30 June 2022

Modelling the $K^+\Sigma^-$ photoproduction with an Isobar model using a novel fitting method

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HYP 2022 PRAGUE

14th International Conference on Hypernuclear and Strange Particle Physics

June 27 – July 1, 2022
Prague, Czech Republic

Outline

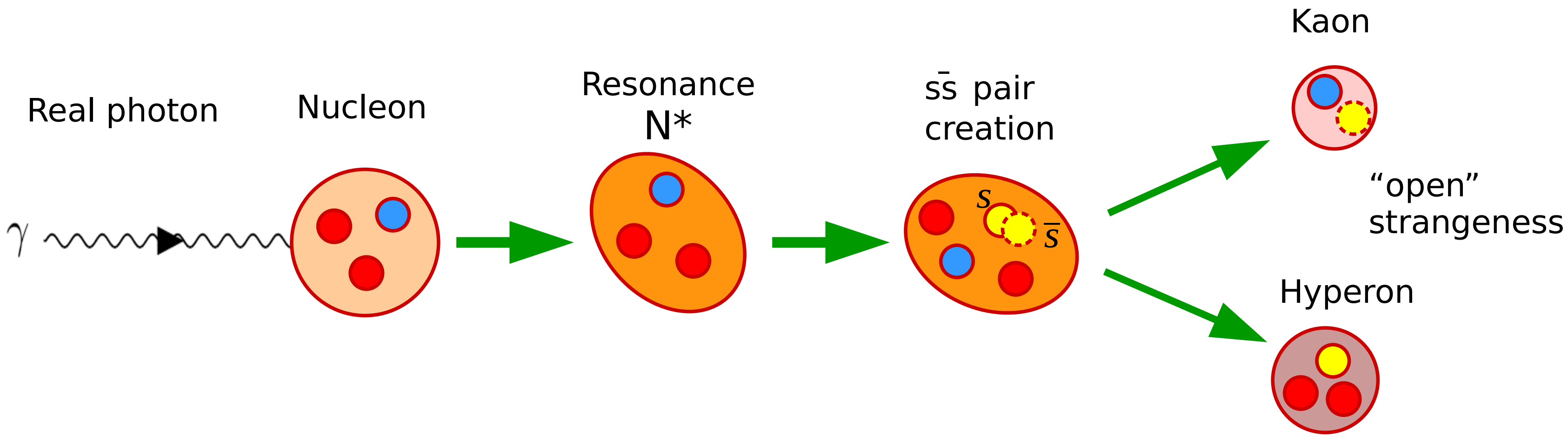
1. Motivation

2. The Isobar model

3. The fitting procedure

4. Numerical results

Photoproduction of Kaons and Hyperons off Nucleons



$$\gamma + p \rightarrow K^{+} + \Lambda$$

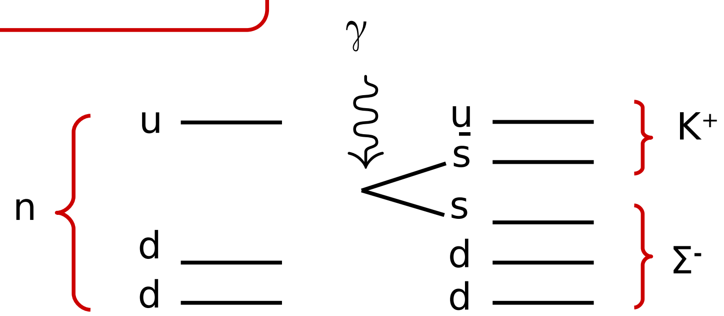
$$\gamma + p \rightarrow K^{+} + \Sigma^{0}$$

$$\gamma + p \rightarrow K^{0} + \Sigma^{+}$$

$$\gamma + n \rightarrow K^{0} + \Lambda$$

$$\gamma + n \rightarrow K^{0} + \Sigma^{0}$$

$$\gamma + n \rightarrow K^{+} + \Sigma^{-}$$ [1, 2]



[1.] P. Bydzovsky et al., Phys. Rev. C 104, 065202 (2021), present work

[2.] N. Zachariou et al., Phys. Lett. B 827, 136985 (2022), new data from CLAS @ JLab (Hall B)

[3.] Figure adapted from: L. De Cruz, PhD Thesis, Ghent University 2012

Motivation

- Use of *regularized* least squares fitting
    - model with fewer parameters → improves the quality of the fits, since: large number of parameters → ordinary χ^2 fitting is problematic similar minima, large *variations* in the parameter values
- + Information criteria
    - automatic selection out of a huge number of possible combinations of candidate resonances (model selection)

[4.] B. Guegan et al., JINST 10 P09002 (2015)
[5,6.] J. Landay et al., Phys. Rev. C 95, 015203 (2017), Phys. Rev. D 99, 016001 (2019)

General features of Isobar models

- interactions described by means of effective Lagrangians
    - effective degrees of freedom: hadrons

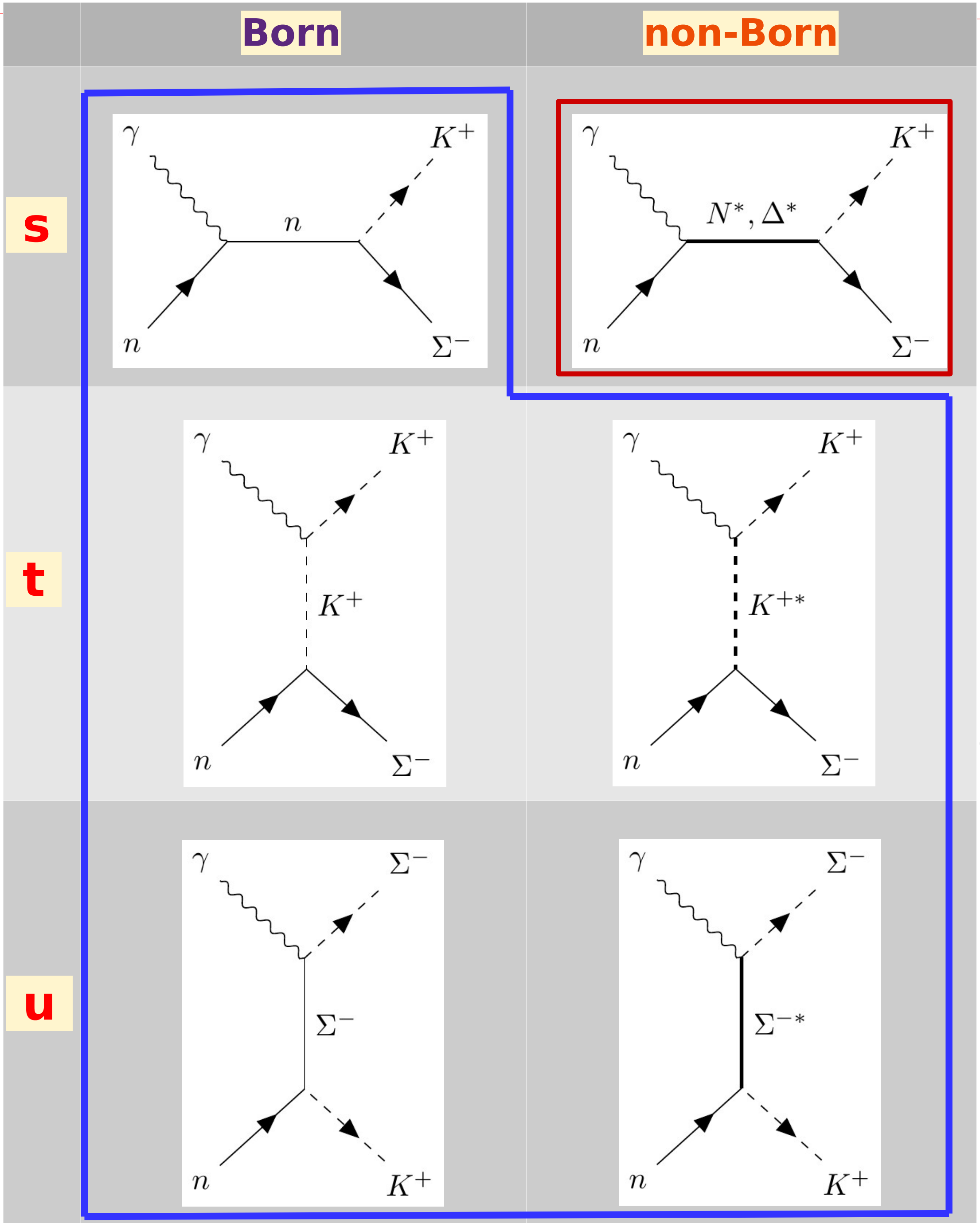
- amplitude = sum of tree-level Feynman diagrams
    - s-, t-, u- channels: exchange of nucleon, kaon, hyperon
    - intermediate state: ground state hadron (Born), resonance (non-Born)

- single-channel: intermediate channels (2nd order) not taken into account → coupling constants: *effective* values

- Saclay-Lyon, MAID & Kaon-MAID, Gent, BS1,2,3[7,8] models

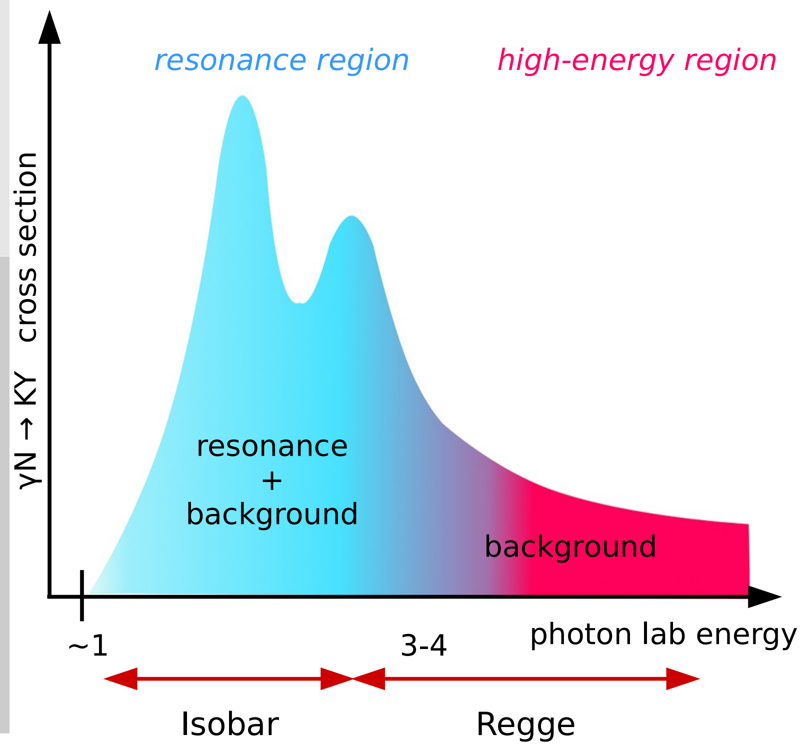
7. D. Skoupil and P. Bydzovsky, Phys. Rev. C 93, 025204 (2016)
8. D. Skoupil and P. Bydzovsky, Phys. Rev. D 97, 025202 (2018)

Tree-level contributions to $n(\gamma,K^+)\Sigma^-$



the only resonant diagrams

rest of diagrams
→ background



Specific features of Isobar model

Hadronic form factors

- hadron internal structure
- mitigate Born terms' contribution to cross sections

$$F_d = \frac{\Lambda_h^4}{\Lambda_h^4 + (x - m_h^2)^2}$$

Λ_h cutoff parameter
x 4-momentum^2,
m_h mass,

of intermediate hadron h

Decay widths

- finite lifetime of resonances
- decay widths Γ, introduced by hand in propagators of s-channel particles

$$\mathcal{P} \sim \frac{1}{q^2 - m^2} \qquad q^2 = s \qquad s - m_R^2 \rightarrow s - m_R^2 + i m_R \Gamma_R \qquad R = N^*, \Delta^*$$

Parameters and observables

Resonances

masses, widths: from PDG

Parameters to fit

- $g_{K\Sigma n}$
- coupling constants of resonances
(= products of E/M and strong c.c.)
- hadron form factor cutoffs

674 data points from: CLAS, LEPS

Observables

differential cross sections

photon beam asymmetries

Tag	Resonance	Mass (MeV)	Width (MeV)
K*	$K^*(892)$	891.7	50.8
K1	$K_1(1270)$	1270	90
N3	$N(1535)\ 1/2^-$	1530	150
N4	$N(1650)\ 1/2^-$	1650	125
N8	$N(1675)\ 5/2^-$	1675	145
N6	$N(1710)\ 1/2^+$	1710	140
N7	$N(1720)\ 3/2^+$	1720	250
P4	$N(1875)\ 3/2^-$	1875	200
P1	$N(1880)\ 1/2^+$	1880	300
Mx	$N(1895)\ 1/2^-$	1895	120
P2	$N(1900)\ 3/2^+$	1920	200
M4	$N(2060)\ 5/2^-$	2100	400
M1	$N(2120)\ 3/2^-$	2120	300
D1	$\Delta(1900)\ 1/2^-$	1860	250

included resonances (14)
based on [3]

Minimization with: MINUIT Library

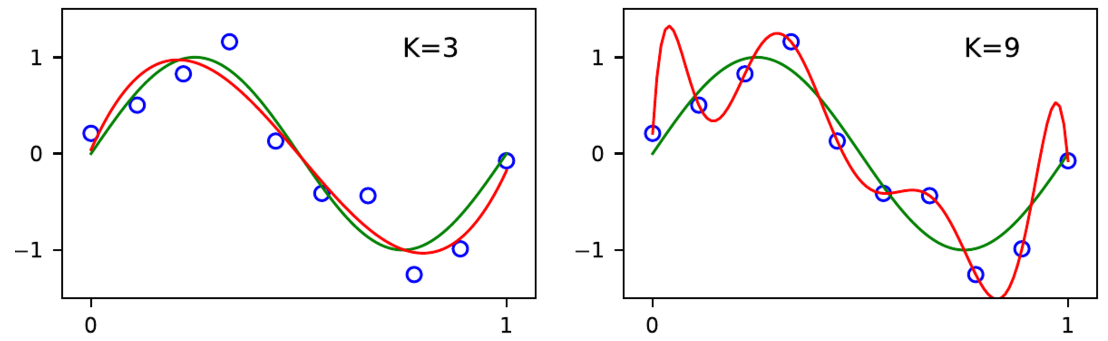
Isobar code available at:

http://www.ujf.cas.cz/en/departments/department-of-theoretical-physics/isobar-model.html

LASSO regularization for parameter selection

Overfitting example

K-th order polynomial fitting



Regularization

Error function

$$\chi^2 = \sum_{i=1}^{N} \left[\frac{y_i - f(x_i, \mathbf{w})}{\sigma_i} \right]^2 \Rightarrow \chi^2_T(\lambda) = \chi^2 + \lambda \sum_{j=1}^{K} |w_j|^q$$

penalty term

λ: regularization parameter

w_j: model parameters

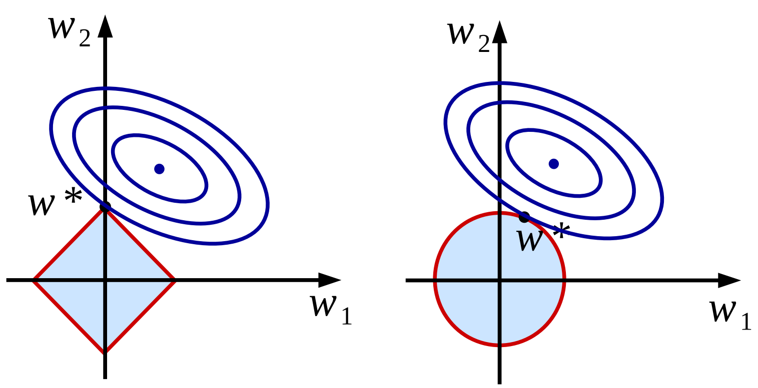
Constrained error minimization

→ reduces parameter values

↑ λ → ↑ reduction

LASSO (**L**east **A**bsolute **S**hrinkage and **S**election **O**perator)

- some parameters become zero ($w^*_1 = 0$)
- λ → how many parameters survive



[9.] T. Hastie et al. The Elements of Statistical Learning, 2nd ed. Springer 2009

Information Criteria (IC)

In the case of LASSO: $\lambda_i \rightarrow \text{model}_i$

For a set of λ values: $\{\lambda_1, \lambda_2, \ldots \lambda_m\}$ ($\rightarrow$ $\{\text{model}_1, \text{model}_2, \ldots \text{model}_m\}$)

Compute:

Akaike IC[10]: $$AIC_i = \chi^2_T(\lambda_i)_{\min} + 2k_i$$

Bayesian IC[11]: $$BIC_i = \chi^2_T(\lambda_i)_{\min} + k_i \ln(N)$$

k_i : number of parameters corresponding to model i

N : number of data points

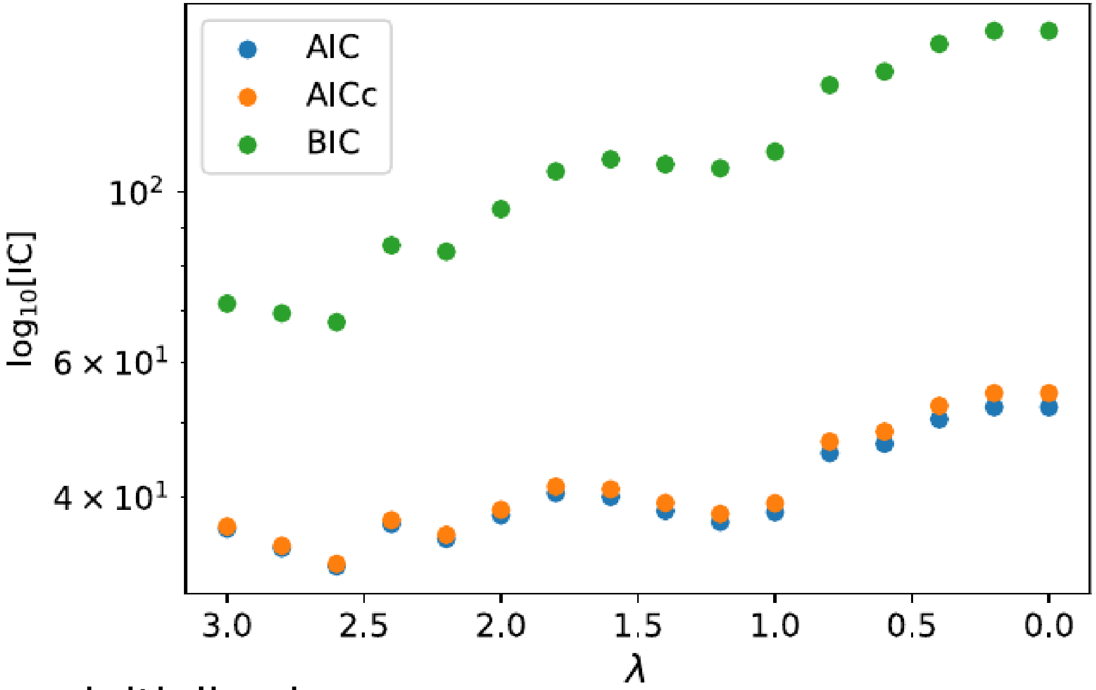
Choose λ_i (model_i) with the minimum AIC, BIC

[both AIC and BIC give similar results, although BIC tends to penalize complexity more]

10. Akaike, IEEE Transactions on Automatic Control, 19 (6) 716 (1974)

11. G. Schwarz, Ann. Stat. 6(2), 461 (1978)

Applying the Information Criteria



Forward selection:

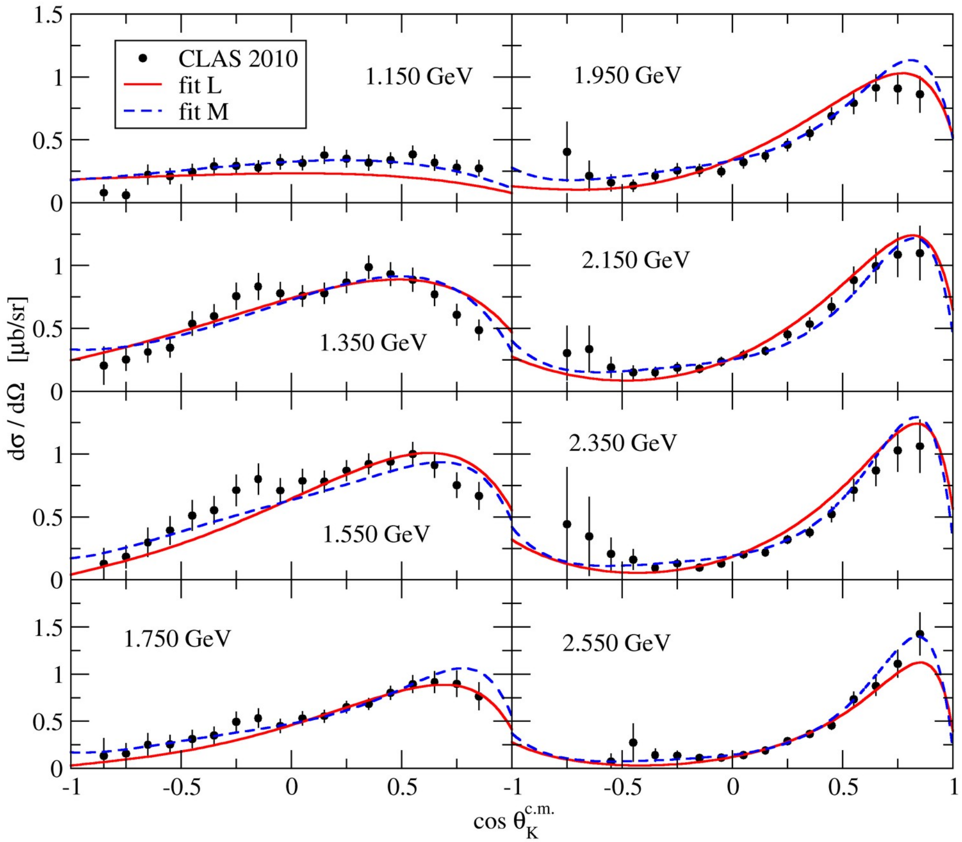
- start with the full model, all parameters initialized with random values and use some λ_{max}
- perform LASSO χ^2 minimization and compute AIC, BIC
- in each run progressively decrease λ and rerun LASSO using the fitted parameter values of the last run as starting values
- repeat until λ_{min} is reached
- optimal λ_{opt} occurs at the minimum of BIC, AIC

Reduction in the number of parameters

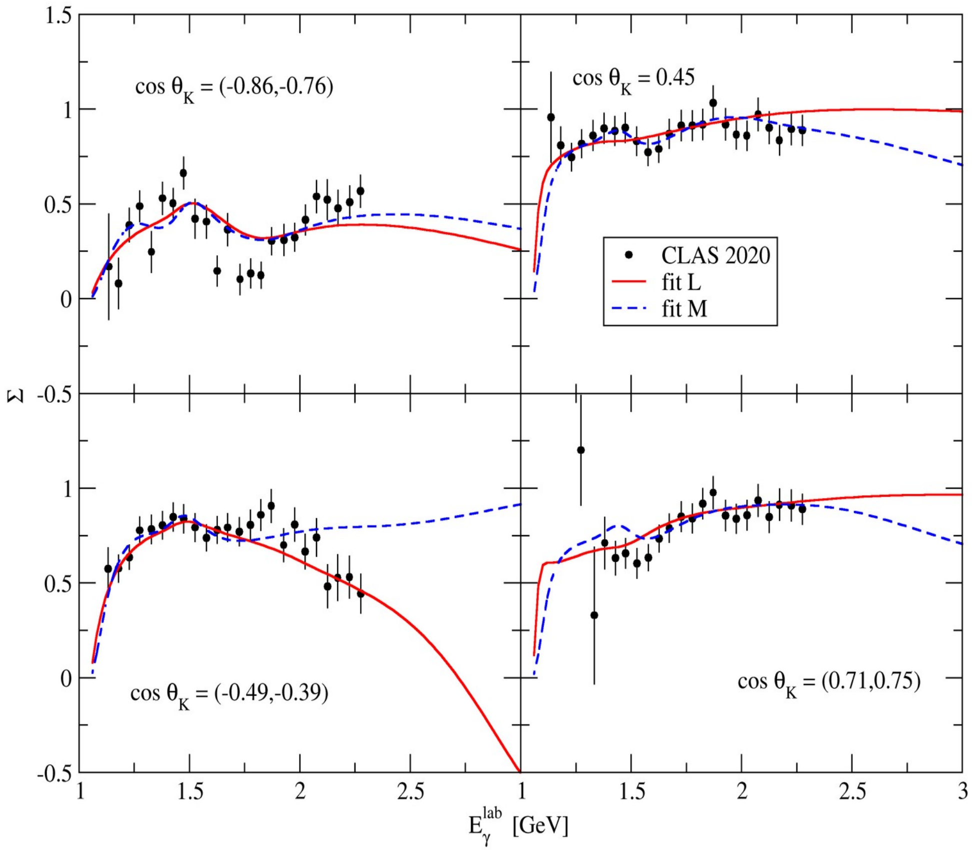
Tag	Resonance	Mass (MeV)	Width (MeV)	Branching ratio $K\Lambda$	Branching ratio $K\Sigma$	Fit M g_1	Fit M g_2	Fit L g_1	Fit L g_2
K*	$K^*(892)$	891.7	50.8			0.366 ± 0.024	1.103 ± 0.198	0.310 ± 0.019	
K1	$K_1(1270)$	1270	90			-1.448 ± 0.189	0.473 ± 0.156		
N3	$N(1535)\ 1/2^-$	1530	150			-0.709 ± 0.071			
N4	$N(1650)\ 1/2^-$	1650	125	0.07	0.00	0.314 ± 0.034		-0.085 ± 0.006	
N8	$N(1675)\ 5/2^-$	1675	145			-0.013 ± 0.001	0.022 ± 0.003	-0.010 ± 0.001	0.003 ± 0.002
N6	$N(1710)\ 1/2^+$	1710	140	0.15	0.01	-0.940 ± 0.093			
N7	$N(1720)\ 3/2^+$	1720	250	0.05	0.00	-0.098 ± 0.017	-0.082 ± 0.002	-0.187 ± 0.004	-0.126 ± 0.002
P4	$N(1875)\ 3/2^-$	1875	200	0.01	0.01	-0.220 ± 0.023	-0.223 ± 0.023	-0.042 ± 0.015	0.025 ± 0.013
P1	$N(1880)\ 1/2^+$	1880	300	0.16	0.14	-0.050 ± 0.064			
Mx	$N(1895)\ 1/2^-$	1895	120	0.18	0.13	-0.063 ± 0.005		0.019 ± 0.002	
P2	$N(1900)\ 3/2^+$	1920	200	0.11	0.05	-0.051 ± 0.005	-0.004 ± 0.001	0.027 ± 0.003	0.010 ± 0.001
M4	$N(2060)\ 5/2^-$	2100	400	0.01	0.03	-0.00001 ± 0.0001	0.003 ± 0.0003	-0.003 ± 0.0001	0.004 ± 0.0002
M1	$N(2120)\ 3/2^-$	2120	300			-0.034 ± 0.014	-0.010 ± 0.013	0.0003 ± 0.001	0.0 ± 0.0001
D1	$\Delta(1900)\ 1/2^-$	1860	250		0.01	0.298 ± 0.028			
D2	$\Delta(1930)\ 5/2^-$	1880	300						
D3	$\Delta(1920)\ 3/2^+$	1900	300						
D4	$\Delta(1940)\ 5/2^-$	1950	400						
S1	$\Sigma(1660)\ 1/2^+$	1660	100						
S2	$\Sigma(1750)\ 1/2^-$	1750	90						
S3	$\Sigma(1670)\ 3/2^-$	1670	60						
S4	$\Sigma(2010)\ 3/2^-$	1940	220						

	M "full" fit	L LASSO + IC fit
number of resonances	14	9
number of parameters	25	17
χ^2 / n.d.f.	2.4	3.2

Results:
differential cross sections & photon beam asymmetries Σ

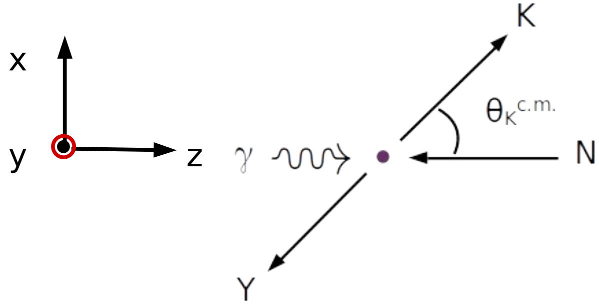


$\theta_K^{c.m.}$: Kaon center-of-mass angle



E^{lab}_{γ}: incident photon energy

fit M: MINUIT
fit L: MINUIT + LASSO + IC



Linearly polarized photons

$$\epsilon^{\lambda=x} \equiv \epsilon^{\parallel} = (0,1,0,0) \qquad \Sigma = \frac{d\sigma^{\perp} - d\sigma^{\parallel}}{d\sigma^{\perp} + d\sigma^{\parallel}}$$

$$\epsilon^{\lambda=y} \equiv \epsilon^{\perp} = (0,0,1,0)$$

more results in presentation of D. Skoupil

Summary

- We modeled $K^+ \Sigma^-$ photoproduction with an Isobar model using regularization (LASSO) in combination with the Akaike and Bayesian information criteria.
- Regularization in a model with many parameters leads to more robust results, less prone to overfitting.
- The combination of the LASSO method with Information Criteria provides a method to choose the best subset of parameters (model).

Thanks to:

P. Bydzovsky, A. Cieply, D. Skoupil, P. Vesely, N Zachariou

Thank you for your attention!

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9). T. Hastie, R. Tibshirani, and J. Friedman, The Elements of Statistical Learning: Data Mining, Inference, and Prediction, 2nd ed. Springer (2009)

10). H. Akaike, A new look at the statistical model identification. IEEE Transactions on Automatic Control, 19 (6): 716–723 (1974).

11). G. Schwarz, Estimating the dimension of a model, Ann. Stat. 6(2), 461-464 (1978).