

Energy spectra of $^{15}_{\Xi}\text{C}$ and $^{12}_{\Xi}\text{Be}$, and ΞN two-body interaction

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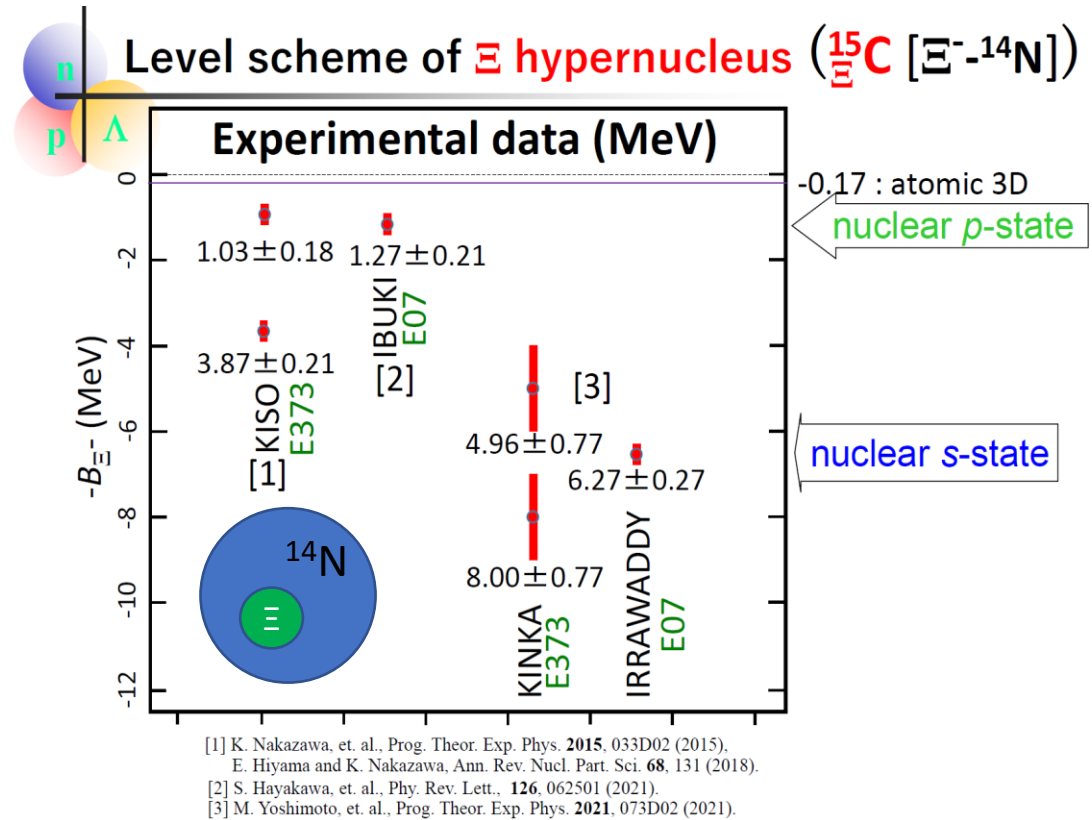
Tingting Sun (Zhenzhou Univ.)



Phys. Rev. C **105**, 044324 (2022)

Information on $N\Xi$ interactions

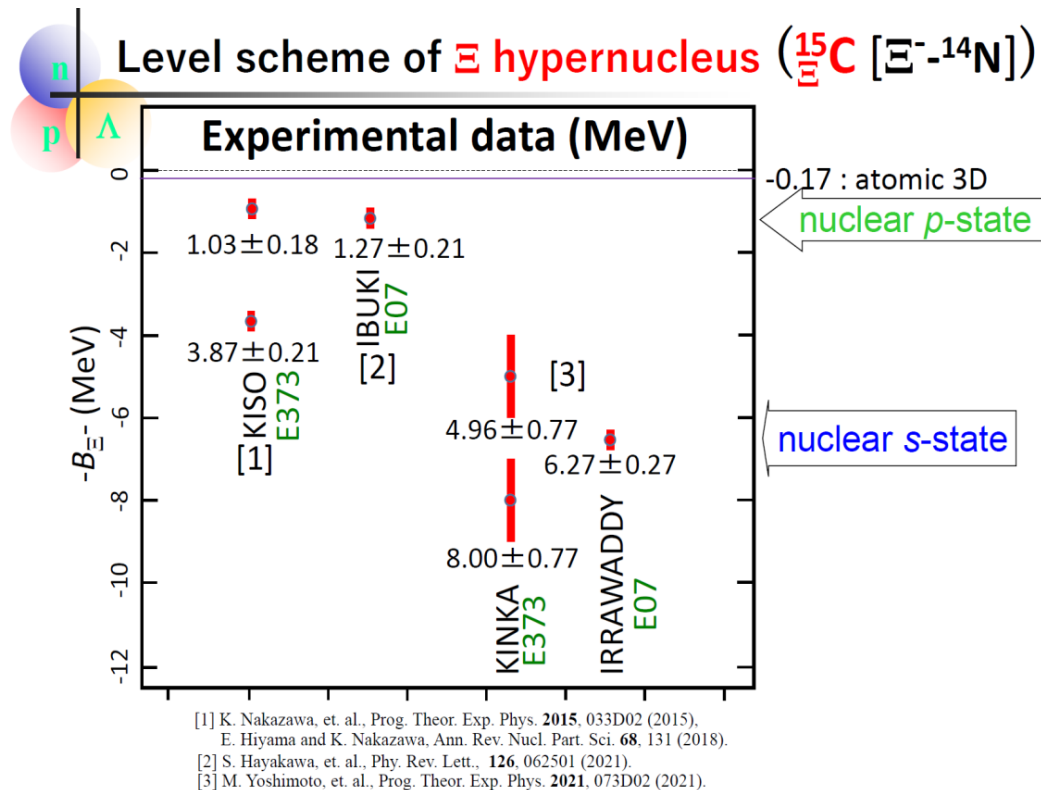
Recent experimental data of ${}_{\Xi}^{15}\text{C} \rightarrow$
 Provide information on $S=-2$ sector of B-B force



Yoshimoto et al., PTEP **2021**, 073D02 (2021).

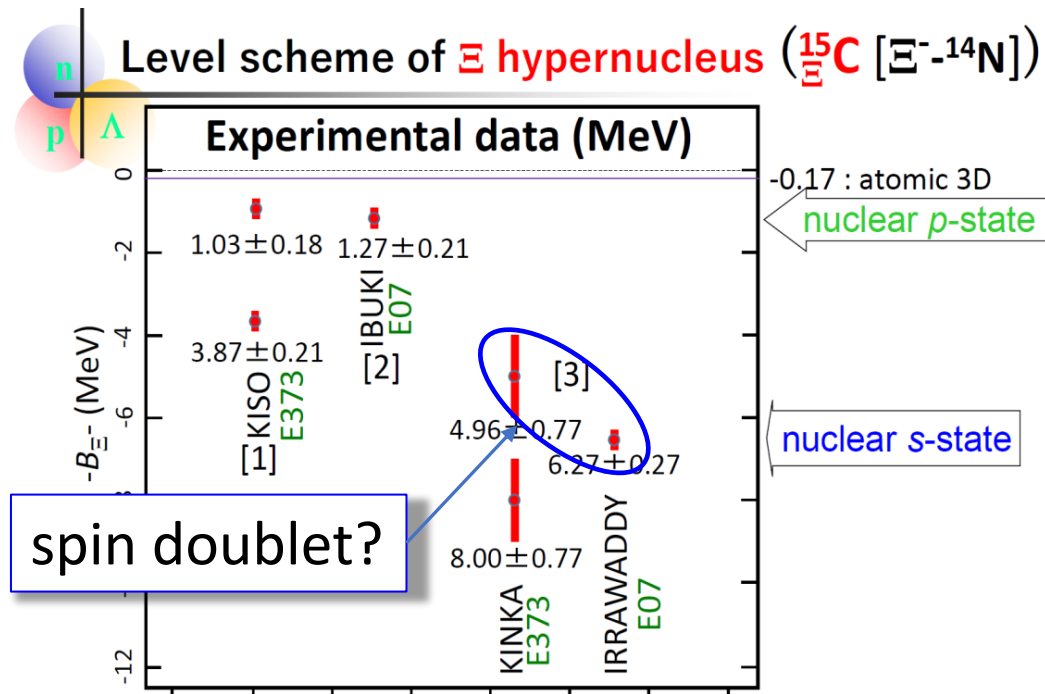
Purpose

- Interpretation of $^{15}_{\Xi}\text{C}$ data
- Estimate of spin-dependent ΞN interaction
- Spectrum $^{12}_{\Xi}\text{Be}$? (future expt. at J-PARC)



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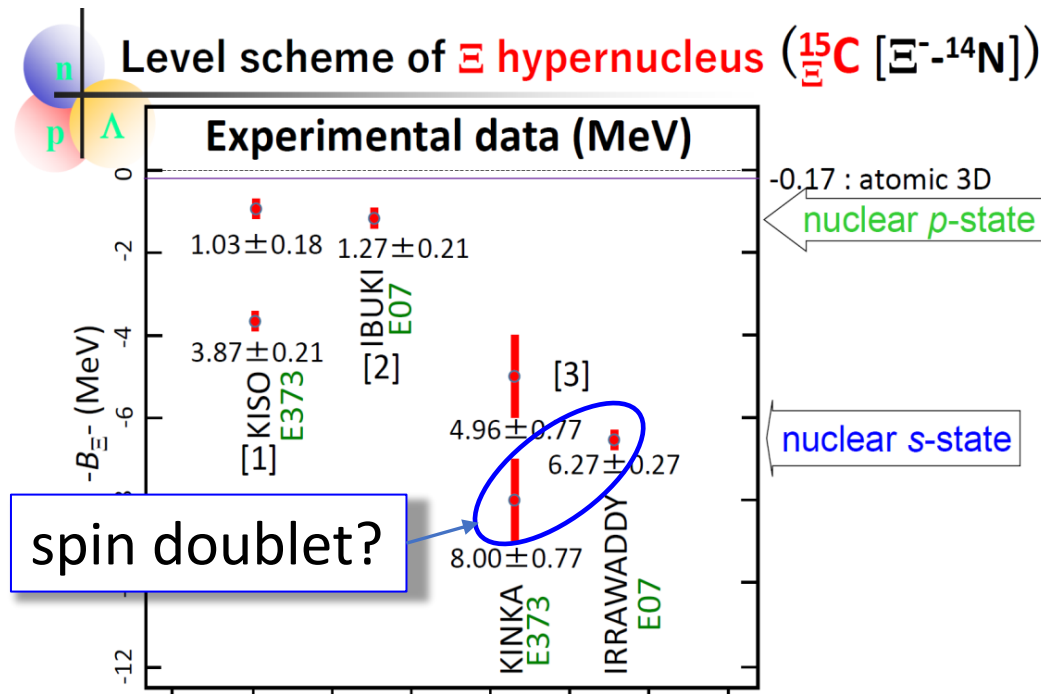
$$V_{N\Xi} \sim \sigma_N \cdot \sigma_{\Xi}$$

[1] K. Nakazawa, et. al., Prog. Theor. Exp. Phys. **2015**, 033D02 (2015), E. Hiyama and K. Nakazawa, Ann. Rev. Nucl. Part. Sci. **68**, 131 (2018).
 [2] S. Hayakawa, et. al., Phy. Rev. Lett., **126**, 062501 (2021).
 [3] M. Yoshimoto, et. al., Prog. Theor. Exp. Phys. **2021**, 073D02 (2021).

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$$V_{\Xi\text{N}} \sim \sigma_{\text{N}} \cdot \sigma_{\Xi}$$

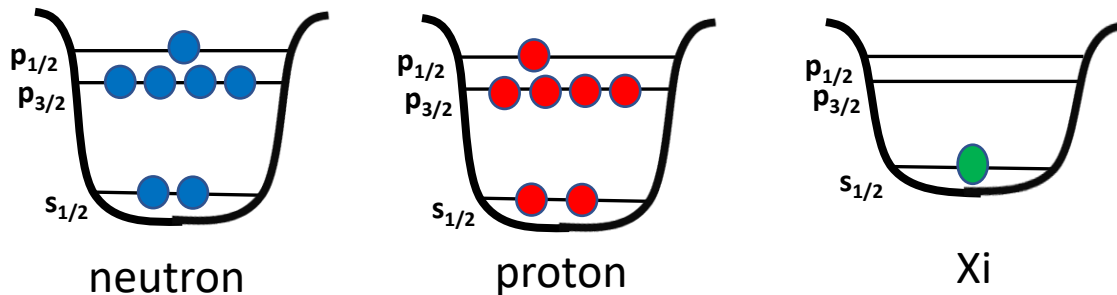


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Method

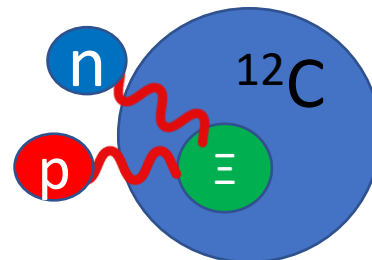
1. Relativistic mean-field (RMF) calculation

→ single-particle levels



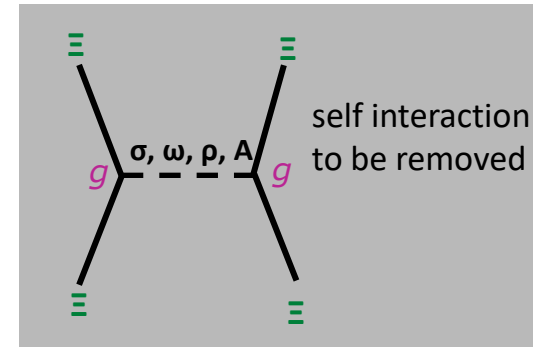
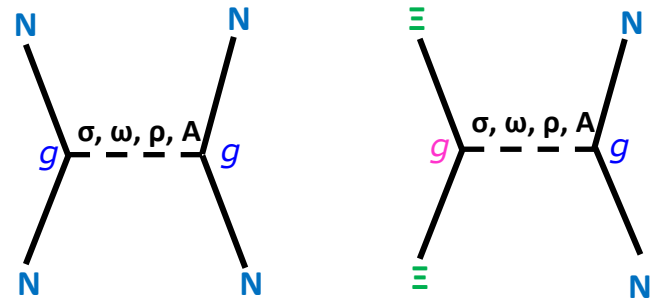
2. N- Ξ residual interaction (simple shell model)

→ energy spectrum of $^{15}_{\Xi}\text{C}$



Relativistic mean field (RMF) theory

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi}_N (i\partial\!\!\!/ - m_N) \psi_N + \bar{\psi}_\Xi (i\partial\!\!\!/ - m_\Xi) \psi_\Xi \\
 & + \frac{1}{2} (\partial_\mu \sigma) (\partial^\mu \sigma) - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{c_3}{3} \sigma^3 - \frac{c_4}{4} \sigma^4 \\
 & - \frac{1}{4} G^{\mu\nu} G_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu + \frac{d_4}{4} (\omega^\mu \omega_\mu)^2 \\
 & - \frac{1}{4} \vec{R}^{\mu\nu} \cdot \vec{R}_{\mu\nu} + \frac{1}{2} m_\rho^2 \vec{\rho}^\mu \cdot \vec{\rho}_\mu + \frac{g_4}{4} (\vec{\rho}^\mu \cdot \vec{\rho}_\mu)^2 \\
 & - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \\
 & - \bar{\psi}_N \left(g_{\sigma N} \sigma + g_{\omega N} \not{\omega} + g_{\rho N} \vec{\rho} \cdot \vec{\tau} + eA \frac{1 - \tau_3}{2} \right) \psi_N \\
 & - \bar{\psi}_\Xi \left(g_{\sigma \Xi} \sigma + g_{\omega \Xi} \not{\omega} + g_{\rho \Xi} \vec{\rho} \cdot \vec{\tau} \right. \\
 & \quad \left. + \frac{f_{\omega \Xi}}{4m_\Xi} G_{\mu\nu} \sigma^{\mu\nu} - A \frac{\tau_3 - 1}{2} \right) \psi_\Xi
 \end{aligned}$$



Nucleon sector: PK1 Long et al., PRC 69 034319 ('04)

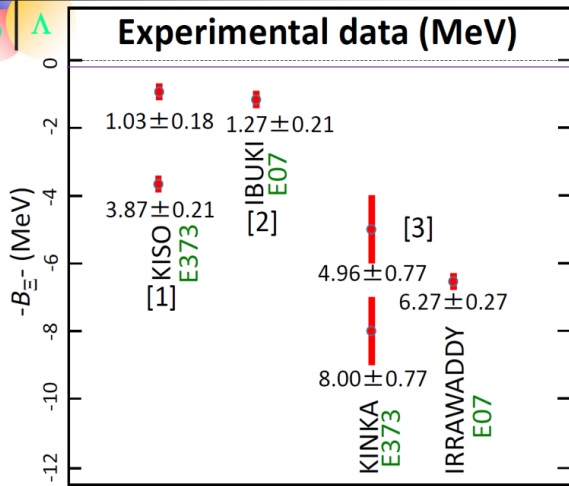
Hyperon sector: Fit to ${}_{\Xi}^{15}\text{C}$ spectrum

Relativistic mean field (RMF) theory

Parameter fitting:

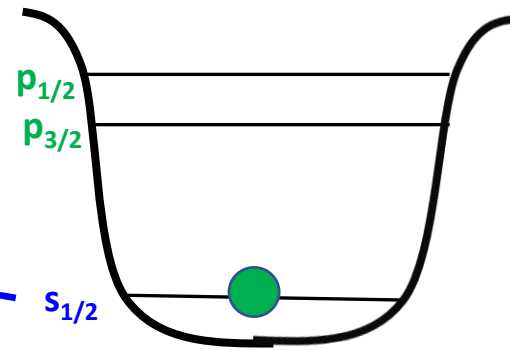
- $g_{\sigma\Xi}, g_{\omega\Xi}$: $B_{\Xi s} \approx 6 \text{ MeV}$, $B_{\Xi p} \approx 1 \text{ MeV}$
- ($f_{\omega\Xi}$: $p_{3/2}$ - $p_{1/2}$ splitting as small as Λ)

Level scheme of Ξ hypernucleus ($^{15}_{\Xi}\text{C}$ [Ξ^{-} - ^{14}N])



-0.17 : atomic 3D
nuclear p-state

nuclear s-state



$$U_{\Xi}(\mathbf{r}) \approx g_{\sigma\Xi}\sigma(\mathbf{r}) + g_{\omega\Xi}\omega(\mathbf{r})$$

[1] K. Nakazawa, et al., Prog. Theor. Exp. Phys. **2015**, 033D02 (2015),
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Relativistic mean field (RMF) theory

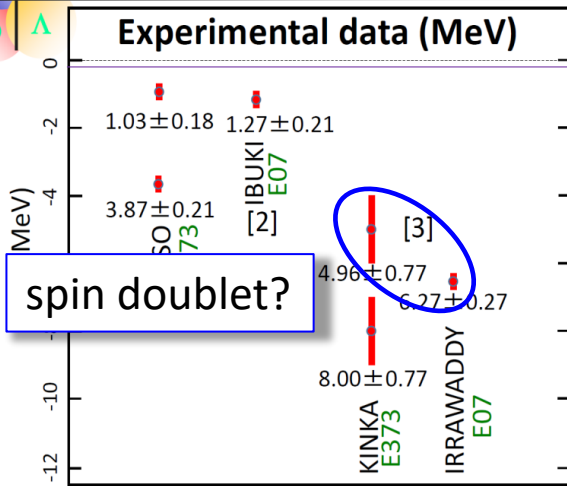
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Coupling	Fitted (a)	Fitted (b)
$g_{\sigma\Xi}/g_{\sigma N}$	0.497	0.497
$g_{\omega\Xi}/g_{\omega N}$	0.5733	0.5663
$f_{\omega\Xi}/g_{\omega\Xi}$	-0.8	-0.8

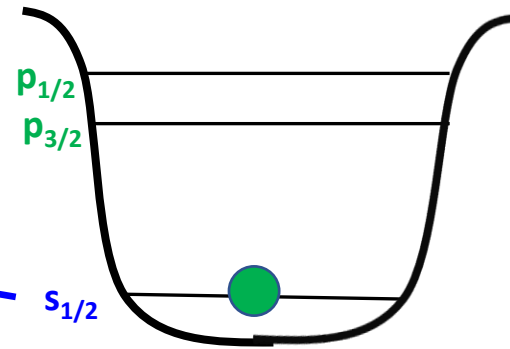
cf. Hu et al., J. Phys. G49, 025104 (2022),
QMF calculation

Level scheme of Ξ hypernucleus ($^{15}_{\Xi}\text{C}$ [$\Xi^{-14}\text{N}$])



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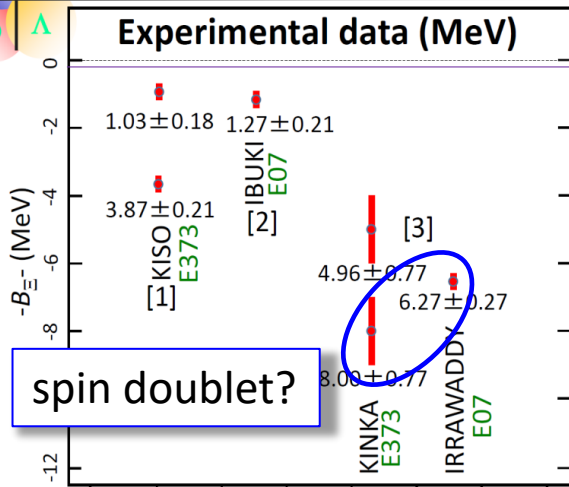
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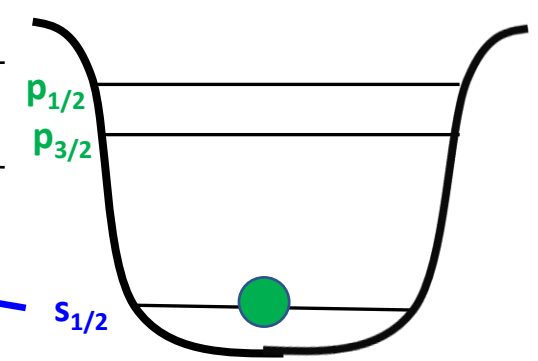
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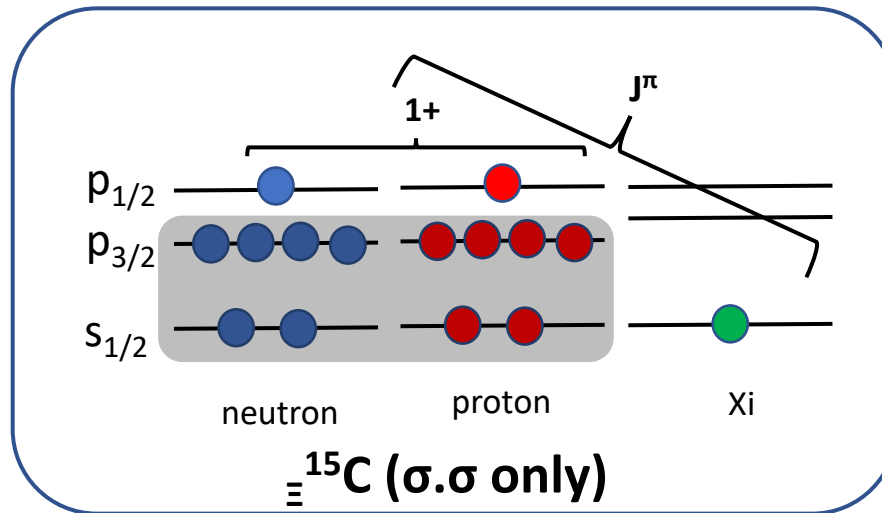
Spin-isospin dependent residual interaction

s-wave interaction

$$V = \sum_{i \in \text{nucleons}} (v_{\sigma} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{\Xi} + v_{\tau} \vec{\tau}_i \cdot \vec{\tau}_{\Xi} + v_{\sigma\tau} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{\Xi} \vec{\tau}_i \cdot \vec{\tau}_{\Xi}) \delta(\mathbf{r}_i - \mathbf{r}_{\Xi})$$

Energy shift due to V (first-order perturbation)

$$\Delta E = \langle V \rangle = \left\langle \left[\left[\nu 1p_{1/2} \pi 1p_{1/2} \right]_{1+} \xi n l_j \right]_{J\pi} \left| V \right| \left[\left[\nu 1p_{1/2} \pi 1p_{1/2} \right]_{1+} \xi n l_j \right]_{J\pi} \right\rangle$$



g.s. doublet (Ξ in $s_{1/2}$):
 $J^{\pi} = 3/2^{+}$ and $1/2^{+}$

Spin-isospin dependent residual interaction

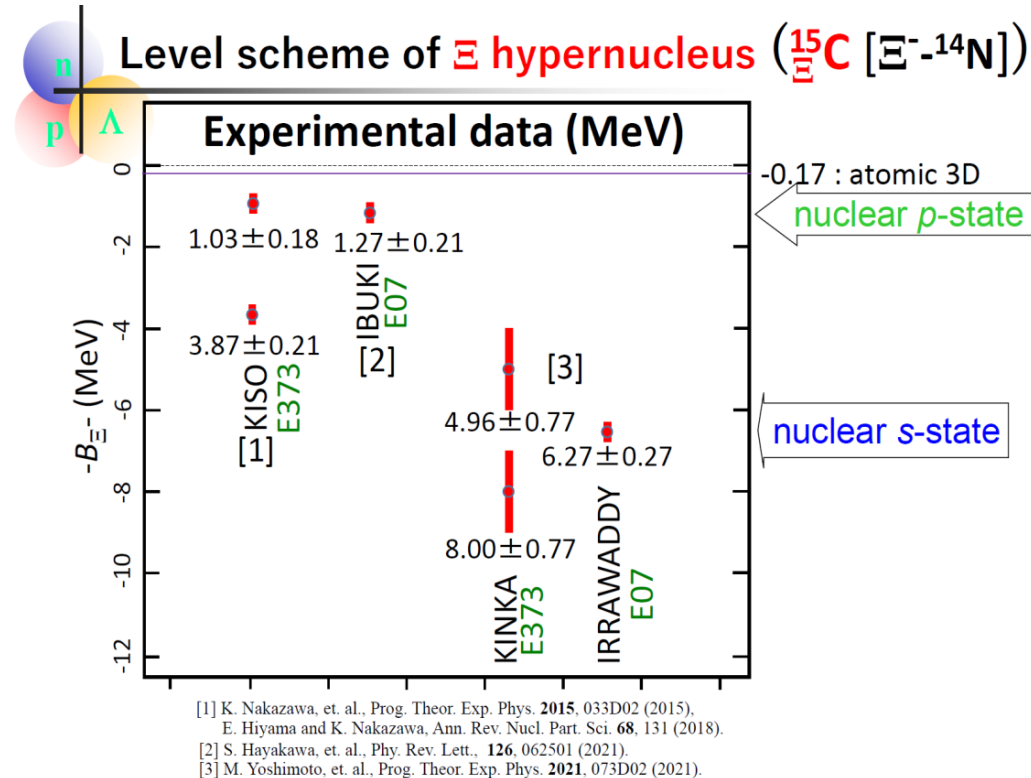
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The strength parameters are estimated based on HAL QCD N- Ξ s-wave potential

Sasaki et al., NPA998, 121737 (2020).

→ Too weak to reproduce
KINKA-IRRAWADDY splitting



Spin-isospin dependent residual interaction

s-wave interaction

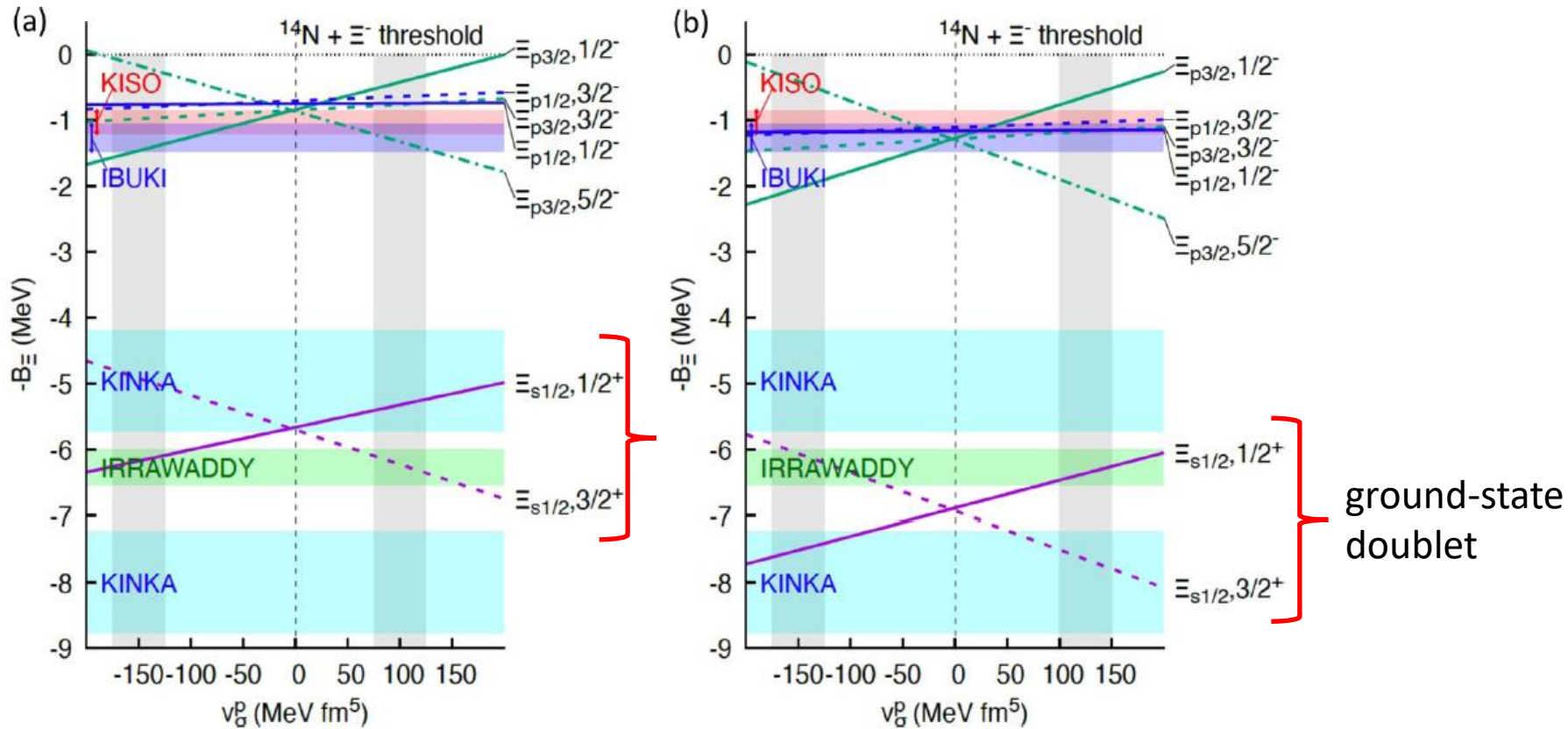
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p-wave interaction

$$V^p = \sum_{i \in \text{nucleons}} v_{\sigma}^p \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_{\Xi} \overleftarrow{\nabla} \cdot \delta(\mathbf{r}_i - \mathbf{r}_{\Xi}) \overrightarrow{\nabla}$$

$$\nabla = \nabla_i - \nabla_{\Xi}$$

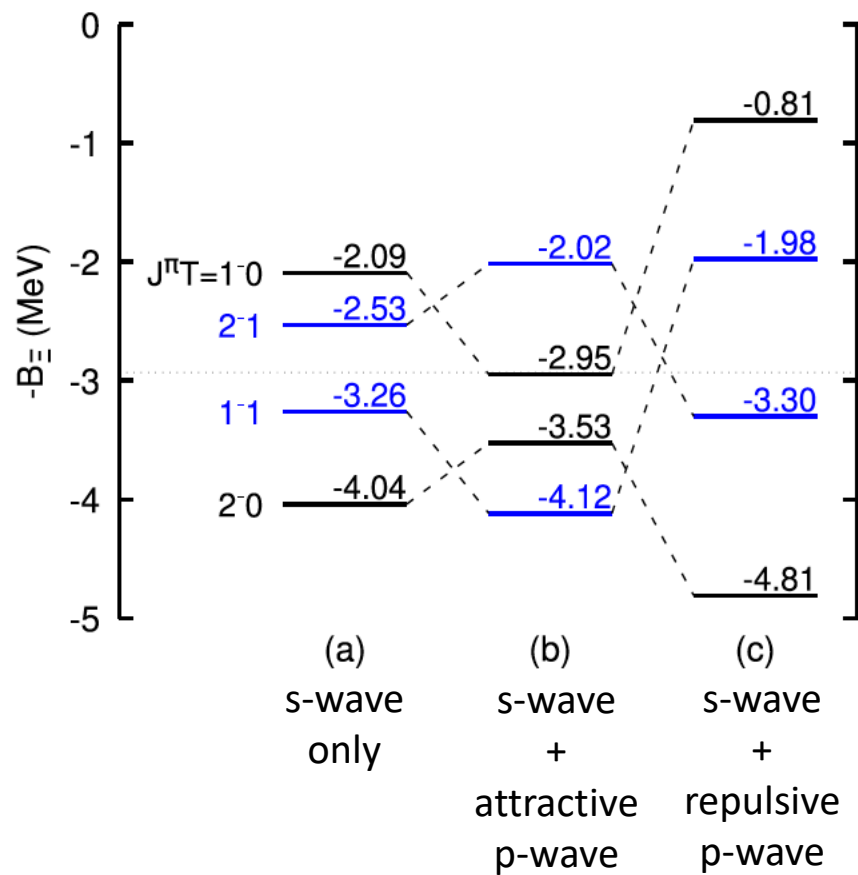
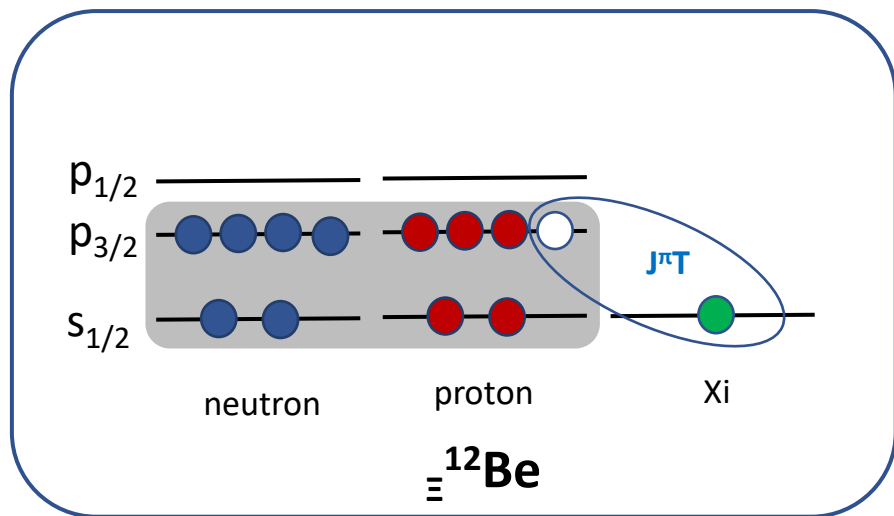
→ Energy spectra of $^{15}_{\Xi}C$ as functions of p -wave interaction strength



$$V^p = \sum_{i \in \text{nucleons}} v_{\sigma}^p \sigma_i \cdot \sigma_{\Xi} \overleftrightarrow{\nabla} \cdot \delta(\mathbf{r}_i - \mathbf{r}_{\Xi}) \overleftrightarrow{\nabla}$$

Both positive (~ 100 MeV.fm 5) and negative (~ -150 MeV.fm 5) v_{σ}^p can reproduce **KINKA** and **IRRAWADDY** as spin-doublet states

Spectrum of $^{12}_{\Xi}\text{Be}$



Summary

Qualitative interpretation of new data of $^{15}_{\Xi}\text{C}$

→ KINKA and IRRAWADDY: g.s. spin doublet

Spin-dependent p-wave $\text{N}\Xi$ interaction is important?

