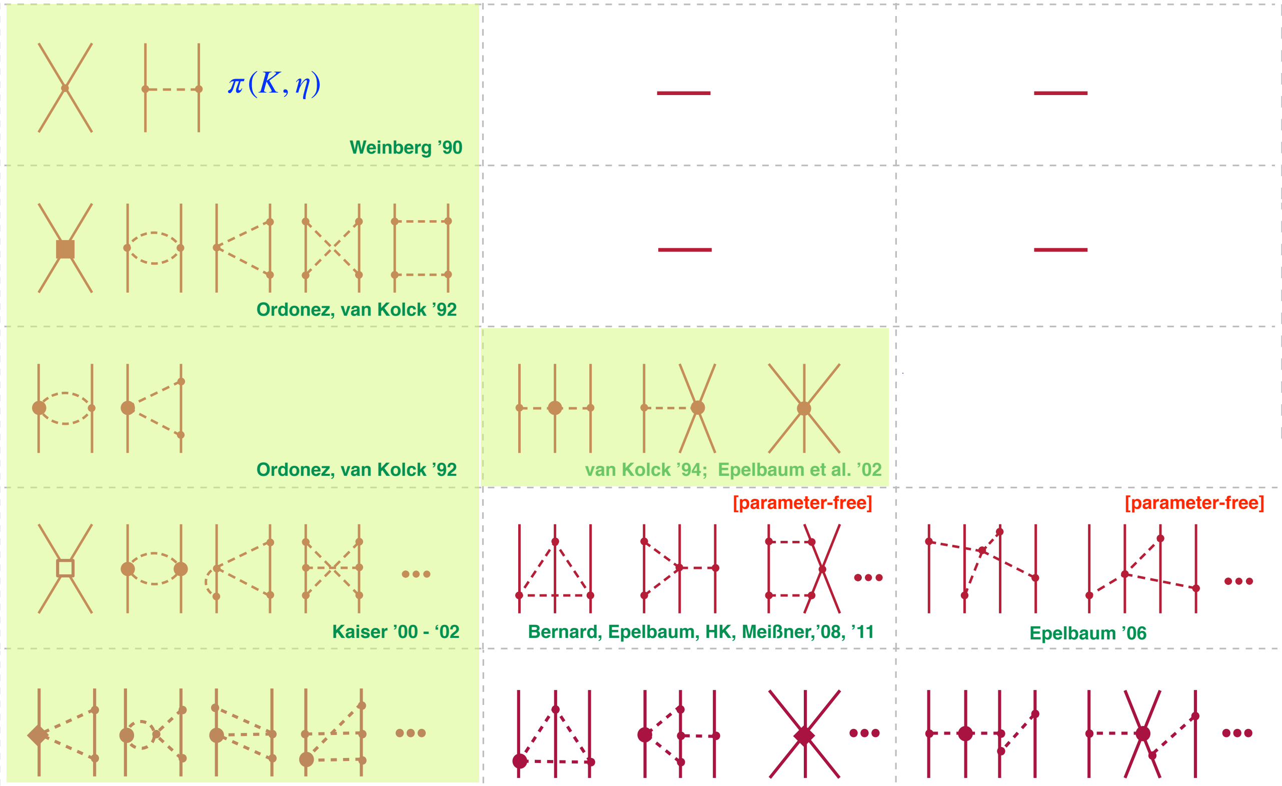
Single- & double-strangeness hypernuclei up to A=8 within chiral EFT

Hoai Le, IAS-4 & IKP-3, Forschungszentrum Jülich, Germany

HYP2022, Prague, Czech Republic, June 27- July 1, 2022

collaborators: Johann Haidenbauer, Ulf-G Meißner, Andreas Nogga

BB interactions in χEFT



	2B force	3B force	4B force	
LO (Q^0)	$\pi(K,\eta)$ Weinberg '90	—	—	2 NN, 5 YN LECs (short range parameters)
NLO (Q^2)	Ordonez, van Kolck '92	—	—	+7 NN, +23 YN LECs
N²LO (Q^3)	Ordonez, van Kolck '92	van Kolck '94; Epelbaum et al. '02		**+2 NNN LECs, +5 ΛNN LECs**
N³LO (Q^4)	Kaiser '00 - '02	[parameter-free] Bernard, Epelbaum, HK, Meißner,'08, '11	[parameter-free] Epelbaum '06	+15 NN LECs
N⁴LO (Q^5)	Entem, Kaiser, Machleidt, Nosyk '15 Epelbaum, HK, Meißner '15	Girlanda, Kievsky, Viviani '11 HK, Gasparyan, Epelbaum '12,'13		+5 NN LECs

(adapted from H. Krebs CD workshop, 18th November 2021)

- LECs are determined via a fit to experiment:
  - ~5000 NN + Nd scattering data + ^{2}H, ^{3}H/^{3}He → NN forces up to N^{4}LO+, 3NF up to N^{2}LO **(see E. Epelbaum talk)**
  - ~36 YN data + $^3_\Lambda$H → YN forces up to NLO (NLO13, NLO19) and N^{2}LO **(see J. Haidenbauer talk)**

Jacobi-NCSM approach

diagonalize the A-body translationally invariant hypernuclear Hamiltonian

$$H = T_{rel} + V^{NN} + V^{YN} + V^{NNN} + V^{YNN} + \Delta M$$

$\Lambda N \leftrightarrow \Sigma N$

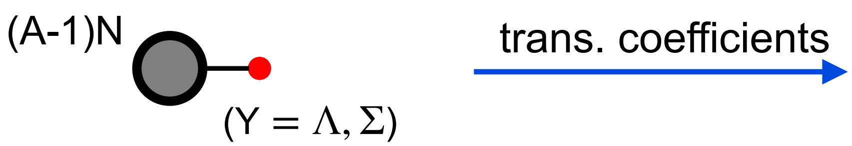
in a finite A-particle harmonic oscillator (HO) basis

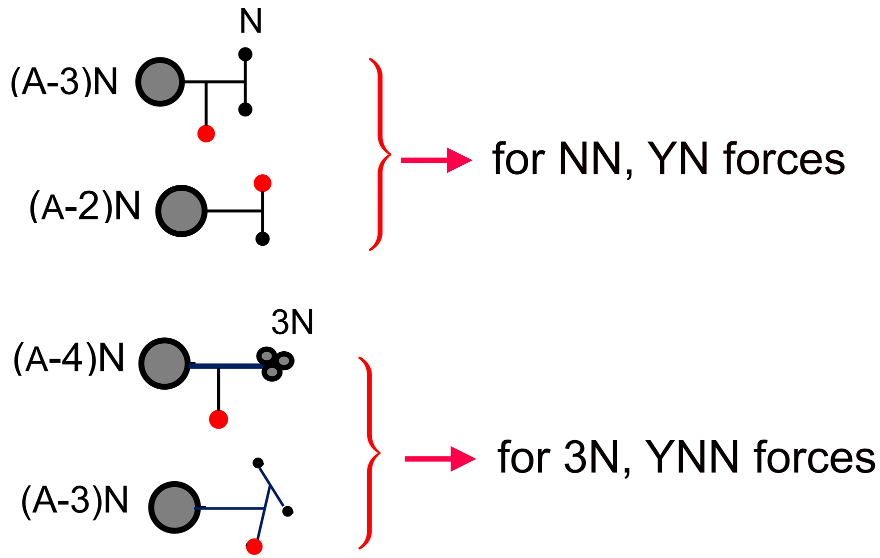
- basis states for $S = -1$ systems:

(A-1)N

$$\left| \text{(A-1)N} - \Lambda(\Sigma) \right\rangle = | \mathcal{N} J T, \underbrace{\mathcal{N}_{A-1} J_{A-1} T_{A-1}}_{antisym.(A-1)N}, \underbrace{n_Y l_Y I_Y t_Y}_{\Lambda(\Sigma)\ state}; (J_{A-1}(l_Y s_Y) I_Y) J, (T_{A-1} t_Y) T \rangle$$

- intermediate bases for evaluating Hamiltonian:



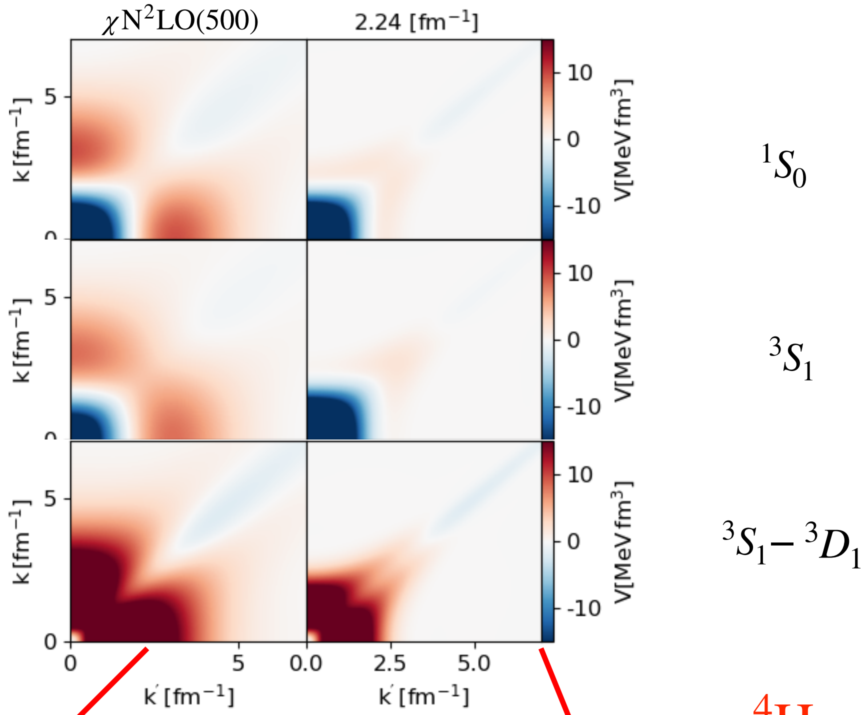


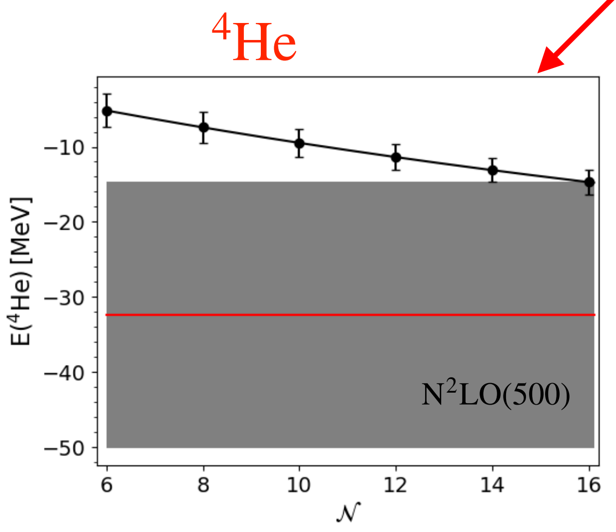
- basis truncation: $\mathcal{N} = \mathcal{N}_{A-1} + 2n_\lambda + \lambda \leq \mathcal{N}_{max}$ $\Rightarrow E_b = E_b(\omega, \mathcal{N}_{max})$

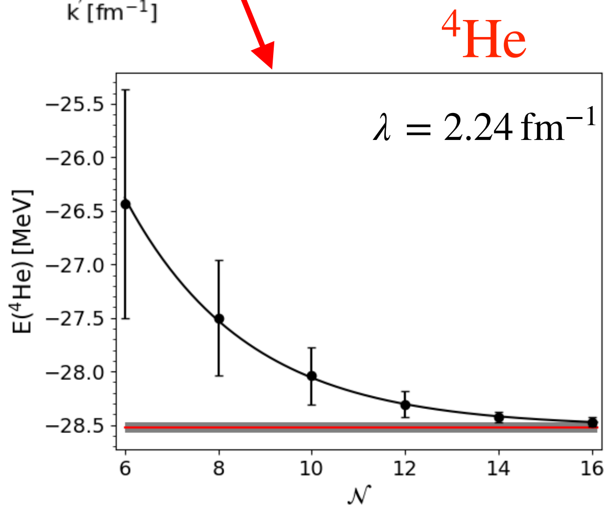
→ extrapolate in ω- and $\mathcal{N}$-spaces to obtain converged results **(HL et al., EPJA (2020))**

Convergence of E with respect to $\mathcal{N}$

- BB interactions contain **short-range and tensor** correlations that couple low- and high-momentum states → NCSM calculations **converge with respect to model space slowly**







Similarity Renormalization Group (SRG)

JÜLICH FORSCHUNGSZENTRUM

Idea: continuously apply unitary transformation to H to suppress off-diagonal matrix elements

→ **observables (binding energies) are conserved due to unitarity of transformation**

F.J. Wegner NPB 90 (2000). S.K. Bogner, R.J. Furnstahl, R.J. Perry PRC 75 (2007)

$$\frac{dV(s)}{ds} = [[T_{rel}, V(s)], H(s)], \qquad H(s) = T_{rel} + V(s) + \Delta M$$

$$V(s) = V_{12}(s) + V_{13}(s) + V_{23}(s) + V_{123}(s), \quad V_{123} \equiv V_{NNN}\ (V_{YNN})$$

- separate SRG flow equations for 2-body and 3-body interactions:

$$\frac{dV^{NN}(s)}{ds} = [[T^{NN}, V^{NN}], T^{NN} + V^{NN}]$$

$$\frac{dV^{YN}(s)}{ds} = [[T^{YN}, V^{YN}], T^{YN} + V^{YN} + \Delta M]$$

$$\begin{aligned}\frac{dV_{123}}{ds} = & [[T_{12}, V_{12}], V_{31} + V_{23} + V_{123}] \\ & + [[T_{31}, V_{31}], V_{12} + V_{23} + V_{123}] \\ & + [[T_{23}, V_{23}], V_{12} + V_{31} + V_{123}] + [[T_{rel}, V_{123}], H_s]\end{aligned}$$

Eqs.(1)

→ SRG-induced 3BFs are generated even if $V_{123}^{\text{bare}} = 0$

⇒ no disconnected terms in $\frac{dV_{123}}{ds}$: avoid delta functions on the right hand side

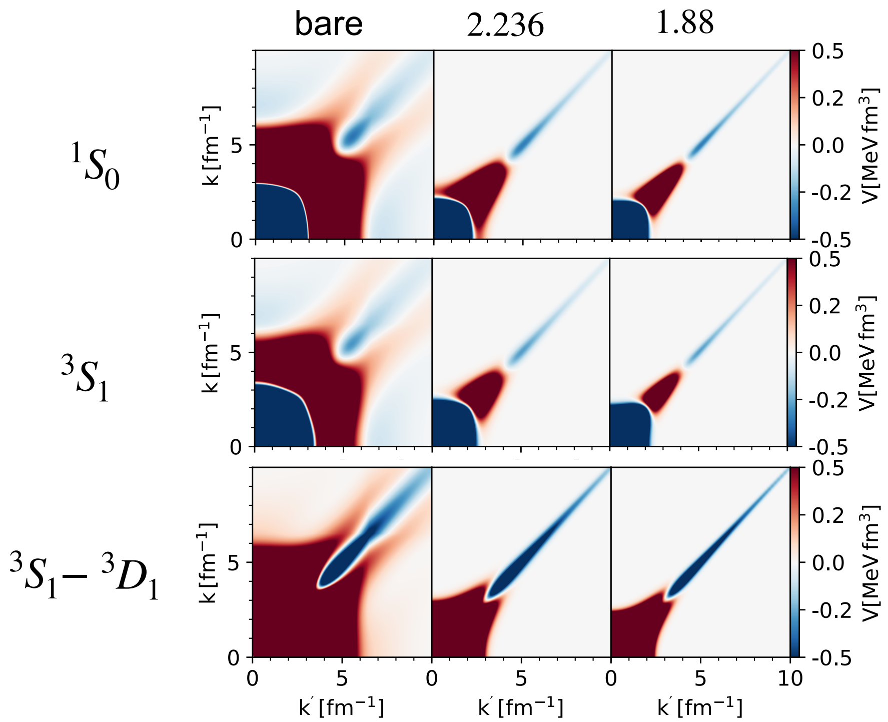
(S.K. Bogner et al PRC75 (2007), K. Hebeler PRC85 (2012))

- Eqs.(1) are solved by projecting on a partial-wave decomposed 3N (YNN) Jacobi-momentum basis

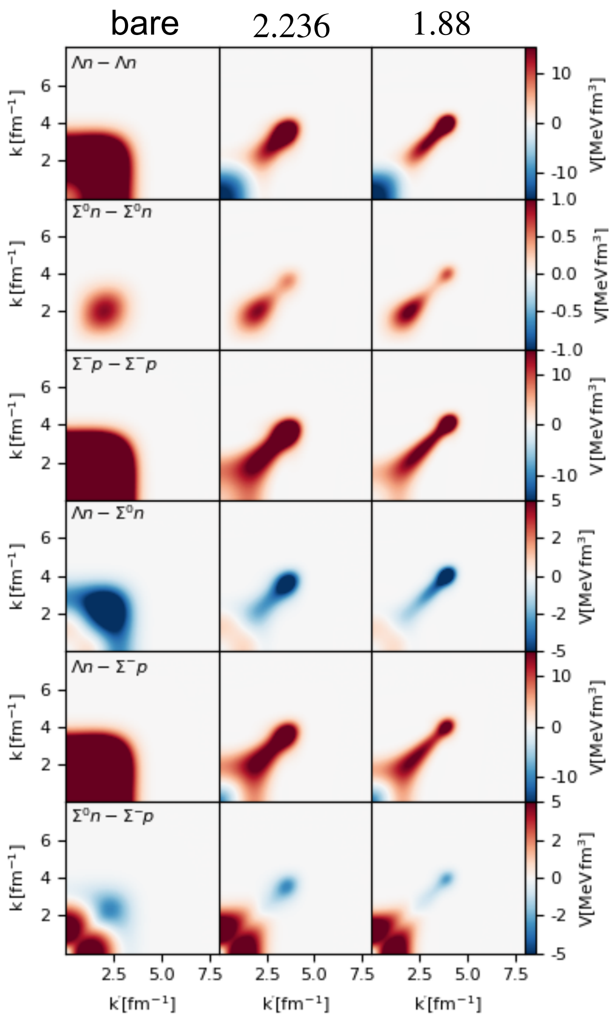
SRG evolution of NN, YN

- $\lambda = (4\mu^2/s)^{1/4}$, $[\lambda] = [p]$: $\lambda \sim$ width of the band-diagonal structure of V in p-space

(S.K. Bogner et al., PRC 75 (2007))



NN: N^{4}LO + (450)

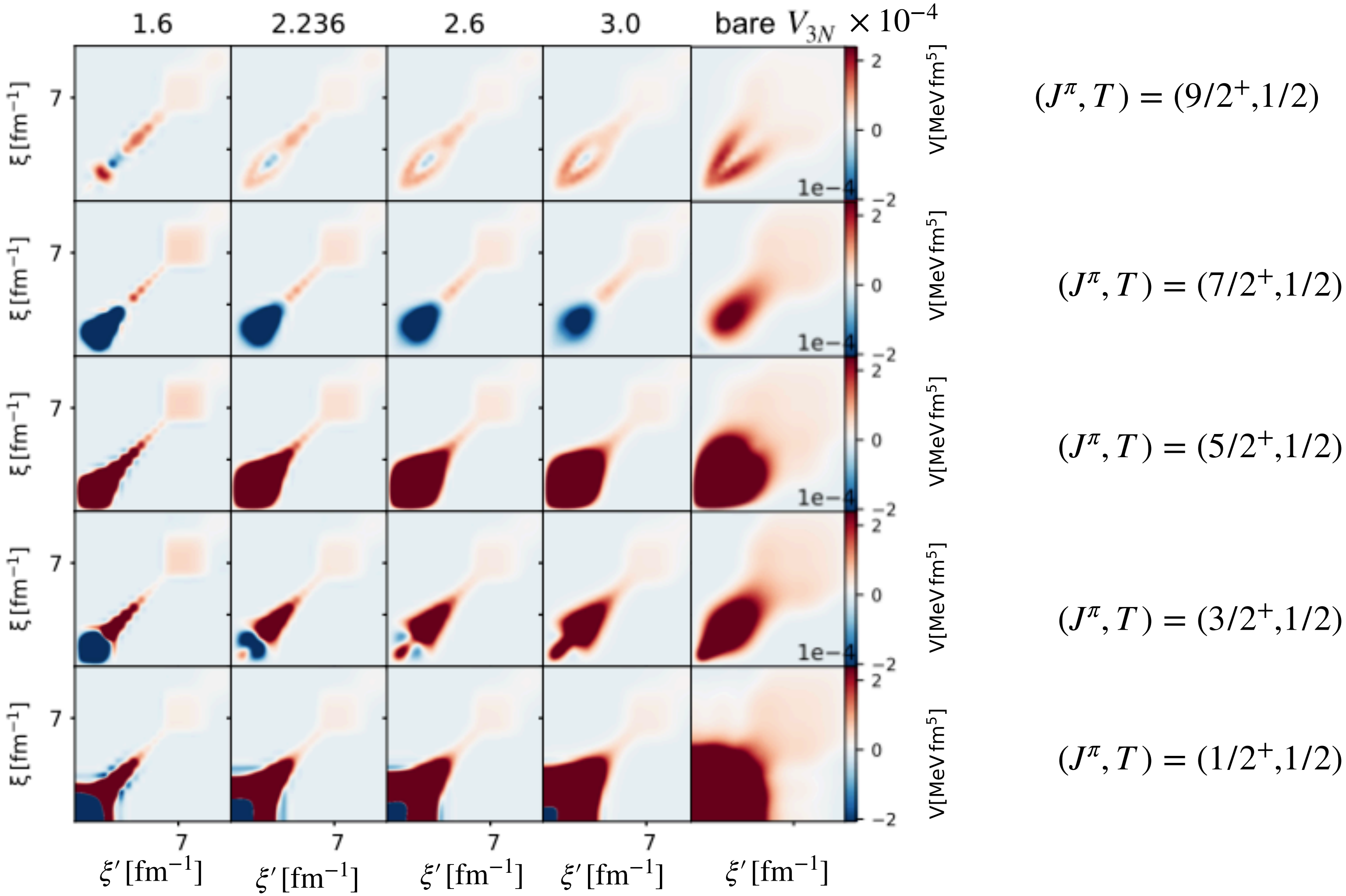


YN: NLO19(500)

SRG evolution of $V^{NNN}(pq\alpha, p'q'\alpha')$

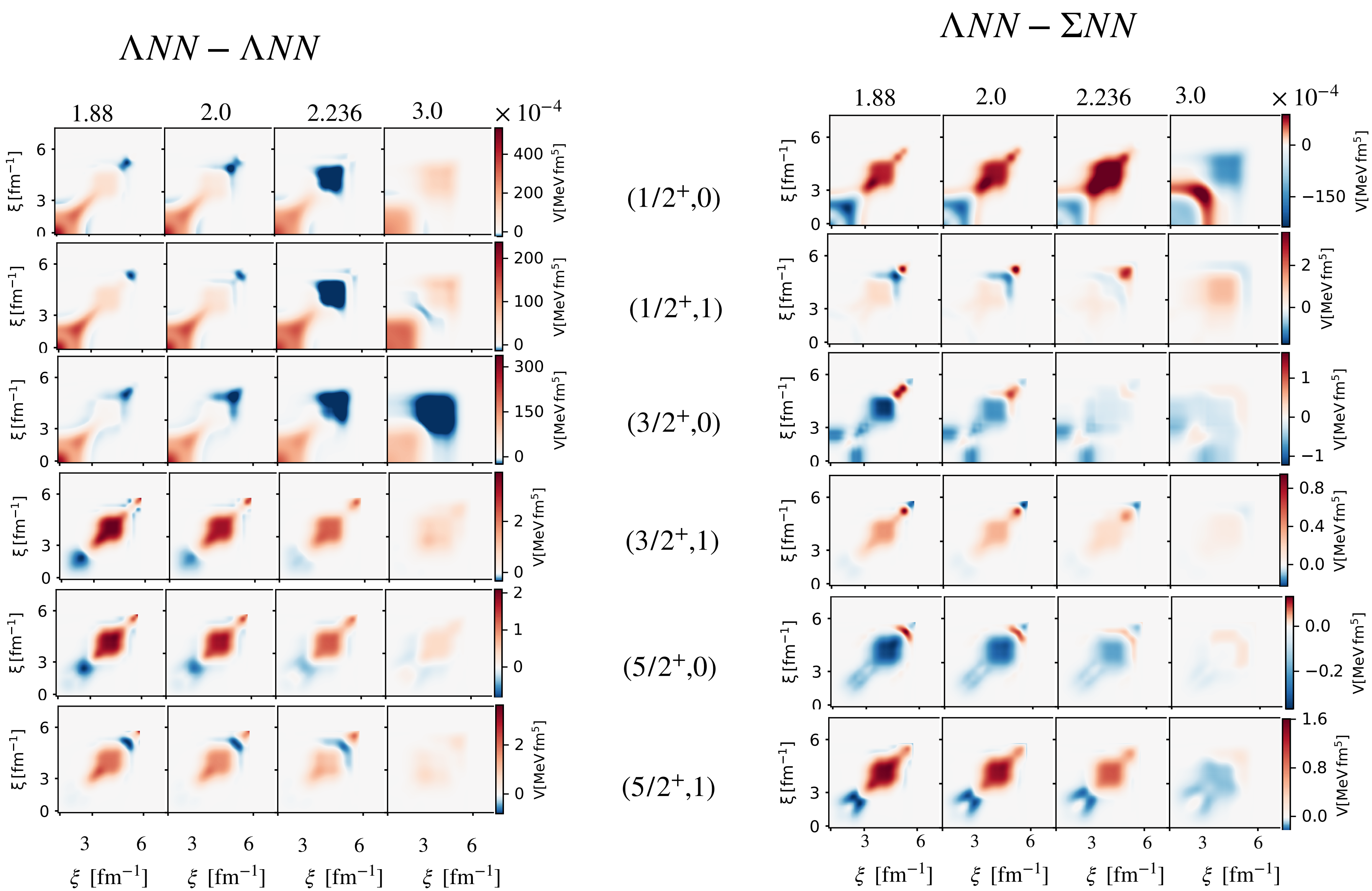
- hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$; $\tan\theta = \frac{2p}{\sqrt{3}q}$ $(\theta = \frac{\pi}{12})$; $\alpha = \alpha' = 1 \Rightarrow V_{123} = V_{123}(\xi', \xi)$

(K. Hebeler PRC85 (2012))



3N: χN^{2}LO(550)

SRG-induced $V^{YNN}(pq\alpha, p'q'\alpha')$

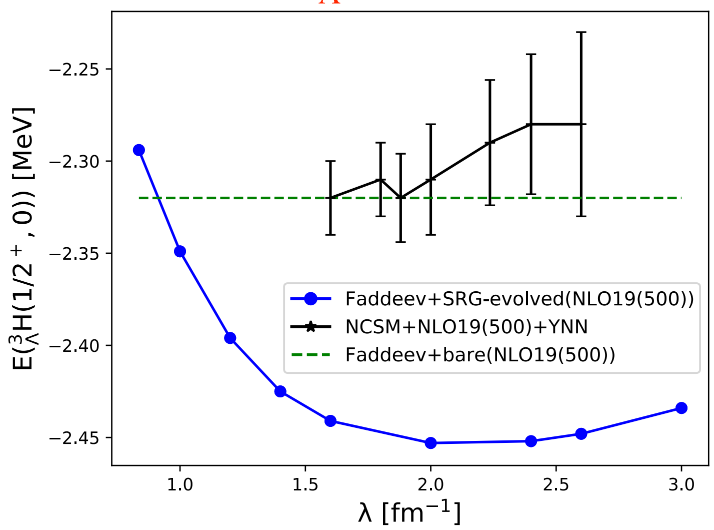


NN: $N^4LO+(450)$; YN: NLO19(500)

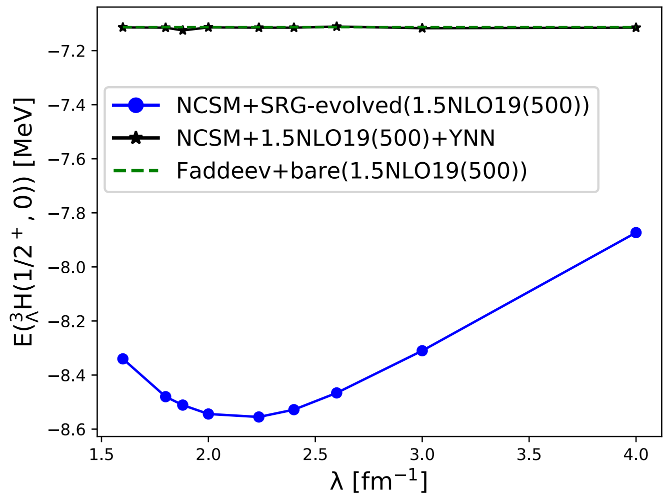
A=3-5 hypernuclei with SRG-induced YNN

JÜLICH FORSCHUNGSZENTRUM

$^{3}_{\Lambda}\mathrm{H}(1/2^{+},0)$



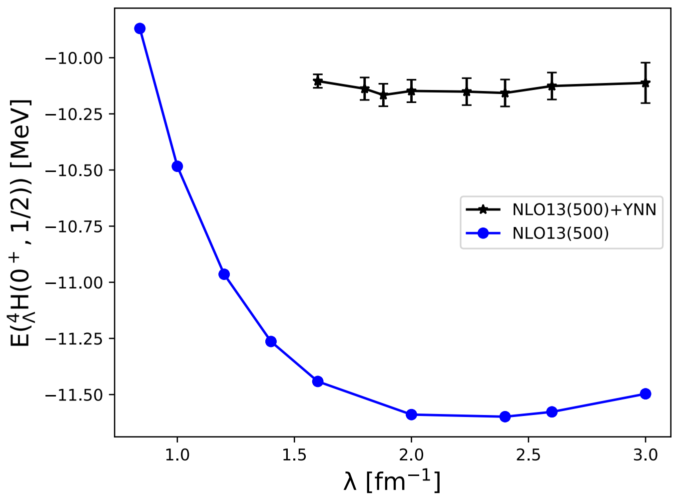
$^{3}_{\Lambda}\mathrm{H}(1/2^{+},0)$



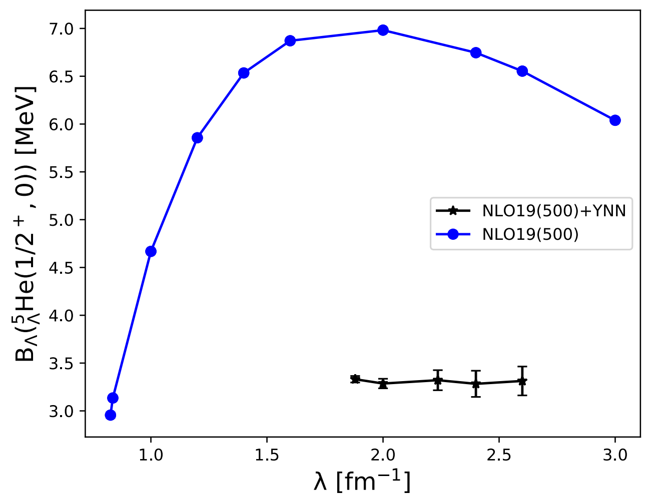
NN:SMS $\mathrm{N^4LO}$+(450)

3N: $\mathrm{N^2LO}$(450)

$^{4}_{\Lambda}\mathrm{H}(0^{+},1/2)$



$^{5}_{\Lambda}\mathrm{He}(1/2^{+},0)$



→ **contributions of SRG-induced YNNN forces to** $B_{\Lambda}(^{4}_{\Lambda}\mathrm{H},\ ^{5}_{\Lambda}\mathrm{He})$ **are negligible**

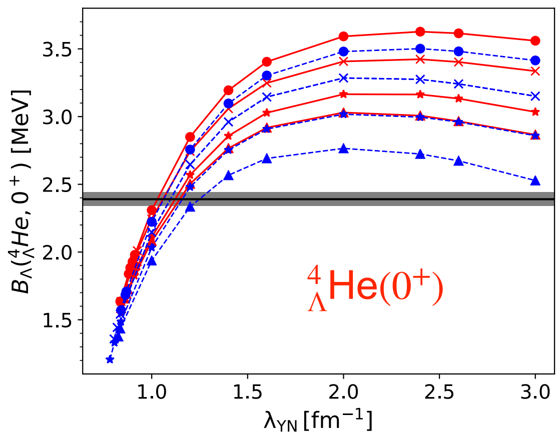
(R. Wirth, R. Roth PRL117 (2016), PRC100 (2019))

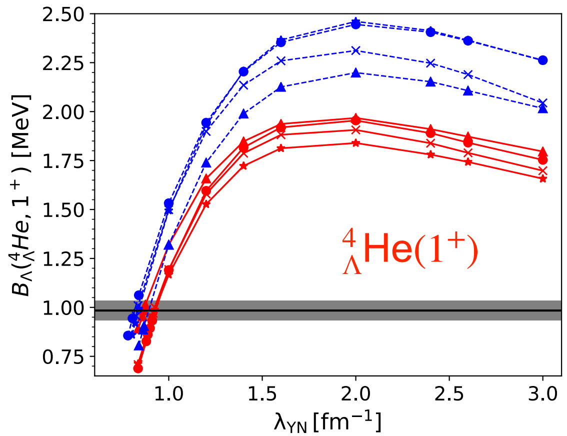
Impact of YN interactions on $B_\Lambda(A \leq 5)$

JÜLICH FORSCHUNGSZENTRUM

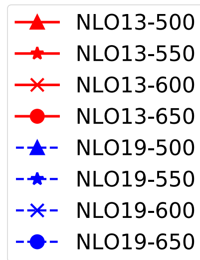
- NLO13 and NLO19 are almost phase equivalent **(J.Haidenbauer et al., NPA 915 2019))**
- NLO13 characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

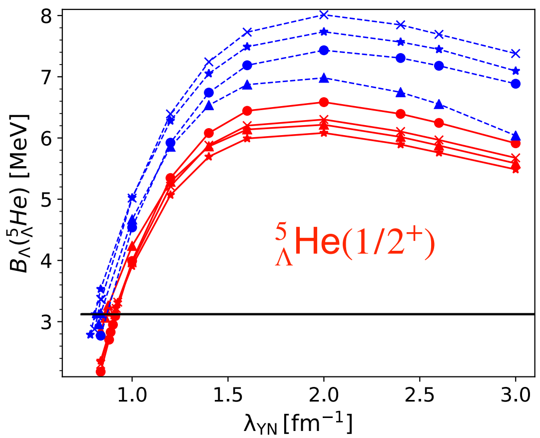
→ **manifest in higher-body observables**





NN:SMS N^{4}LO+(450)





$$ {}^4_\Lambda\text{He}\ (0^+): \quad \tilde{V}_{\Lambda N} \approx \frac{1}{2} V^s_{\Lambda N} + \frac{1}{2} V^t_{\Lambda N} $$

$$ {}^4_\Lambda\text{He}\ (1^+): \quad \tilde{V}_{\Lambda N} \approx \frac{1}{6} V^s_{\Lambda N} + \frac{5}{6} V^t_{\Lambda N} $$

$$ {}^5_\Lambda\text{He}: \quad \tilde{V}_{\Lambda N} \approx \frac{1}{4} V^s_{\Lambda N} + \frac{3}{4} V^t_{\Lambda N} $$

(HL et al., EPJA (2020))

- $B_\Lambda(\text{NLO19}) > B_\Lambda(\text{NLO13})$ → possible contribution of chiral YNN force

Impact of YN interactions on $B_\Lambda(A \leq 5)$

JÜLICH FORSCHUNGSZENTRUM

- NLO13 and NLO19 are almost phase equivalent (**J.Haidenbauer et al., NPA 915 2019)**)
- NLO13 characterised by a stronger $\Lambda N - \Sigma N$ transition potential (especially in $^3S_1$)

→ **manifest in higher-body observables**

	${}^4_\Lambda\mathrm{H}$ 0^+	${}^4_\Lambda\mathrm{H}$ 1^+	${}^5_\Lambda\mathrm{He}$ $1/2^+$
NLO13(500)	1.551 ± 0.007	0.823 ± 0.003	2.22 ± 0.06
NLO19(500)	1.514 ± 0.007	1.27 ± 0.009	3.32 ± 0.03
Exp.	2.16 ± 0.08	1.07 ± 0.08	3.12 ± 0.02

NN:SMS $\mathrm{N^4LO}$+(450)

+3N: $\mathrm{N^2LO}$(450)

+SRG-induced YNN

→ • chiral YNN force should contribute to B_Λ in ${}^4_\Lambda\mathrm{H}(0^+,1^+)$, ${}^5_\Lambda\mathrm{He}$ differently

- using decuplet saturation scheme: 2LECs (1LEC if only ΛNN is considered) can be fitted to

$B_\Lambda({}^4_\Lambda\mathrm{H}/{}^4_\Lambda\mathrm{He}(0^+,1^+))$ **or** $B_\Lambda({}^4_\Lambda\mathrm{H/He}(0^+),\ {}^5_\Lambda\mathrm{He}(1/2^+))$

CSB in A=7 isotriplet: ${}^{7}_{\Lambda}$He, ${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$Be

JÜLICH FORSCHUNGSZENTRUM

	YN	ΔT	ΔNN_{nucl}	ΔYN_{nucl}	Δ CSB
(${}^{7}_{\Lambda}$Be, ${}^{7}_{\Lambda}$Li*)	NLO13	10	-49	27	-12(30)
	CSB1	**10**	**-49**	**7**	**-32(30)**
	CSB2	5	-63	306	248(30)
	NLO19	8	-48	30	-10(30)
	CSB1	**7**	**-48**	**32**	**-10(30)**
	CSB2	8	-60	171	119(30)
	Gal[1]				-17
	Exp.[2]				-100 ± 90
(${}^{7}_{\Lambda}$Li*, ${}^{7}_{\Lambda}$He)	NLO13	9	-23	41	27(30)
	CSB1	**9**	**-23**	**29**	**6(30)**
	CSB2	6	-29	311	288(30)
	NLO19	8	-23	50	35(30)
	CSB1	**8**	**-22**	**51**	**37(30)**
	CSB2	8	-28	176	156(30)
	Exp.[2]				-20 ± 230

(HL, J. Haidenbauer, U-G. Meißner and A. Nogga in preparation)

- **CSB1** fixed to:

  $\Delta E(0^+, A=4) = 233 \pm 92$

  $\Delta E(1^+, A=4) = -83 \pm 94$

- CSB2 fixed to:

  $\Delta E(0^+, A=4) = 350 \pm 50$

  $\Delta E(1^+, A=4) = 240 \pm 80$

J. Haidenbauer et al., FBS 62 (2021)

[1] **A. Gal PLB 744 (2015)**

[2] **E. Botta et al., NPA 960 (2017)**

NN:SMS N^{4}LO+(450)

→ • **CSB1** results for A=4 are in line with the presently extracted CSB(${}^{4}_{\Lambda}$H/${}^{4}_{\Lambda}$He) **(see J. Haidenbauer talk)**

- **CSB1** fits reproduce CSB in A=7 isotriplet

CSB in A=8 doublet: ${}^{8}_{\Lambda}\mathrm{Be}$, ${}^{8}_{\Lambda}\mathrm{Li}$

JÜLICH FORSCHUNGSZENTRUM

NN:SMS N^{4}LO+(450)

	YN	ΔT	ΔNN_{nucl}	ΔYN_{nucl}	Δ CSB
$({}^{8}_{\Lambda}\mathrm{Be}, {}^{8}_{\Lambda}\mathrm{Li})$	NLO13	15	-10	54	59(50)
	CSB1	**14**	**-10**	**186**	**190(50)**
	CSB2	6	-24	297	279(50)
	NLO19	6	-12	53	47(50)
	CSB1	**6**	**-12**	**164**	**158(50)**
	CSB2	13	-19	183	177(50)
	Hiyama[3]				160
	Gal[1]	11	(-81)	119	49
	Exp[2]				40 ± 60

(HL, J. Haindenbauer, U-G. Meißner and A. Nogga in preparation)

- **CSB1** fixed to:

  $\Delta E(0^+, A=4) = 233 \pm 92$

  $\Delta E(1^+, A=4) = -83 \pm 94$

- CSB2 fixed to:

  $\Delta E(0^+, A=4) = 350 \pm 50$

  $\Delta E(1^+, A=4) = 240 \pm 80$

J. Haidenbauer et al., FBS 62 (2021)

[1] **A. Gal PLB 744 (2015)**

[2] **E. Botta et al., NPA 960 (2017)**

[3] **E. Hiyama et al., PRC 80 (2009)**

- **CSB1** fits lead to a larger CSB in A=8 doublet as compared to experiment

→ experimental CSB result for A=8 could be **larger than** 40 ± 60 **keV**?

CSB estimated for A=4 could **still be too large** or have **different spin-dependence**?

Results for A=4-7 Ξ hypernuclei

JÜLICH FORSCHUNGSZENTRUM

(HL, J. Haidenbauer, U.-G. Meißner, A. Nogga, EPJA 57 (2021))

	NLO(500)		others	
	B_Ξ [MeV]	Γ [MeV]	B_Ξ [MeV]	Γ [MeV]
${}^{4}_{\Xi}\mathrm{H}(1^+,0)$	0.48 ± 0.01	0.74	0.36(16)(26)[1]	0.06[1]
			10.20[2]	0.89[2]
${}^{4}_{\Xi}\mathrm{n}(0^+,1)$	0.71 ± 0.08	0.2	3.55[2]	0.43[2]
${}^{4}_{\Xi}\mathrm{n}(1^+,1)$	0.64 ± 0.11	0.01	10.11[2]	0.03[2]
${}^{4}_{\Xi}\mathrm{H}(0^+,0)$	-	-	-	-
${}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{1}{2})$	2.16 ± 0.10	0.19	1.7[3]	0.2[3]
			2.0[4]	0.45[4]
${}^{7}_{\Xi}\mathrm{H}(\frac{1}{2}^+,\frac{3}{2})$	3.50 ± 0.39	0.2	3.15[5]	0.02[5]
			1.8[6]	2.64[6]

NN:SMS $\mathrm{N^4LO}$+(450)

[1] HAL QCD (t/a=12)

[2] Nijmegen ESC08c
E.Hiyama et al., PRL 124 (2020)

[3] K. Myint, Y. Akaishi PTPS 117 (1994)

[4] E. Friedman, A. Gal PLB 820(2021)

[5] HAL QCD (t/a=11)

[6] Nijmegen ESC04d
H. Fujioko APFB2021, March (2021)

- employ YY NLO500; $\Xi\mathrm{N}-\Lambda\Lambda$ coupling is effectively incorporated into the strength of $V_{\Xi\mathrm{N}-\Xi\mathrm{N}}$
- $\Xi^- p$ Coulomb interaction contributes ~ 200, 600 and 400 keV to $NNN\Xi$, ${}^{5}_{\Xi}\mathrm{H}$, ${}^{7}_{\Xi}\mathrm{H}$

Results for A=4-7 Ξ hypernuclei

(HL, J. Haidenbauer, U.-G. Meißner, A. Nogga, EPJA 57 (2021)

	$\langle V^{S=-2} \rangle$ [MeV]					E [MeV]
	$^{11}S_0$	$^{31}S_0$	$^{13}S_1$	$^{33}S_1$	total	
$^{4}_{\Xi}\mathrm{H}(1^+,0)$	-1.95	0.02	-0.7	-2.31	-5.21	-8.97
$^{4}_{\Xi}\mathrm{n}(0^+,1)$	-0.6	0.25	-0.004	-0.74	-1.37	-9.07
$^{4}_{\Xi}\mathrm{n}(1^+,1)$	-0.02	0.16	-0.13	-1.14	-1.30	-9.0
$^{4}_{\Xi}\mathrm{H}(0^+,0)$	-0.002	0.08	-0.01	-0.006	-0.11	-6.94
$^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$	-0.96	0.94	-0.58	-3.63	-4.88	-31.43
$^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$	-1.23	1.79	-0.79	-6.74	-8.04	-33.22

→ ΞN attraction in $^{33}S_1$ is essential for the binding of A=4-7 Ξ-hypernuclei

Estimate partial-wave contributions

JÜLICH FORSCHUNGSZENTRUM

- **Assumption:**
  - no particle conversion contributing
  - clear core-Ξ structure, both core nucleons and Ξ are in s-wave states
- **A=3 system:**

$$ {}^{3}_{\Xi}\mathrm{H}(\frac{1}{2}^{+},\frac{1}{2}): \ \tilde{V}_{\Xi N} \approx \frac{3}{16} V^{11S_0}_{\Xi N} + \frac{9}{16} V^{31S_0}_{\Xi N} + \frac{1}{16} V^{13S_1}_{\Xi N} + \frac{3}{16} V^{33S_1}_{\Xi N} $$

$$ {}^{3}_{\Xi}\mathrm{H}(\frac{3}{2}^{+},\frac{1}{2}): \ \tilde{V}_{\Xi N} \approx \frac{1}{4} V^{13S_1}_{\Xi N} + \frac{3}{4} V^{33S_1}_{\Xi N} $$

- **A=4 system:**

$$ {}^{4}_{\Xi}\mathrm{H}(1^{+},0): \ \tilde{V}_{\Xi N} \approx \frac{1}{6} V^{11S_0}_{\Xi N} + \frac{1}{3} V^{13S_1}_{\Xi N} + \frac{1}{2} V^{33S_1}_{\Xi N} $$

$$ {}^{4}_{\Xi}\mathrm{H}(0^{+},1): \ \tilde{V}_{\Xi N} \approx \frac{1}{6} V^{11S_0}_{\Xi N} + \frac{1}{3} V^{31S_0}_{\Xi N} + \frac{1}{2} V^{33S_1}_{\Xi N} $$

$$ {}^{4}_{\Xi}\mathrm{H}(1^{+},1): \ \tilde{V}_{\Xi N} \approx \frac{1}{6} V^{31S_0}_{\Xi N} + \frac{1}{6} V^{13S_1}_{\Xi N} + \frac{2}{3} V^{33S_1}_{\Xi N} $$

$$ {}^{4}_{\Xi}\mathrm{H}(0^{+},0): \ \tilde{V}_{\Xi N} \approx \frac{1}{2} V^{31S_0}_{\Xi N} + \frac{1}{2} V^{13S_1}_{\Xi N} $$

- **A=5 system:**

$$ {}^{5}_{\Xi}\mathrm{H}(\frac{1}{2}^{+},\frac{1}{2}): \ \tilde{V}_{\Xi N} \approx \frac{1}{16} V^{11S_0}_{\Xi N} + \frac{3}{16} V^{31S_0}_{\Xi N} + \frac{3}{16} V^{13S_1}_{\Xi N} + \frac{9}{16} V^{33S_1}_{\Xi N} $$

Summary

study ${}^{4}_{\Lambda}\mathrm{H}(0^+,1^+)$, ${}^{5}_{\Lambda}\mathrm{He}$ **hypernuclei using chiral 2B & 3N interactions + SRG-induced YNN**

- contribution of SRG-induced YNNN force is negligible
- YN NLO13 & NLO19 potentials predict different results for ${}^{4}_{\Lambda}\mathrm{H}(1^+)$, ${}^{5}_{\Lambda}\mathrm{He}$

→ YNN force is needed in order to properly describe light hypernuclei (work in progress)

study CSB in A=7 isotriplet and A=8 doublet

- **CSB1** fits reproduce experimental results for A=4 & 7 systems

  but lead to a somewhat larger than the experimental CSB for the ${}^{8}_{\Lambda}\mathrm{Be}$, ${}^{8}_{\Lambda}\mathrm{Li}$ doublet

investigate A=4-7 Ξ hypernuclei using NLO(500) ΞN **potential**

- found 3 loosely bound states $(1^+,0)$, $(0^+,1)$, $(1^+,1)$ in $NNN\Xi$;

  ${}^{5}_{\Xi}\mathrm{H}(1/2^+,1/2)$, ${}^{7}_{\Xi}\mathrm{H}(1/2^+,3/2)$ are more tightly bound

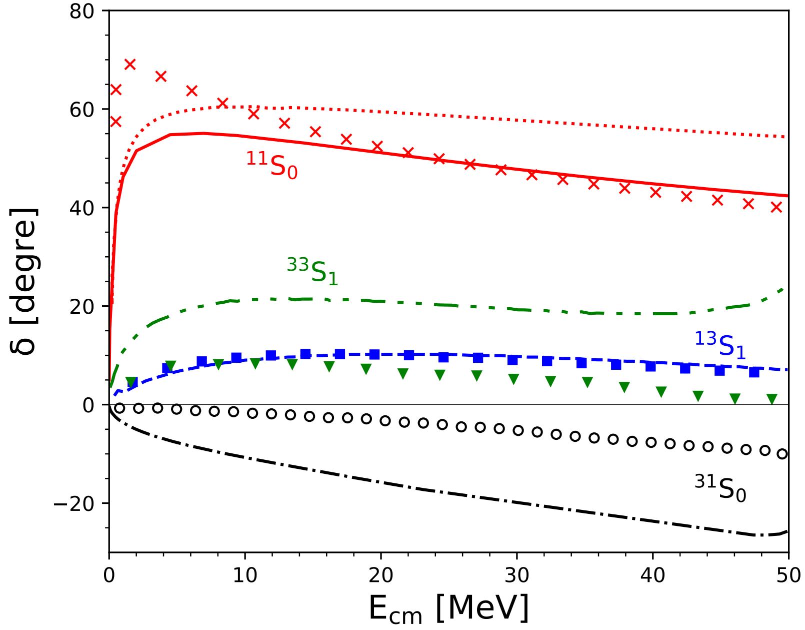
Thank you for the attention!

	NLO19(500) ($\lambda = 0.837$)	NLO13(650) ($\lambda = 0.913$)	Exp.		Hiyama [8]
${}^{7}_{\Lambda}$Be	5.44 ± 0.03	5.58 ± 0.03	5.16 ± 0.08		5.21
${}^{7}_{\Lambda}$Li*	5.49 ± 0.04	5.65 ± 0.03	5.26 ± 0.03	5.53 ± 0.13	5.28
${}^{7}_{\Lambda}$He	5.43 ± 0.06	5.63 ± 0.05		5.55 ± 0.1	5.36

	${}^{8}_{\Lambda}$Be	${}^{8}_{\Lambda}$Li
NLO13(600)		7.04 ± 0.08
NLO13(600)CSB1	7.13 ± 0.05	6.99 ± 0.09
NLO13(600)CSB2	7.56 ± 0.05	7.23 ± 0.04
Exp.	6.84 ± 0.05	6.80 ± 0.03
Hiyama	6.72	6.80

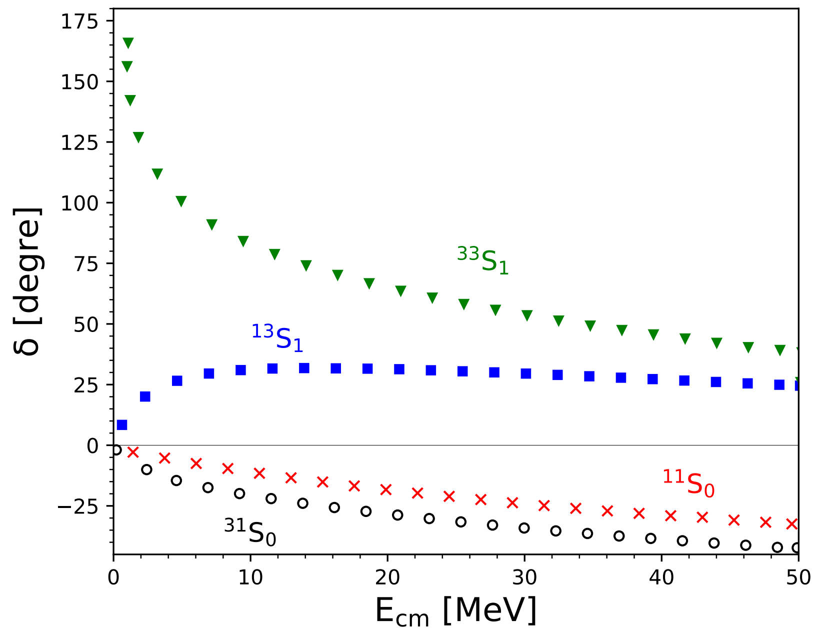
ΞN phase shifts predicted by modern interactions

NLO(500) & HAL QCD*



(* E. Hiyama et al. PRL 124, 092501 (2020))

Nijmegen ESC08c*



(* E. Hiyama et al PRL 124, 092501 (2020))

- $^{11}S_0$ is rather attractive in NLO and HAL QCD, but repulsive in ESC08c
- $^{33}S_1$ is strongly attractive in ESC08c (lead to a ΞN bound state), it is only moderately (weakly) attractive in NLO (HAL QCD)