{-0.364}$	$R_{\rm eff}$
$\Omega\Omega$ dibaryon	0.337	0.252	$1.56^{+1.95}_{-0.773}$	$1.56^{+1.22}_{-0.626}$	$R_{\rm eff}$
$^3_\Lambda$H	0.157	0.295	$1.35^{+0.532}_{-0.366}$	$1.35^{+1.25}_{-0.605}$	$R_{\rm int}$
^{4}He dimer	0.0757	0.0560	$1.08^{+0.177}_{-0.152}$	$1.08^{+0.128}_{-0.114}$	$R_{\rm eff}$

Estimated X_l by previous relation

$N\Omega$ dibaryon :

$X_l = 1.40 - 0.364 > 1$

definition of $X : 0 \leq X \leq 1$

Range correction is important for $X(3872)$ and $N\Omega$.

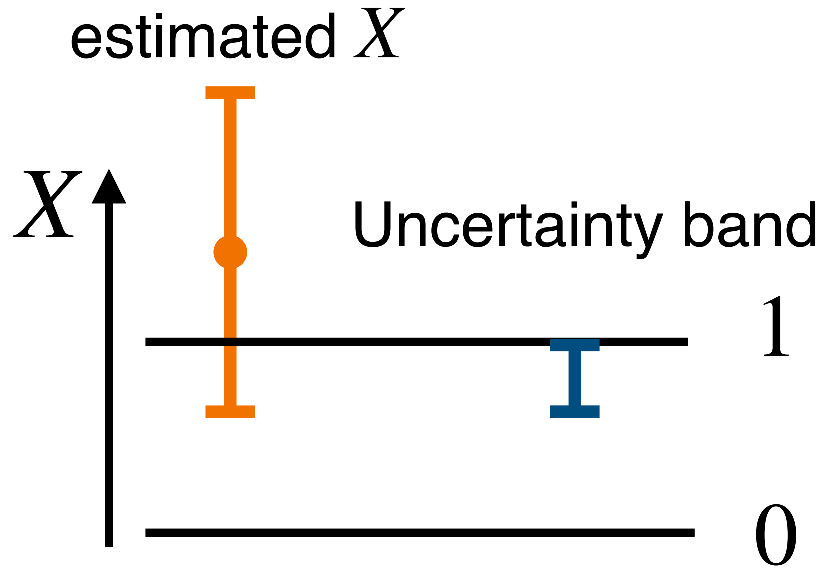
Application

Uncertainty band of X

$$\bar{X}_u(\xi) = \min\{X_u(\xi), 1\},$$

$$\bar{X}_l(\xi) = \max\{X_l(\xi), 0\}\ .$$

→ $\bar{X}_l(\xi) \leq X \leq \bar{X}_u(\xi)$



bound state	compositeness X
d	$0.74 \leq X \leq 1$
$X(3872)$	$0.53 \leq X \leq 1$
$D^*_{s0}(2317)$	$0.81 \leq X \leq 1$
$D_{s1}(2460)$	$0.55 \leq X \leq 1$
$N\Omega$ dibaryon	$0.80 \leq X \leq 1$
$\Omega\Omega$ dibaryon	$0.79 \leq X \leq 1$
$^3_\Lambda\mathrm{H}$	$0.74 \leq X \leq 1$
$^4\mathrm{He}$ dimer	$0.93 \leq X \leq 1$

- composite dominant : $0.5 < X$

- $X(3872)$ model calculation

M. Takizawa and S. Takeuchi, PTEP 2013, 093D01 (2013), arXiv:1206.4877.

$$|X(3872)\rangle = c_1\,|c\bar{c}\rangle + c_2\,|D^0\bar{D}^{*0}\rangle + c_3\,|D^+D^{*-}\rangle$$

$$|c_2|^2 = X, \quad 0.759 \leq X \leq 0.897$$

→ Consistent with the model calculation

Conclusion

T. Kinugawa, T. Hyodo, arXiv:2205.08470[hep-ph]
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- Weak-binding relation : observable → compositeness (X)

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

- We study the range correction to weak-binding relation.

- Improved weak-binding relation by redefinition of $R_{\rm typ}$:

$$R_{\rm typ} = \max\left\{R_{\rm int}, |r_e|, \cdots\right\}$$

- We find the region where only the improved weak-binding relation can be applied.

- The range correction is important for the reasonable estimation of X.

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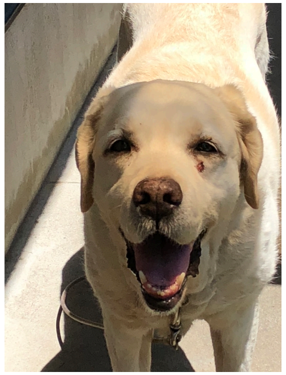

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Numerical calculation

$$a_0 = R\left\{\frac{2X}{1+X} + \mathcal{O}\left(\frac{R_{\rm typ}}{R}\right)\right\}$$

When dose the weak-binding relation work?

Estimation with correction terms ($\xi \equiv R_{\rm typ}/R$): Y. Kamiya and T. Hyodo, PTEP 2017, 023D02 (2017).

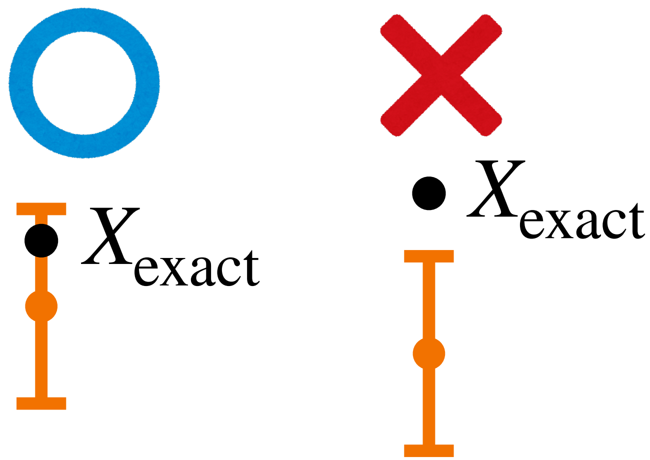
Central value: $X_c = \dfrac{a_0/R}{2 - a_0/R}$

$$X_{\rm upper}(\xi) = \frac{a_0/R}{2 - a_0/R} + \xi,\ X_{\rm lower}(\xi) = \frac{a_0/R}{2 - a_0/R} - \xi\,.$$

Weak-binding relation works when…

$$X_{\rm lower} < X_{\rm exact} < X_{\rm upper}$$

→ Validity condition



Numerical calculation

Effective range model ($R_{\rm int} \neq 0$)

$$f(k;\lambda_0,\rho_0,\Lambda) = \left[-\frac{1}{a_0} + \frac{r_e}{2}k^2 + \mathcal{O}\left(\frac{R_{\rm int}}{R}\right) - ik\right]^{-1}$$ (two length scales r_e and $R_{\rm int}$)

$$1/f(k = i/R) = 0$$

- $r_e \neq 0$ (range correction): $\xi_{r_e} = |r_e/R|$ → Uncertainty from r_e

  $r_e < 0$ (effective range model)

- $R_{\rm int} \neq 0$: $\xi_{\rm int} = R_{\rm int}/R$. → Uncertainty from $R_{\rm int}$

- $X_{\rm exact} = 1$

→ We search for the region of r_e and $R_{\rm int}$ in which validity condition are satisfied.

Precision of X

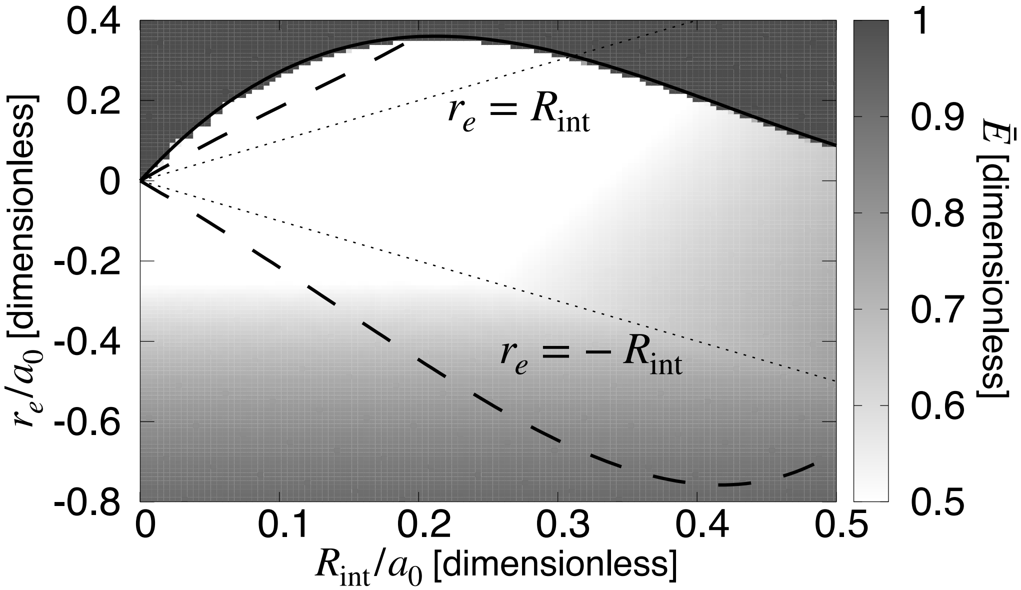
Precision of X estimated by the weak-binding relations ?

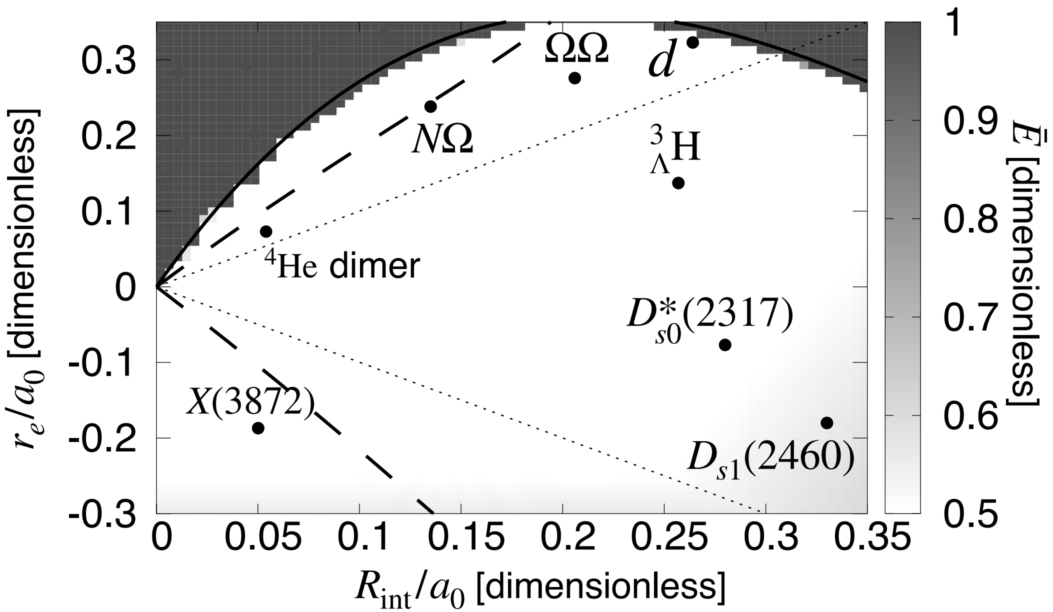
Uncertainty band $\bar{E}$

$$\bar{E} = \bar{X}_u - \bar{X}_l, \quad \bar{X}_u = \min\{X_u, 1\}, \quad \bar{X}_l = \max\{X_l, 0\},$$

→ We require $\bar{E} \lesssim 0.5$ for meaningful estimation

Uncertainty band $\bar{E}$ in $R_{\rm int}/a_0$ - r_e/a_0 plane





Another numerical calculation

Resonance model ($X_{\rm exact} \leq 1$)

$$X_{\rm exact}^{-1} = 1 + 16\pi\kappa \frac{g_0^2}{\left\{ (-\kappa^2 - \nu_0)\left(\frac{8\pi}{1 - \frac{2}{\pi}\Lambda} + \frac{g_0^2}{\nu_0} \right) - g_0^2 \right\}^2},$$

$$f(k)^{-1} = -\frac{8\pi}{m}\left(\lambda_0 + \frac{g_0^2}{E - \nu_0} \right)^{-1} - \frac{2}{\pi}\Lambda - ik,$$

$$r_e = -\frac{16\pi g_0^2}{m^2\nu_0^2}\left(-\frac{g_0^2}{\nu_0} + \lambda_0 \right)^{-2}$$

< 0 (Wigner bound)

Applicable region of the weak-binding relations ($ma_0^2\nu_0 = -0.5$)