

Λ Polarization (P_Λ) Measurement of the $\pi^- p \rightarrow K^0 \Lambda$ Reaction in J-PARC E40 Experiment

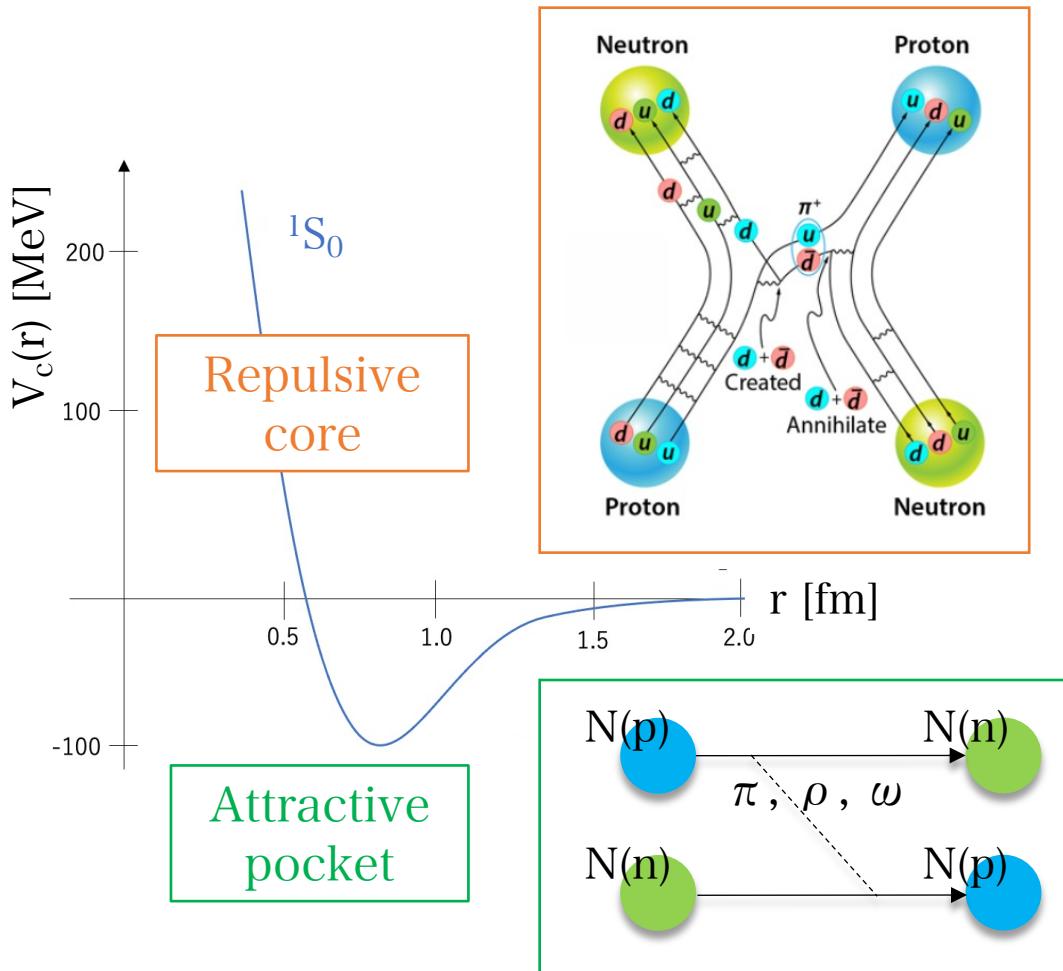
Tamao SAKAO (Tohoku University.),
for the J-PARC E40, P86 Collaborations.

HYP2022, June 27th - July 1st, 2022

BB Interactions for Realistic Nuclear Force

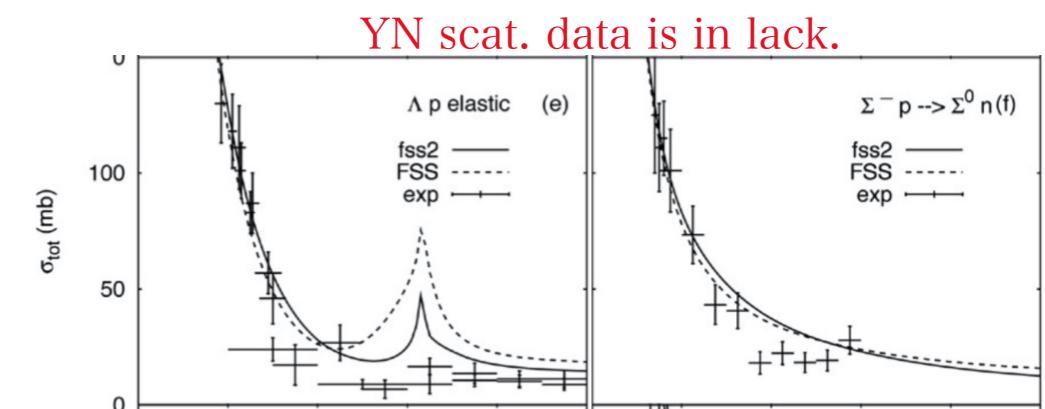
Nuclear physics

- NN potentials are the foundation.
 - Based on NN scat. data.

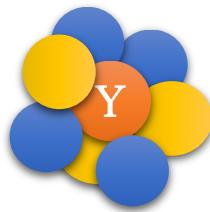


Hypernuclear physics

- To expand the nuclear force study.
- BB int. models:
 - Nijmegen model (Boson exchange)
 - Quark cluster model
 - χ EFT
 - ... etc.
- To update the theories (from exp.)
 - Precise two (few-body) YN (YNN) data
 - To make a more realistic YN int.
 - Accurate B_A & level scheme data
 - Compare the realistic BB int.



Experimental Approach for YN Interaction

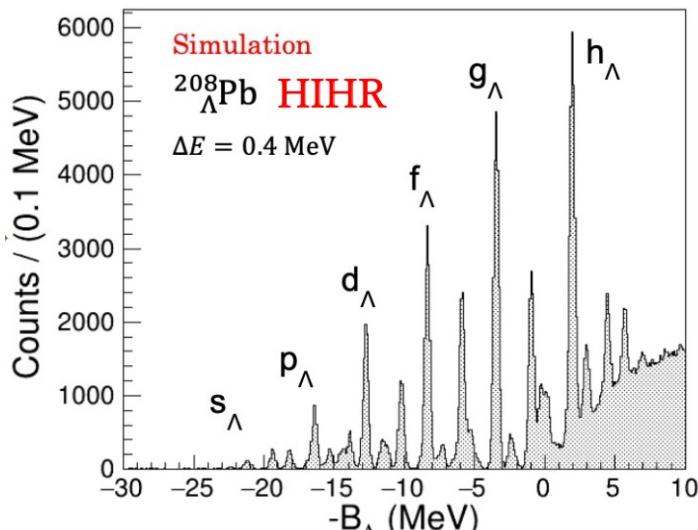


Hypernuclear

- In J-PARC, Jefferson Lab, KEK ...

In the future

- Accurate B_A & level scheme data.
 - J-PARC (K1.1 & HIHR),
J-Lab, Emulsion ...

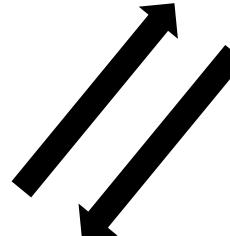
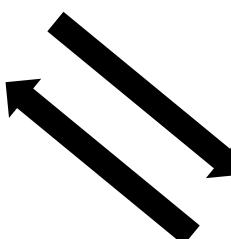


YN scattering

Hypertriton

- Precise two (few)-body YN (YNN) data.
- To get a more realistic two (few)-body int.
- From a scattering exp. side
 - The J-PARC E40 exp. pioneered the YN scattering technique!!

→ The new Λp scat. exp. is under the plan.



BB int. models

- Realistic YN int. by two (few)-body data
- Many-body int. based on density dependence

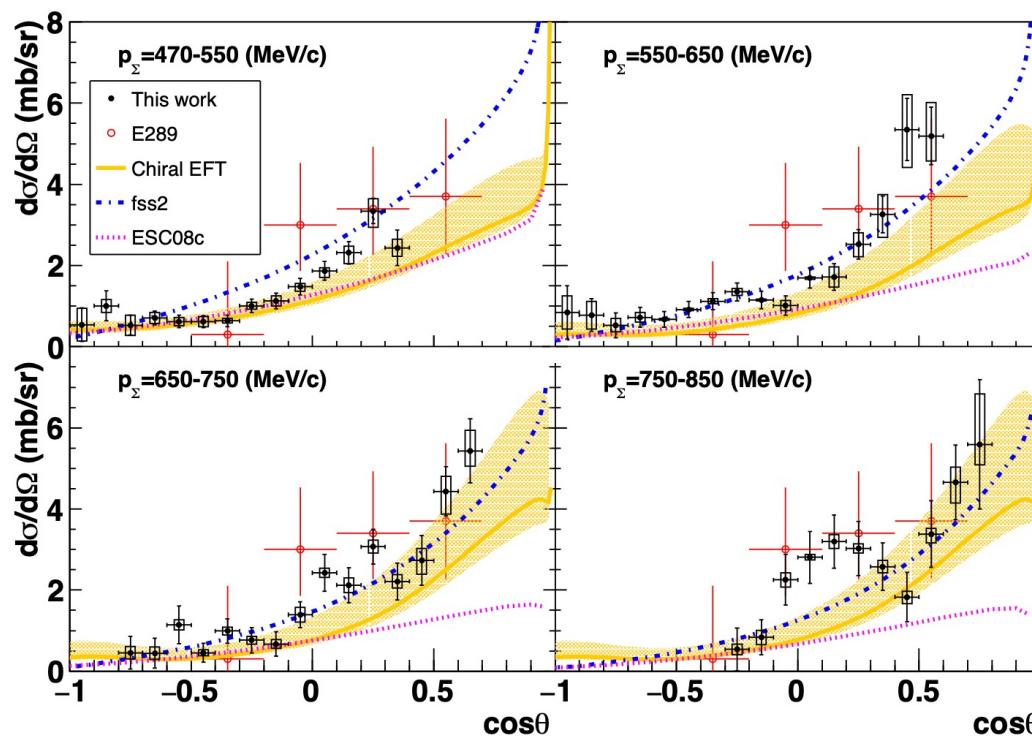
Toward the Next Λp Scattering Experiment in J-PARC

J-PARC E40 exp. (Σp scat.)

- High-statistics YN scattering method.
- The first-time Σp $d\sigma/d\Omega$ data.

K. Miwa et al., PRC 104, 045204 (2021)

Differential cross sections of $\Sigma^- p$ scattering



Next Λp scattering exp. (J-PARC P86)

- $d\sigma/d\Omega$ in 0.4 – 0.8 GeV/c.
- Spin observables:
 - Analyzing power, A_y

$$A_y = \frac{\pi}{2P_\Lambda} \frac{\left(\frac{d\sigma}{d\Omega}\right)_L - \left(\frac{d\sigma}{d\Omega}\right)_R}{\left(\frac{d\sigma}{d\Omega}\right)_L + \left(\frac{d\sigma}{d\Omega}\right)_R}$$

- Depolarization, D^{yy}

$$P_{\Lambda p \text{ scat.}} = \frac{P + \mathbf{D}_y^y P_\Lambda \cos \phi}{1 + P P_\Lambda \cos \phi} = \frac{2 N_U - N_D}{\alpha N_U + N_D}$$

* P_Λ : Λ' polarization.

* ϕ : Angle b/w Λ and Λ' spin axis.

Toward the Next Λp Scattering Experiment in J-PARC

J-PARC E40 exp. (Σp scat.)

- High-statistics YN scattering method.
- The first-time Σp $d\sigma/d\Omega$ data.

The $\pi^- p \rightarrow K^0 \Lambda$ reaction data
as a by-product

Next Λp scattering exp. (J-PARC P86)

- $d\sigma/d\Omega$ in 0.4 – 0.8 GeV/c.
- Spin observables:
 - Analyzing power, A_y

$$A_y = \frac{\pi}{2P_\Lambda} \frac{\left(\frac{d\sigma}{d\Omega}\right)_L - \left(\frac{d\sigma}{d\Omega}\right)_R}{\left(\frac{d\sigma}{d\Omega}\right)_L + \left(\frac{d\sigma}{d\Omega}\right)_R}$$

- Depolarization, D^{y_y}

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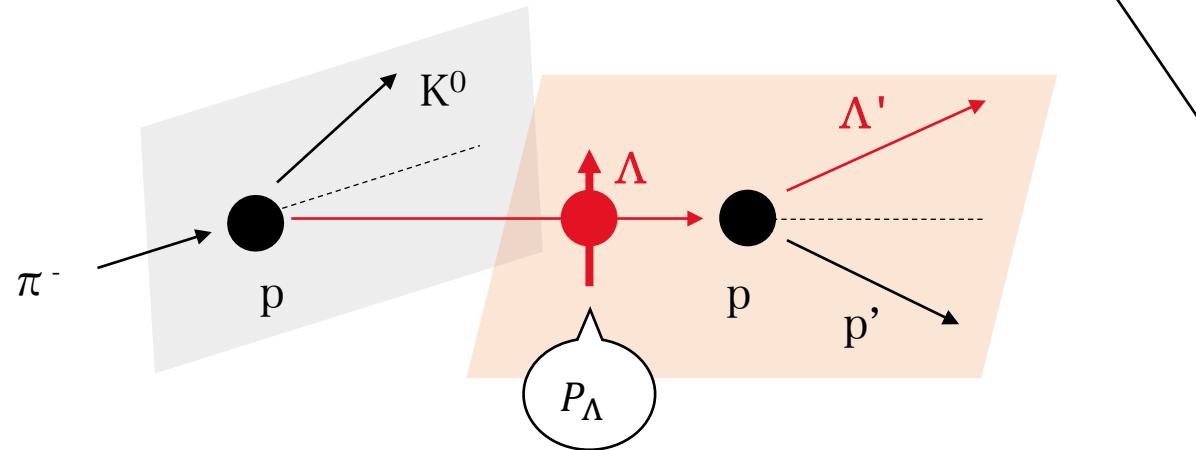
* P_Λ : Λ' polarization.
* ϕ : Angle b/w Λ and Λ' spin axis.

Feasibility study for P86

J-PARC P86 Exp. (Λp Scat.)

Points

- Λ is produced by the $\pi^- p \rightarrow K^0 \Lambda$ reaction.
- $d\sigma/d\Omega$, A_y , and D^{yy}

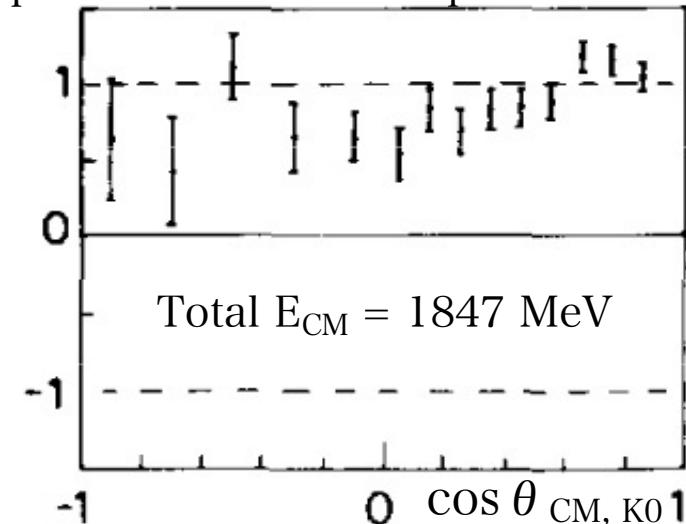


Λ produced by the $\pi^- p \rightarrow K^0 \Lambda$ reaction
should be polarized 100%!!

The past data of Λ polarization (P_Λ)

- P_Λ is ~ 100% in the K^0 -forward region.
- Low statistics.

Λ polarization in the $\pi^- p \rightarrow K^0 \Lambda$ reaction



R. D. Baker et al., Nucl. Phys. B 141, 29 (1978).

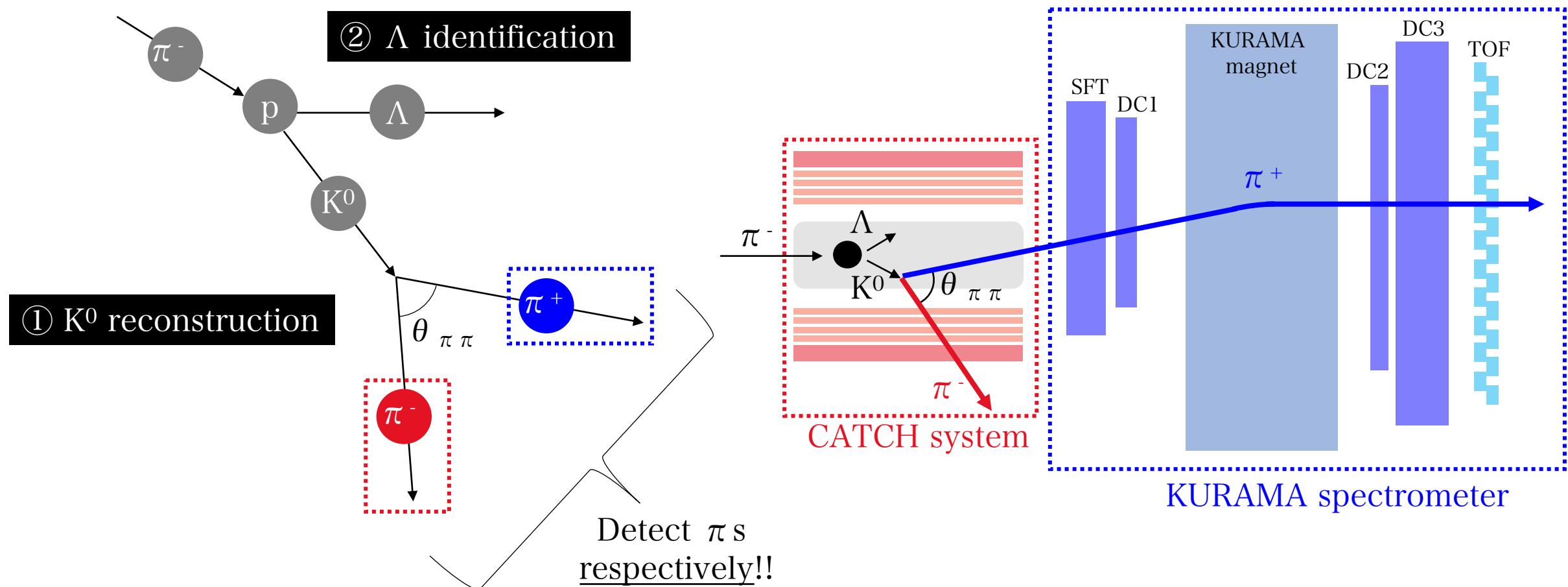
P_Λ Analysis

- Verifies if the Λ beam is polarized 100% or not.
- With the $\pi^- p \rightarrow K^0 \Lambda$ reaction data (E40 by-product data).

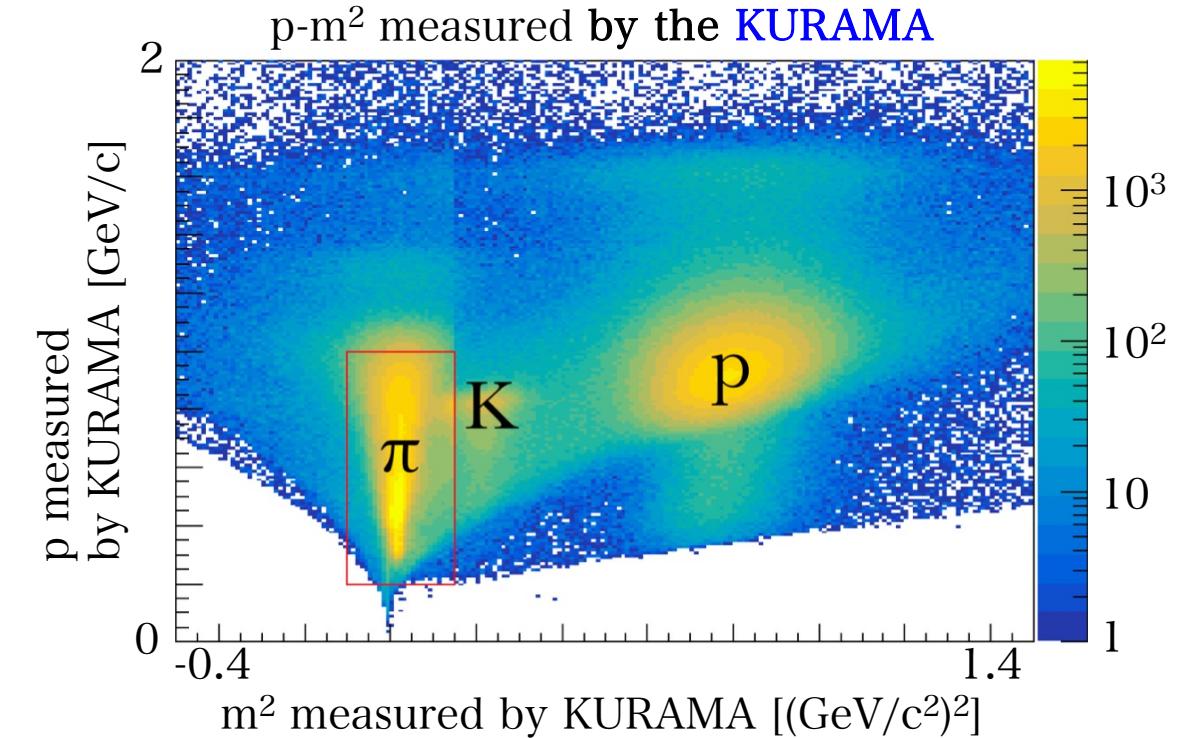
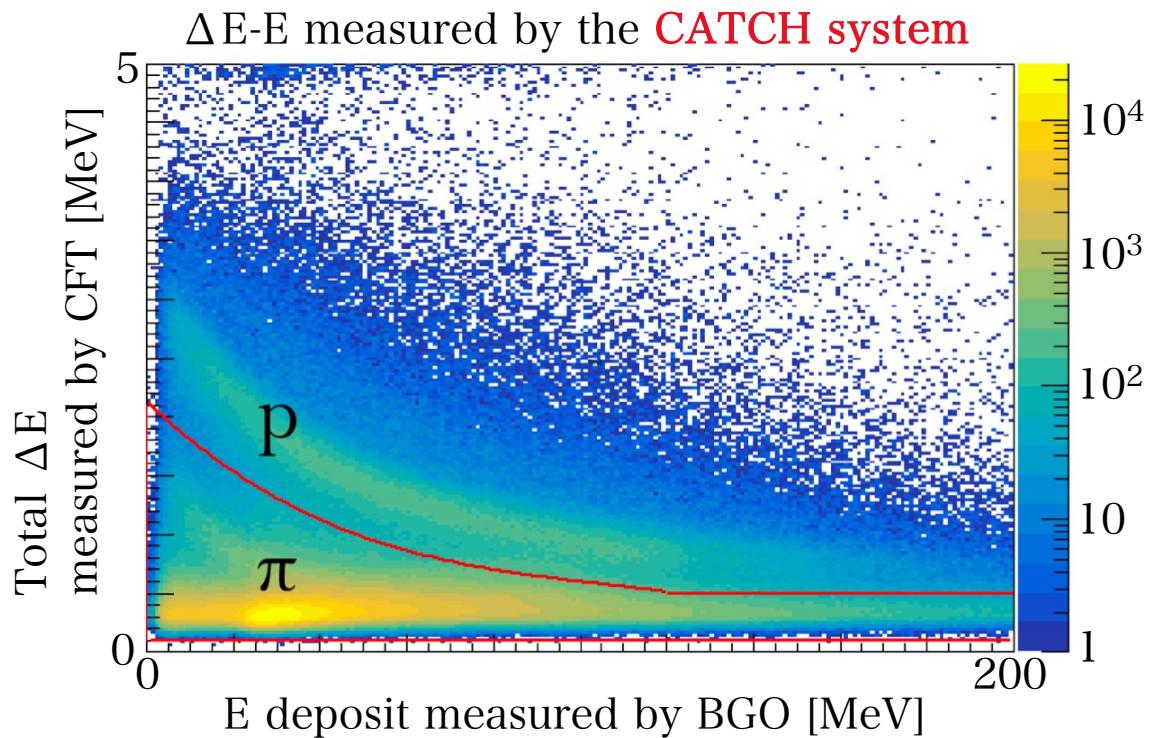
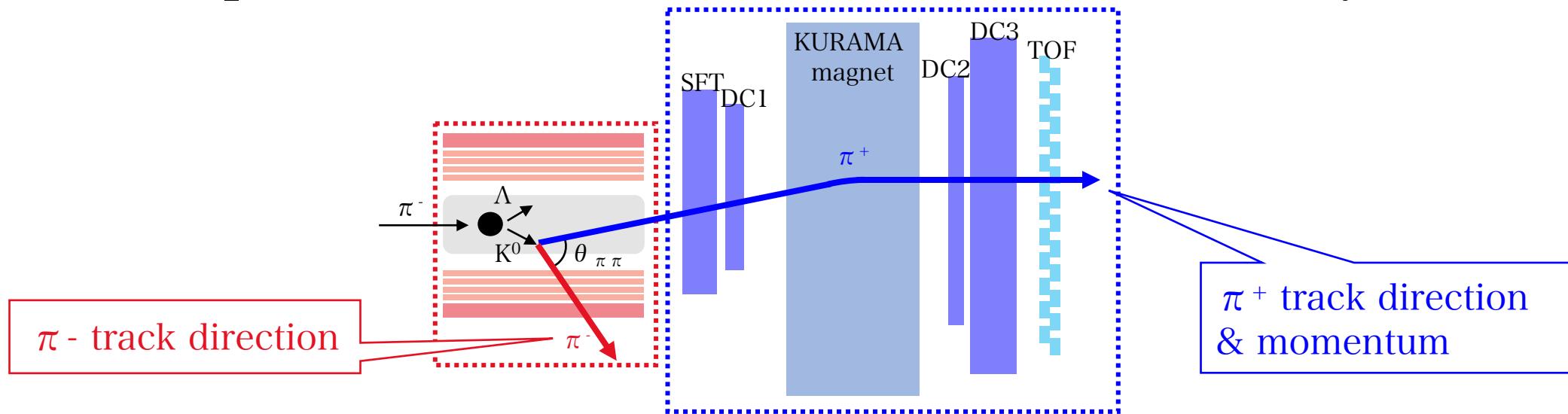
K⁰ Reconstruction in the E40

In the E40 by-product data analysis as the feasibility study of P86

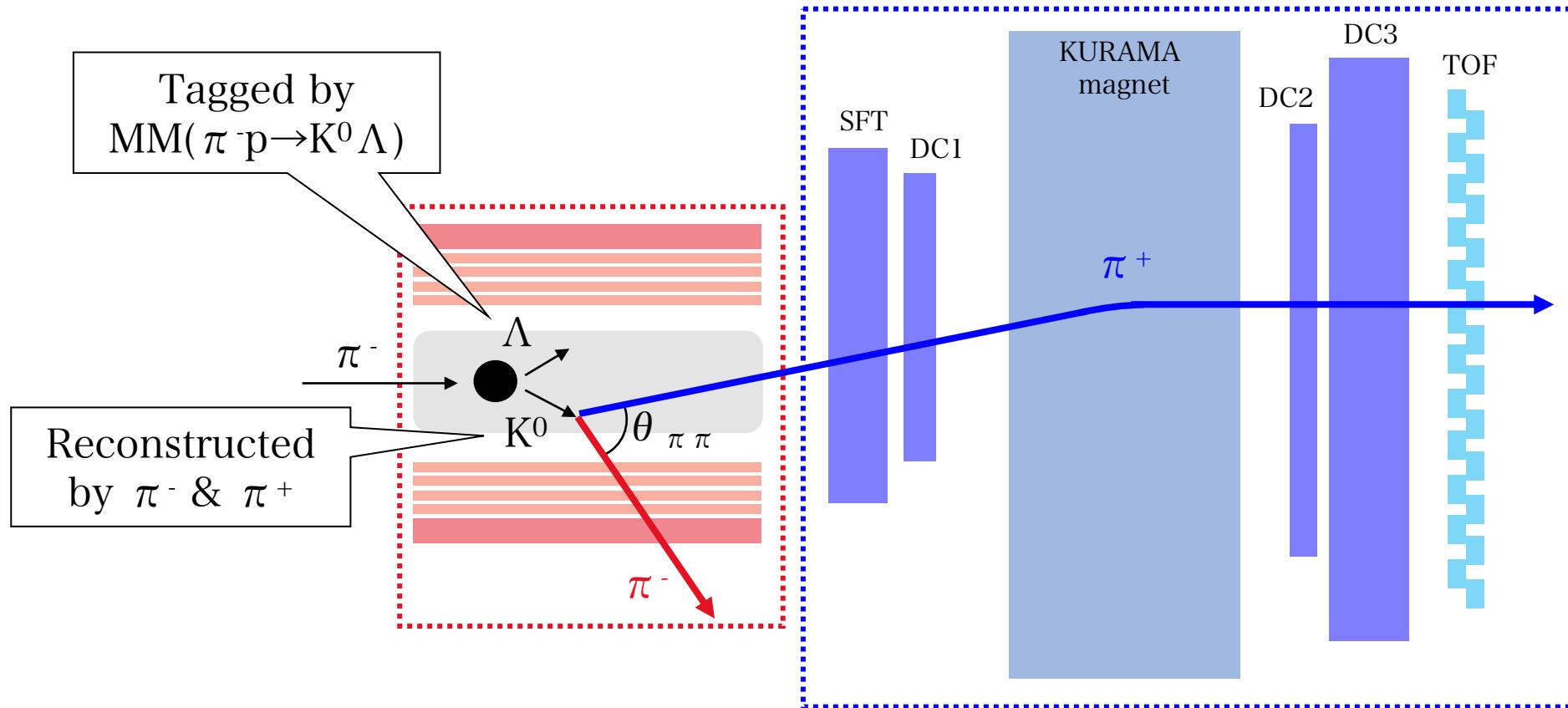
- Apply the P86 detection assuming the E40 setup.



Respective Detection of the $K^0 \rightarrow \pi^- \pi^+$ Decay



Λ Tagging After the Particle ID



To measure the MM($\pi^- p \rightarrow K^0 X$)

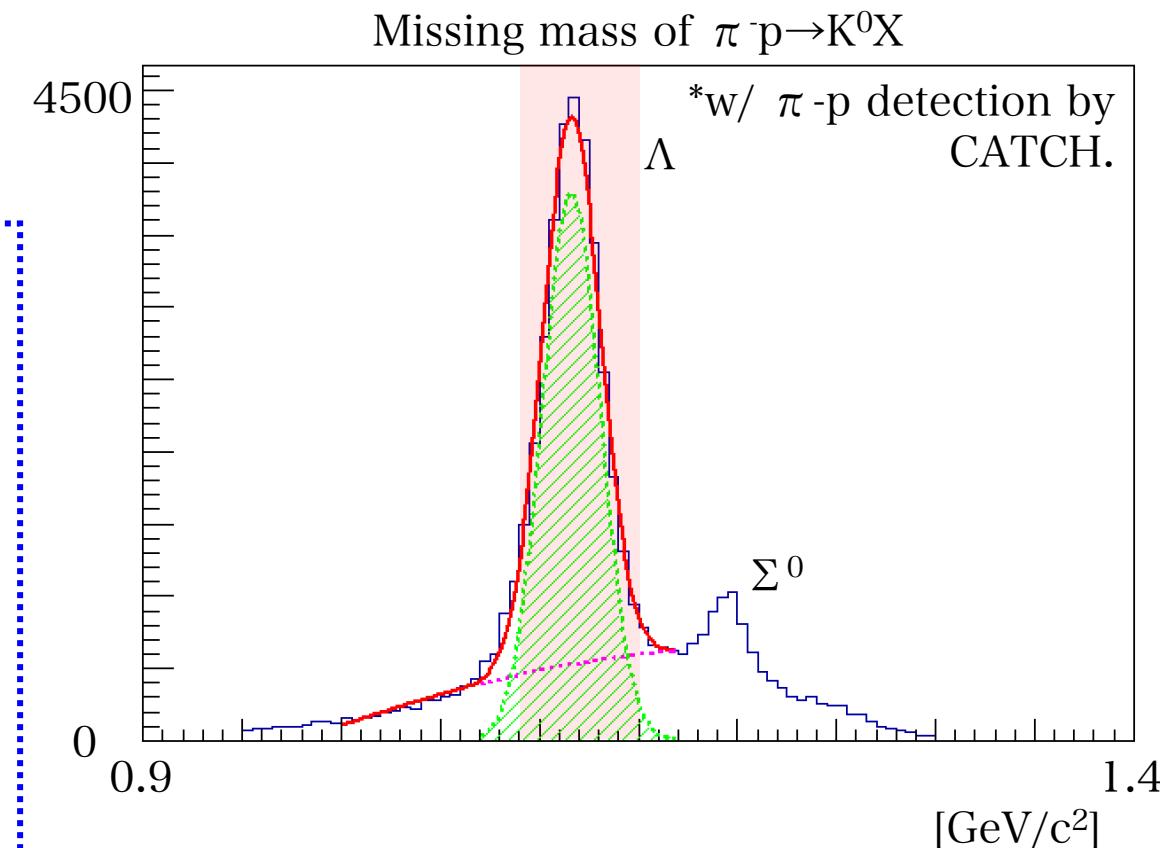
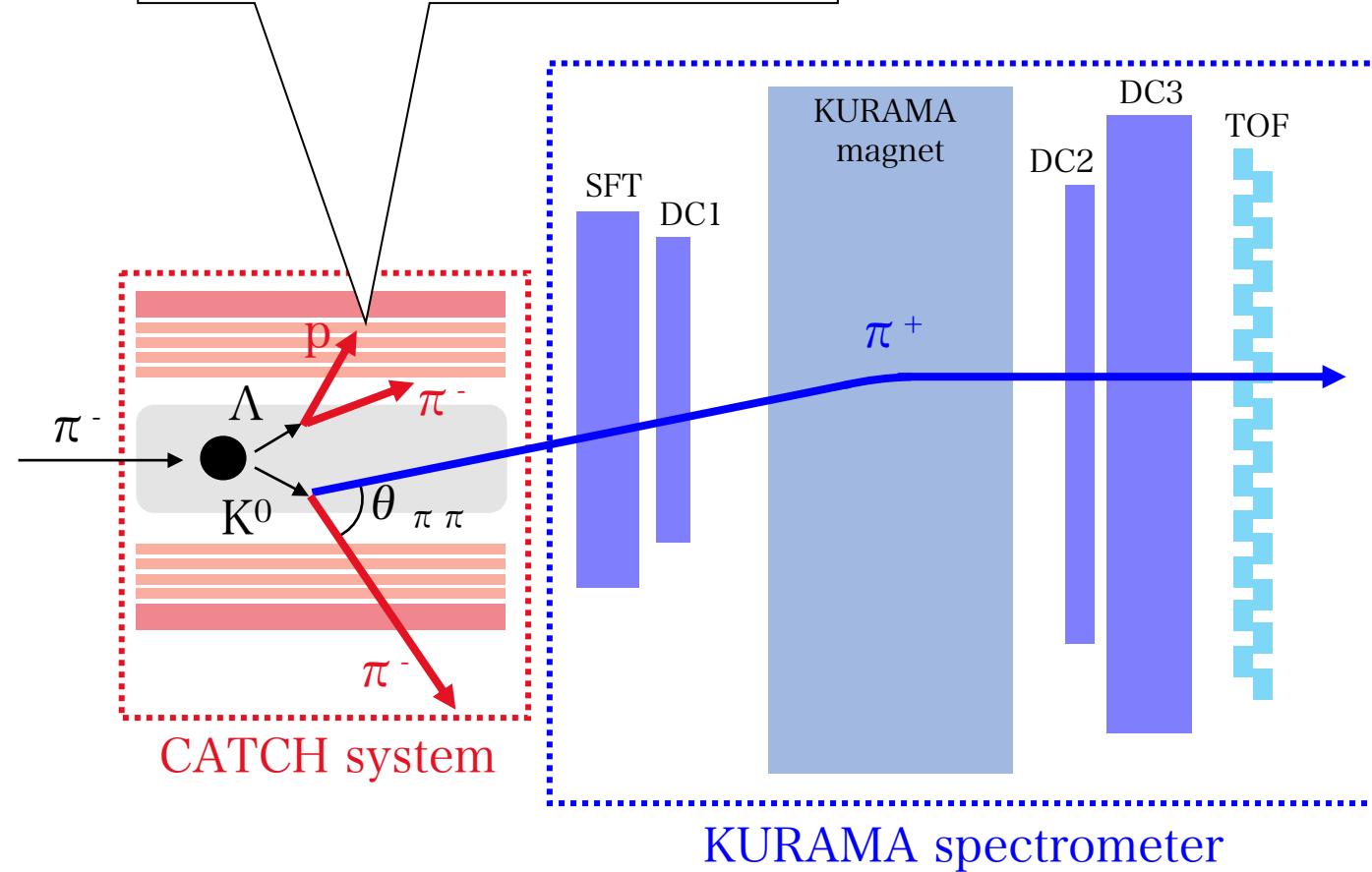
1. Calculate p_{π^-} by assuming $K^0 \rightarrow \pi^+ \pi^-$ decay
by using the measured p_{π^+} and the opening angle $\theta_{\pi\pi}$.
2. Calculate the missing mass of $\pi^- p \rightarrow K^0 X$.

If the K^0 decay assumption is correct, the produced Λ is tagged in the MM histogram.

Λ Tagging Performance for P_Λ Analysis in E40

10

For P_Λ analysis, we required the $\Lambda \rightarrow \pi^- p$ decay.



Λ tagging performance for P_Λ analysis

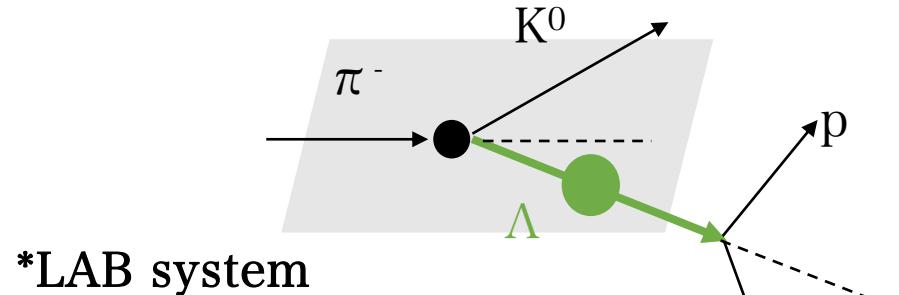
Λ yield: 2.80×10^4

SN ratio: 2.67

Toward P_Λ Analysis by Using E40 Data

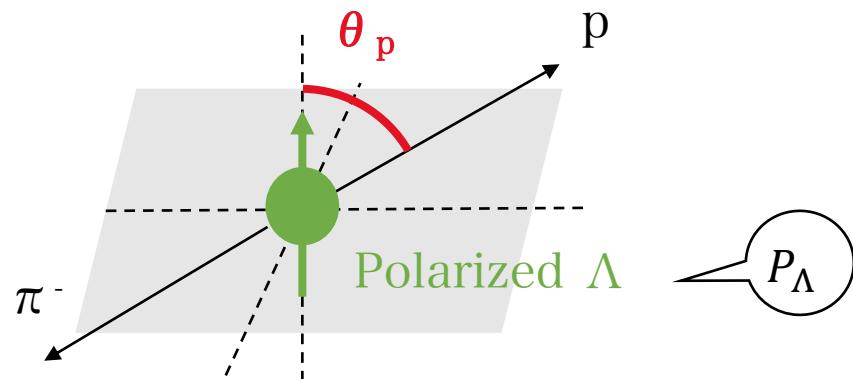
P_Λ analysis method

1. Detect p from the $\Lambda \rightarrow \pi^- p$ decay.

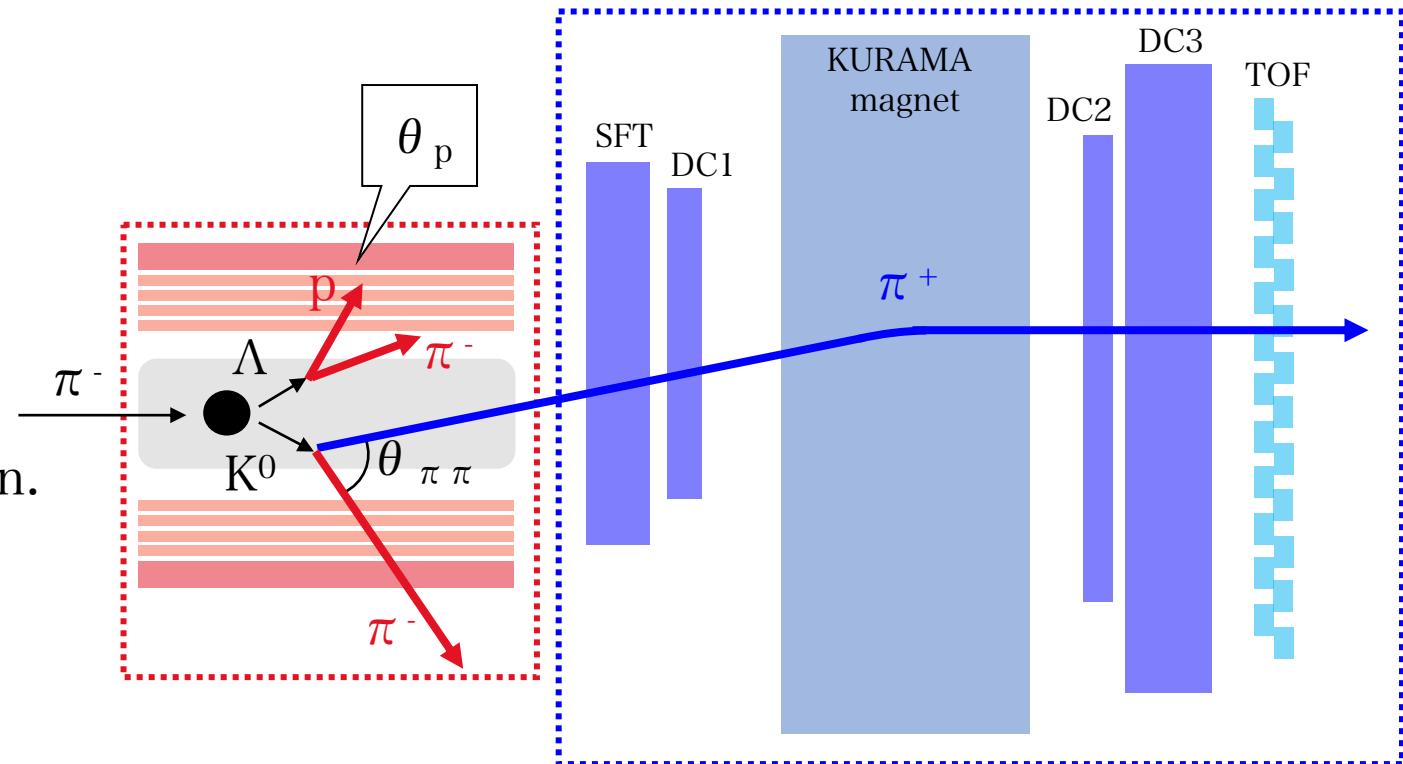


2. Measure θ_p in the rest of Λ .

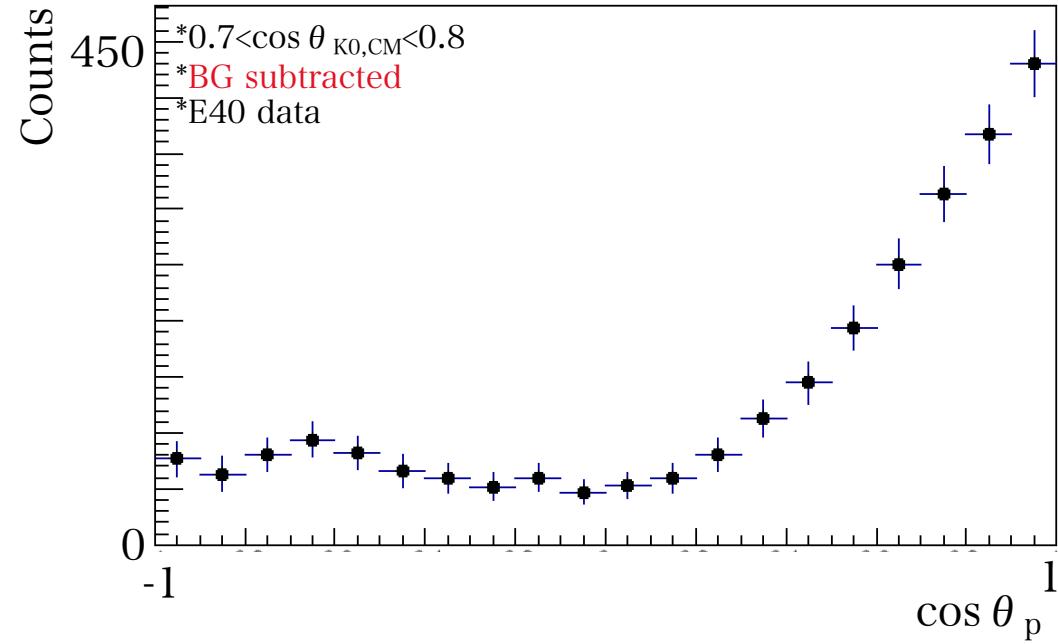
* θ_p : Angle b/w the Λ spin & p direction.



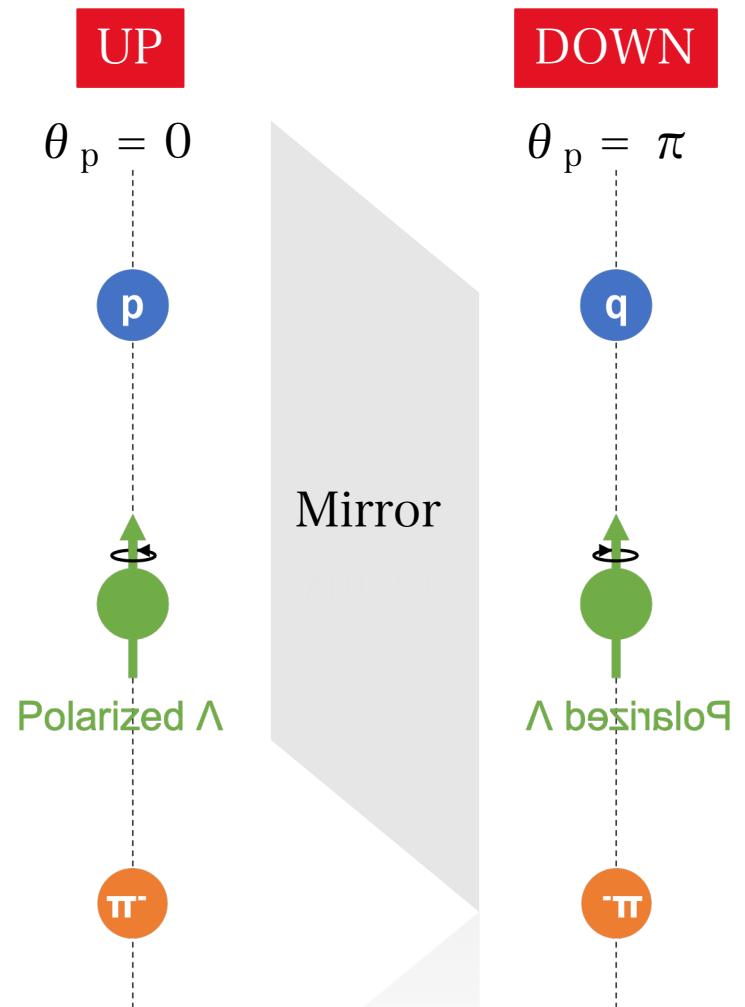
3. Analyze the θ_p distribution (Up-Down asymmetry).



BG Suppression & Up-Down Asymmetry of $\cos \theta_p$

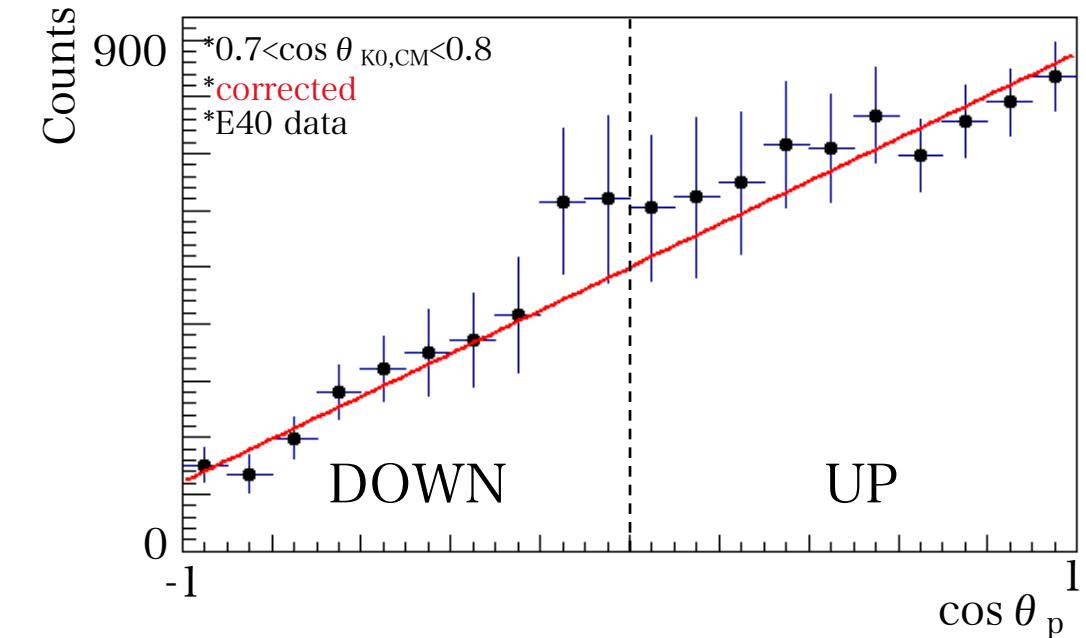
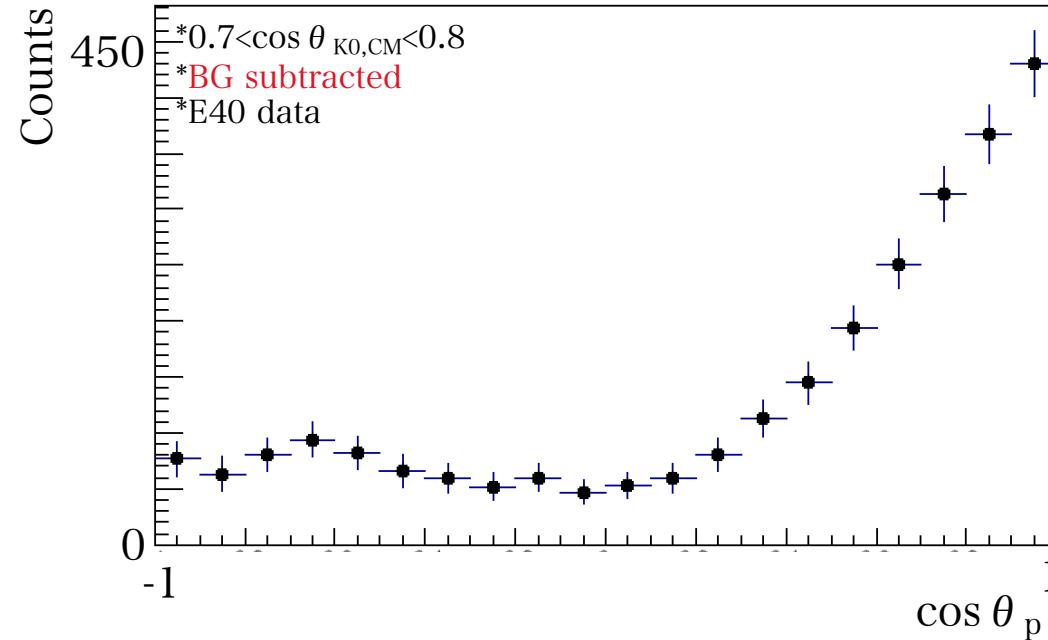


Due to the parity violation of the $\Lambda \rightarrow \pi^- p$ decay,
 θ_p distribution becomes asymmetric (Up-Down asymmetry).

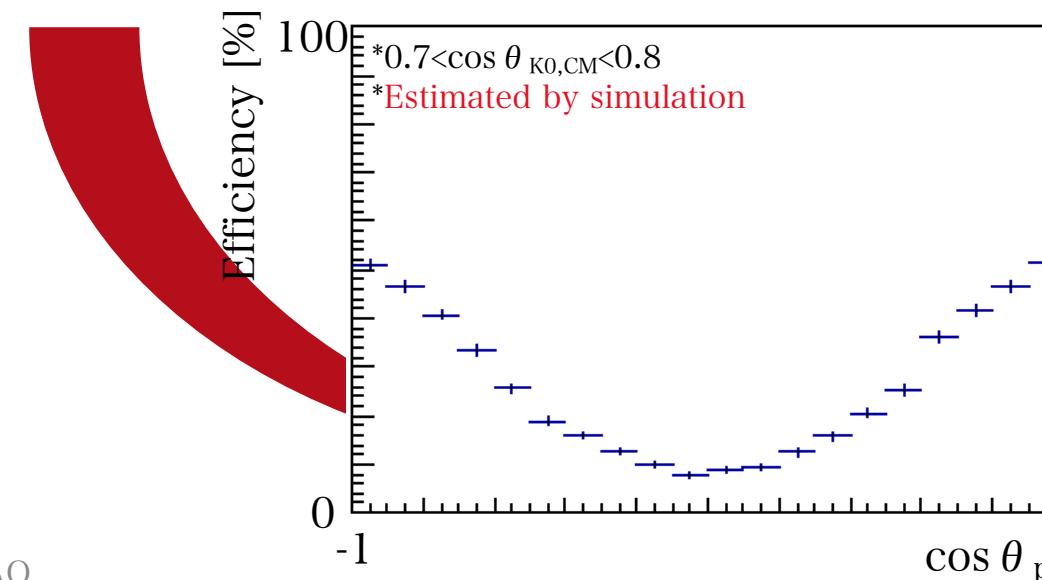


Two decay modes have different amplitudes.

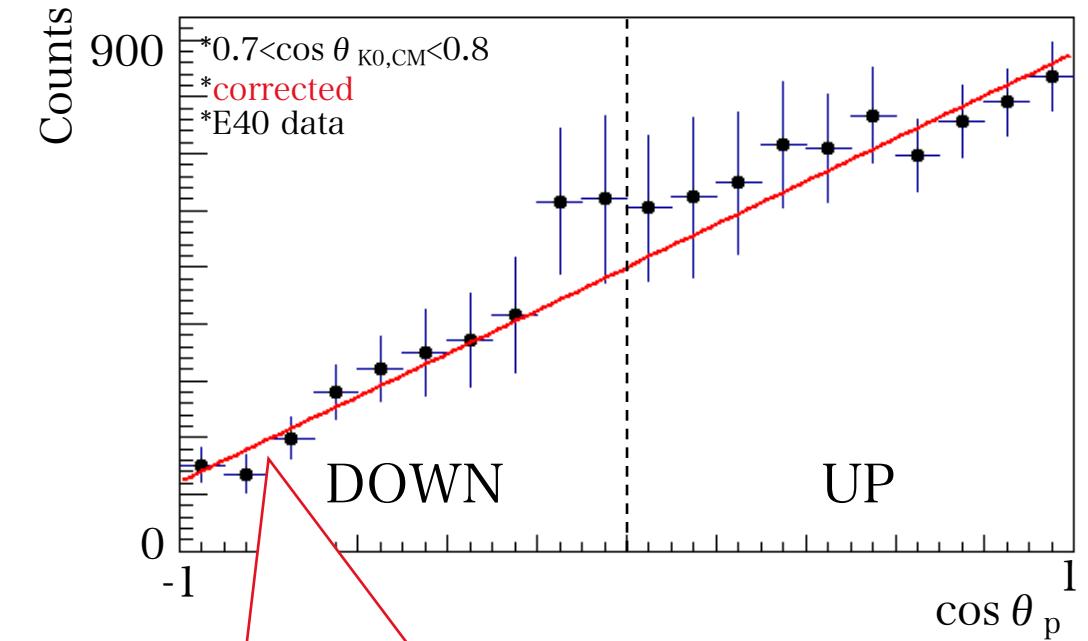
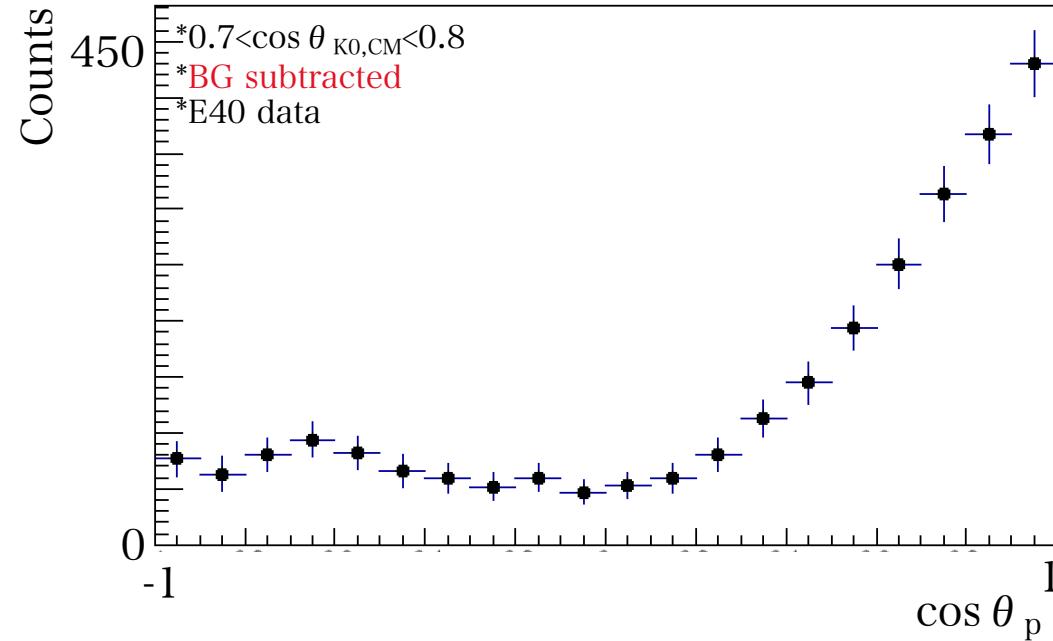
BG Suppression & Up-Down Asymmetry of $\cos \theta_p$



P_A = the slope b/w the data of $\cos \theta_p = -1$ & 1.



BG Suppression & Up-Down Asymmetry of $\cos \theta_p$



P_A fitting function:

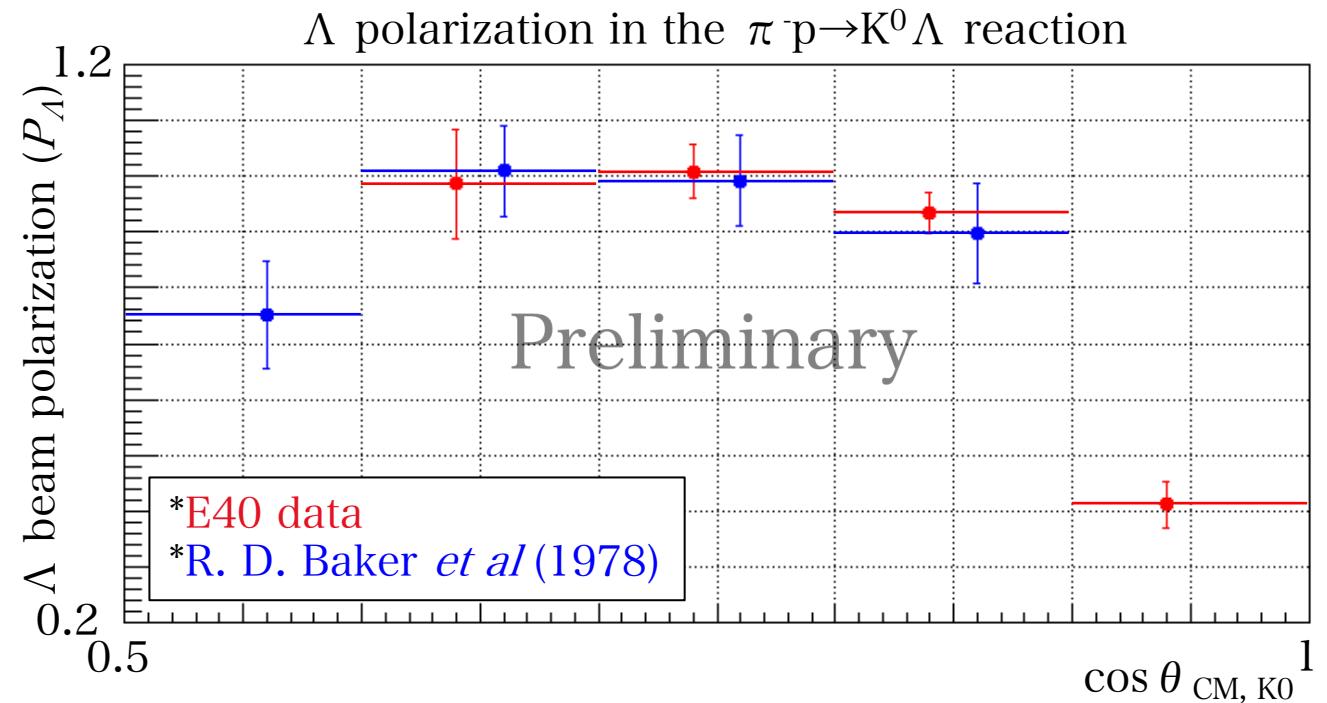
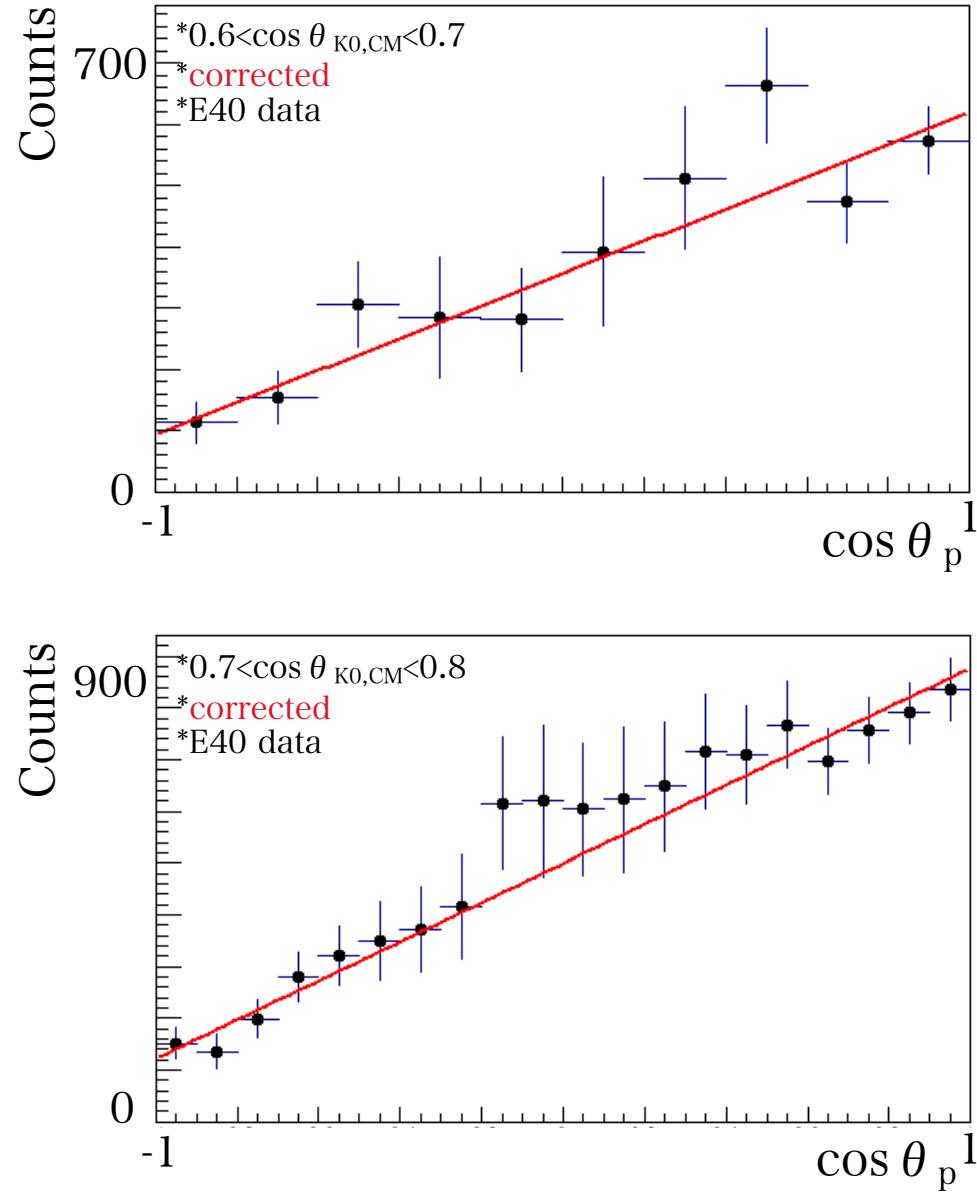
$$\frac{1}{N_0} \frac{dN}{dcos\theta_p} = \frac{1}{2} (1 + \alpha P_A \cos \theta_p)$$

α : asymmetry parameter

(= $0.750 \pm 0.009 \pm 0.004$ [2]),

[2] M. Ablikim et al. (2019)

Measured P_A in E40 & Prospect



Achievement

1. 1st step for establishing the P_A analysis was done.
2. High polarization of Λ beam was confirmed.
($P_A = 1.009 \pm 0.049$ in $0.7 < \cos \theta_{K0,CM} < 0.8$ region)
3. The Λp spin observable measurement is possible in the J-PARC P86.

Summary

- Λ tagging method by using the $\pi^- p \rightarrow K^0 \Lambda$ reaction was established.
- High polarization of Λ beam was confirmed by J-PARC E40.
 - $P_\Lambda = 1.009 \pm 0.049$ in $0.7 < \cos \theta_{K^0, CM} < 0.8$ region.
- By selecting such K^0 angular range, we can get a highly polarized Λ beam.
 - Λp spin observables (A_y, D^{y_y}) measurement is possible in the J-PARC P86.

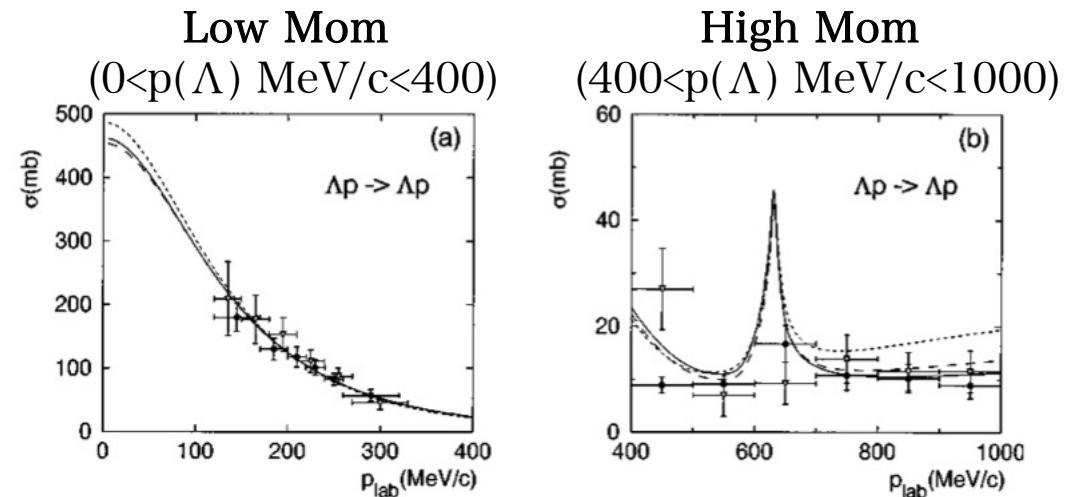
Backup

Physics Motivation

Many Theoretical Models (Nijmegen & Quark Model)

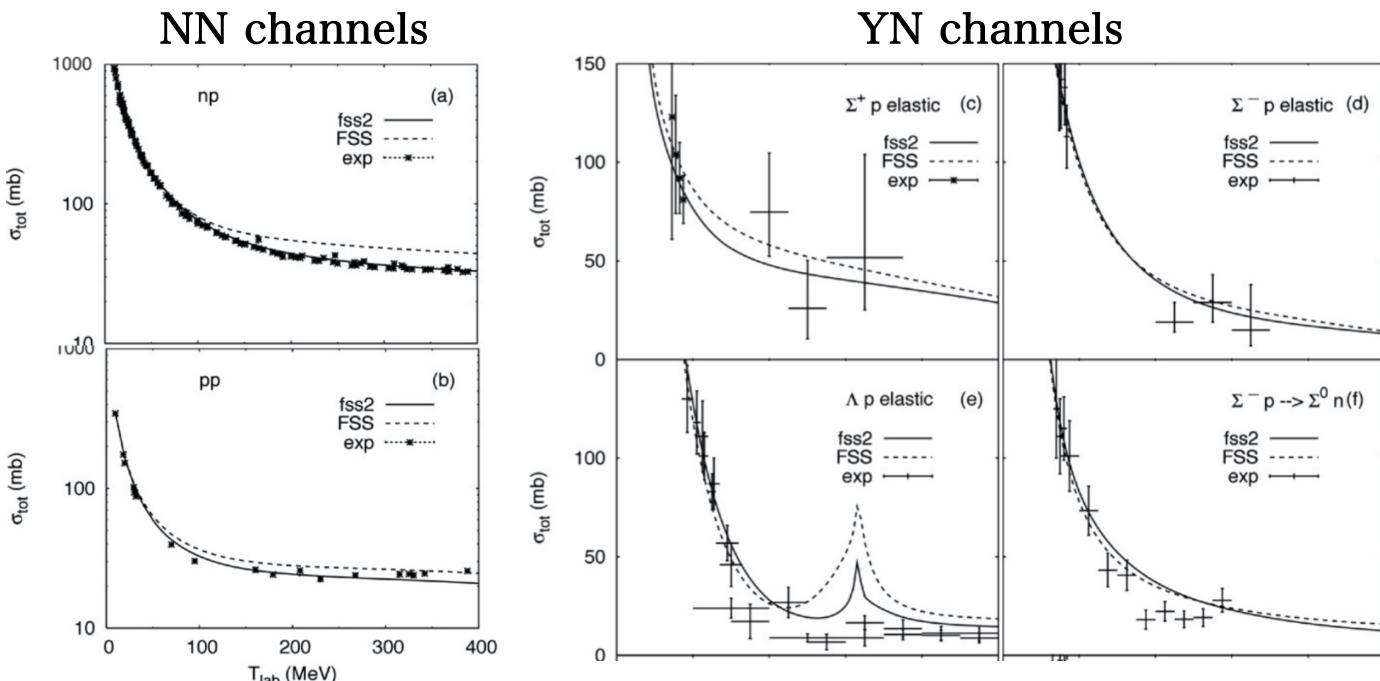
Nijmegen (Extended) Soft-Core

- **Soft-core OBE.**
 - i. Pseudoscalar mesons (π , η , η' , K)
 - ii. Vector mesons (ρ , ϕ , ω , K^*)
 - iii. Scalar mesons ($a_0(980)$, $f_0(975)$, $f_0(760)$, $\kappa(880)$)
 - iv. Pomeron (2-gluon) & tensor meson)
- **Central, spin-spin, spin-orbit & CSB int. were included.**
Th. A. Rijken et al., Phys. Rev. C 59, 21 (1999)



Quark Model (fss2)

- **Quark-model BB int.**
in the 2-cluster RGM (Resonating Group Method) kernels.
 - i. Kinetic-energy operator
 - ii. Effective meson-exchange potential
- A. MULLER-GROELING et al., Nucl. Phys. A513 (1990) 557-583

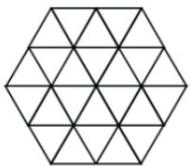


Many Theoretical Models (Lattice QCD)

Lattice QCD

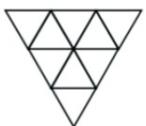
- Numerical QCD simulation.
- Very strong calculation power.
- $SU(3)_f$ limit affects mass dependence.

27



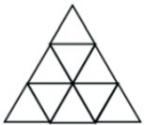
NN ($I=1$)
 ΣN ($I=3/2$), $\Sigma N - \Lambda N$ ($I=1/2$)
 $\Sigma\Sigma$ ($I=2$), $\Xi N - \Sigma\Sigma - \Lambda N$ ($I=1/2$), $\Xi N - \Sigma\Sigma - \Lambda\Lambda$ ($I=0$)
 $\Xi\Sigma$ ($I=3/2$), $\Xi\Sigma - \Xi\Lambda$ ($I=1/2$)
 $\Xi\Xi$ ($I=1$)

10



ΣN ($I=3/2$)
 $\Xi N - \Sigma\Sigma - \Lambda N$ ($I=1$)
 $\Xi\Sigma - \Xi\Lambda$ ($I=1/2$)
 $\Xi\Xi$ ($I=0$)

10*



NN ($I=0$)
 $\Sigma N - \Lambda N$ ($I=1/2$)
 $\Xi N - \Xi\Lambda$ ($I=1$)
 ΞN ($I=3/2$)

8s



$\Sigma N - \Lambda N$ ($I=1/2$)
 $\Xi N - \Lambda N$ ($I=1$), $\Xi N - \Sigma\Sigma - \Lambda\Lambda$ ($I=0$)
 $\Xi N - \Xi\Lambda$ ($I=1/2$)

8a

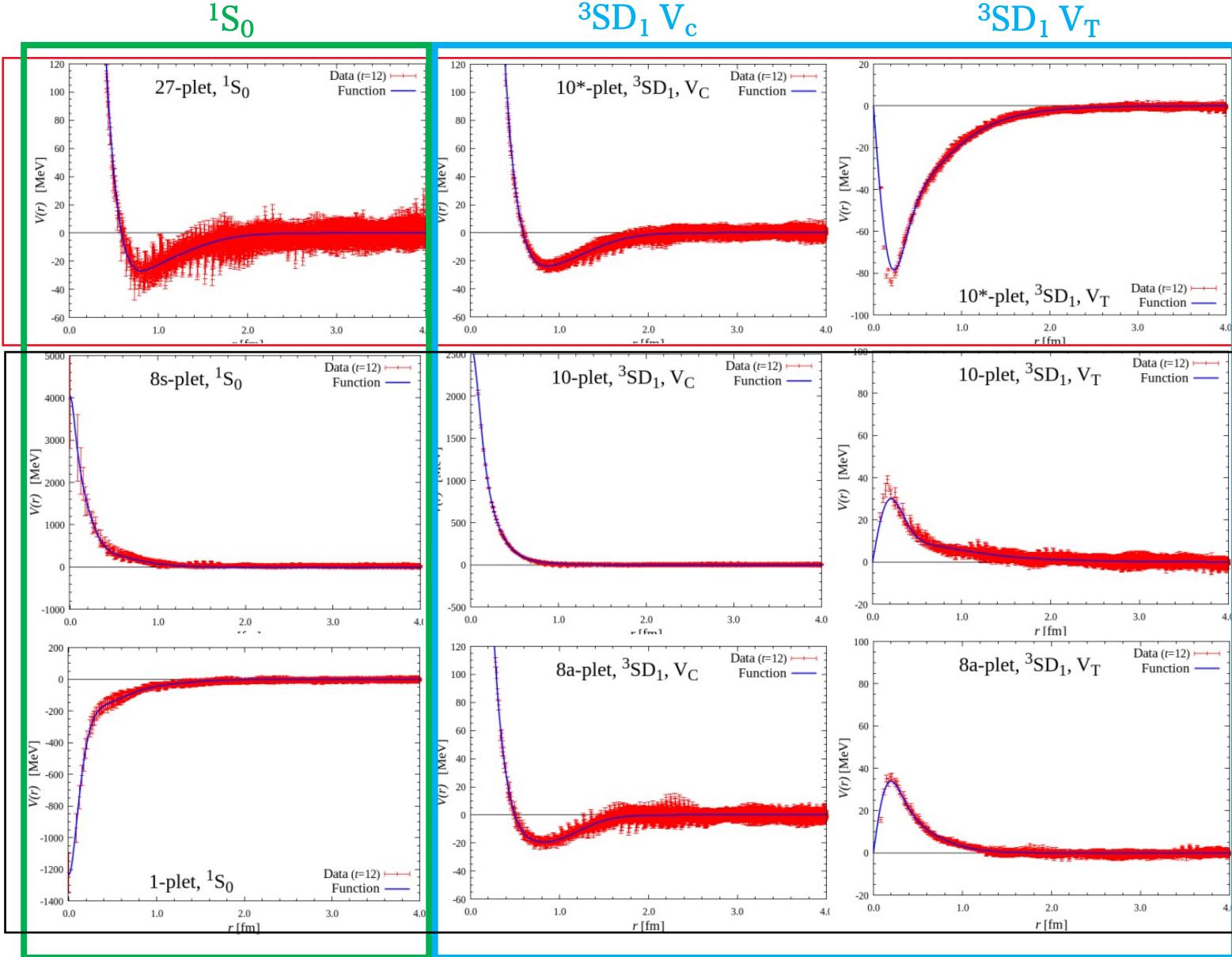


$\Sigma N - \Lambda N$ ($I=1/2$)
 $\Xi N - \Sigma\Sigma - \Sigma N$ ($I=1$), ΞN ($I=0$)
 $\Xi\Sigma - \Xi\Lambda$ ($I=1/2$)
 $\Xi N - \Sigma\Sigma - \Lambda\Lambda$ ($I=0$)

1

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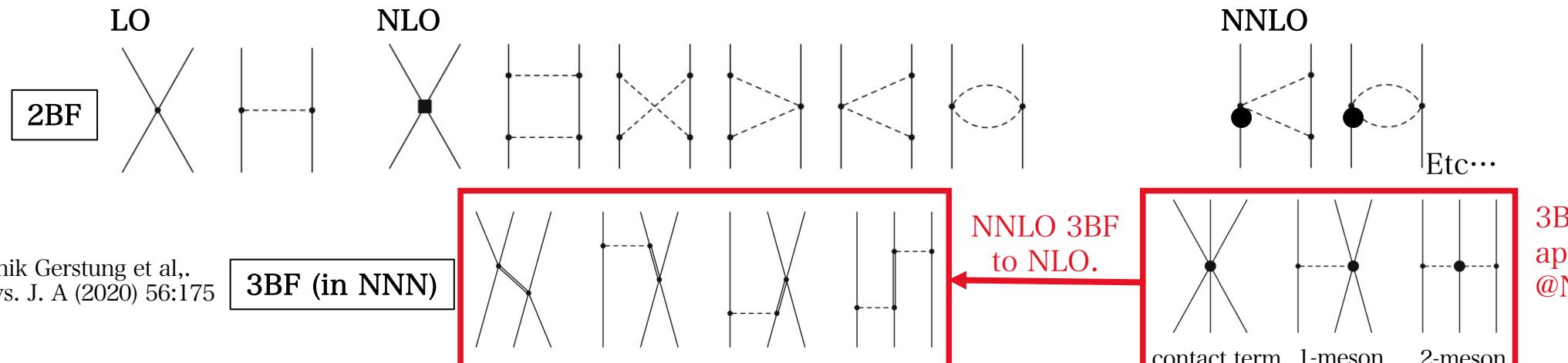
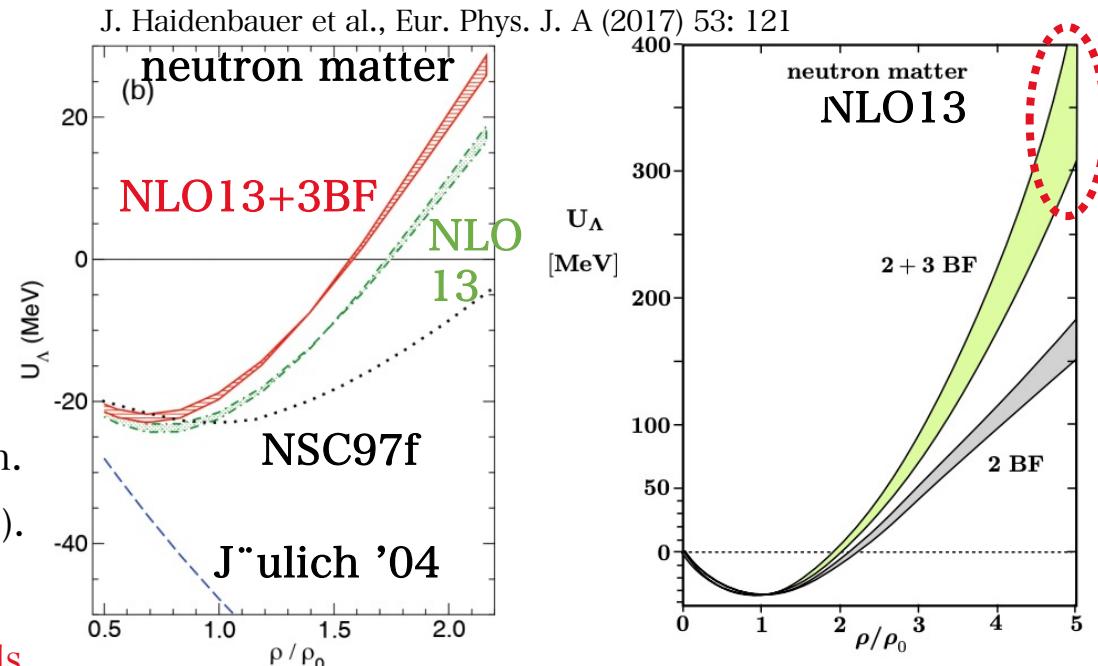
NN interaction



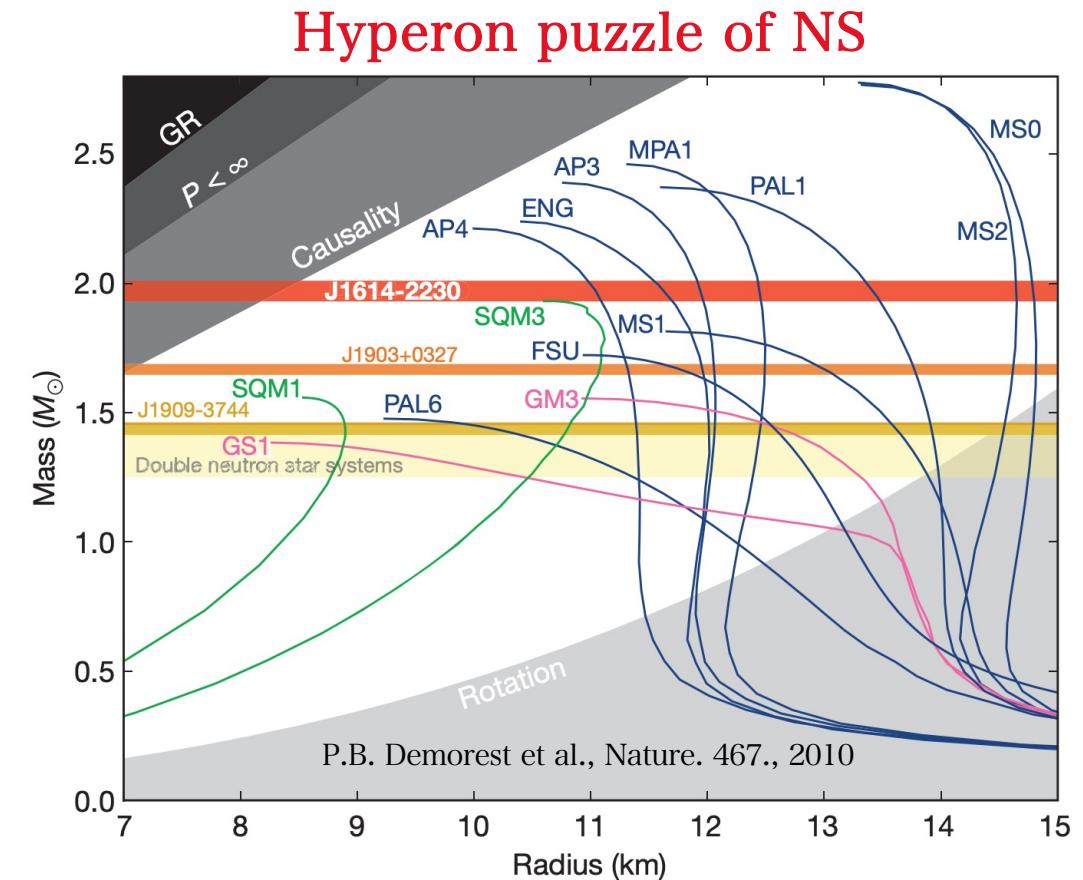
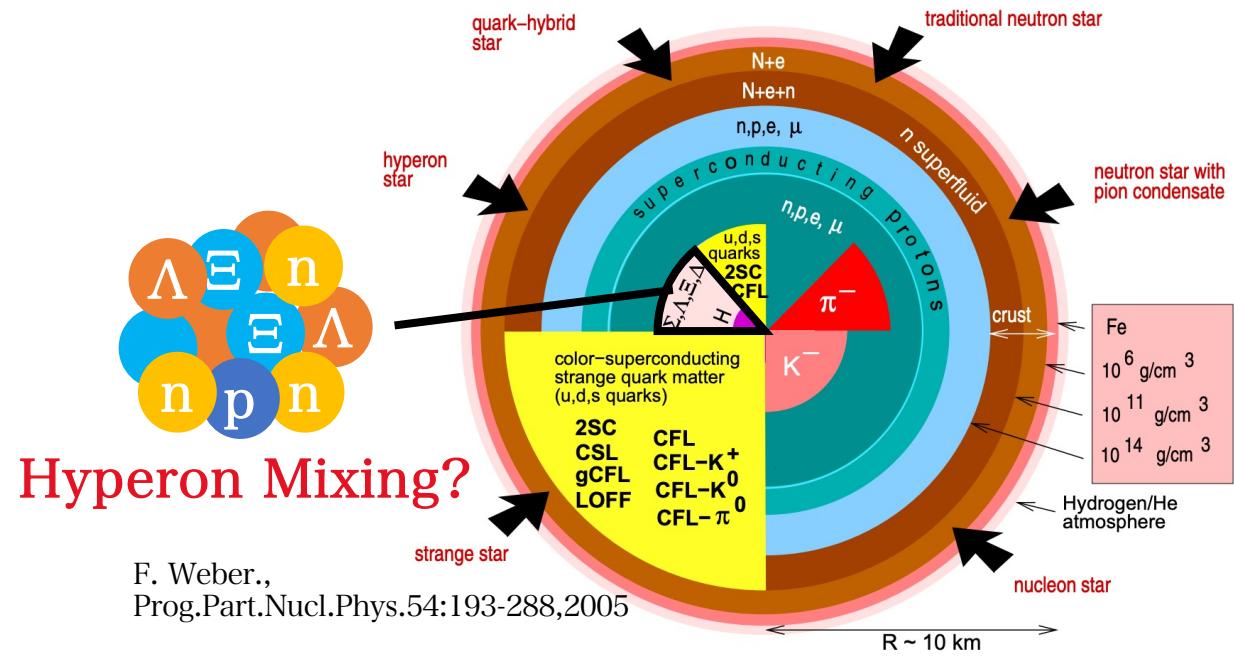
Many Theoretical Models (Chiral Effective Model)

Chiral EFT

- NLO includes ...
 - i. One-pseudoscalar octet meson exchange
 - ii. Two-pseudoscalar octet meson exchange
 - iii. Four-baryon contact term
- NNLO 3BF (in NNN)
 - Can be replaced into NLO w/ the decuplet saturation approximation.
→ Only three parameters (1 coupling length & 2 coupling constants).
- NNLO 3BF (in Λ NN, Σ NN, Λ NN \rightleftharpoons Σ NN)
 - It can be translated into Effective Density-Dependence YN potentials.
→ Accurate B_A data should restrict the Low Energy Constants (LEC) introduced in this model.



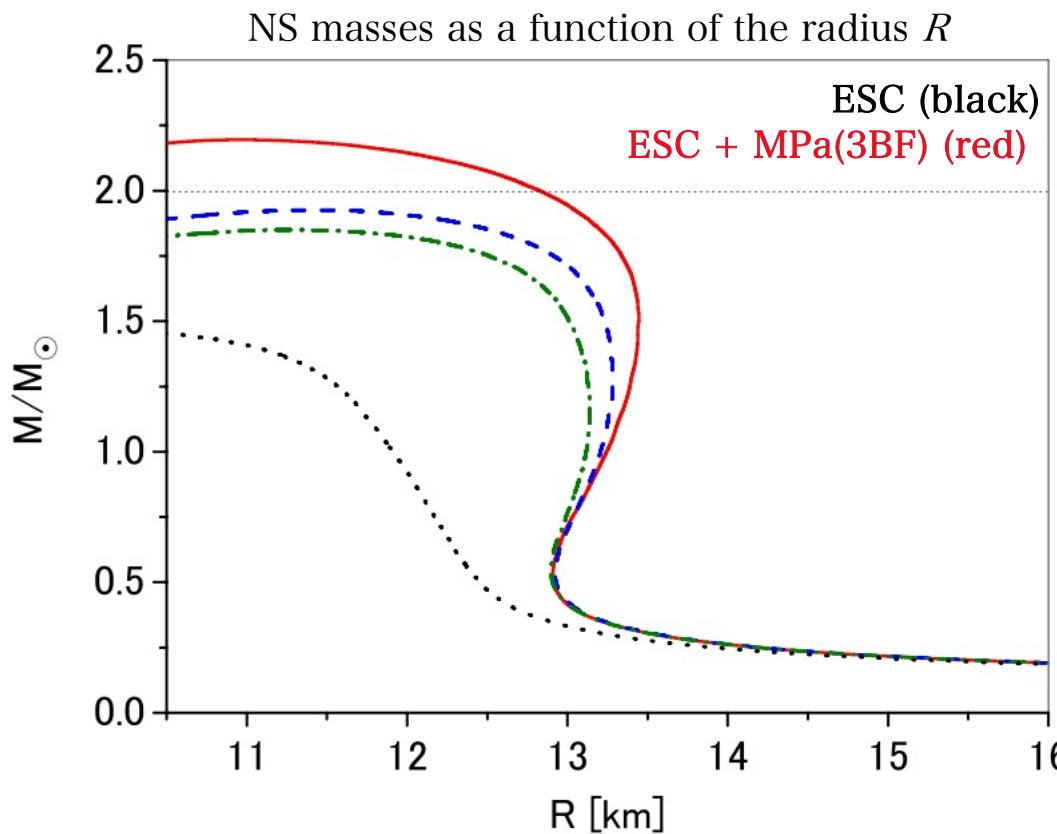
Recent YN Interaction Study



Hyperon Puzzle of NS & Λ N, Λ NN Interaction

Hyperon Puzzle of NS

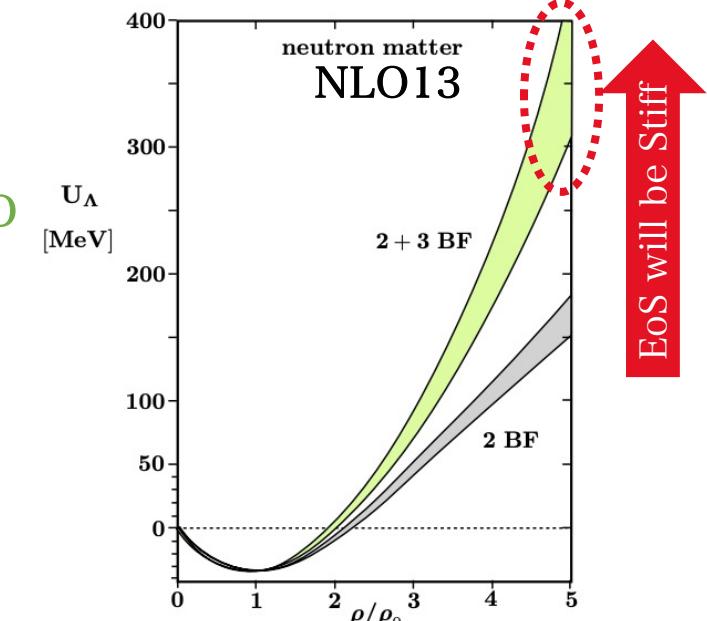
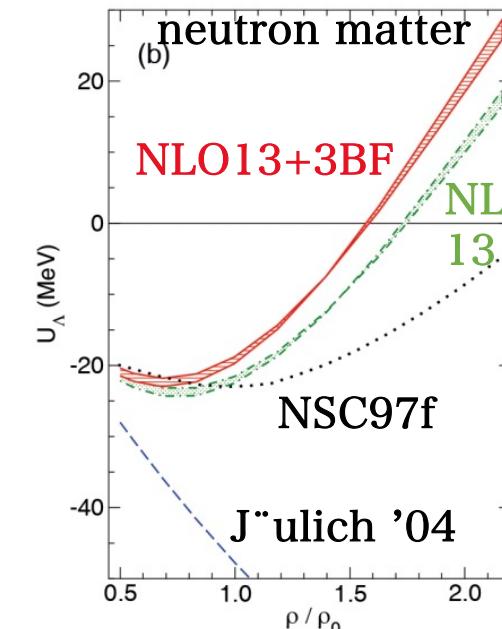
- Hyperon mixing: soft EoS
- $2M_\odot$ NS data: stiff EoS



Y. Yamamoto, T. Furumoto, N. Yasutake, and Th. A. Rijken
Phys. Rev. C 90, 045805 – Published 30 October 2014

How to support the massive NS w/ hyperons??

- 3BF gives repulsion in the high-density region.



J. Haidenbauer et al., Eur. Phys. J. A (2017) 53: 121

- Λ N density dependence study is necessary
(by B_Λ data from hypernuclear exp.).
→ Λ NN force strength study

Hypernuclear Exp.
Accurate B_Λ measurement

Models based on YN Scattering Exp.
Realistic Λ N interaction

Approach for BB Interaction & Spin-Dependent Terms

S-wave B_8B_8 interactions of NN, ΣN & ΛN channels

$B_8B_8(I)$	spin-singlet	spin-triplet
$NN(I = 0)$	—	$(\mathbf{10}^*)$
$NN(I = 1)$	$(\mathbf{27})$	—
$\Sigma N(I = 1/2)$	$\frac{1}{\sqrt{10}}[(3(\mathbf{8s}) - (\mathbf{27}))]$	$\frac{1}{\sqrt{2}}[(\mathbf{8a}) + (\mathbf{10}^*)]$
$\Sigma N(I = 3/2)$	$(\mathbf{27})$	$(\mathbf{10})$
ΛN	$\frac{1}{\sqrt{10}}[(\mathbf{8s}) + 3(\mathbf{27})]$	$\frac{1}{\sqrt{2}}[-(\mathbf{8a}) + (\mathbf{10}^*)]$

Past NN scattering

J-PARC E40
(Σp scattering)

New Λp scattering
@J-PARC

Restricting
YN int.

w/ more restricted YN int.,
study more precisely.

T matrix
of YN scattering

$$\mathbf{M} = V_c + V_\sigma(\mathbf{s}_a \cdot \mathbf{s}_b) + V_{SLS}(\mathbf{s}_a + \mathbf{s}_b) \cdot \mathbf{L} + V_{ALS}(\mathbf{s}_a - \mathbf{s}_b) \cdot \mathbf{L} + V_T([\mathbf{s}_a \otimes \mathbf{s}_b]^{(2)} \cdot \mathbf{Y}_2(\hat{\mathbf{r}}))$$

Analyzing power
of YN scattering

$$A_y(a) = -\frac{4\sqrt{2}}{N_R} \text{Im} \left\{ \underbrace{U_\alpha^*(S_\alpha + S_\beta)}_{\text{Central}} + \underbrace{\frac{1}{4} U_\beta^*(-S_\alpha + S_\beta)}_{\text{ALS}} - \underbrace{\frac{1}{2} T_\alpha^*(-S_\alpha + S_\beta)}_{\text{SLS}} \right\} \quad \text{※No polarized target}$$

- Spin-dependent terms of ΛN interaction

can be constrained by spin observable measurement.

- ~ 100% polarized Λ beam is an advantage.

Polarization phenomena in hyperon-nucleon scattering
S. Ishikawa, M. Tanifuji, Y. Iseri, and Y. Yamamoto, Phys. Rev. C 69, 034001 – Published 4 March 2004

Approach for BB Interaction & Spin-Dependent Terms

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$\Sigma N(I = 3/2)$	$(\mathbf{27})$	$(\mathbf{10})$
ΛN	$\frac{1}{\sqrt{10}}[(\mathbf{8s}) + 3(\mathbf{27})]$	$\frac{1}{\sqrt{2}}[-(\mathbf{8a}) + (\mathbf{10}^*)]$

Past NN scattering

J-PARC E40
(Σp scattering)

New Λp scattering
@J-PARC

Restricting
YN int.

w/ more restricted YN int.,
study more precisely.

T matrix
of YN scattering

$$\mathbf{M} = V_c + V_\sigma(\mathbf{s}_a \cdot \mathbf{s}_b) + V_{SLS}(\mathbf{s}_a + \mathbf{s}_b) \cdot \mathbf{L} + V_{ALS}(\mathbf{s}_a - \mathbf{s}_b) \cdot \mathbf{L} + V_T([\mathbf{s}_a \otimes \mathbf{s}_b]^{(2)} \cdot \mathbf{Y}_2(\hat{\mathbf{r}}))$$

Depolarization
of YN scattering

$$D_i^j(a) = \frac{1}{N_R} \text{Tr}[\mathbf{M} \sigma_i(a) \mathbf{M}^\dagger \sigma_j(a)].$$

$$\begin{aligned} D_y^y(a) = & \frac{4}{N_R} \text{Re} \left\{ \frac{1}{2\sqrt{3}} \left(U_0 + \frac{1}{\sqrt{3}} U_1 \right)^* U_1 + \frac{1}{2} \left(U_0 - \frac{1}{\sqrt{3}} U_1 \right)^* \right. \\ & \times \left(\frac{1}{\sqrt{6}} T_1 + T_3 \right) - S_1^* S_2 + \frac{1}{2} |S_3|^2 \right. \\ & \left. - \frac{1}{\sqrt{6}} T_1^* \left(\frac{1}{\sqrt{6}} T_1 - T_3 \right) - \frac{1}{2} |T_2|^2 \right\}, \end{aligned} \quad (\text{C2})$$

Polarization phenomena in hyperon-nucleon scattering
S. Ishikawa, M. Tanifuji, Y. Iseri, and Y. Yamamoto, Phys. Rev. C 69, 034001 – Published 4 March 2004

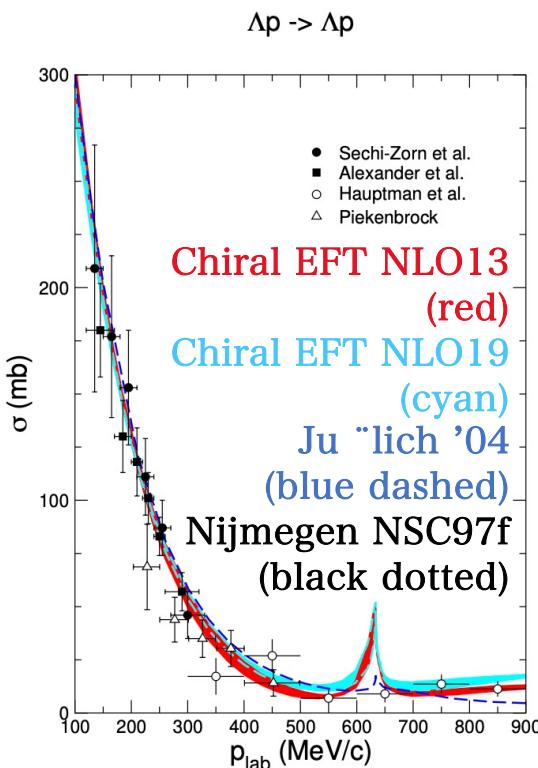
Key to Restrict YN Interaction Models = Λp Differential Observables

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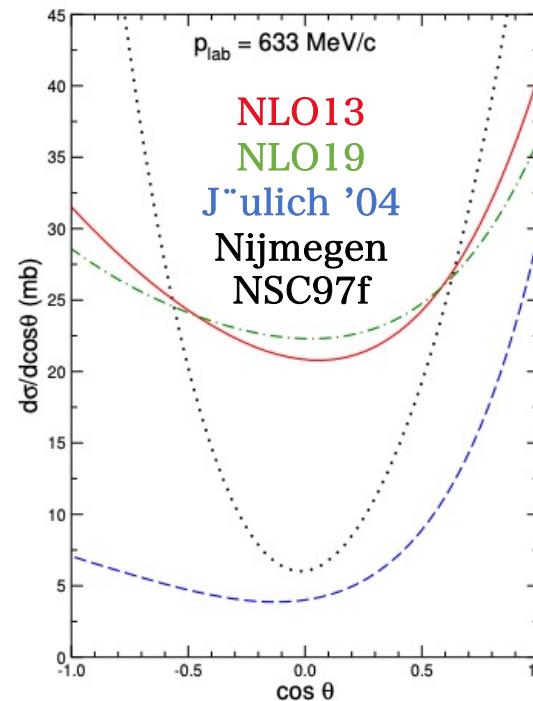
- YN Interaction Models

- Soft-core OBE (Nijmegen)
- Quark-model interaction (fss2)
- Chiral EFT

etc...



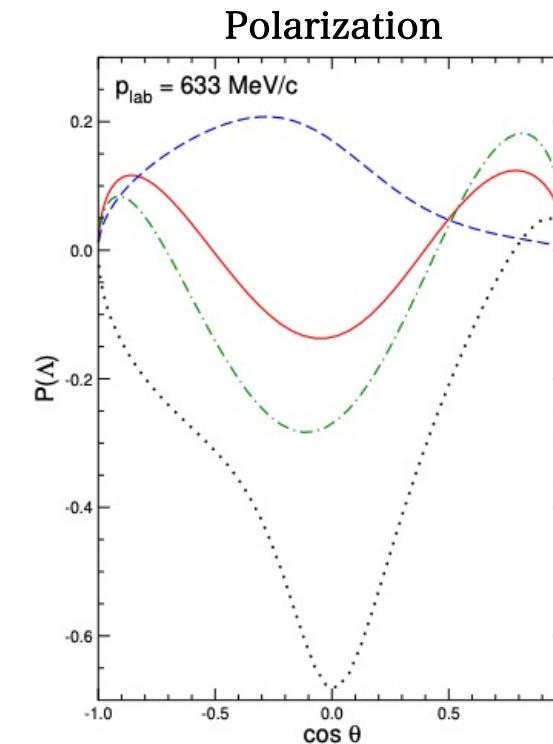
$\Lambda p \rightarrow \Lambda p$ differential cross section



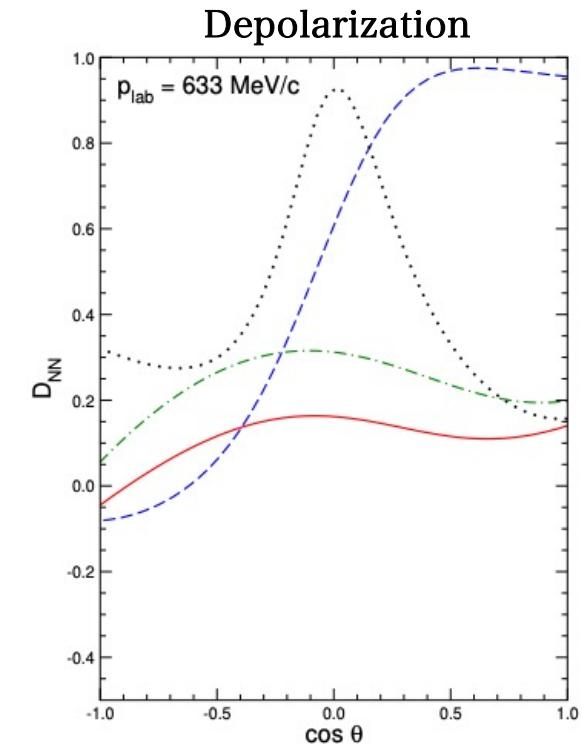
To restrict models

We want to measure them
w/ $\sim 100\%$ polarized Λ beam.

Λp differential observables



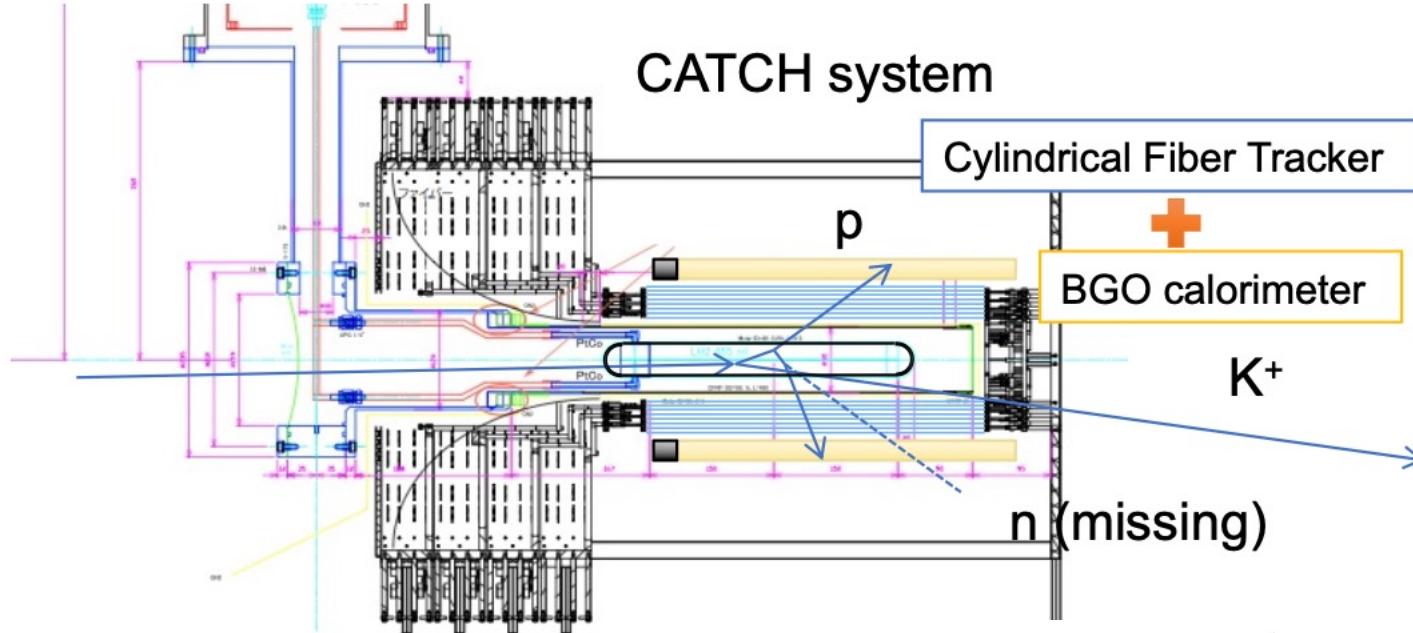
Polarization



Depolarization

J-PARC E40

J-PARC E40 (Σ p scat.) Analysis Method



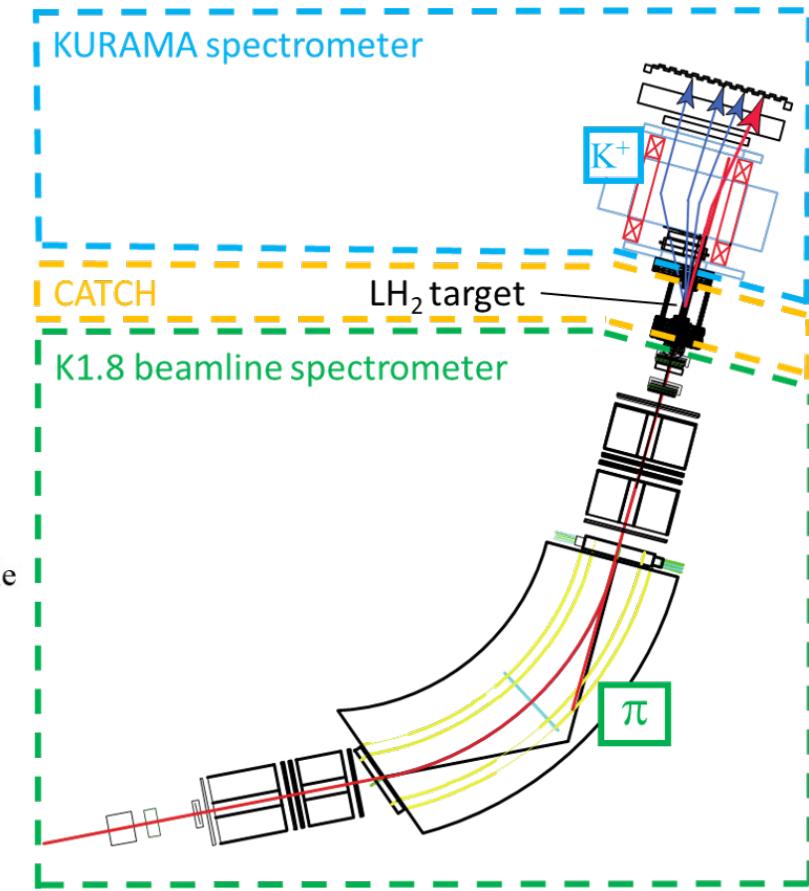
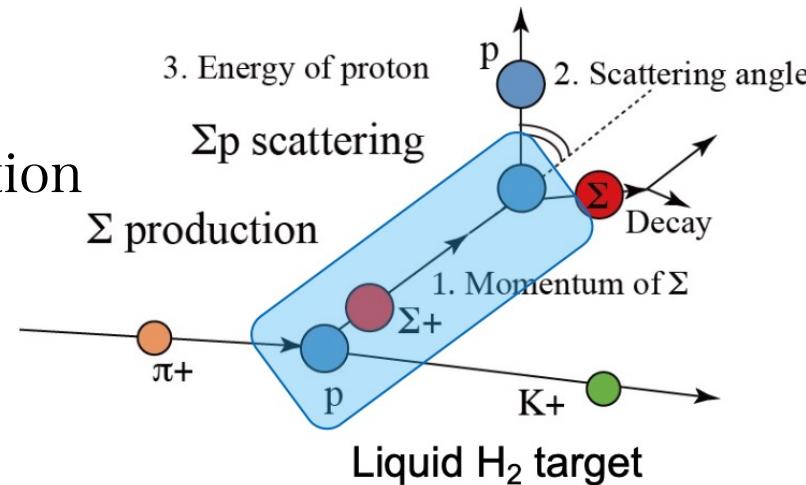
Σ production in LH_2 Target

- Tagged by the $\pi^\pm p \rightarrow K^\pm X$ reaction

- $\Sigma^+:$ 65 M

- $\Sigma^-:$ 17 M

$\sim 1\%$ of data where



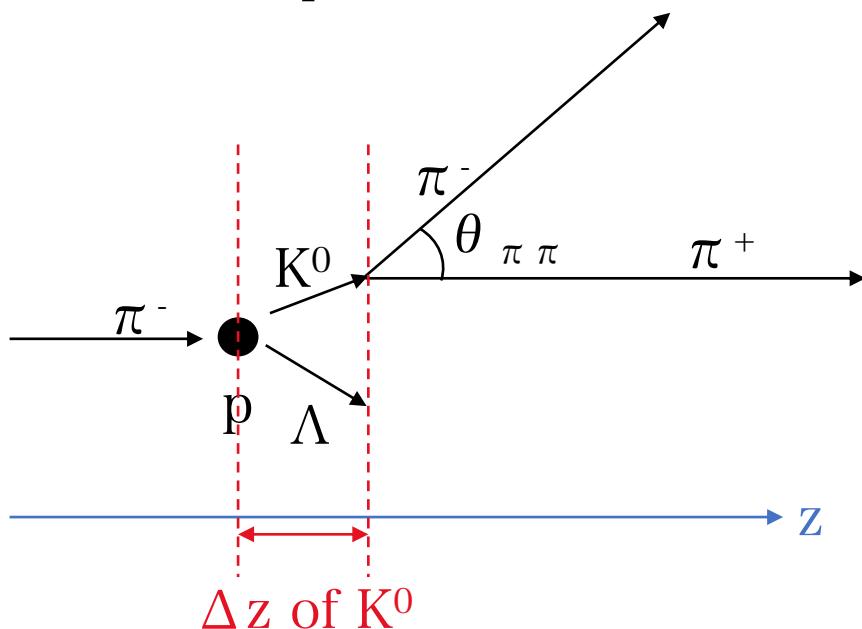
BG Suppression

BG Suppression in P_Λ Analysis

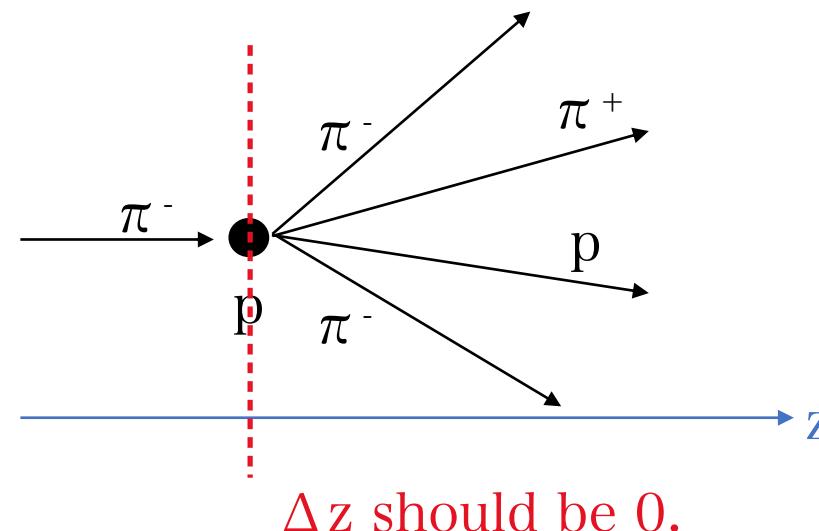
Point

- Δz ($\pi \pi$ vertex – primary vertex) of BG and Λ production are different from each other.

The $\pi^- p \rightarrow K^0 \Lambda$ reaction



The main BG (multi- π production)

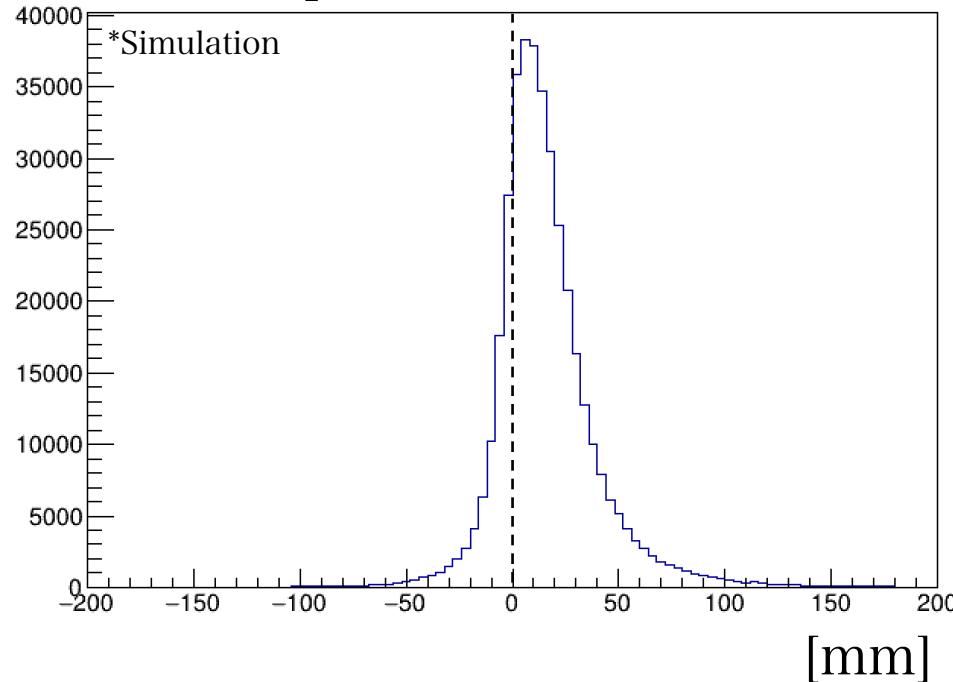


BG Suppression in P_Λ Analysis

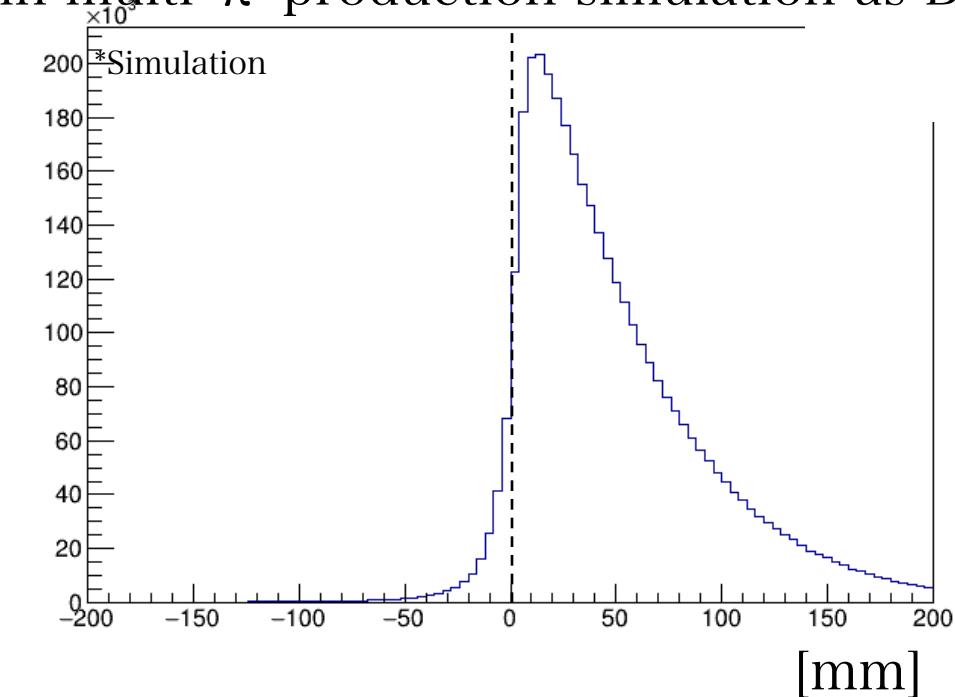
Point

- Δz ($\pi \pi$ vertex – primary vertex) of BG and Λ production are different from each other.

Calculated $\Delta z(K0)$
in Λ production simulation



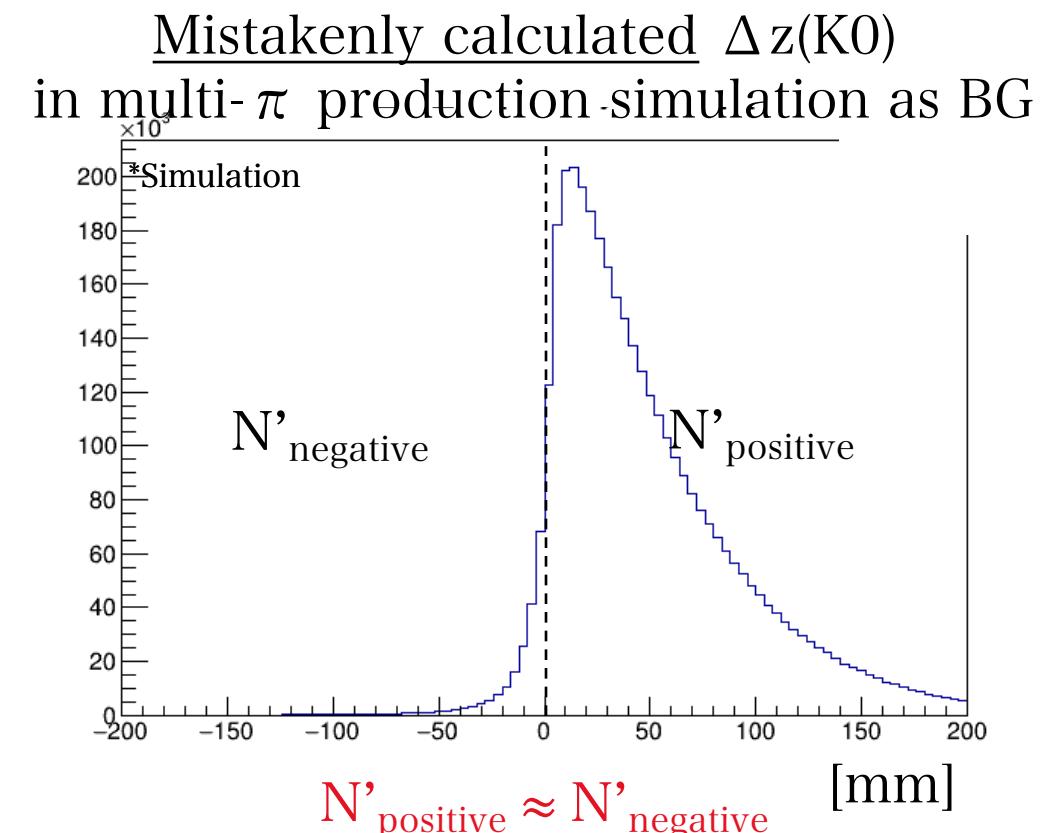
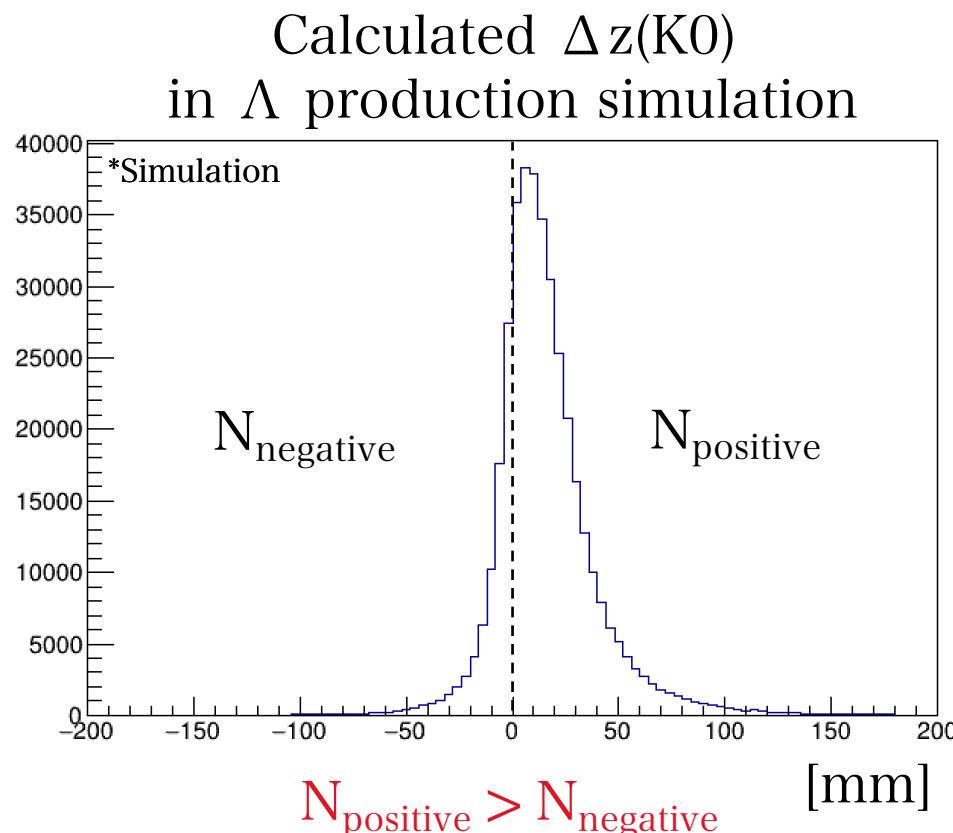
Mistakenly calculated $\Delta z(K0)$
in multi- π production-simulation as BG



BG Suppression in P_Λ Analysis

Δz (z-flight length) cut

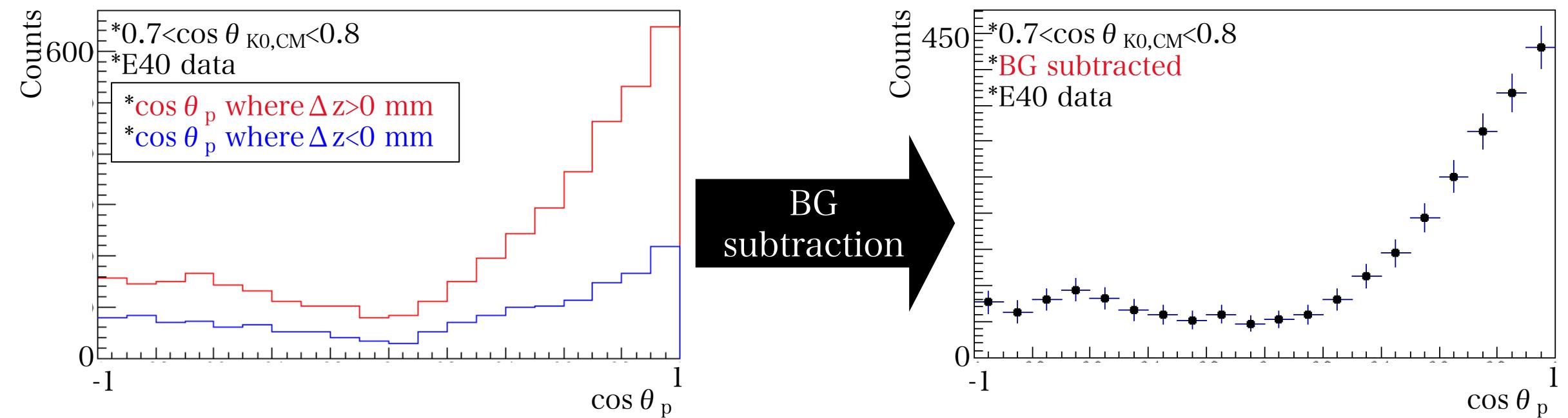
- Set cut value = 0 mm.



BG Suppression in P_A Analysis

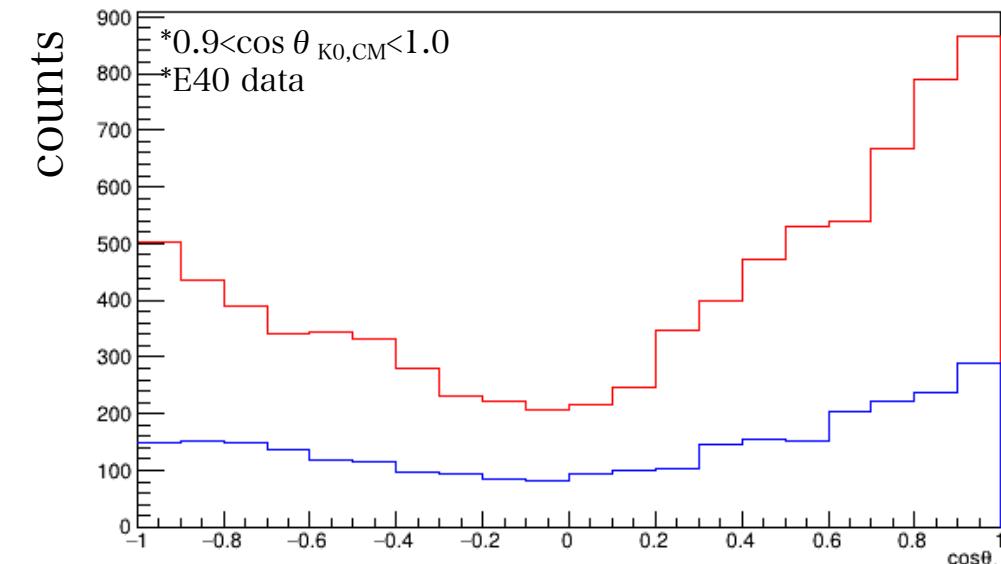
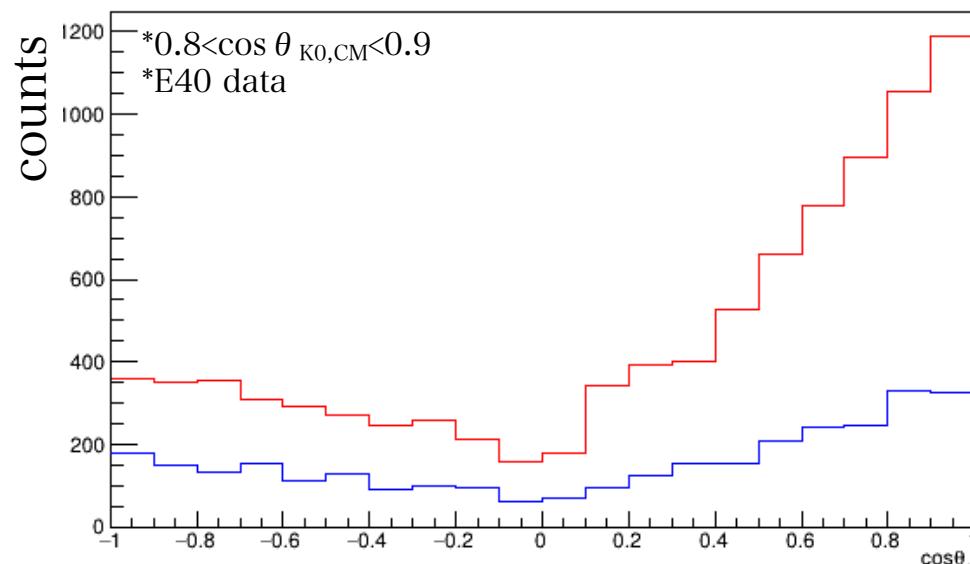
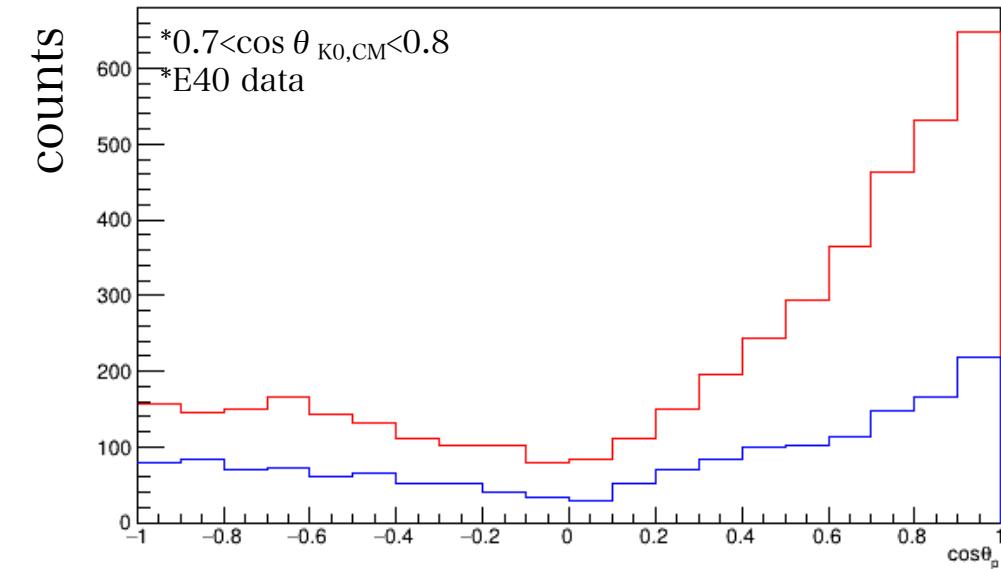
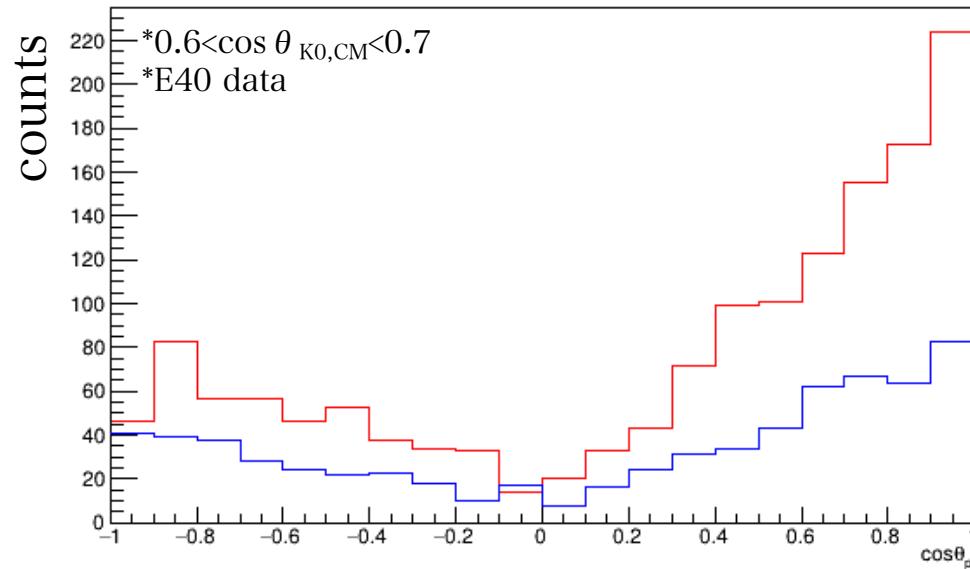
Δz (z-flight length) cut

1. Set cut value = 0 mm.
2. Define $\cos \theta_p$ where $\Delta z > 0$ mm & $\cos \theta_p$ where $\Delta z < 0$ mm.
3. By calculating the difference b/w them, BG was subtracted.



* $\cos \theta_p$ where $\Delta z > 0$ mm
 * $\cos \theta_p$ where $\Delta z < 0$ mm

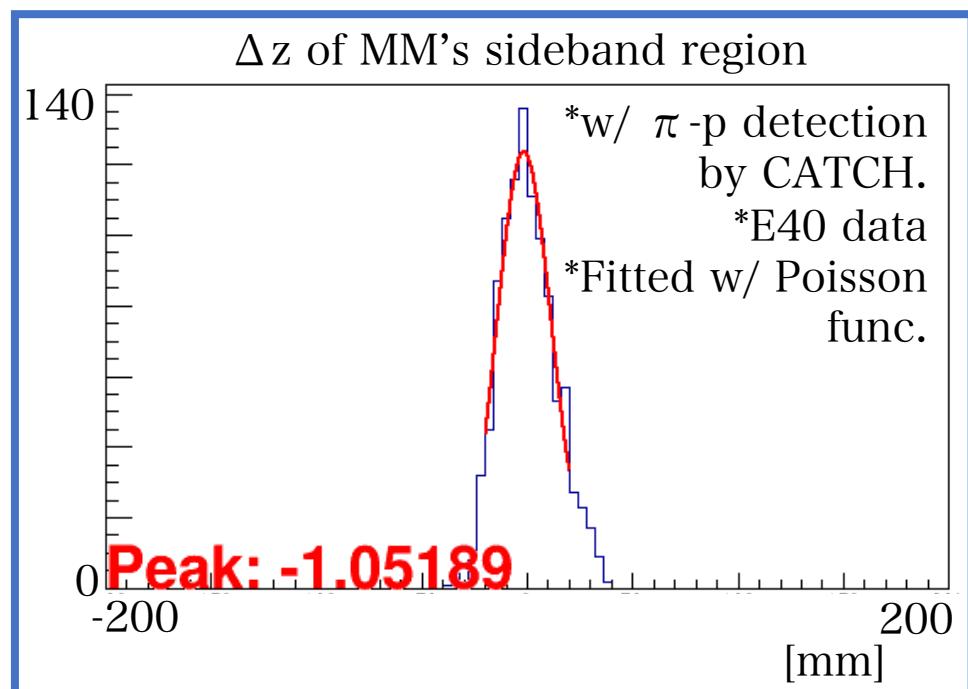
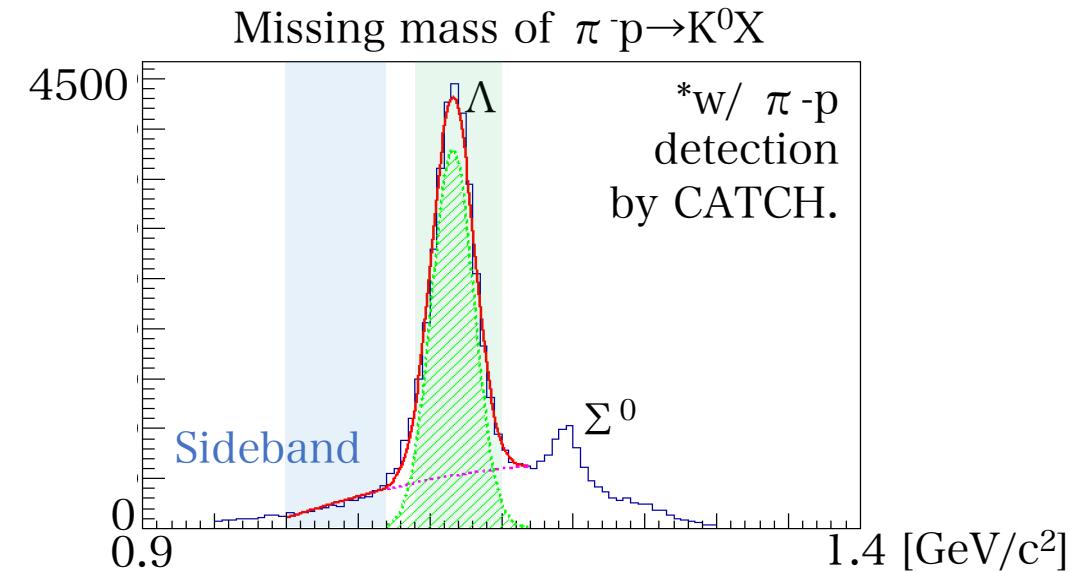
BG Suppression in P_A Analysis



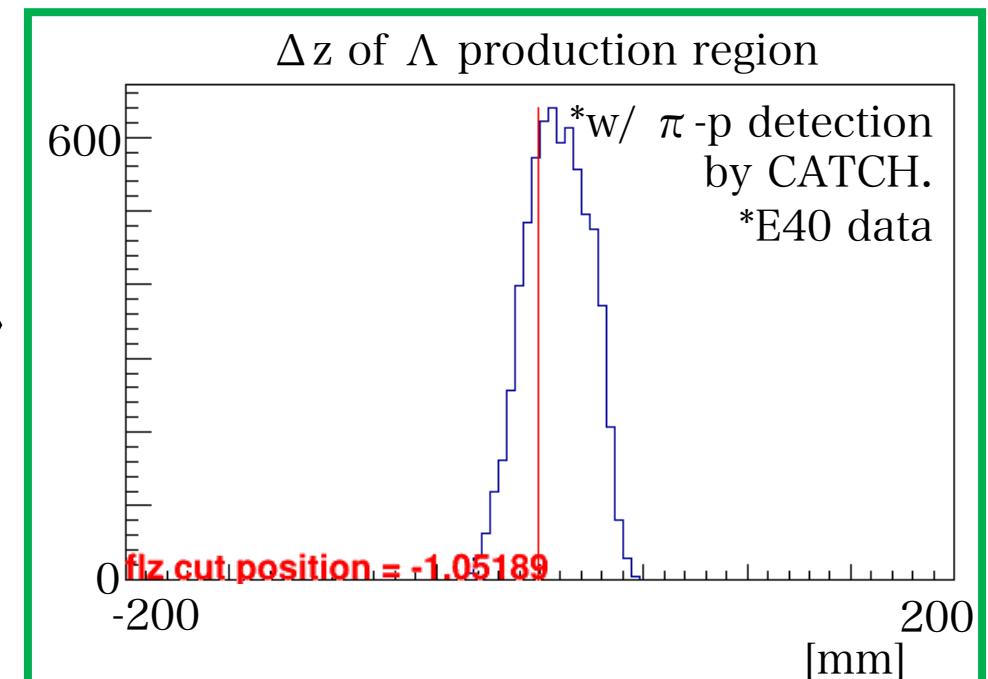
Outlook of the BG Suppression

Estimation of BG structure from MM's sideband

- To reproduce BG more precisely.
- Use peak positions of MM's sideband as the Δz cut value.



Set
the cut value

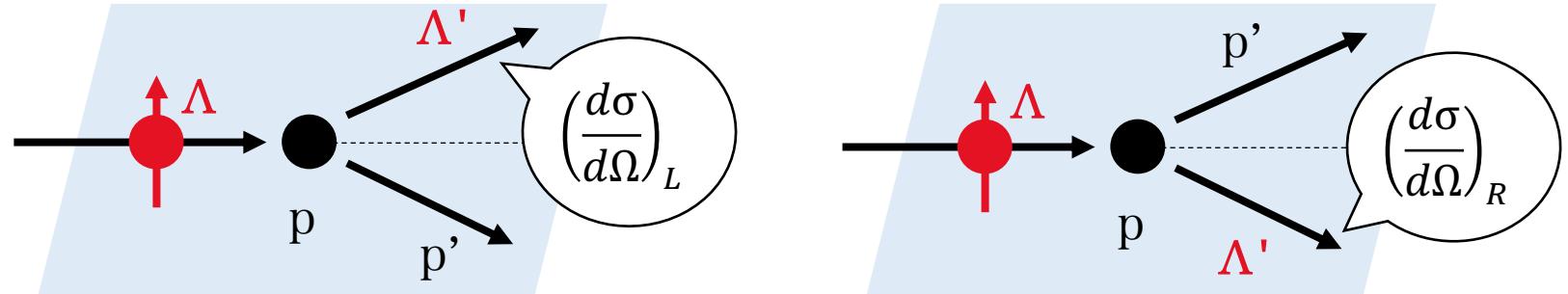


P_A Calculation

Λ Beam Polarization & Spin Observables

Analyzing Power:

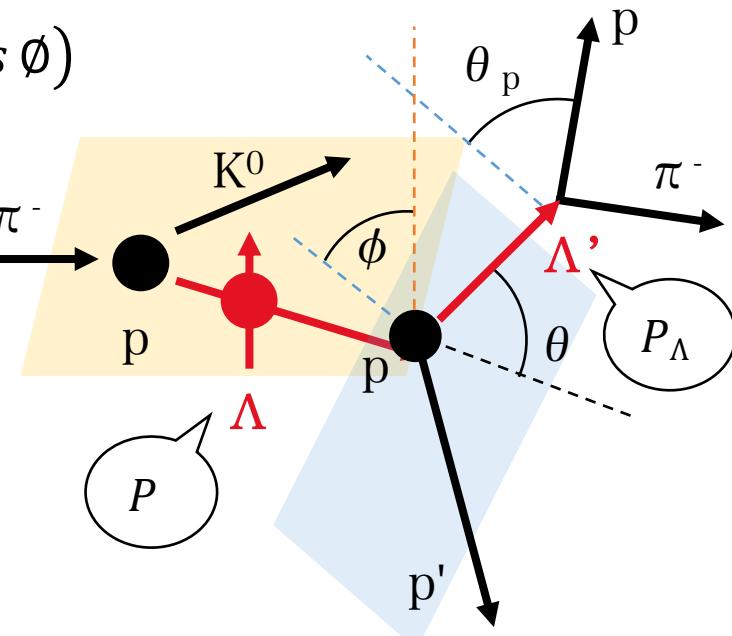
$$A_y = \frac{\pi}{2P_\Lambda} \frac{\left(\frac{d\sigma}{d\Omega}\right)_L - \left(\frac{d\sigma}{d\Omega}\right)_R}{\left(\frac{d\sigma}{d\Omega}\right)_L + \left(\frac{d\sigma}{d\Omega}\right)_R}$$



Differential Cross Section:

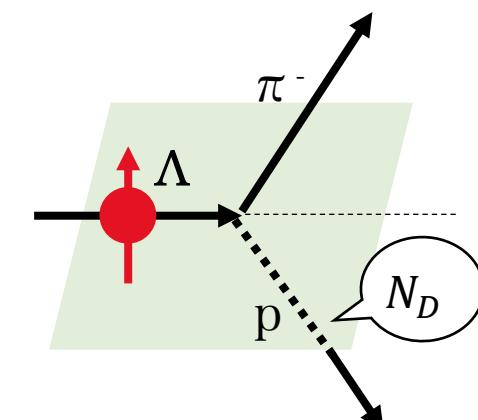
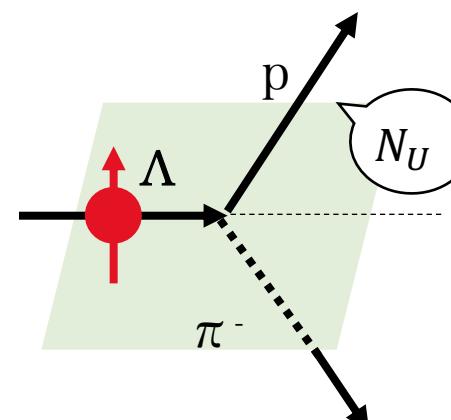
$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_0 (1 + A_y(\theta)P_\Lambda \cos \phi)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{2} \left(\left(\frac{d\sigma}{d\Omega}\right)_L + \left(\frac{d\sigma}{d\Omega}\right)_R \right)$$



Depolarization:

$$P_{\Lambda p \text{ scat.}} = \frac{P + D_y^y P_\Lambda \cos \phi}{1 + PP_\Lambda \cos \phi} = \frac{2N_U - N_D}{\alpha N_U + N_D}$$



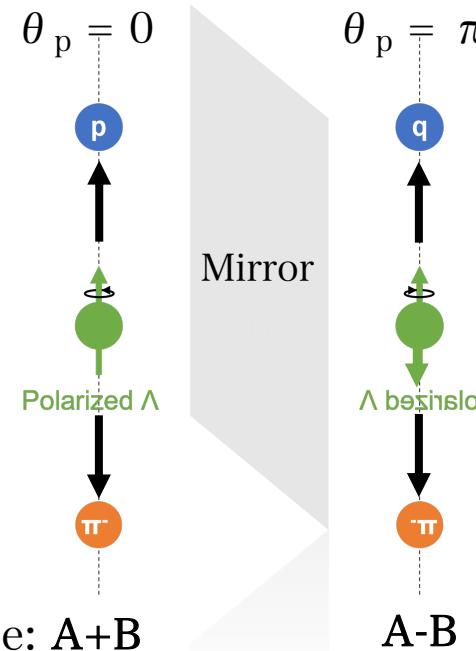
P_Λ Measurement Method w/ the $\Lambda \rightarrow p \pi^-$ Decay

Parity violation ($P_p=+1$, $P_\pi=-1$)

Amplitudes of the π -N system

- $S_{1/2}$: A
- $P_{1/2}$: B

→ Two decay modes show
different amplitudes.



$$\frac{1+\alpha}{1-\alpha} = \frac{|A+B|^2}{|A-B|^2}$$

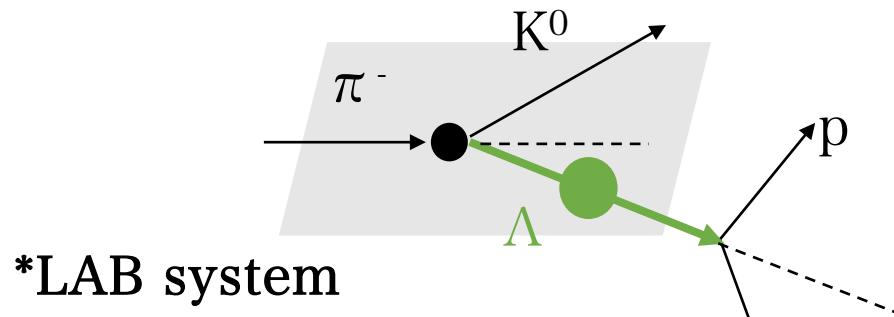
The scattering angle (θ_p) will be asymmetric.

U/D asymmetry in $\cos \theta_p$ appears.

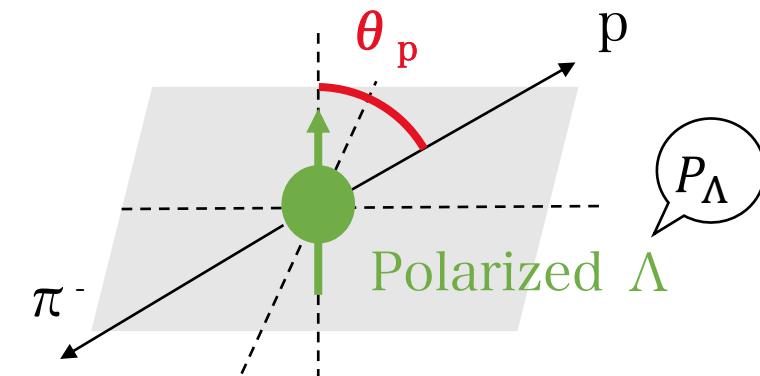
General Partial Wave Analysis of the Decay of a Hyperon of Spin $\frac{1}{2}$
T. D. Lee and C. N. Yang
Phys. Rev. 108, 1645 – Published 15 December 1957

P_Λ analysis method

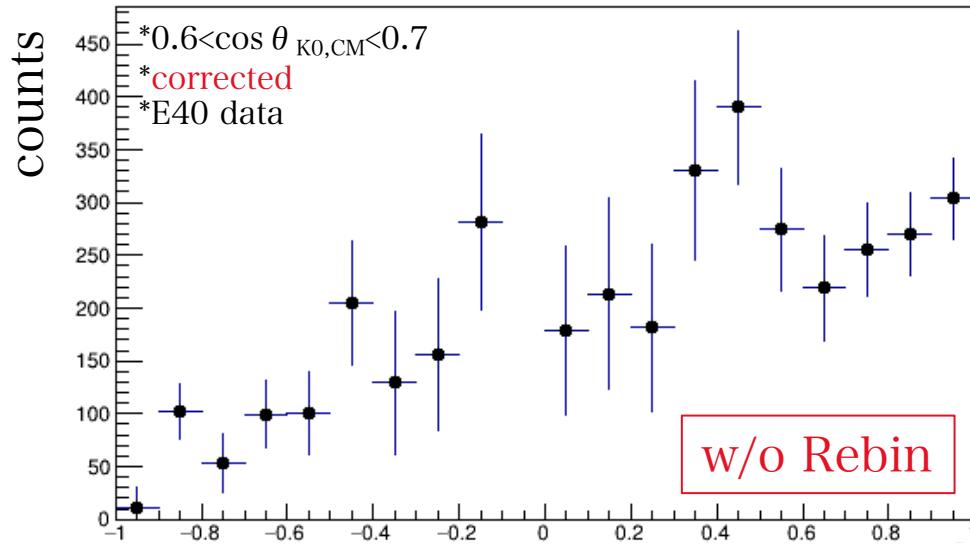
1. Detect p from the $\Lambda \rightarrow \pi^- p$ decay.



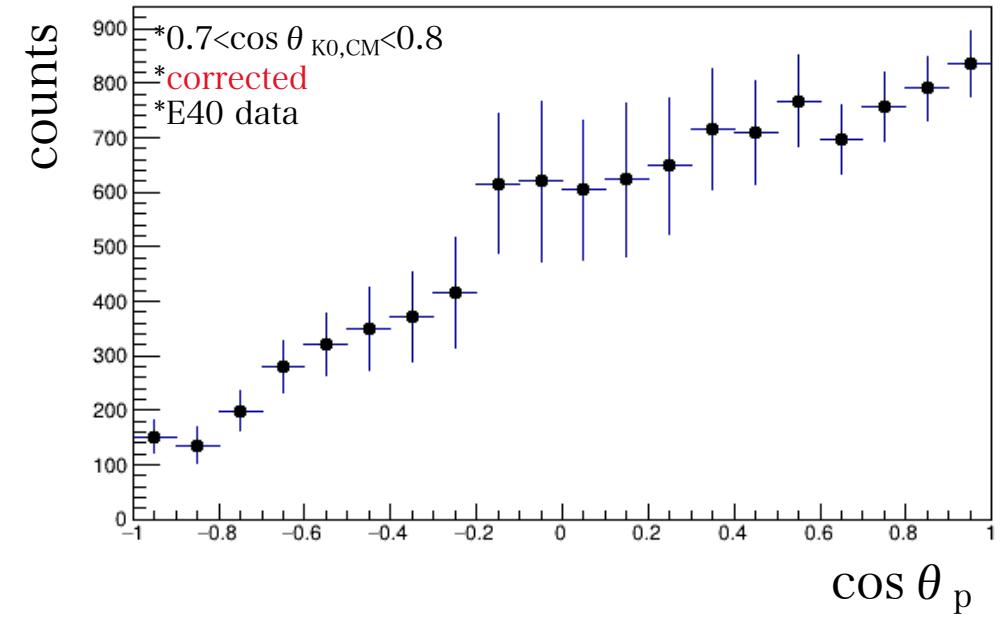
2. Measure θ_p in the rest of Λ .



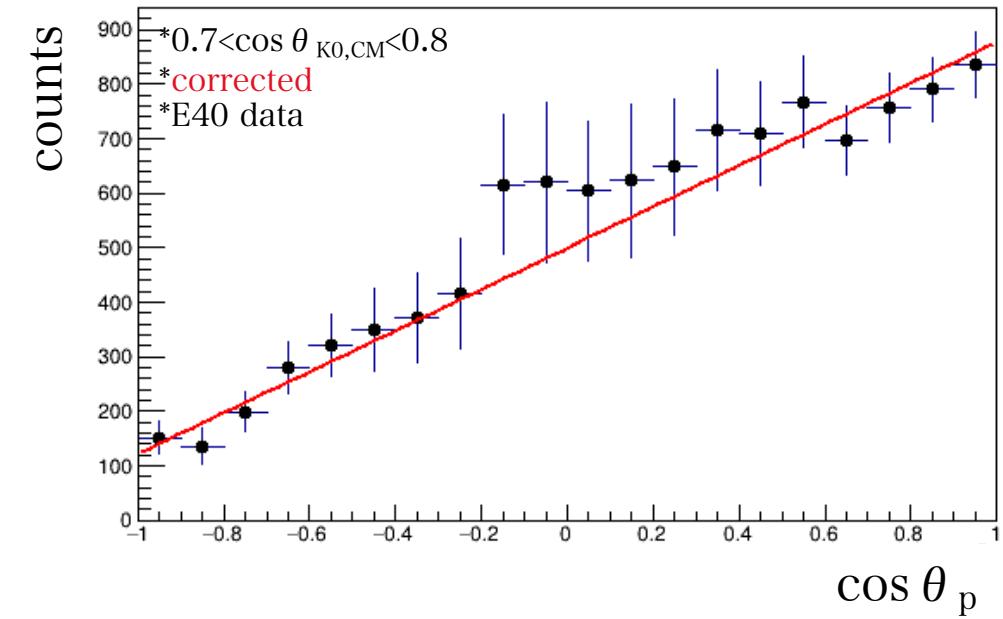
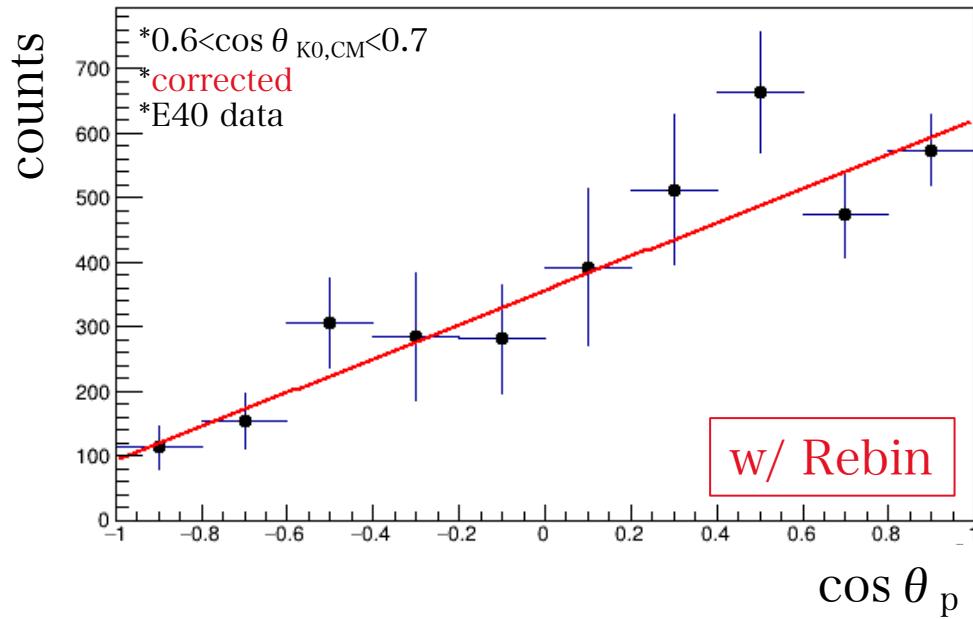
Measured $\cos \theta_p$ in E40



↑ The fitting line is pulled down by the bins
whose entry is accidentally lower than others.



Measured $\cos \theta_p$ in E40



P_A Calculation (Two-parameter fitting)

The reference equation:

$$\frac{1}{N_0} \frac{dN}{d\cos\theta_p} = \frac{1}{2} (1 + \alpha \mathbf{P}_A \cos\theta_p) \quad {}^*\varepsilon_{N,cor} = N_{cor} \sqrt{\left(\frac{\varepsilon_N}{N}\right)^2 + \left(\frac{\varepsilon_{eff}}{eff}\right)^2}$$

To Calculate P_A

- Fitting

$$f(x) = \frac{p_0}{2} (1 + p_1 \cos\theta_p) \leftarrow \text{Two parameters}$$

$$\mathbf{P}_A = \frac{p_1}{\alpha}$$

- Error

$$\varepsilon_{P_A} = \mathbf{P}_A \sqrt{\left(\frac{\varepsilon_{p_1}}{p_1}\right)^2 + \left(\frac{\varepsilon_\alpha}{\alpha}\right)^2} \quad \text{*The systematic error will be estimated.}$$

- The systematic error

- Will be estimated by studying the cut condition's effect for P_A .

Efficiency Estimation for Every K^0 Angle

