Analysis of coupled-channel potentials with quark and hadron degrees of freedom

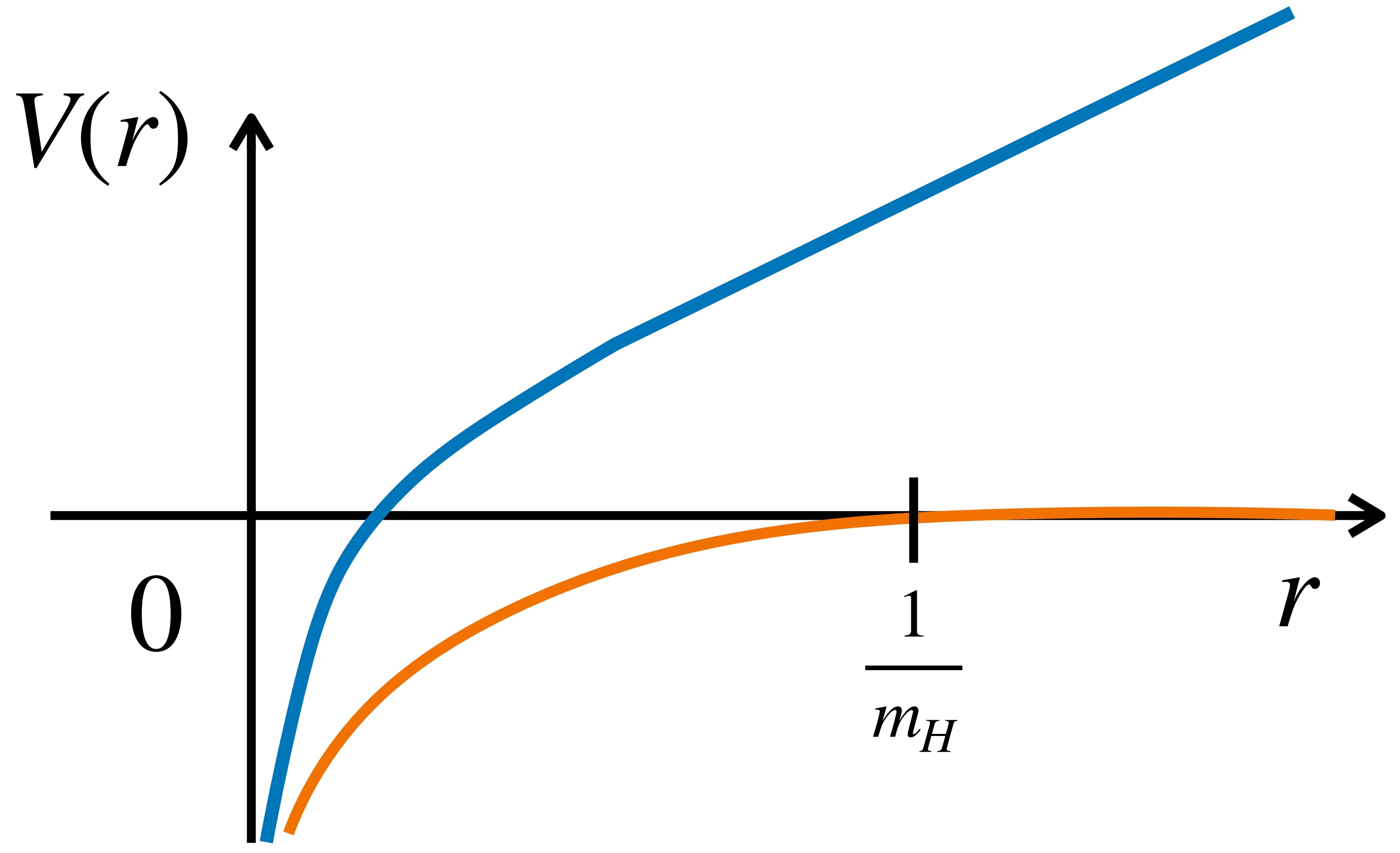
Ibuki Terashima (Tokyo Metropolitan University)
Tetsuo Hyodo (Tokyo Metropolitan University)

TOKYO METROPOLITAN UNIVERSITY
東京都立大学

Motivation

$$V^{\bar{c}c}(r) = -\frac{A}{r} + \sigma r + V_0 \xrightarrow[r\to\infty]{} \infty$$: Confinement potential

$$V^{\bar{D}D}(r) = K_{\bar{D}D}\frac{\exp[-m_H r]}{r} + \cdots \xrightarrow[r\to\infty]{} 0$$: Scattering potential



Recent topics

- experimental observation of **exotic charmonia X, Y, Z**
- **Lattice QCD** for potentials
  - ➡ $V^{\bar{c}c}(r)$ and $V^{\bar{D}D}(r)$ have been calculated independently

We study

- **Channel couplings** between $\bar{c}c$ and $\bar{D}D$ potentials
- Properties of **effective potentials** by eliminating one of the channels

Formulation

Hamiltonian H with coupled-channel between $\bar{c}c$ and $\bar{D}D$:

Kinetic energy — Threshold Energy — Transition potential

$$H = \begin{pmatrix} T^{\bar{c}c} & 0 \\ 0 & T^{\bar{D}D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{\bar{c}c} & V^t \\ V^t & V^{\bar{D}D} \end{pmatrix}$$

→ Defined as local potentials

Eliminate $\bar{D}D$ to obtain effective Hamiltonian $H^{\bar{c}c}_{\mathrm{eff}}(E)$ with $H^{\bar{c}c}_{eff}(E)|\Psi^{\bar{c}c}\rangle = E|\Psi^{\bar{c}c}\rangle$[1]

$$H^{\bar{c}c}_{\mathrm{eff}}(E) = T^{\bar{c}c} + \underline{V^{\bar{c}c} + V^t G^{\bar{D}D}(E) V^t}$$

$$G^{\bar{D}D}(E) = (E - T^{\bar{D}D} - \Delta - V^{\bar{D}D} + i0^+)^{-1}$$

Effective potential $\hat{V}^{\bar{c}c}_{eff}$

(w/o approximation)

In the same way, eliminating $\bar{c}c$

$$H^{\bar{D}D}_{\mathrm{eff}}(E) = T^{\bar{D}D} + \Delta + V^{\bar{D}D} + V^t G^{\bar{c}c}(E) V^t$$

$$G^{\bar{c}c}(E) = (E - T^{\bar{c}c} - V^{\bar{c}c} + i0^+)^{-1}$$

[1] H. Feshbach, Ann. Phys. **5**, 357 (1958); ibid., **19**, 287 (1962)

Result

Effects of channel coupling

$$\propto F_n^*(\boldsymbol{r}') \cdot F_n(\boldsymbol{r})$$

$$\langle \boldsymbol{r}'_{\bar{D}D} | V_{\rm eff}^{\bar{D}D}(E) | \boldsymbol{r}_{\bar{D}D} \rangle = V^{\bar{D}D}\delta(\boldsymbol{r}' - \boldsymbol{r}) + \sum_n \frac{\langle \boldsymbol{r}'_{\bar{D}D} | V^t | \phi_n \rangle \langle \phi_n | V^t | \boldsymbol{r}_{\bar{D}D} \rangle}{E - E_n}$$

$(T^{\bar{c}c} + V^{\bar{c}c} | \phi_n \rangle = E_n | \phi_n \rangle)$

discrete eigenstates by $V^{\bar{c}c}$

$$\propto f(\boldsymbol{r}') \cdot f(\boldsymbol{r})$$

$$\langle \boldsymbol{r}'_{\bar{c}c} | V_{\rm eff}^{\bar{c}c}(E) | \boldsymbol{r}_{\bar{c}c} \rangle = V^{\bar{c}c}\delta(\boldsymbol{r}' - \boldsymbol{r}) + \int d\boldsymbol{p} \frac{\langle \boldsymbol{r}'_{\bar{c}c} | V^t | \boldsymbol{p}_{\rm full} \rangle \langle \boldsymbol{p}_{\rm full} | V^t | \boldsymbol{r}_{\bar{c}c} \rangle}{E - E_{\boldsymbol{p}} + i0^+}$$

$(T^{\bar{D}D} + \Delta + V^{\bar{D}D} | \boldsymbol{p}_{\rm full} \rangle = E_p | \boldsymbol{p}_{\rm full} \rangle)$

continuous eigenstates by $V^{\bar{D}D}$

- Both potentials have non-locality regardless of the nature of $\hat{V}^t$
- Both potentials have energy dependence

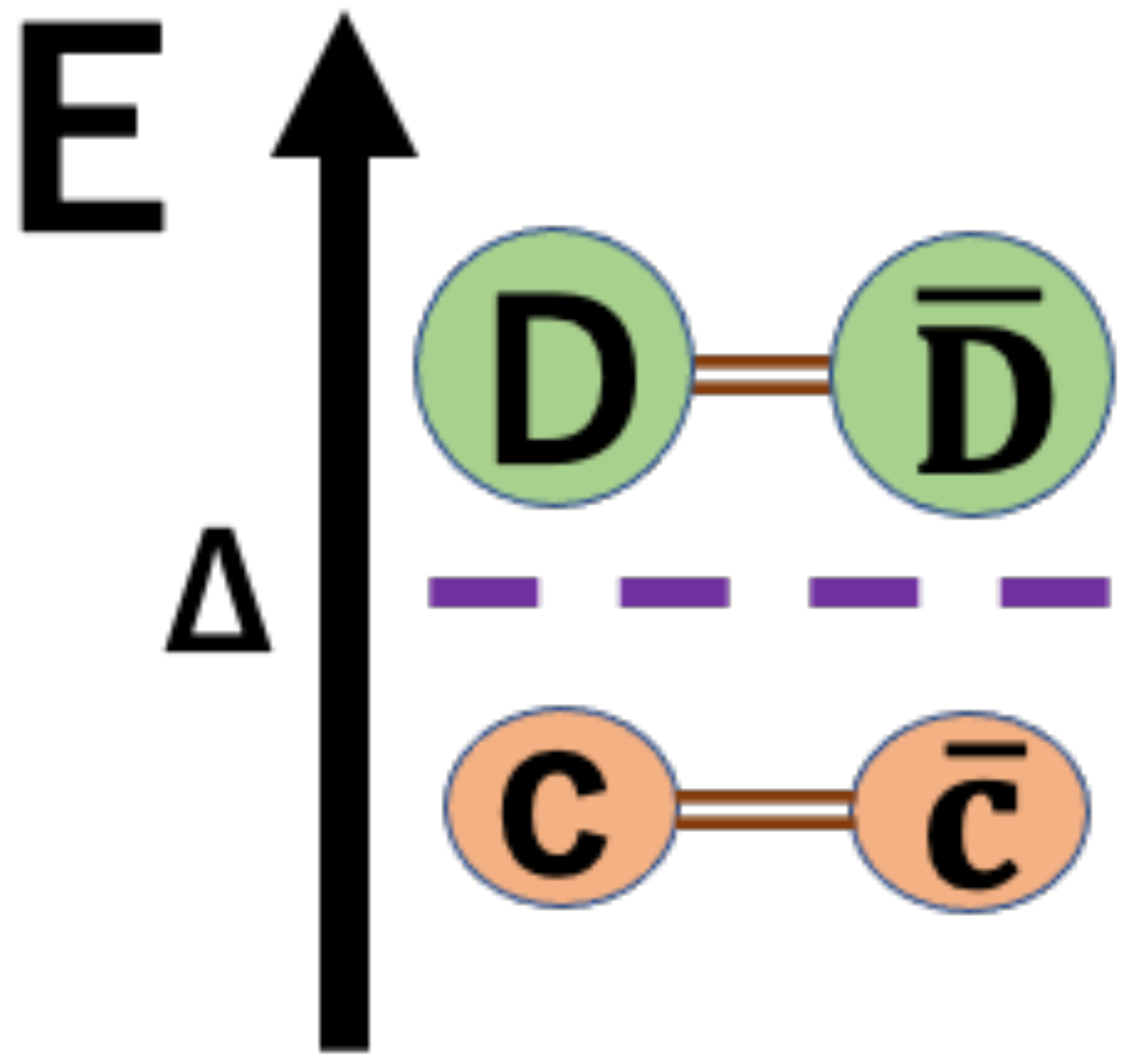
Result

Details of inter-quark potential $\hat{V}^{\bar{c}c}_{\rm eff}(E)$

$$\langle \boldsymbol{r}'_{\bar{c}c} | V^{\bar{c}c}_{\rm eff}(E) | \boldsymbol{r}_{\bar{c}c} \rangle = V^{\bar{c}c}\delta(\boldsymbol{r}' - \boldsymbol{r}) + \int d\boldsymbol{p} \frac{\langle \boldsymbol{r}'_{\bar{c}c} | V^t | \boldsymbol{p}_{\rm full} \rangle \langle \boldsymbol{p}_{\rm full} | V^t | \boldsymbol{r}_{\bar{c}c} \rangle}{E - E_{\boldsymbol{p}} + i0^+} \qquad (E_{\boldsymbol{p}} \geq \Delta (\in \mathbb{R}))$$

$\hat{V}^{\bar{c}c}_{\rm eff}(E)$ has an additional feature

- When $E \geq \Delta$, $\hat{V}^{\bar{c}c}_{\rm eff}(E)$ has an imaginary part because of a pole at $E = E_{\boldsymbol{p}}$
- This represents the decay process of $\bar{c}c$ into $\bar{D}D$ properly



Result

$\hat{V}^{\bar{c}c}_{\text{eff}}(E)$ with derivative expansions

To represent $\hat{V}^{\bar{c}c}_{\text{eff}}(E)$ as a local potential, performing a Fourier transformation and a derivative expansion for $(E - E_{\boldsymbol{p}} + i0^+)^{-1}$ at $|\boldsymbol{p}|^2$

$$\langle \boldsymbol{r}'_{\bar{c}c} | V^{\bar{c}c}_{\text{eff}}(E) | \boldsymbol{r}_{\bar{c}c} \rangle = V^{\bar{c}c}(\boldsymbol{r})\delta(\boldsymbol{r}' - \boldsymbol{r}) + \int d\boldsymbol{p} e^{i\boldsymbol{p}\cdot(\boldsymbol{r}'-\boldsymbol{r})} V^t(\boldsymbol{r}') V^t(\boldsymbol{r}) \left[\frac{1}{E - \Delta} + O(|\boldsymbol{p}|^2) \right]$$

$$= V^{\bar{c}c}(\boldsymbol{r})\delta(\boldsymbol{r}' - \boldsymbol{r}) + \underbrace{\boxed{\frac{[V^t(\boldsymbol{r})]^2}{E - \Delta}\delta(\boldsymbol{r}' - \boldsymbol{r})}}_{\in \mathbb{R}} + O(\nabla^2)$$

➡ Since the expanded potential does **not** have an imaginary part, this expansion may break the physical nature of potential

Summary

- Considering **channel couplings** between $V^{\bar{c}c}(r)$ and $V^{\bar{D}D}(r)$
- By eliminating one of the channels, $V_{\rm eff}^{\bar{D}D}(E)$ and $\hat{V}_{\rm eff}^{\bar{c}c}(E)$ have **non-locality** and **energy dependence**
- $\hat{V}_{\rm eff}^{\bar{c}c}(E)$ in more detail,
    - $\hat{V}_{\rm eff}^{\bar{c}c}(E)$ has an **imaginary part** above the threshold Δ

  ➡ This may not be realized by finite terms in the derivative expansion

Any questions are welcome!