ΛΛ pairing effects in spherical and deformed multi-Λ hyperisotopes

Xian-Rong Zhou

East China Normal University, Shanghai, China

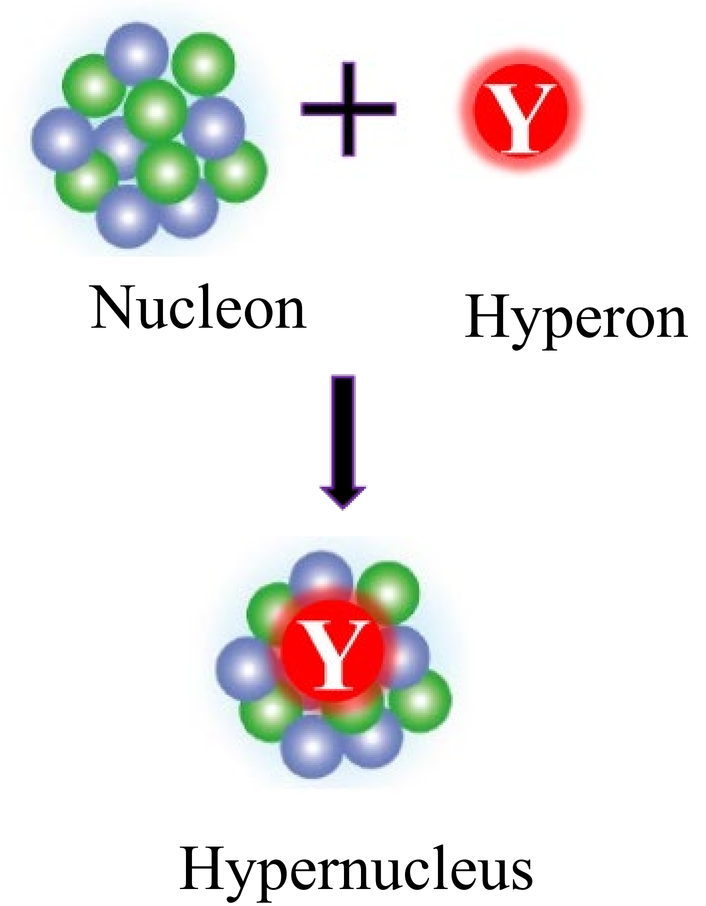
27th June -1st July, HYP2022, Prague, Czech Republic

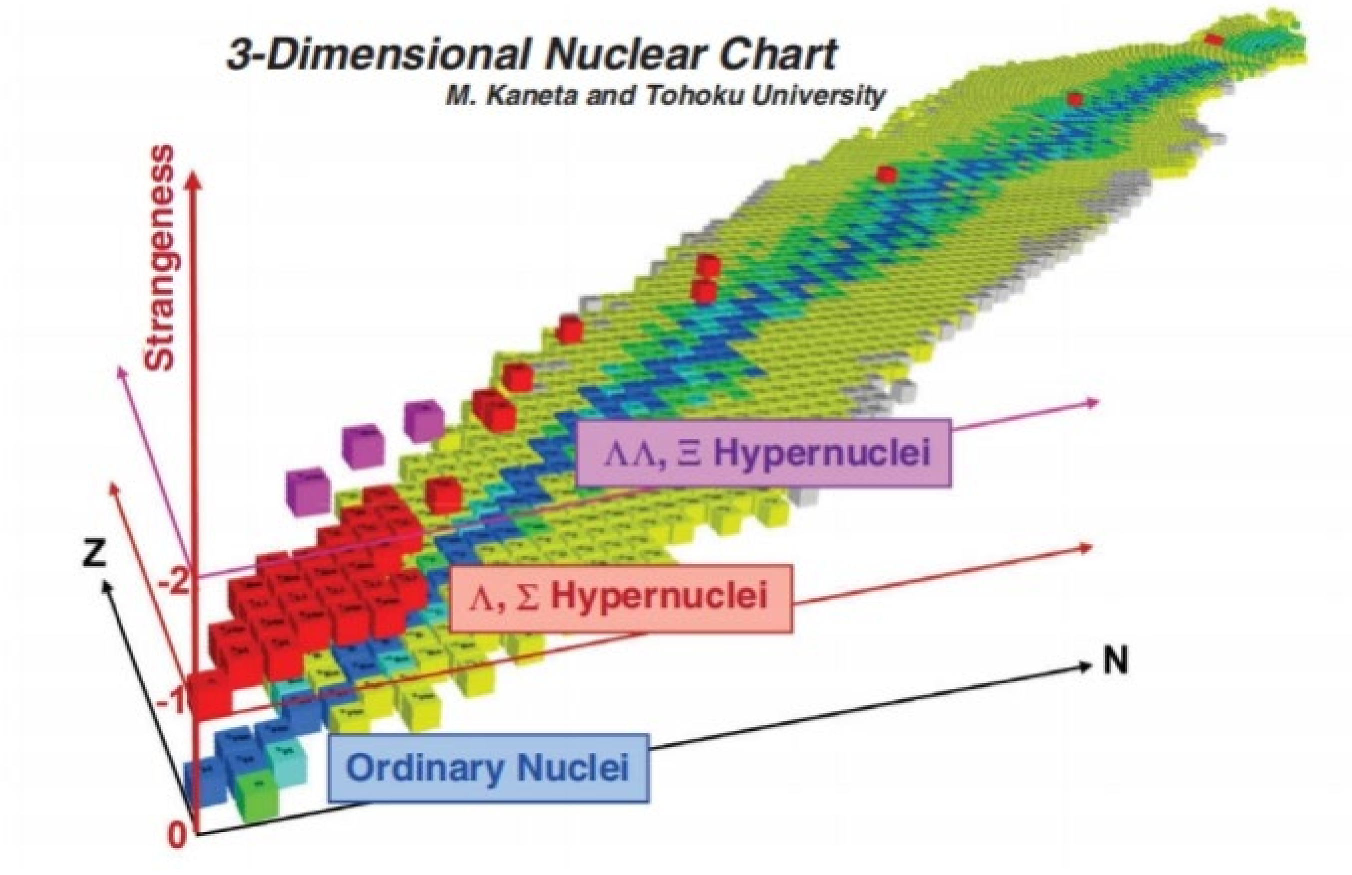
Outline

- Introduction
- Extended Skyrme-Hartree-Fock (SHF) model
- Pairing force and BCS approximation
- ΛΛ pairing effects
- Conclusions

Hypernuclei

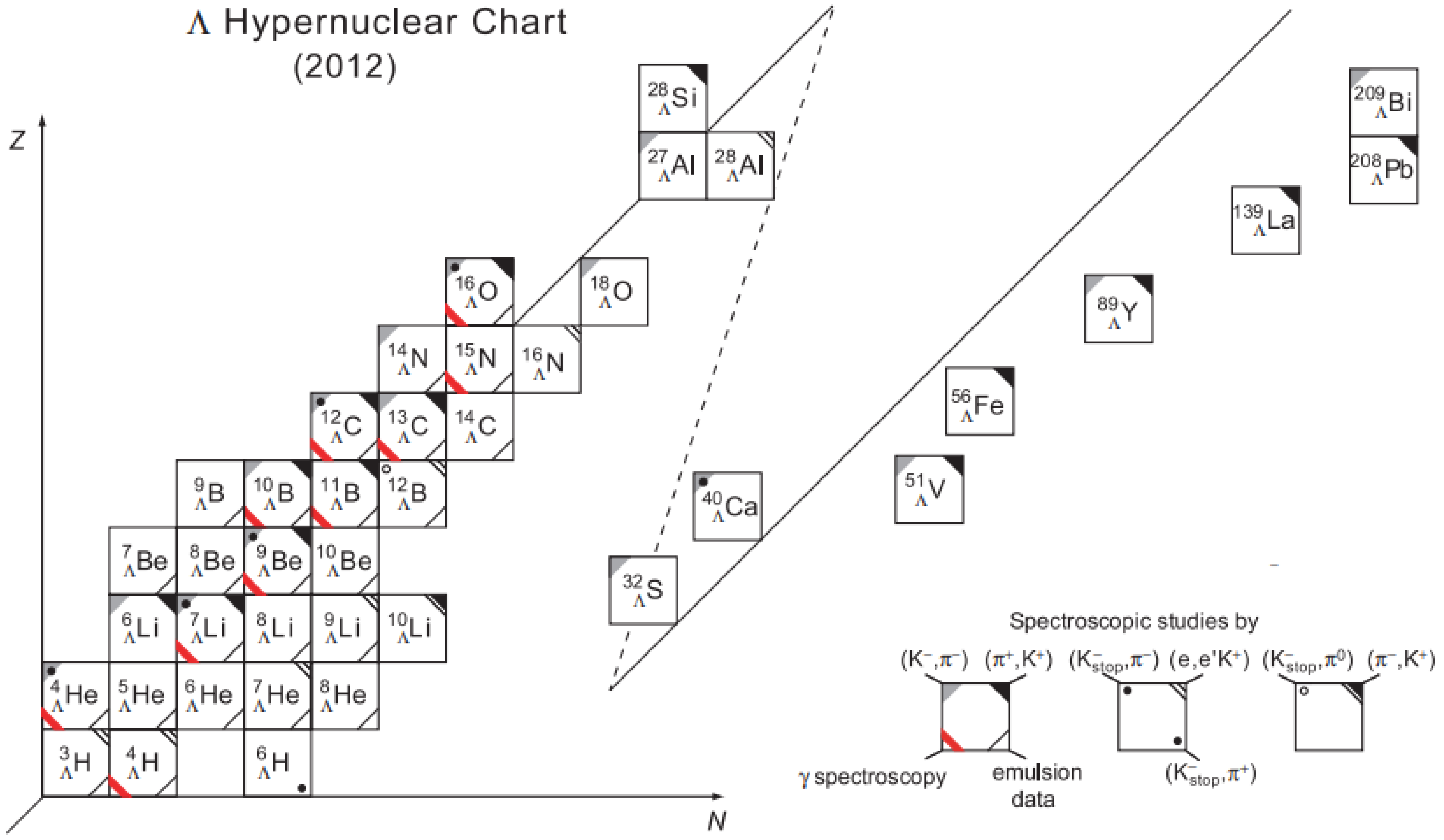
Hypernucleus





H. Tamura, Prog. Theor. Exp. Phys. 1, 02B012 (2012)

Experimental progress of single-Λ hypernuclei



H. Tamura, Prog. Theor. Exp. Phys. 2012, 02B012.

Experimental progress of double-Λ hypernuclei

The only uniquely identified double-Λ hypernucleus.

Event	${}^{A}_{\Lambda\Lambda}Z$	$\bar{B}_{\Lambda}({}^{A-1}_{\Lambda}Z)$	$B^{exp}_{\Lambda\Lambda}$
E373-Nagara	${}^{6}_{\Lambda\Lambda}He$	3.12 ± 0.02	6.91 ± 0.16
E373-DemYan	${}^{10}_{\Lambda\Lambda}Be$	6.71 ± 0.04	14.94 ± 0.13
E176-G2	${}^{11}_{\Lambda\Lambda}Be$	8.86 ± 0.11	17.53 ± 0.71
E373-Hida	${}^{11}_{\Lambda\Lambda}Be$	8.86 ± 0.11	20.83 ± 1.27
E373-Hida	${}^{12}_{\Lambda\Lambda}Be$	10.02 ± 0.05	22.48 ± 1.21
E176-E2	${}^{12}_{\Lambda\Lambda}B$	10.09 ± 0.05	20.02 ± 0.78
E176-E4	${}^{13}_{\Lambda\Lambda}B$	11.27 ± 0.06	23.4 ± 0.7

Reproduced well by the shell model, which confirms the interpretations of the corresponding emulsion events.

The E373-HIDA event does not allow any reasonable assignment, for which one of the alternative interpretations of E176 might be more suitable.

A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016).

Theoretical models for Λ hypernuclei

◻ Relativistic Mean Field model

T.-T. Sun et al, Phys. Rev. C 96, 044312 (2017).
B. Bhowmick et al, Eur. Phys. J. A 50, 125 (2014).
Y.-T. Rong, Phys. Rev. C 104, 054321 (2021).

◻ Brueckner-Hartree-Fock model

J. Cugnon et al, Phys. Rev. C 62, 064308 (2000).
E. Khan et al, Phys. Rev. C 92, 044313 (2015).
H.-J. Schulze et al, Phys. Rev. C 88, 024322 (2013).

◻ Cluster model

E. Hiyama et al, Phys. Rev. C 66, 024007 (2002).
E. Hiyama et al, Prog. Part. Nucl. Phys. 63, 339 (2009)
……..

◻ Few-body calculation

Hiyama & Kamimura et al, PRC 66, 024007 (2002).
Hiyama & Kamimura et al, PRL 104, 212502 (2010).

◻ Beyond-mean-field approach

H. Mei et al, Phys. Rev. C 91, 064305 (2015).
J.-W. Cui et al, Phys. Rev. C 95, 024323 (2017).
X. Y. Wu et al, Phys. Rev. C 95,034309 (2017).

◻ Skyrme-Hartree-Fock model

X-R.Zhou et al, PRC 76 034312(2007)
H-J. Schulze et al, PRC 90 047301(2014)

◻ Shell Model

A. Gal, D.J. Millener, PLB701,342-345(2011)
D. Gazda, A. Gal, PRL 116, 122501(2016)
……

ΛΛ interaction

- The existence of an extra binding associated with a double-hyperon system implies that the interaction is at least marginally attractive, opening the possibility of multistrange systems with a higher number of hyperons.

- A large variety of phenomena were predicted for such nuclei during the 1980s and the early 1990s. However, these studies assumed very attractive hyperon-hyperon interactions.

- In 2017, Margueron et al, gave these ΛΛ interacyion parameters within a density functional approach by fitting $B_{\Lambda\Lambda}$=6.91$\pm$0.61 for ${}^{6}_{\Lambda\Lambda}He$.

J. Margueron, E. Khan, and F. Gulminelli, Phys. Rev. C 96, 054317 (2017).

ΛΛ pairing effects in hypernuclear matter

- **Model:** Relativistic Hartree-Bogoliubov (RHB) model.

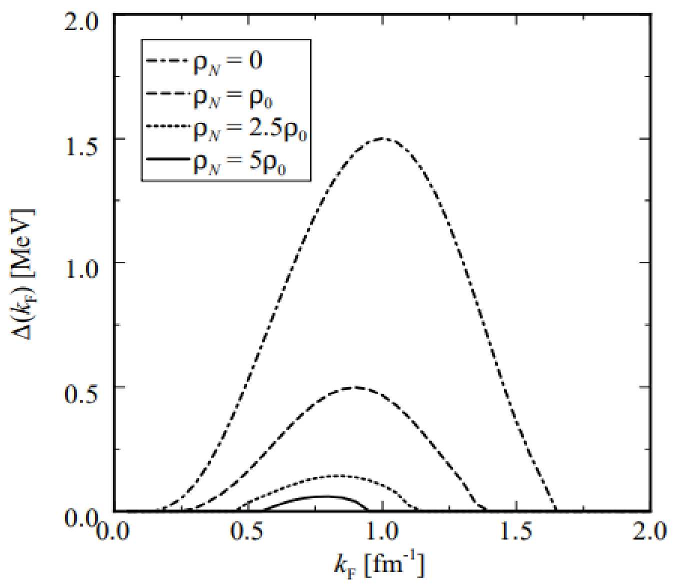


FIG. 1. ΛΛ pairing gap at the Fermi surface of Λ hyperons, for pure neutron background densities $\rho_N=0$, ρ_0, $2.5\rho_0$, and $5\rho_0$. The coupling ratio $\alpha_{\sigma*}=0.5$ is used.

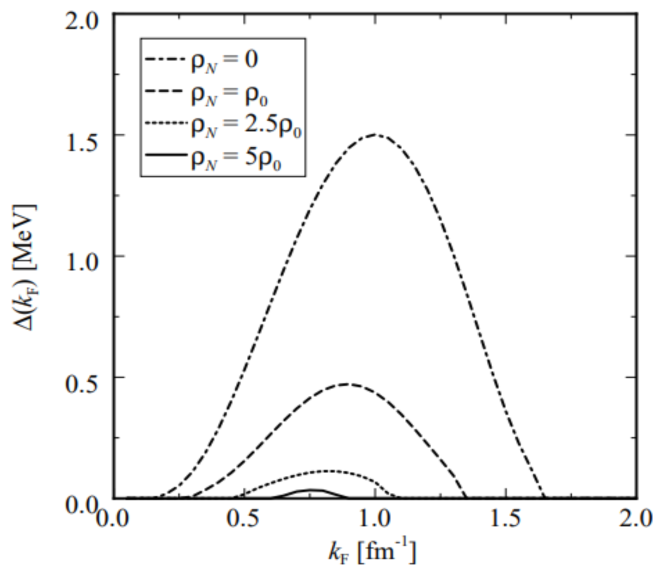


FIG. 5. ΛΛ pairing gap at the Fermi surface of Λ hyperons, for nucleon background densities $\rho_N=0$, ρ_0, $2.5\rho_0$, and $5\rho_0$. The coupling ratio $\alpha_{\sigma*}=0.5$ is used.

- **Conclusion:** It is found that at background density ρ_N=2.5ρ_0 the ΛΛ pairing gap is very small, and that a denser background makes it rapidly suppressed.

T. Tanigawa, M. Matsuzaki, and S. Chiba, Phys. Rev. C 68, 015801 (2003).

ΛΛ pairing effects in hypernuclear matter

- **Model:** Relativistic mean field (RMF) model

- **Conclusion:** To examine the $^{1}S_{0}$ pairing gap of Λ hyperons, they employ several ΛΛ interactions based on the Nijmegen models and used in double-Λ hypernuclei studies. It is found that the maximal pairing gap obtained is a few tenths of a MeV.

Y. N. Wang and H. Shen, Phys. Rev. C 81, 025801 (2010).

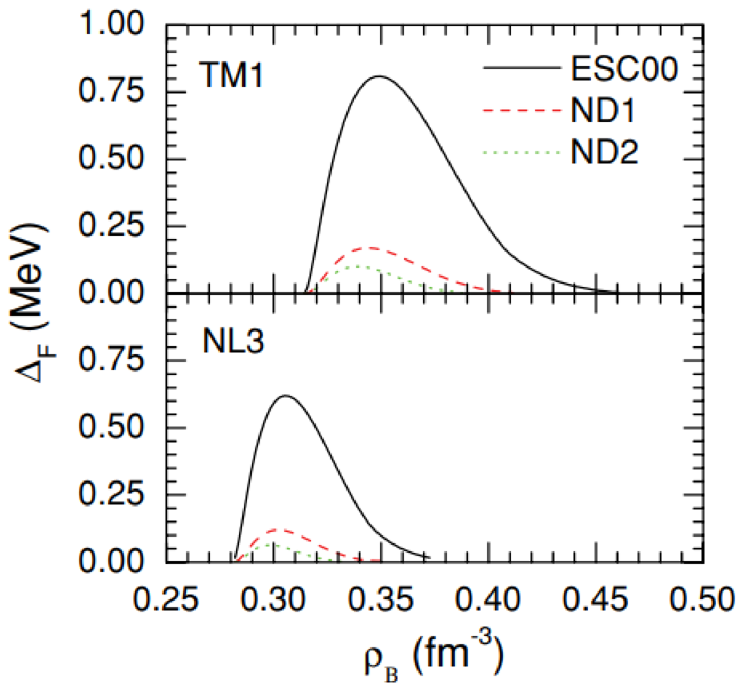
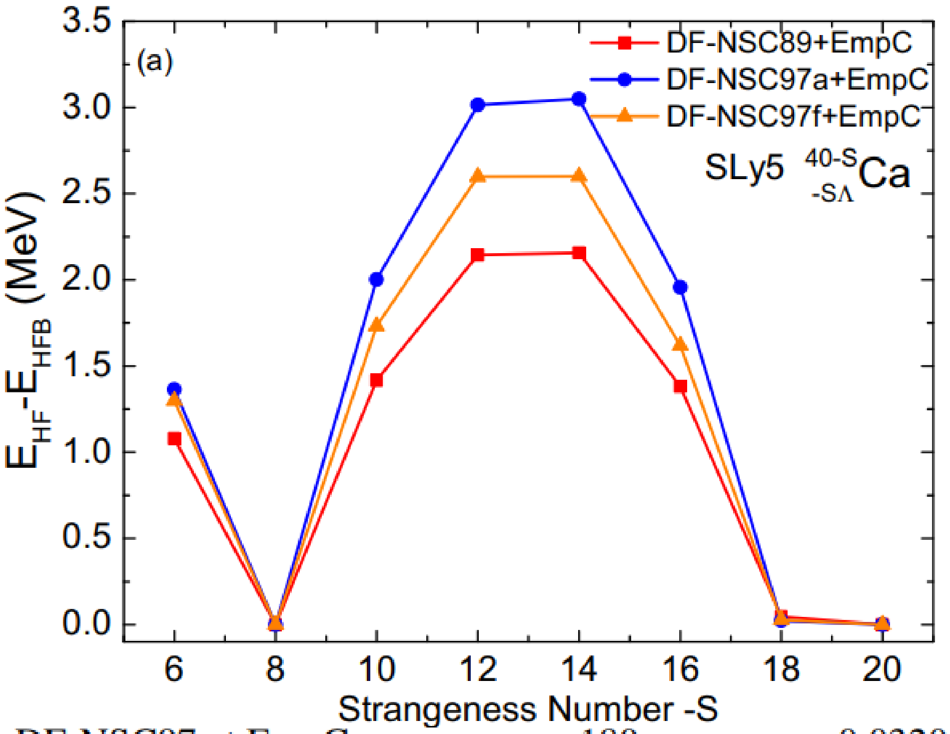
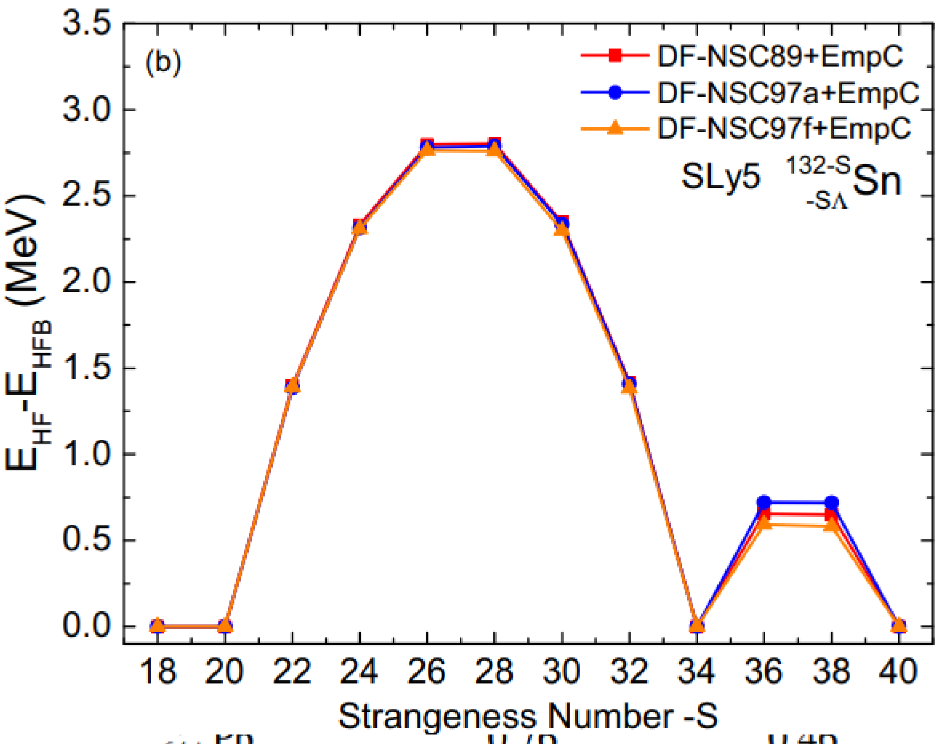


FIG. 2. (Color online) $^{1}S_{0}$ pairing gap of Λ hyperons at the Fermi surface Δ_F as a function of baryon density ρ_B in neutron star matter with the ND1, ND2, and ESC00 potentials: (top) TM1 and (bottom) NL3.

Pairing effects in hypernuclear matter have been explored within the BCS approximation, with so far inconclusive results, even regarding the mere existence of pairing.

ΛΛ pairing effects in hypernuclei within HFB model





each force sets.

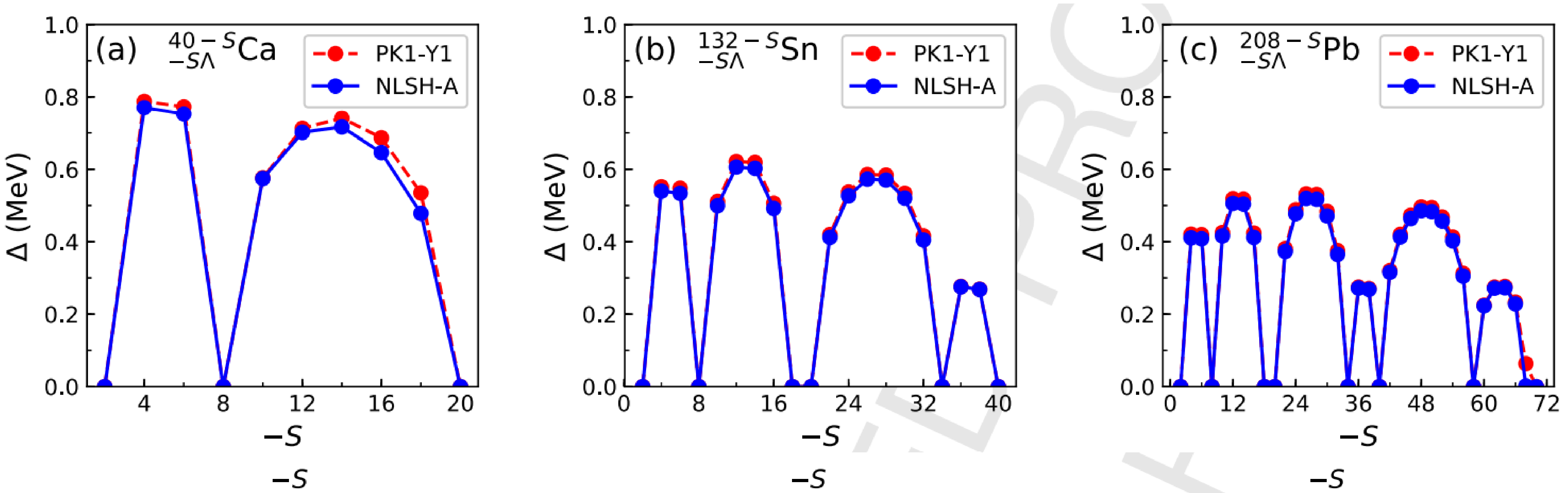
						Pairing gap in uniform matter [28] (MeV)
						0.42
						0.44
						0.43
						0.43
						0.50
						0.45
						0.47
DF-NSC97a+EmpC	−180	0.0320	$_{64\Lambda}$Pb	0.70	0.46	0.45
DF-NSC97f+EmpC	−220	0.0270	$^{272}_{64\Lambda}$Pb	0.71	0.44	0.40

- An upper bound for the prediction of the ΛΛ pairing gap and its effects in hypernuclei were provided.
- The condensation energy is predicted to be about 3 MeV as a maximum value.

H. Guven, K. Bozkurt, E. Khan, and J. Margueron, Phys. Rev. C 98, 014318 (2018).

ΛΛ pairing effects in hypernuclei within RHB model

- **Model: Relativistic Mean-Field (RMF) Model**



- It is revealed that $-S = 2, 8, 20, 34, 40$ and 58 are magic or semi-magic numbers for Λs. The Δ_Λ decreases when the mass number of the core nucleus increasing.

Y.-T. Rong, P. Zhao, and S.-G. Zhou, Phys. Lett. B 807, 135533 (2020).

Motivation

- We extend the method of determining the pairing strength proposed in the RHB model to the nonrelativistic SHF model and apply it to the multi- Ca, Sn, and Pb hypernuclei to compare with the results obtained by HFB and RHB.

- We use that pairing force and study the pairing effects in multi- isotopes of the typical deformed-core nuclei ^{24}Mg, ^{56}Fe, and ^{104}Zr in detail.

Extended SHF Model

- **Extended SHF Model：** SHF model + ΛN interaction + ΛΛ interaction

- **Total energy of a hypernucleus in extended 2D SHF:**

$$E = \int d^3 \boldsymbol{r} \varepsilon(\boldsymbol{r}), \quad \varepsilon = \varepsilon_{NN} + \varepsilon_{\Lambda N} + \varepsilon_{\Lambda\Lambda}$$

- **2D extended SHF equation:**

$$\left[\nabla \cdot \frac{1}{2m_q^*(\boldsymbol{r})} \nabla - V_q(\boldsymbol{r}) + iW_q(\boldsymbol{r}) \cdot (\nabla \times \boldsymbol{\sigma})\right] \phi_q^k(\boldsymbol{r}) = e_q^k \phi_q^k(\boldsymbol{r})$$

1D: J. Cugnon, A. Lejeune, and H.-J. Schulze, Phys. Rev. C 62, 064308 (2000).
2D: X.-R. Zhou, H.-J. Schulze et al, Phys. Rev. C 76, 034312 (2007).

Mean Field

$$V_N = V_N^{\mathrm{SHF}} + \frac{\partial \varepsilon_{N\Lambda}}{\partial \rho_N} + \frac{\partial}{\partial \rho_N}\left(\frac{m_\Lambda}{m_\Lambda^*(\rho_N)}\right)\left(\frac{\tau_\Lambda}{2m_\Lambda} - \frac{3}{5}\frac{\rho_\Lambda(3\pi^2\rho_\Lambda)^{2/3}}{2m_\Lambda}\right),$$

$$V_\Lambda = \frac{\partial(\varepsilon_{N\Lambda} + \varepsilon_{\Lambda\Lambda})}{\partial \rho_\Lambda} - \left(\frac{m_\Lambda}{m_\Lambda^*(\rho_N)} - 1\right)\frac{(3\pi^2\rho_\Lambda)^{2/3}}{2m_\Lambda}.$$

$$\varepsilon_{N\Lambda}(\rho_N,\rho_\Lambda) = -(\varepsilon_1 - \varepsilon_2\rho_N + \varepsilon_3\rho_N^2)\rho_N\rho_\Lambda + (\varepsilon_4 - \varepsilon_5\rho_N + \varepsilon_6\rho_N^2)\rho_N\rho_\Lambda^{5/3},$$

$$\varepsilon_{\Lambda\Lambda}(\rho_\Lambda) = -\varepsilon_7\rho_\Lambda^2\Theta(N_\Lambda > 1),$$

- **NN Interaction parameters： Skyrme force SLy5** [M. Bender et al, Rev. Mod. Phys. 75, 121 (2003).]
- **ΛN Interaction parameters： Nijmegen interactions NSC89, NSC97a , NSC97f** [H.-J. Schulze and T. Rijken, Phys. Rev. C 88, 024322 (2013).]

In 2017, Margueron et al, gave these parameters by fitting $B_{\Lambda\Lambda}$=6. 91±0. 61 for $^{6}_{\Lambda\Lambda}He$.

TABLE II. Prescription EmpC. We present the values of the parameters $\alpha_1^{\Lambda\Lambda}$, the resulting bond energy in He $\Delta B_{\Lambda\Lambda}(A=6)$ in MeV, and the ratio of the Λ density in He to the saturation density (ρ_0).

Pot. ΛN Pot. ΛΛ	DF-NSC89 EmpC	DF-NSC97a EmpC	DF-NSC97f EmpC
$\alpha_1^{\Lambda\Lambda}$ (MeV fm^3)	22.81	21.12	33.25
$\Delta B_{\Lambda\Lambda}(6)^{HF}$ (MeV)	1.00	0.99	1.01
$\rho_\Lambda(6)/\rho_0$	0.137	0.148	0.094

J. Margueron, E. Khan, and F. Gulminelli, Phys. Rev. C 96,054317 (2017).

Pairing force

- **Pairing force [1]:**

NN part:

$$V_q(\boldsymbol{r}_1, \boldsymbol{r}_2) = -V_0^{(q)} \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$V_0^{(N)} = 323 \text{ MeV fm}^3$$ **[1]**

ΛΛ part:

$$V_q(\boldsymbol{r}_1, \boldsymbol{r}_2) = -V_0^{(q)} \delta(\boldsymbol{r}_1 - \boldsymbol{r}_2)$$

$$V_0^{(\Lambda)} = 144 \text{ MeV fm}^3$$ **[2]**

- **Pairing energies [1]:** $E^q_{\text{pair}} = \frac{1}{4} \int d^3\boldsymbol{r} G_q(\boldsymbol{r}) \chi_q^*(\boldsymbol{r}) \chi_q(\boldsymbol{r}), \quad \chi_q(\boldsymbol{r}) = -2 \sum_{k \in \Omega_q, k>0} u_q(k) v_q(k) \left|\phi_q^k(\boldsymbol{r})\right|^2$

- **Average pairing gap [1]:** $\Delta_q \equiv \frac{\sum_{k \in \Omega_q} f_q(k) u_q(k) v_q(k) \Delta_q(k)}{\sum_{k \in \Omega_q} f_q(k) u_q(k) v_q(k)}$

[1] M. Bender, K. Rutz, P. G. Reinhard, and J. A. Maruhn, Eur. Phys. J. A 8, 59 (2000).
[2] Y.-T. Rong, P. Zhao, and S.-G. Zhou, Phys. Lett. B 807, 135533 (2020).

BCS approximation

BCS gap equation:

$$2\left(\varepsilon_\mu-\lambda\right)v_\mu-G\left(\sum_v u_\nu v_\mu\right)\left(u_\mu^2-v_u^2\right)/u_\mu=0$$

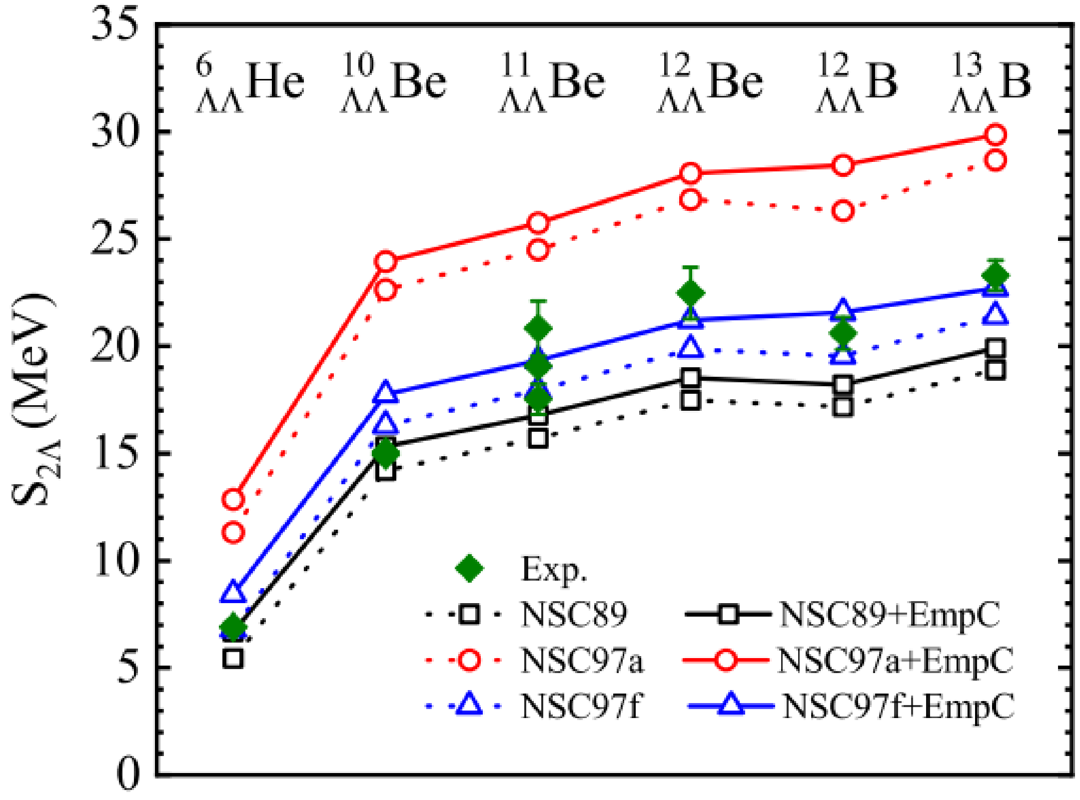
$$\Big\downarrow \qquad v_\mu^2+\mu_\mu^2=1 \quad \varepsilon_\mu=-G v_\mu^2$$

$$\mu_\mu^2=\frac{1}{2}[1+\frac{\varepsilon_\mu^{'}-\lambda}{(\varepsilon_\mu^{'}-\lambda)^2+\Delta}],$$

$$v_\mu^2=\frac{1}{2}[1-\frac{\varepsilon_\mu^{'}-\lambda}{(\varepsilon_\mu^{'}-\lambda)^2+\Delta}].$$

M. Bender, K. Rutz, P. G. Reinhard, and J. A. Maruhn, Eur. Phys. J. A 8, 59 (2000).

Examine ΛN and ΛΛ interaction

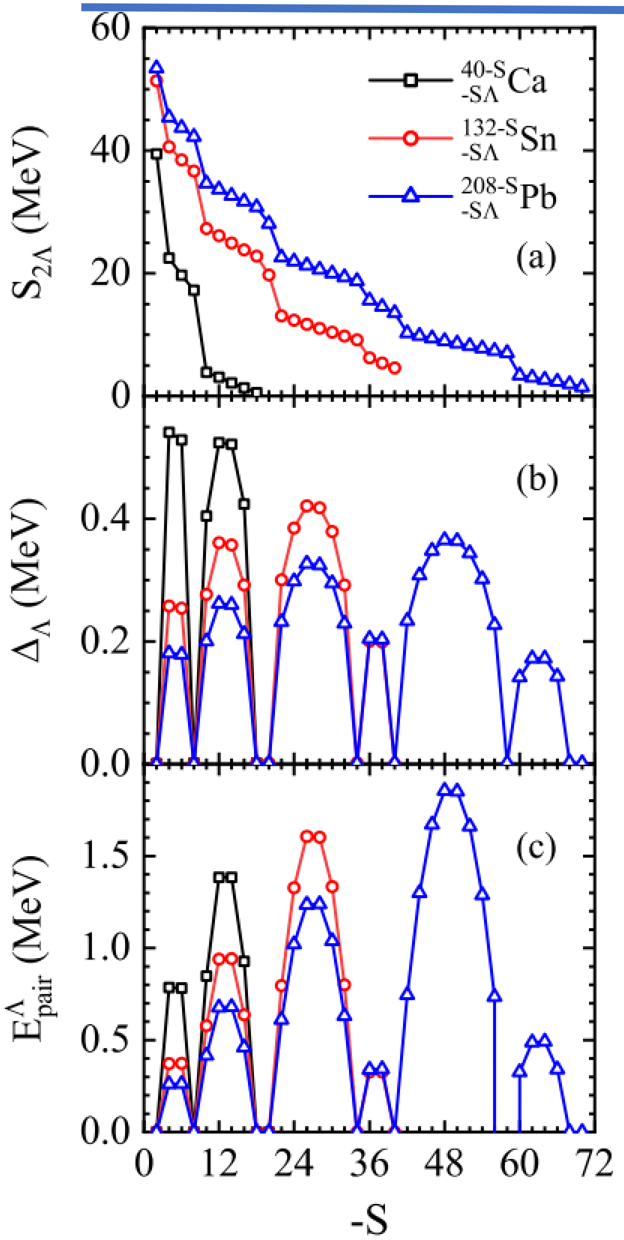


- When including the ΛΛ interaction (EmpC prescription), the $B_{\Lambda\Lambda}$ values increase due to the attractive ΛΛ interaction, which is fit to the NAGARA event.

- The NSC97f + EmpC parameter set gives the best description for the experimental data.

Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

ΛΛ pairing effects in spherical hypernuclei



2Λ separation energy

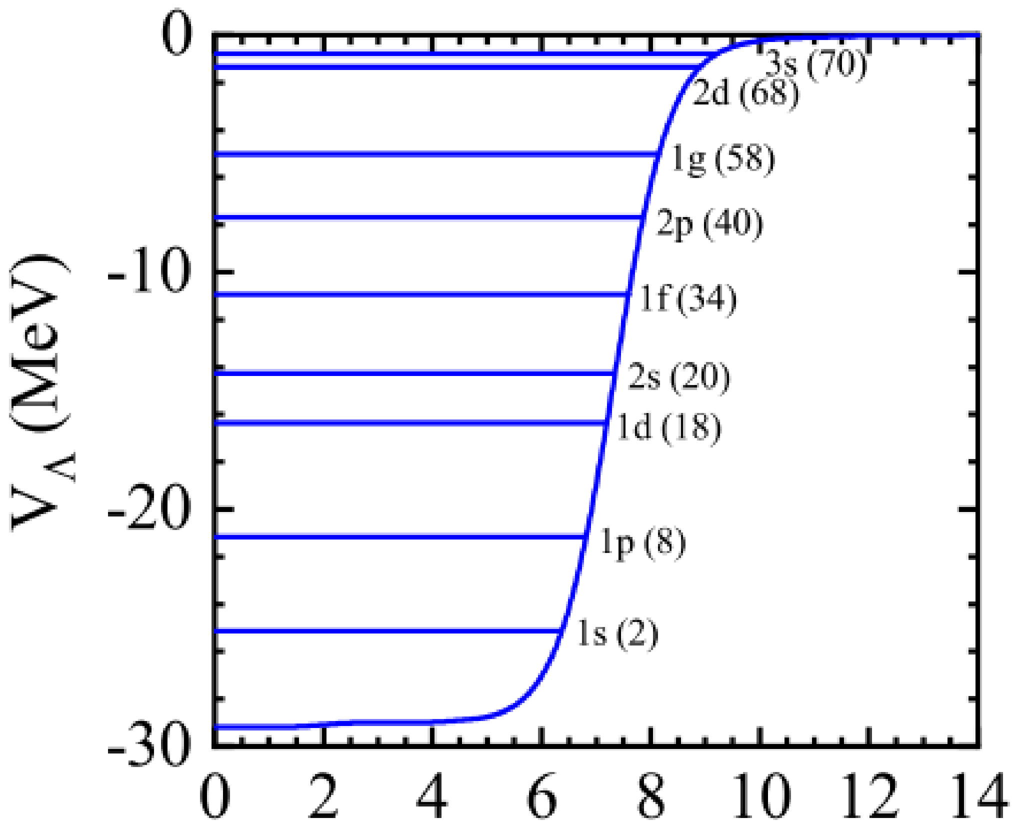
Pairing gaps

Pairing energy

- The occurrences of magic numbers −S = 2, 8, 18, 20, 34, 58, 68, and 70, which are attributed to a Woods-Saxon-like Λ hyperon potential.
- Due to the absence of relevant Λ spin-orbit forces, the individual shells are fairly large and therefore allow strong pairing correlations for hyperisotopes located close to their center.

Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

ΛΛ pairing effects in spherical hypernuclei

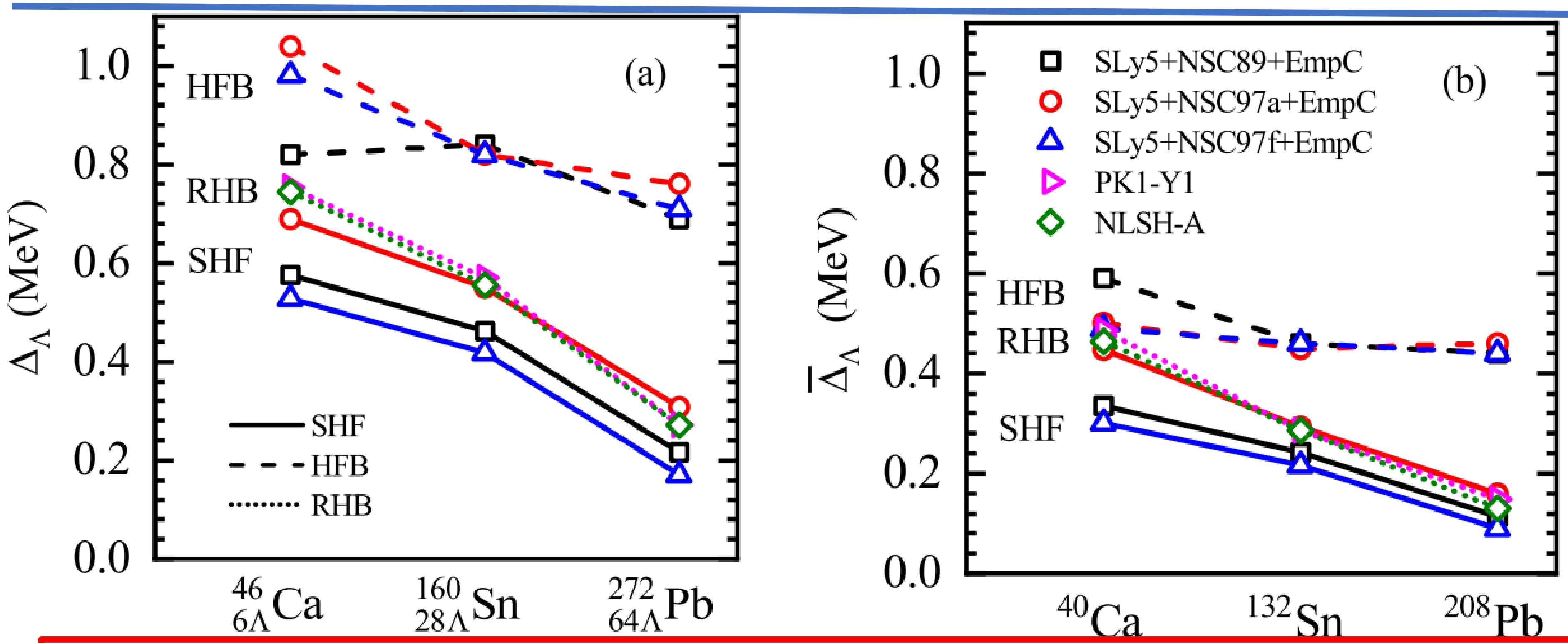


Λ hyperon potential for $^{278}{}_{70\Lambda}$Pb

Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

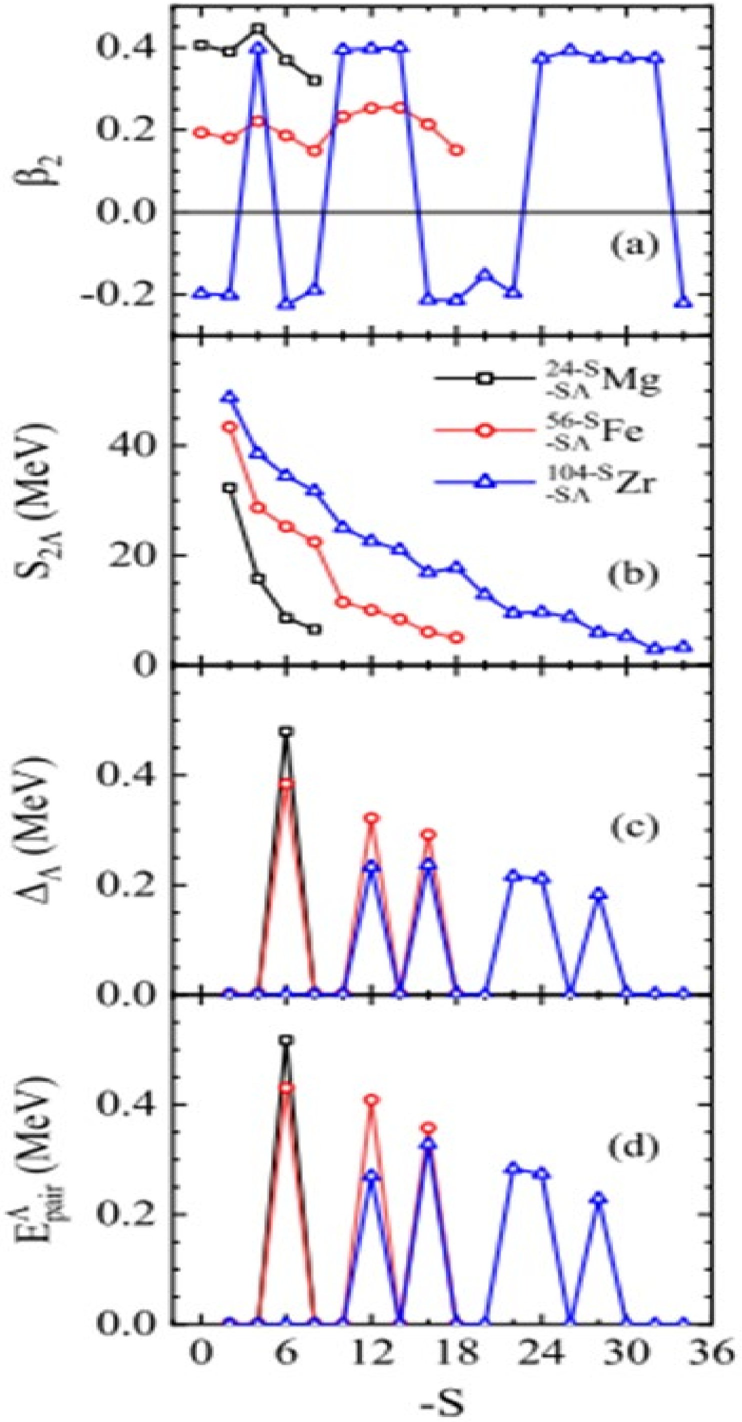
- The reason for the additional shell closures at −S = 18 and 68 in SHF : Because of a lacking spin-orbit term, the magic numbers of hyperons are not the same as for nucleons (2, 8, 20, 28, 50, 82, ...).
- V_Λ is a Woods-Saxon-like potential, which results in the corresponding shell structure as also in HFB due to the same mean-field potential of hyperons.
- V_Λ obtained in the RHB calculation is similar to a harmonic oscillator potential and leads to slightly different shell closures.
- SHF shells at −S = 18 and 20 and at 68 and 70 are nearly degenerate in energy.

Comparison of Λ gaps and gap average in SHF, HFB, RHB model



Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

- The gaps in HFB are larger than those in SHF and RHB due to the larger pairing strength employed.
- With the same treatment of the pairing strength, the SHF and RHB results are very similar, in particular for the SHF NSC97a model.
- The differences between the NSC89/97a/97f models in SHF and RHB are caused by different s.p. properties.

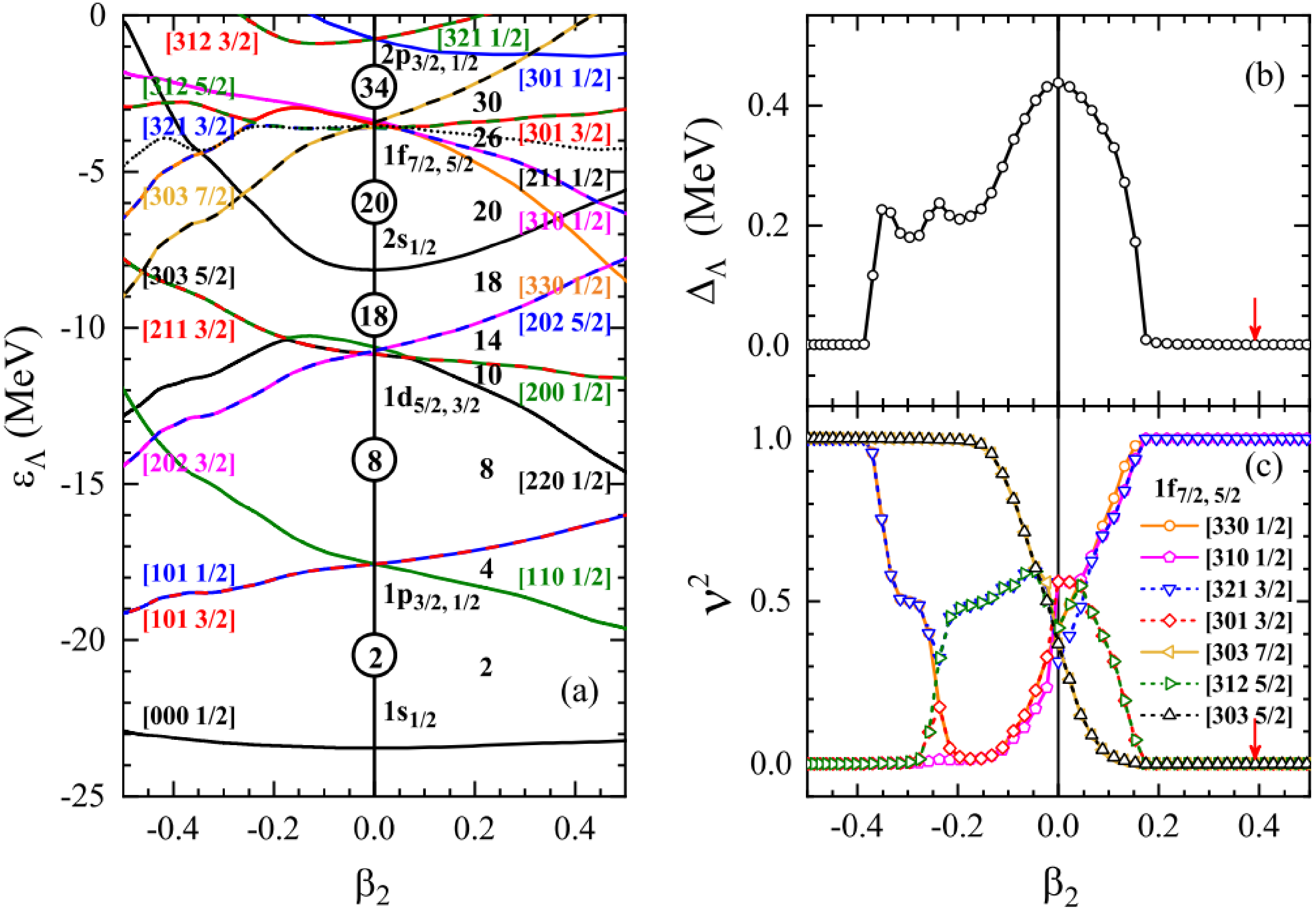


ΛΛ pairing effects in deformed hypernuclei

- For the deformed hyperisotopes, more possible Λ hyperon magic numbers are found. All deformed hyperisotopes show vanished pairing gap energy at $-S$ = 2, 4, 8, 10, 14, 18, 20, 22, 26, 30, 32, and 34, and the Zr hyperisotopes in addition at $-S$ = 6.
- This provides the evidence for the appearance of the new possible magic numbers $-S$ = 4, 6, 10, 14, 26, 30, and 32 in the deformed hyperisotopes.

Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

ΛΛ pairing effects in deformed hypernuclei



- The possible magic numbers in deformed hypernuclei are sensitive to β_2 due to the possible crossing of some s.p. levels.
- The pairing effect depends on the s.p. level density around the Fermi surface. The pairing gap for prolate deformation vanishes gradually because the levels become more separated at larger deformation.
- In general, deformation always leads to a weakening or complete suppression of pairing.

Jing Guo, C. F. Chen, Xian-Rong Zhou, Q. B. Chen, and H.-J. Schulze, Phys. Rev. C 105, 034322 (2022).

Conclusions

- For the spherical hyperisotopes, the occurrences of magic numbers $-S = 2, 8, 18, 20, 34, 58, 68$, and 70 are evinced by the sudden drop of 2Λ separation energies and the vanished average pairing gaps and pairing energies due to a Woods-Saxon-like Λ hyperon potential.
- For the deformed hyperisotopes, more possible hyperon magic numbers $-S = 4, 6, 10, 14, 22, 26, 30$, and 32 corresponding to vanishing pairing appear. All possible hyperon magic numbers of the deformed hyperisotopes are sensitive to β_2.
- The current work predicts similar pairing gap results as a RHB approach, but smaller ones than those in HFB approach due to a smaller pairing strength. In all cases, the hyperon gaps are much smaller than the nucleonic ones, and in deformed nuclei the pairing correlations are even weaker.

Thank you for your attention!