Chiral Effective Theory of Diquarks and Application to Heavy Hadron Spectrum

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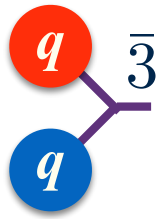
Diquark

- **Diquark:** **the simplest *colorful cluster* in hadrons**

  "bound" qq state

  color $3 \otimes 3 = \bar{3} \oplus 6$ **spin :** $\frac{1}{2} \otimes \frac{1}{2} = 0 \oplus 1$



- **S-wave color 3bar diquarks: S(0+) and A(1+)**

- **Spin dependent force from magnetic gluon exchange predicts strong attraction in S(0+).**

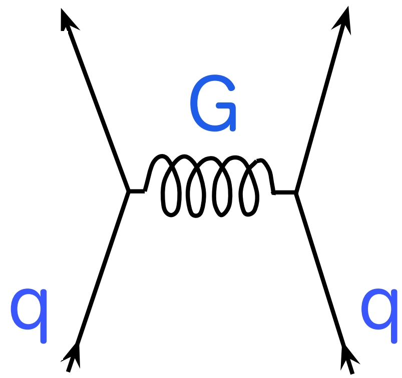
  Color-Magnetic Interaction

$$\Delta_{\rm CM} \equiv \langle -\sum_{i<j} (\vec{\lambda}_i \cdot \vec{\lambda}_j)(\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle$$

  S(0+) color 3bar $\Delta_{\rm CM} = -8$

  A(1+) color 3bar $\Delta_{\rm CM} = +8/3$

  M(A)-M(S) = (2/3) [M(Δ)- M(N)] ~ 200 MeV



Diquark in Lattice QCD

- **Hess, Karsch, Laermann, Wetzorke, PR D58, 111502 (1998)**
  quench, Landau gauge fixed
  $M(0^+) \sim 694$ MeV, $M(1^+) \sim 810$ MeV
- **Alexandrou, de Forcrand, Lucini, PRL 97, 222002 (2006)**
  From Qqq system, quench, gauge invariant
  $M(1^+) - M(0^+) \sim 200\text{-}220$ MeV, $R(S) \sim 1$ fm
- **Babich, et al., PR D76, 074021 (2007)**
  quench, Landau gauge
  $M(1^+) - M(0^+) \sim 162$ MeV, $M(0^+) - 2m_q \sim -200$ MeV
- **Yujiang Bi, et al., Chinese Physics C40 (2016) 073106**
  full QCD, Landau gauge
  $M(1^+) - M(0^+) \sim 290$ MeV, $M(0^+) - m_q \sim 310$ MeV

Diquark in Heavy Baryons

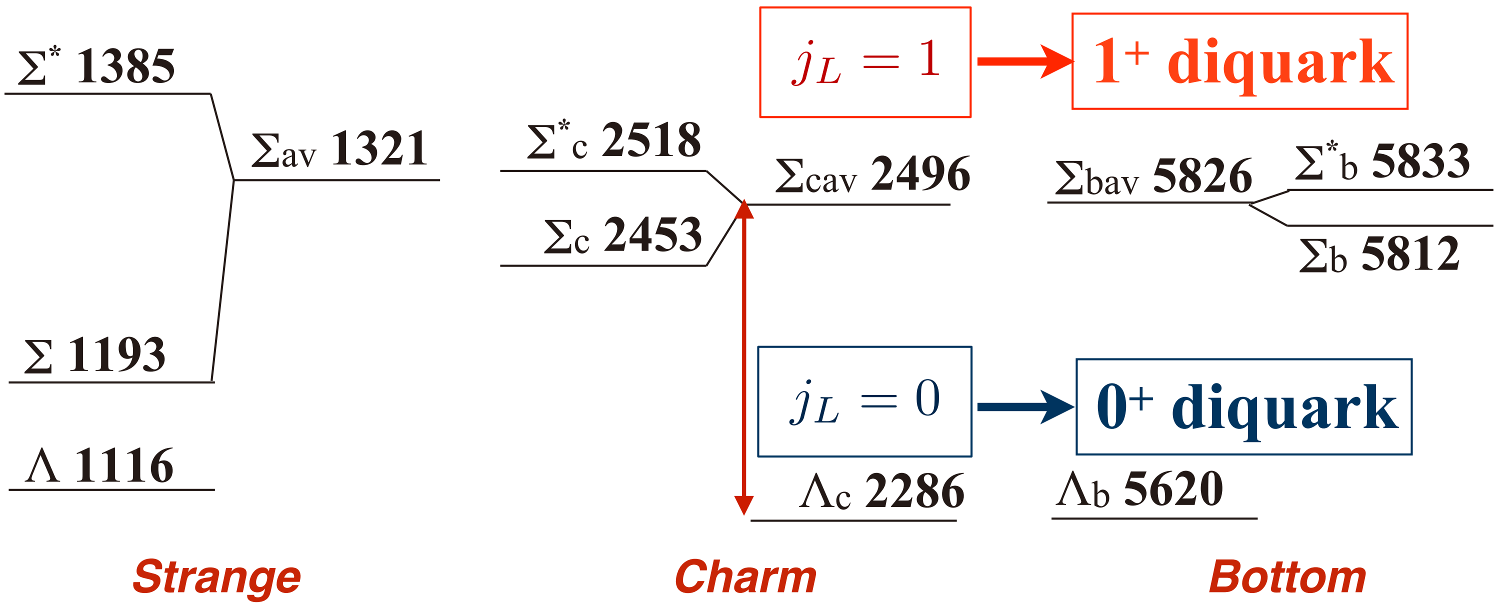
HQ spin symmetry $[S_Q, H] = O\left(\frac{1}{m_Q}\right)$

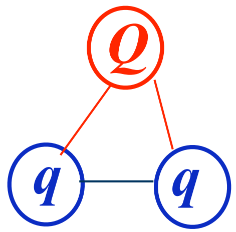
Q

q

$\vec{J} = \vec{S}_Q + \vec{j}_L \qquad \vec{j}_L = \vec{S}_q + \vec{L}_q$

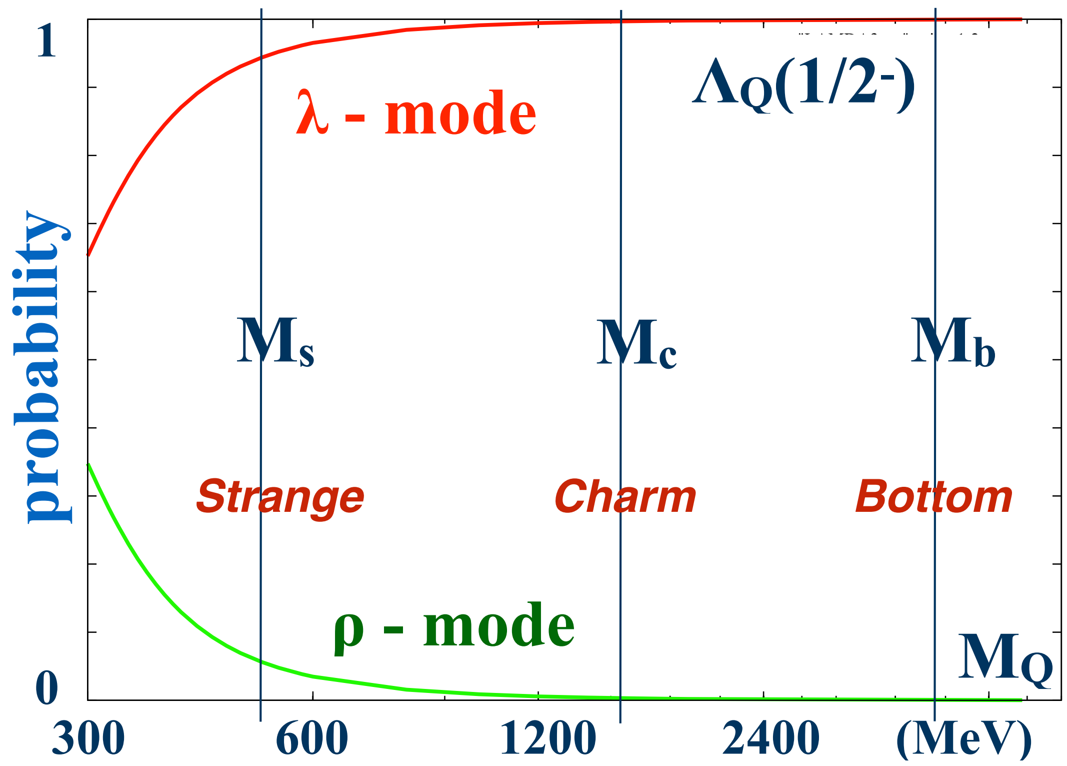
$J = j_L \pm \frac{1}{2}$ **states are degenerate in the HQ limit.**

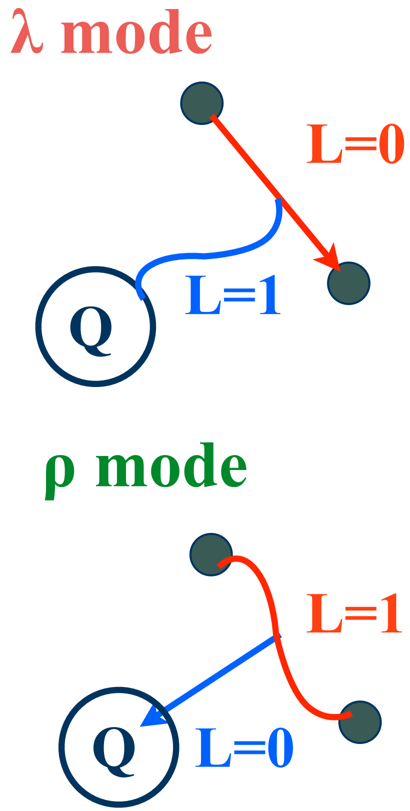




P-wave excited states: from s to c/b

- **Probabilities of λ and ρ modes *v.s.* heavy quark mass in the lowest P-wave $\Lambda_Q(1/2^-)$ state**

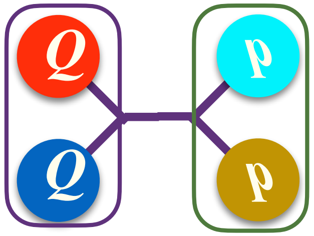


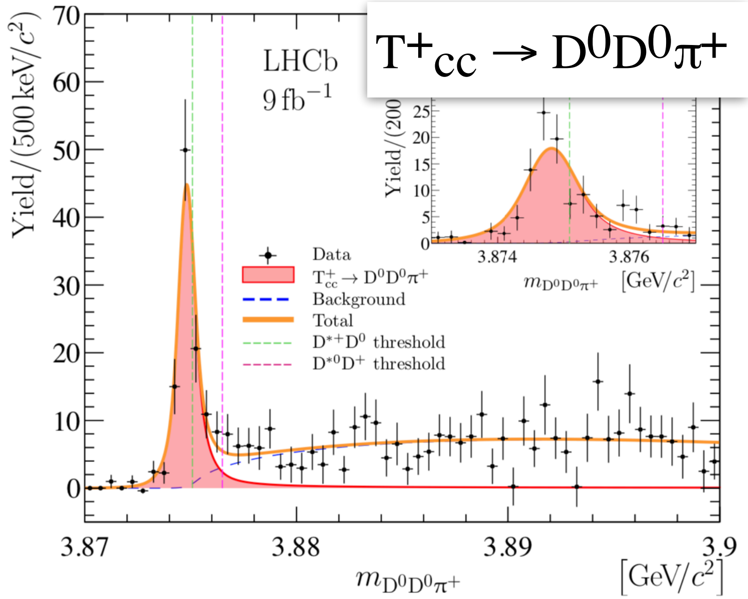


Quark model calculation by Yoshida, et al., PRD 92, 114029 (2015)

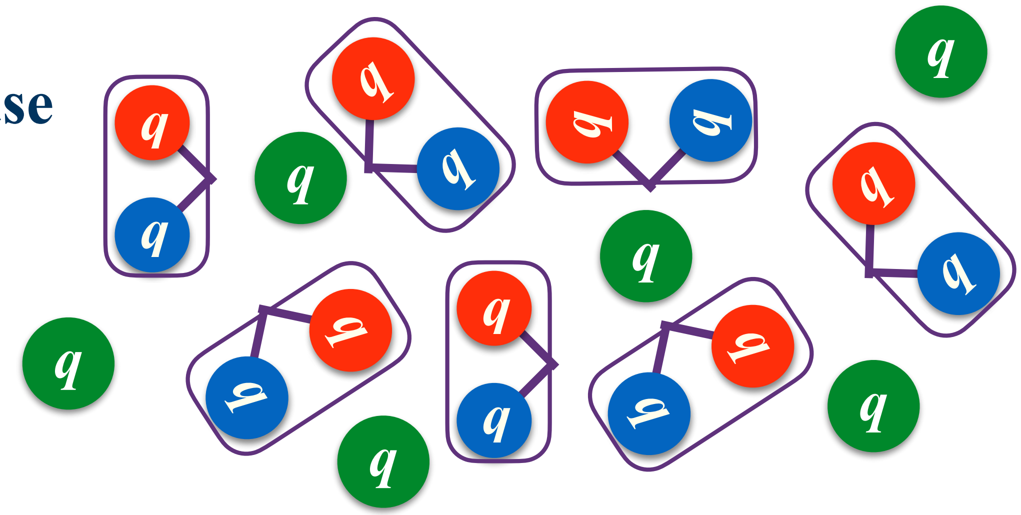
Diquarks in exotic hadrons/matter

- Diquark may form doubly heavy tetraquark bound states.
  $T_{QQ} = QQ\ (q^{bar}q^{bar})$





- Diquarks may form BE condensate in dense hadronic matter.
  => color-superconducting phase



Chiral Effective Theory of Diquarks

- **Goal: to explore properties of *light diquarks* under SU(3)×SU(3) chiral symmetry and answer questions such as**

What are the chiral partners of diquarks and their implications to hadron spectroscopy?

How do we observe the chiral properties of diquarks?

How do diquarks behave in matter, where chiral symmetry is partially restored?

- **Some previous works on ChET for diquarks:**
  D.K. Hong, et al., *PL B596 (2004) 191, IJMP A27 (2012) 1250051*.
  Non-linear chiral Diquark effective theory for penta/tetraquarks
  T. Hatsuda, M. Tachibana, N. Yamamoto, G. Baym, *PRL 97, 122001 (2006), PR D76, 074001 (2007)*.
  Chiral/Diquark effective theory and the axial anomaly in dense QCD
  Y. Kawakami, M. Harada, *PR D97 (2018) 114024, PR D99 (2019) 094016*.
  Chiral effective theory of Single Heavy Baryons (HQ symmetry)

Chiral Effective Theory of Diquarks

M. Harada, Y.R. Liu, M.O., K. Suzuki, "*Chiral effective theory of diquarks and $U_A(1)$ anomaly*", Phys. Rev. D101, 054038 (2020)

Chiral effective Lagrangian for the scalar (0^+) and psudoscalar (0^-) diquarks

Y. Kim, E. Hiyama, M.O., K. Suzuki, "*Spectrum of singly heavy baryons from a chiral effective theory of diquarks*", Phys. Rev. D102, 014004 (2020)

Diquark-heavy-quark model of singly heavy baryons with scalar diquark

Y. Kawakami, M. Harada, M.O., K. Suzuki, "*Suppression of decay widths in singly heavy baryons induced by the $U_A(1)$ anomaly*", Phys. Rev. D102, 114004 (2020)

Goldberger-Treiman relation and decays of the P-wave excited state of Λ_c baryon

Y. Kim, Y.R. Liu, M.O., K. Suzuki, "*Heavy baryon spectrum with chiral multiplets of scalar and vector diquarks*", Phys. Rev. D 104, 054012 (2021)

Chiral effective theory of axialvector (1^+) and vector (1^-) diquarks and its application to the spectrum of singly heavy baryons

Y. Kim, M.O., K. Suzuki, "*Doubly heavy tetraquarks in a chiral-diquark picture*", Phys. Rev. D 105, 074021 (2022)

Chiral effective theory applied to QQ+diquark systems

Chiral Effective Theory of Diquarks

- **Linear representation of chiral SU(3)$_R$ x SU(3)$_L$**

$q^a_{\alpha i}$ a (color), α (Dirac), i (flavor)

$$q^a_{iR} = P_R\, q^a_i, \quad q^a_{iL} = P_L\, q^a_i \qquad P_{R,L} \equiv \frac{1 \pm \gamma_5}{2}$$

$$q_R \to U_R\, q_R = (U_R)_{ij} q_{jR}, \quad U_R \in SU(3)_R$$

$$q_L \to U_L\, q_L = (U_L)_{ij} q_{jL}, \quad U_L \in SU(3)_L$$

- **Scalar chiral diquarks (color 3^{bar})**

$d^a_{iR} \equiv \epsilon_{ijk}(q^T_{jR}\, C\, q_{kR})^{\bar{3}}$ Right scalar diquark, chiral $(\bar{3},1)$, color $\bar{3}$

$d^a_{iL} \equiv \epsilon_{ijk}(q^T_{jL}\, C\, q_{kL})^{\bar{3}}$ Left scalar diquark, chiral $(1,\bar{3})$, color $\bar{3}$

- **Parity eigenstates: 0$^+$, 0$^-$ diquarks**

$$S^a_i = d^a_{iR} - d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\gamma_5\, q_k)^{\bar{3}}$$

$$P^a_i = d^a_{iR} + d^a_{iL} = \epsilon_{ijk}(q^T_j\, C\, q_k)^{\bar{3}}$$

$$(\bar{3},1) + (1,\bar{3})$$

Chiral Effective Theory of Diquarks

M. Harada, Y.R. Liu, M.O., K. Suzuki, PR D101, 054038 (2020)

$$\mathcal{L} = \mathcal{D}_\mu d_{R,i} \left(\mathcal{D}^\mu d_{R,i}\right)^\dagger + \mathcal{D}_\mu d_{L,i} \left(\mathcal{D}^\mu d_{L,i}\right)^\dagger$$

$$-m_0^2(d_{R,i}d_{R,i}^\dagger + d_{L,i}d_{L,i}^\dagger)$$

chiral invariant mass term

$$-\frac{m_1^2}{f}(d_{R,i}\Sigma_{ij}^\dagger d_{L,j}^\dagger + d_{L,i}\Sigma_{ij} d_{R,j}^\dagger)$$

$U_A(1)$ anomaly

CSB mass terms

$$-\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d_{L,n}^\dagger + d_{L,k}\Sigma_{\ell i}^\dagger\Sigma_{mj}^\dagger d_{R,n}^\dagger)$$

$$+\frac{1}{4}\mathrm{Tr}\left[\partial^\mu\Sigma^\dagger\partial_\mu\Sigma\right] + V(\Sigma). \qquad \Sigma_{ij} \equiv \sigma_{ij} + i\pi_{ij}$$

- For the SSB vacuum $\langle\Sigma\rangle = f$, the mass term of the right and left diquarks are given by

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix} \qquad \text{for } \langle\Sigma_{ij}\rangle = f\delta_{ij}$$

Chiral Effective Theory of Diquarks

- **The mass eigenstates are given by**

$$M^2 = \begin{pmatrix} m_0^2 & m_1^2 + m_2^2 \\ m_1^2 + m_2^2 & m_0^2 \end{pmatrix}$$

Scalar diquark

$$S_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a - d_{L,i}^a)$$

$$\longrightarrow M(0^+) = \sqrt{m_0^2 - m_1^2 - m_2^2},$$

Pseudo-scalar diquark

$$P_i^a = \frac{1}{\sqrt{2}}(d_{R,i}^a + d_{L,i}^a)$$

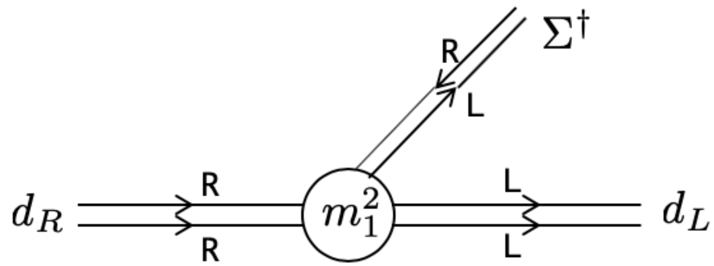
$$\longrightarrow M(0^-) = \sqrt{m_0^2 + m_1^2 + m_2^2},$$

$U_A(1)$ anomaly

- **$U_A(1)$ anomaly in the diquark effective theory**

$$-\frac{m_1^2}{f}(\underline{d_{R,i}\Sigma^\dagger_{ij}d^\dagger_{L,j}} + d_{L,i}\Sigma_{ij}d^\dagger_{R,j})$$

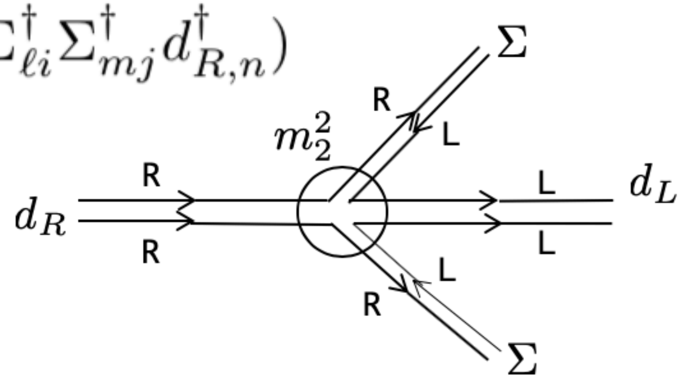
3 left quarks and 3 right antiquarks
flavor antisymmetric
induces anomalous singlet current



$$\partial_\mu J_A^{\mu 0} = \frac{3m_1^2}{2}(S\lambda_0 P^\dagger - P\lambda_0 S^\dagger)$$

- **non-anomalous term**

$$-\frac{m_2^2}{2f^2}\epsilon_{ijk}\epsilon_{\ell mn}(d_{R,k}\Sigma_{\ell i}\Sigma_{mj}d^\dagger_{L,n} + d_{L,k}\Sigma^\dagger_{\ell i}\Sigma^\dagger_{mj}d^\dagger_{R,n})$$



Chiral Effective Theory of Diquarks

SU(3) breaking and inverse mass hierarchy

$$A \equiv \frac{f_s}{f_\pi}\left(1+\frac{m_s}{g_s f_s}\right) \sim \frac{5}{3}$$

i=3 (ud)

$$M_3(0^+) = \sqrt{m_0^2 - Am_1^2 - m_2^2}, \quad M_3(0^-) = \sqrt{m_0^2 + Am_1^2 + m_2^2}.$$

i=1,2 (ds), (us)

$$M_1(0^+) = M_2(0^+) = \sqrt{m_0^2 - m_1^2 - Am_2^2}, \quad M_1(0^-) = M_2(0^-) = \sqrt{m_0^2 + m_1^2 + Am_2^2},$$

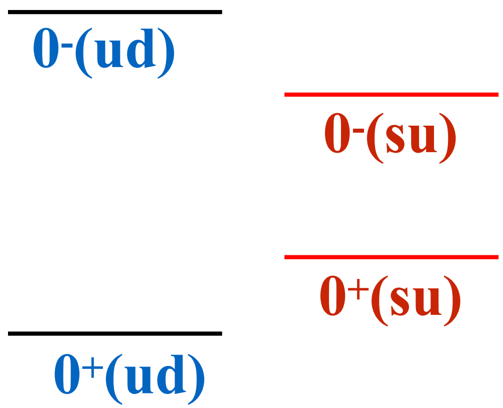
Inverse Mass Hierarchy due to $U_A(1)$ anomaly

$$M_1(0^+)^2 + M_1(0^-)^2 = M_3(0^+)^2 + M_3(0^-)^2$$

$$M_1(0^+)^2 - M_3(0^+)^2 = M_3(0^-)^2 - M_1(0^-)^2 > 0$$

$$M_3(0^-) > M_1(0^-)$$

(ds), (us) (ud)

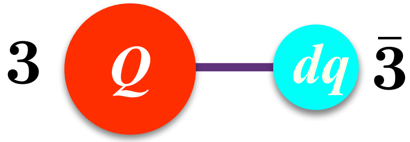


M. Harada, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D101, 054038 (2020)

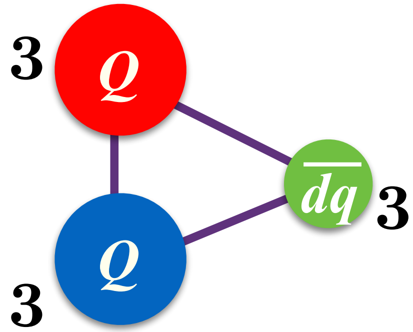
Diquark-Heavy-Quark model

- **Single-Heavy-Baryon with a *Q-dq* potential:**

$$V(r) = -\frac{\alpha}{r} + \lambda r + C,$$



- **Double-Heavy-Tetraquarks with *Q-Q* and *Q-dq*^{bar} potentials**



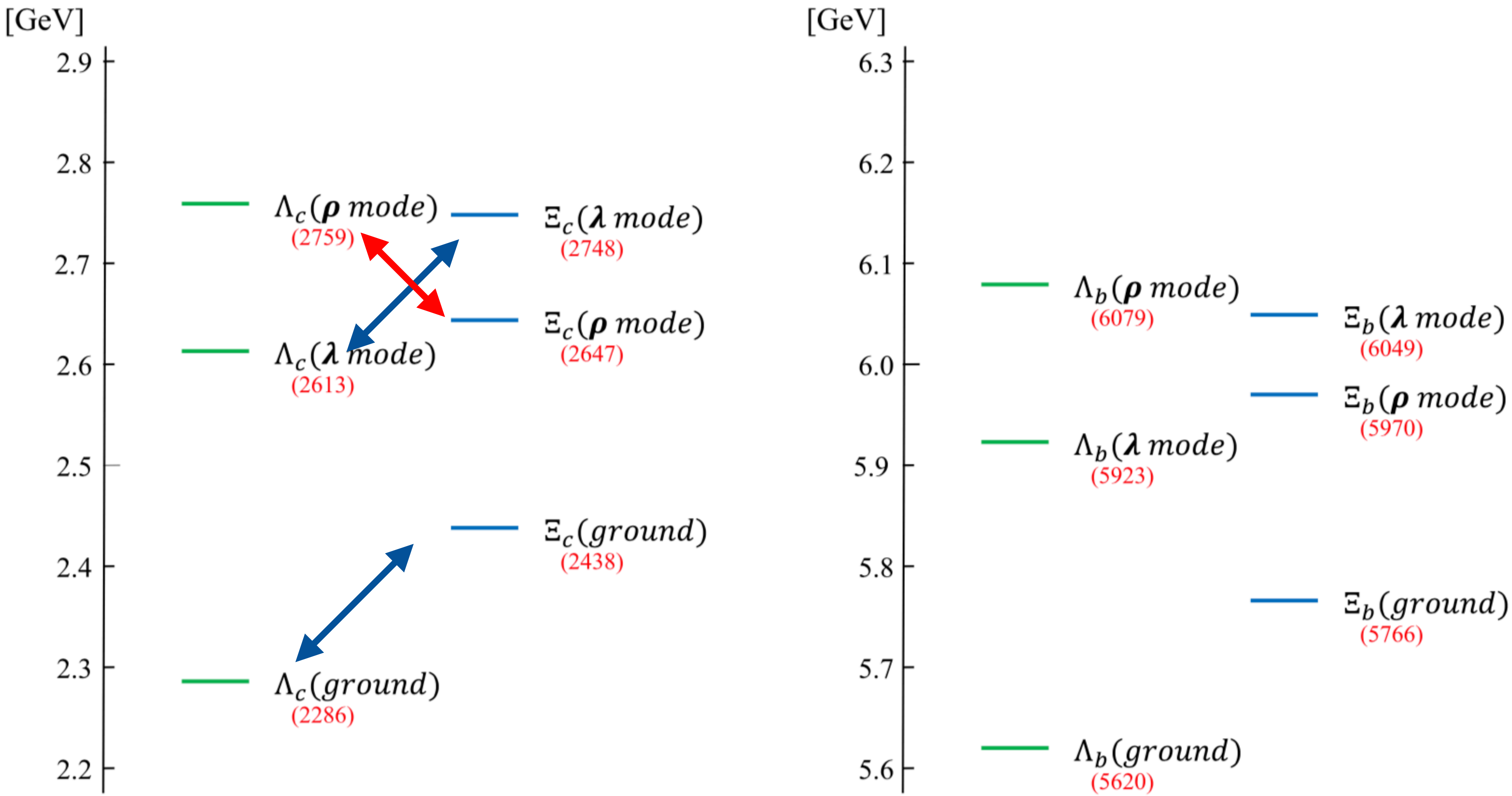
α	$\lambda(\mathrm{GeV}^2)$	$C_c(\mathrm{GeV})$	$C_b(\mathrm{GeV})$	$M_c(\mathrm{GeV})$	$M_b(\mathrm{GeV})$
$(2/3)\times 90/\mu$	0.165	-0.58418362	-0.58829590	1.750	5.112

B. Silvestre-Brac, C. Semay, Z. Phys. C 59, 457 (1993)
T. Yoshida, E. Hiyama, A. Hosaka, M. Oka, K. Sadato, PR D 92, 114029 (2015)

$M_{(ud)}(0^+) = 725$ MeV $M_{(ud)}(0^-) = 1265$ MeV
$M_{(us)}(0^+) = 906$ MeV $M_{(us)}(0^-) = 1142$ MeV
$M_{(qq)}(1^+) = 974$ MeV $M_{(qq)}(1^-) = 1447$ MeV
$M_{(qs)}(1^+) = 1116$ MeV $M_{(ss)}(1^+) = 1242$ MeV

Inverse mass hierarchy for Baryons

Y. Kim, E. Hiyama, M. O., K. Suzuki, Phys. Rev. D 102, 014004 (2020)



Axialvector/Vector Diquarks

Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

- **The $1^+/1^-$ diquarks in (3,3) representation**

$$d^{\mu a}_{ij} \equiv \epsilon_{abc}(q^{bT}_{iL}\, C\gamma^\mu\, q^c_{jR}) = \epsilon_{abc}(q^{bT}_{jR}\, C\gamma^\mu\, q^c_{iL}) \quad \text{chiral (3,3) vector diquark}$$

$$d^{\mu a}_{V[ij]} = d^{\mu a}_{ij} - d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_{i}\, C\gamma^\mu\gamma^5\, q^c_{j}) \quad \text{Vector } 1^- \text{ diquark, flavor } \bar{3}$$

$$d^{\mu a}_{A\{ij\}} = d^{\mu a}_{ij} + d^{\mu a}_{ji} = \epsilon_{abc}(q^{bT}_{i}\, C\gamma^\mu\, q^c_{j}) \quad \text{Axial-vector } 1^+ \text{ diquark, flavor } 6$$

$$d^\mu \longrightarrow U_L d^\mu U_R^T, \quad (3,3) \qquad d^{\mu\dagger} \longrightarrow U_R^{T\dagger} d^\mu U_L^\dagger \quad (\bar{3},\bar{3})$$

$$\mathcal{L} = -\frac{1}{2}\mathrm{Tr}[F^{\mu\nu}F^\dagger_{\mu\nu}] + m_0^2\mathrm{Tr}[d^\mu d^\dagger_\mu] + \frac{m_1^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger d^\mu \Sigma^T d^{\dagger T}_\mu] + \frac{2m_2^2}{f_\pi^2}\mathrm{Tr}[\Sigma^\dagger \Sigma d^{\mu T} d^{\dagger T}_\mu]$$

$$F^{\mu\nu} = D^\mu d^\nu - D^\nu d^\mu$$

- **All the terms are chiral and $U_A(1)$ invariant.**

Axialvector/Vector Diquarks

- **Using the masses of the 6-irrep single charm baryons, we determine the diquark masses.**

Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

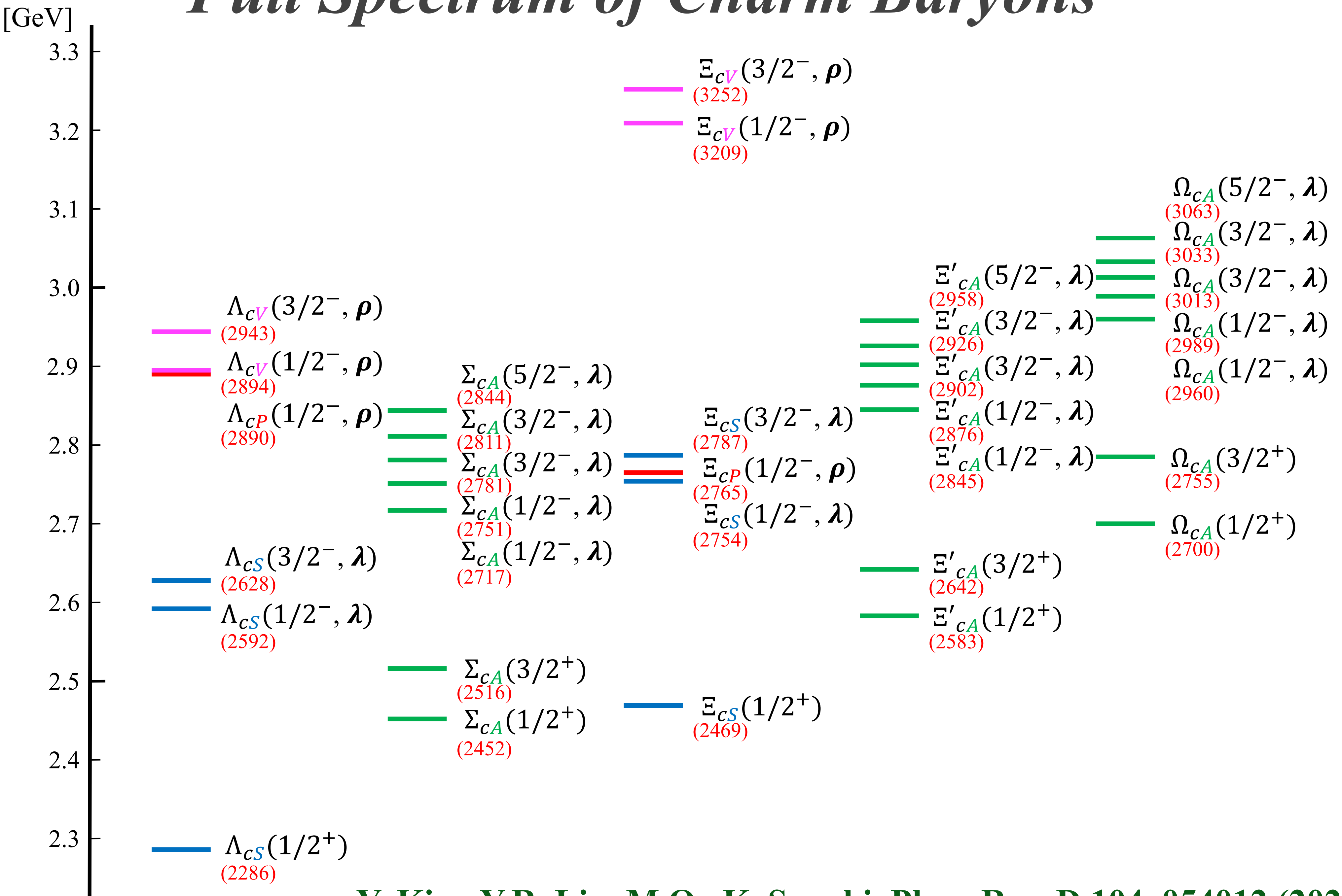
$$[M_{qq}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2,$$
$$[M_{qs}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2 + \epsilon(m_{V1}^2 + 2m_{V2}^2)$$
$$[M_{ss}(1^+)]^2 = m_{V0}^2 + m_{V1}^2 + 2m_{V2}^2 + 2\epsilon(m_{V1}^2 + 2m_{V2}^2)$$
$$[M_{qq}(1^-)]^2 = m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2,$$
$$[M_{qs}(1^-)]^2 = m_{V0}^2 - m_{V1}^2 + 2m_{V2}^2 + \epsilon(-m_{V1}^2 + 2m_{V2}^2)$$
$$\epsilon = A - 1 = \frac{f_s}{f_\pi}\left(1 + \frac{m_s}{g_s f_s}\right) - 1 \sim \frac{2}{3}$$

$M_{qq}(1^+)$ (MeV)	973.41
$M_{qs}(1^+)$ (MeV)	1115.98
$M_{ss}(1^+)$ (MeV)	1242.29
$M_{qq}(1^-)$ (MeV)	1446.72
$M_{qs}(1^-)$ (MeV)	1776.10
m_0^2 (MeV^2)	$(707.60)^2$
m_1^2 (MeV^2)	$-(756.79)^2$
m_2^2 (MeV^2)	$(713.99)^2$

- **The diquark masses satisfy the generalized “Gell-Mann-Okubo” mass formula approximately.**

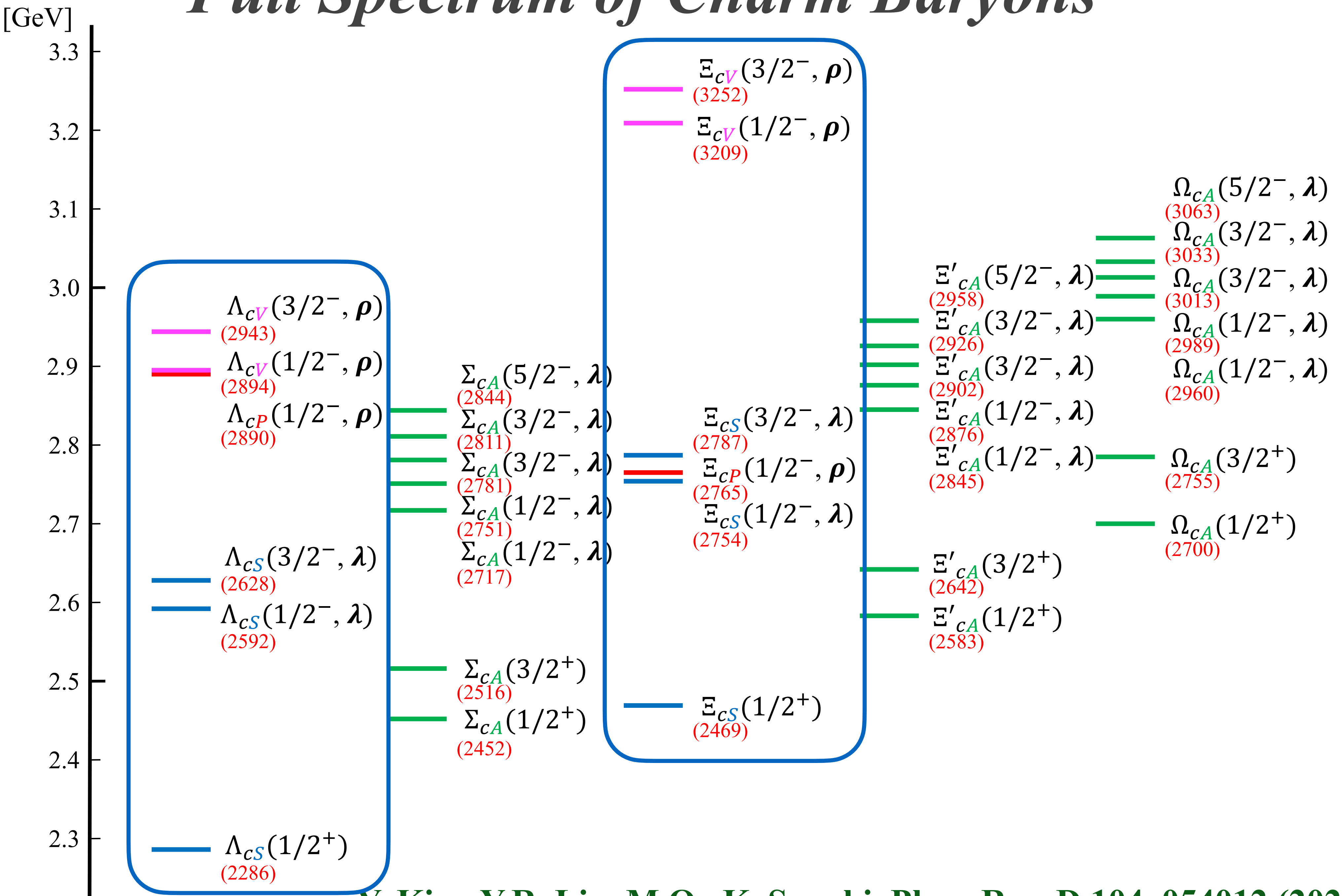
$$M_{ss}^2(1^+) - M_{qs}^2(1^+) = M_{qs}^2(1^+) - M_{qq}^2(1^+)$$

Full Spectrum of Charm Baryons



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Full Spectrum of Charm Baryons



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

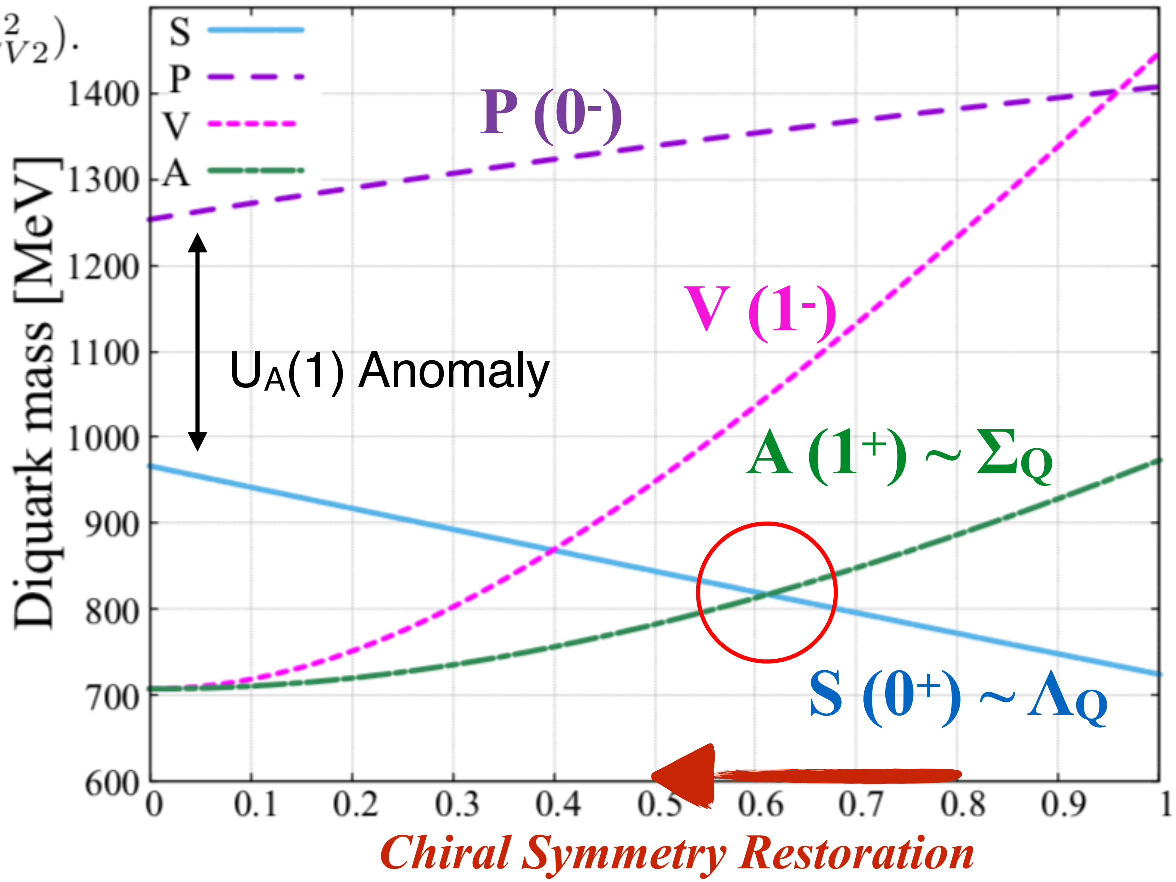
Mass crossing of S(0^+) and A(1^+)

- **Mass crossing of the 0+ and 1+ diquarks under chiral restoration**

$$M(0^+) = \sqrt{m_{S0}^2 - (x+\epsilon)m_{S1}^2 - x^2 m_{S2}^2},$$
$$M(1^+) = \sqrt{m_{V0}^2 + x^2(m_{V1}^2 + 2m_{V2}^2)}.$$

$$m_{S0}^2 = (1031\ \text{MeV})^2$$
$$m_{S1}^2 = (606\ \text{MeV})^2$$
$$m_{S2}^2 = -(274\ \text{MeV})^2$$

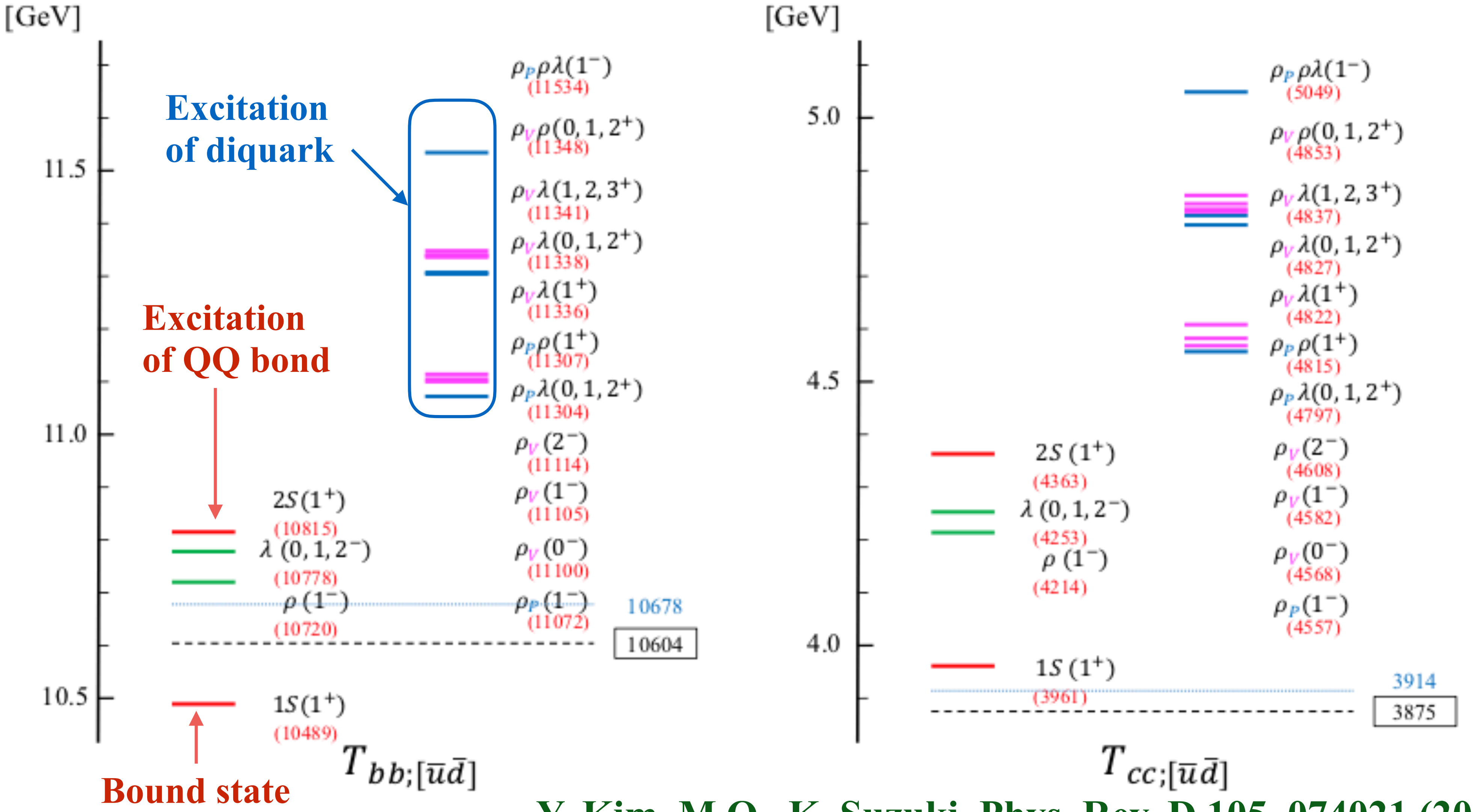
$$m_{V0}^2 = (708\ \text{MeV})^2$$
$$m_{V1}^2 = -(760\ \text{MeV})^2$$
$$m_{V2}^2 = (714\ \text{MeV})^2$$



Y. Kim, Y.R. Liu, M.O., K. Suzuki, Phys. Rev. D 104, 054012 (2021)

Doubly Heavy Tetraquarks

- T_{QQ} states with I=0 (Flavor 3)



Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

Comparison: masses of 1S ground states with prior researches

[1] Q. Meng, E. Hiyama, A. Hosaka, M. Oka, P. Gubler, and K.U. Can, T.T. Takahashi, and H.S. Zong, "Stable double-heavy tetraquarks: spectrum and structure", Phys. Lett. B, 814.136095. (2021).
[2] Eric Braaten, Li-Ping He, and Abhishek Mohapatra, "Masses of doubly heavy tetraquarks with error bars," Phys. Rev. D 103, 016001 (2021).
[3] E.J. Eichten and C. Quigg, "Heavy-quark symmetry implies stable heavy tetraquark mesons $QQ\bar{q}\bar{q}$", Phys. Rev. Lett. 119, 202002 (2017).
[4] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, "Lattice prediction for Deeply Bound Doubly Heavy Tetraquarks", Phys. Rev. Lett. 118, 142001 (2017).
[5] P. Junnarkar, N. Mathur and M. Padmanath "A study of doubly heavy tetraquarks in Lattice QCD", Phys. Rev. D 99, 034507 (2019).

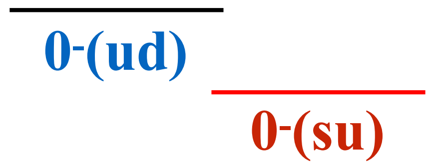
※Unit: MeV

Particle ($\mathbf{1^+}$)	$T_{bb;[\bar{u}\bar{d}]}$	$T_{bb;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$	$T_{cc;[\bar{u}\bar{d}]}$	$T_{cc;[\bar{d}\bar{s}]([\bar{s}\bar{u}])}$
(Threshold)	**10604**	**10692**	**3876**	**3977**
Ref. [1]	10444 (-160)	10625 (-67)	3865 (-11)	. . .
Ref. [2]	10471 (-133)	10644 (-48)	3947 (+71)	4124 (+147)
Ref. [3]	10482 (-122)	10643 (-49)	3978 (+102)	4156 (+179)
Ref. [4]	10415 (-189)	10594 (-98)	. . .	. . .
Ref. [5]	10461 (-143)	10605 (-87)	3853 (-23)	3969 (-8)
This work	**10489 (-115)**	**10664 (-28)**	**3961 (+85)**	**4141 (+164)**

Y. Kim, M.O., K. Suzuki, Phys. Rev. D 105, 074021 (2022)

Conclusion

- **Chiral effective theories of Scalar/Pseudoscalar diquarks and Axialvector/Vector diquarks are formulated.**
- **$U_A(1)$ anomaly is found to give the inverse mass hierarchy in the pseudoscalar diquark spectrum.**



- **Spectrum of Single Heavy Baryon is calculated based on the chiral picture of diquarks. Inverse mass hierarchy appears in ρ-mode excited states, and make the Λ_Q and Ξ_Q spectra largely different.**
- **Under chiral restoration, we find the mass crossing of the scalar and axialvector diquarks. This may give significant effects in the behaviors of heavy baryons in dense matter.**
- **Applying to doubly heavy tetraquarks, we find a bound T_{bb}, and series of excited states. Excitations of the *bb* relative motion lie below the excitation of light diquarks, which is consistent with the four-quark calculation.**