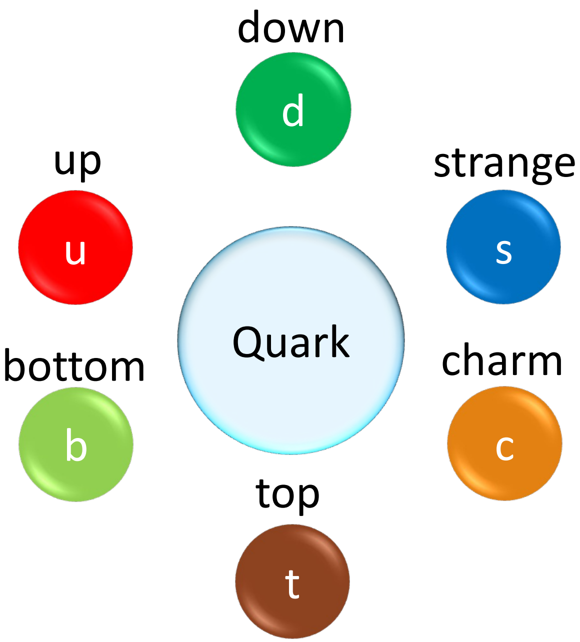
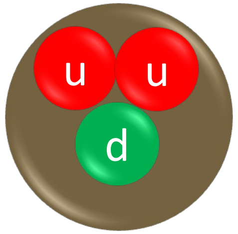
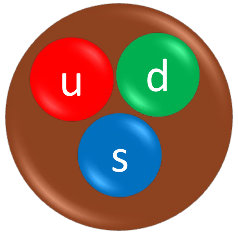
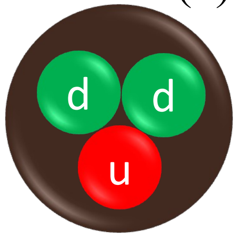
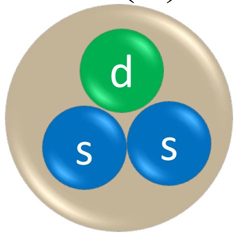
Contents

1. Introduction

Hypernucleus

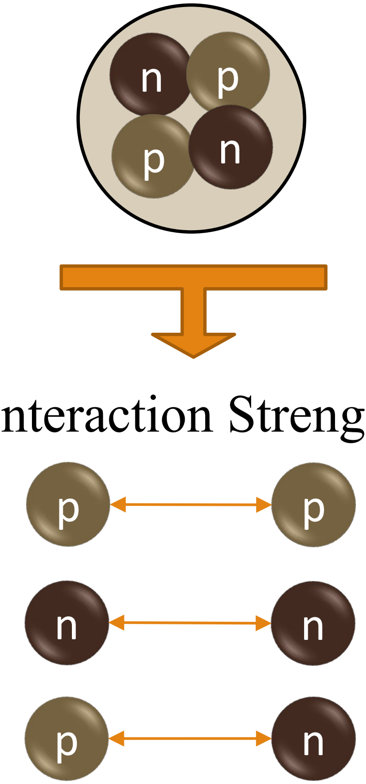
Six types of quarks



Baryons	
Nucleons	Hyperons
Proton (p) 	Lambda (Λ) 
Neutron (n) 	Xi (Ξ) 

✓ Hypernucleus is a nucleus which is composed of one or more hyperons in addition to nucleons.

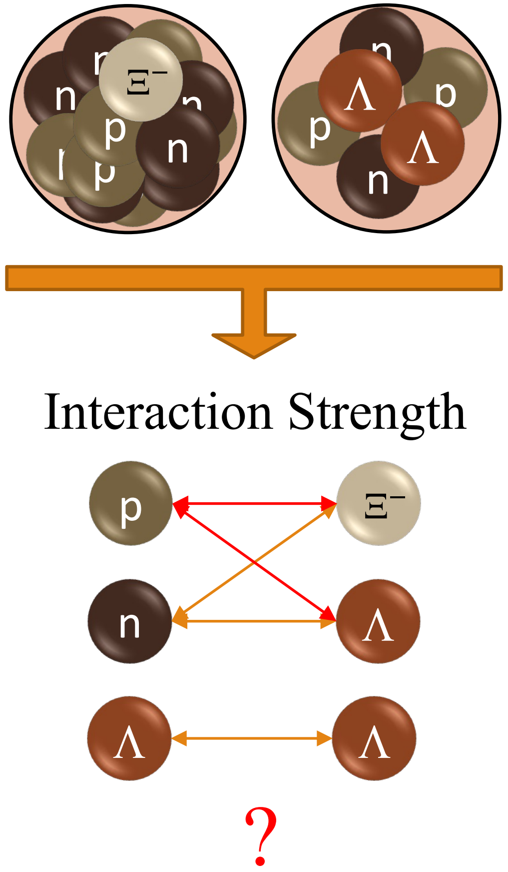
Normal nucleus



Attractive nuclear force

Already known

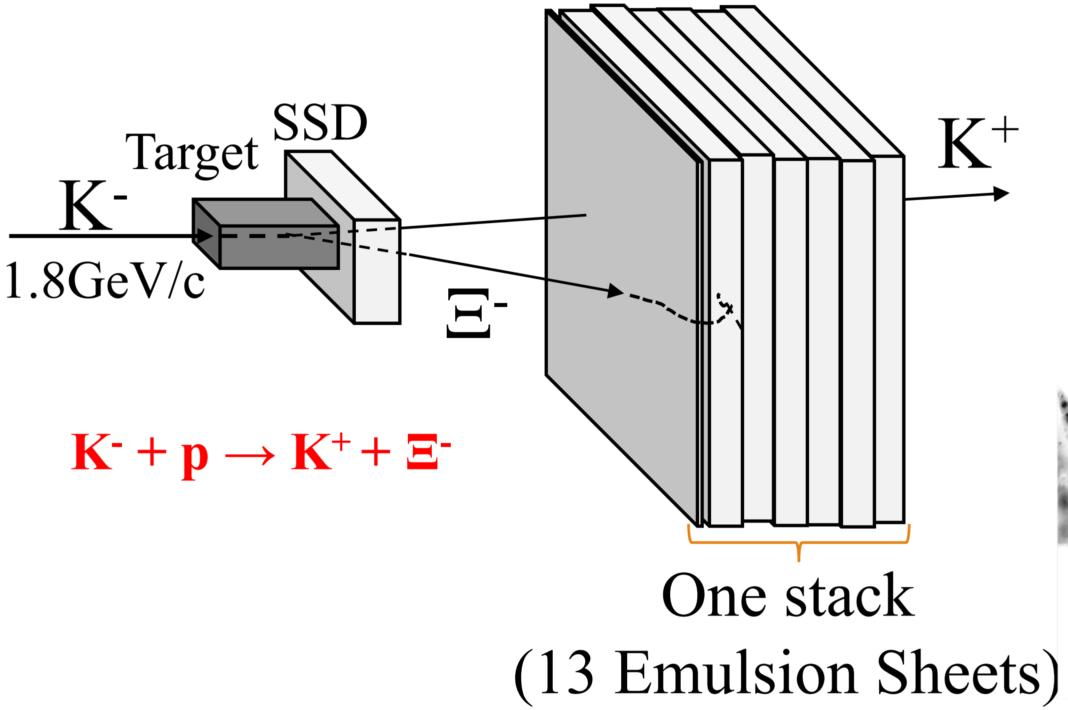
Hypernucleus



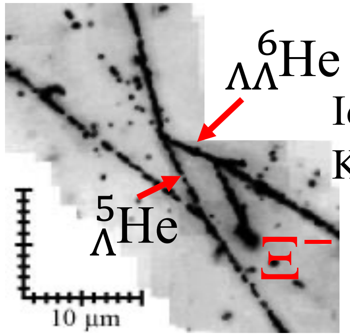
Our study

Experimental Setup for J-PARC E07

- ✓ To understand the characteristics of baryon-baryon interaction, especially, Ξ^--N and Λ-Λ interactions.

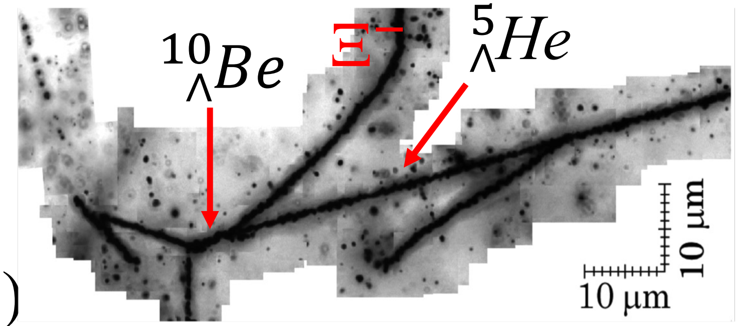


$K^- + p \rightarrow K^+ + \Xi^-$

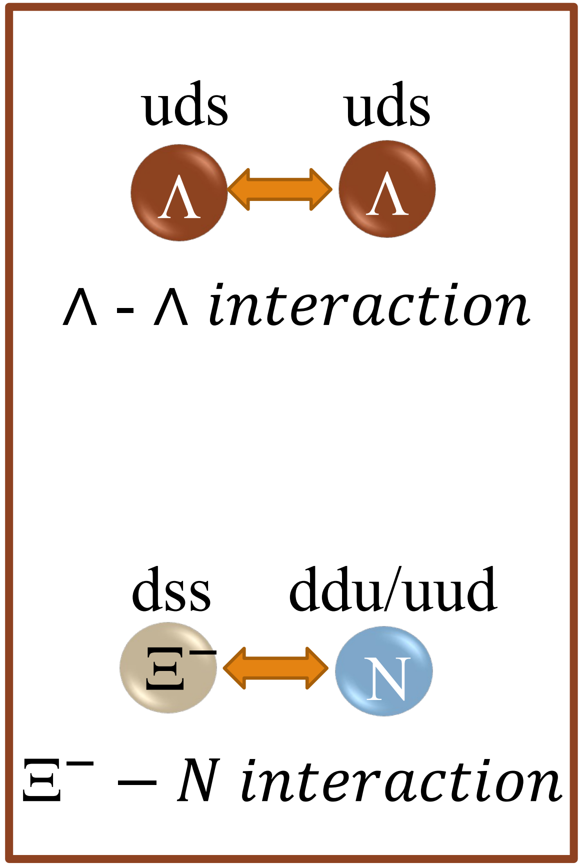


Identified events from KEK E373 experiment

NAGARA Event[1]



KISO Event[2,3]



- ✓ E07 experiment aimed to detect approximately 100 double hypernuclei.
- ✓ 33 events were detected from the first period of analysis.

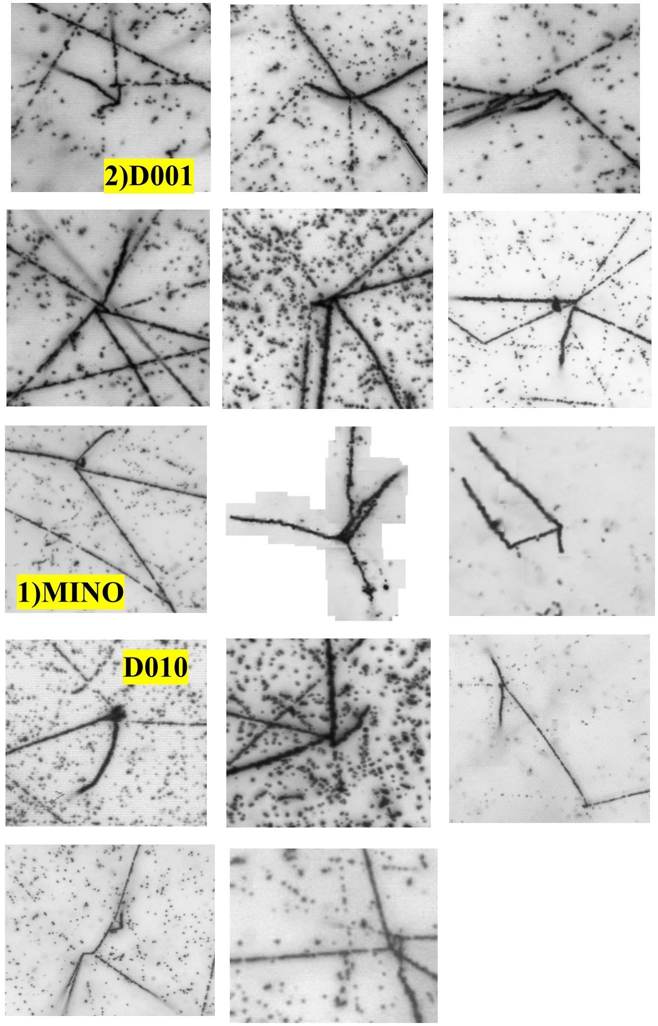
---

1) J. K. Ahn et al., Phys. Rev. C, **88**, 014003 (2013)
2) K. Nakazawa et al., Prog. Theor. Exp. Phys., 033D02 (2015)
3) E. Hiyama and K. Nakazawa, Annu. Rev. Nucl. Part. Sci., 68, 131 (2018)

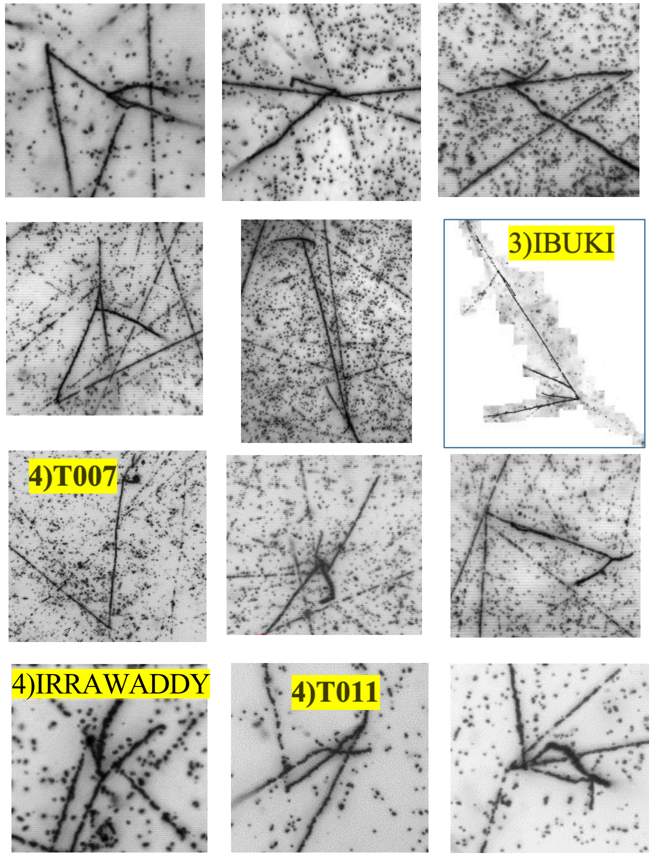
Observed double hypernucler events form J-PARC E07 experiment

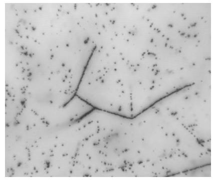
- ✓ 33 events were detected from first period of analysis.

14 Double-Λ hypernuclear events

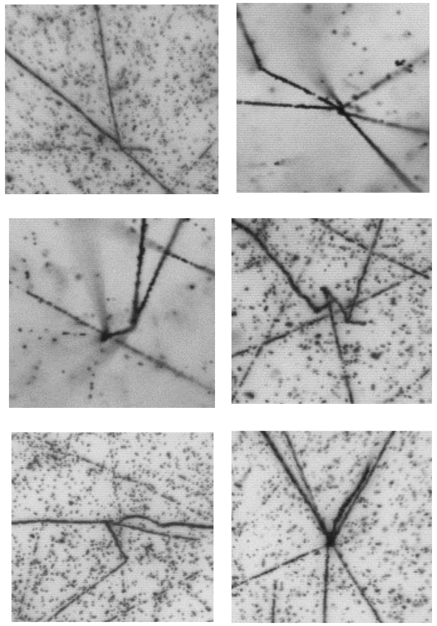


13 twin single-Λ hypernuclear events





6 others



Some of them are already analyzed and published.

1. H. Ekawa et al., Prog. Theor. Exp. Phys., 021D02 (2019)
2. A. N. L. Nyaw, et al., BSPIJ, Vol.30, No.2 022-25 (2020)
3. S. H. Hayakawa et al. [J-PARC E07 Collaboration], *Phys. Rev. Lett.* 126 062501 (2021)
4. M. Yoshimoto et al.Prog. Theor. Exp. Phys., Vol. 2021, Issue 7, 073D02 (2021)

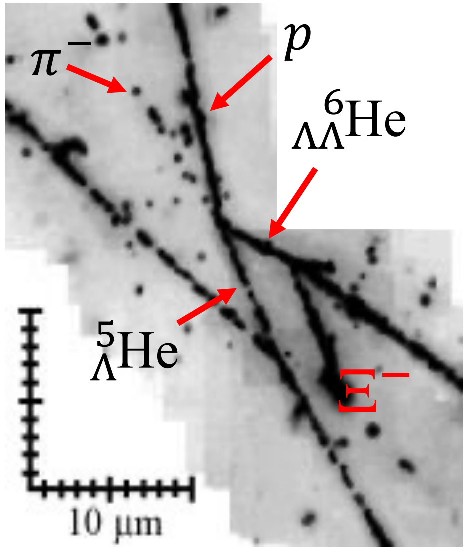
2. Motivation

How to measure masses of double hypernuclei?

Λ-Λ interaction energy ⟺ $\Delta B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) - 2B_{\Lambda}({}^{A-1}_{\Lambda}Z)$

$$B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) = M({}^{A-2}Z) + 2M(\Lambda) - M({}^{A}_{\Lambda\Lambda}Z)$$

NAGARA Event



$${}^{12}C + \Xi^- \to {}^{6}_{\Lambda\Lambda}He + {}^{4}He + t$$
$$\hookrightarrow {}^{5}_{\Lambda}He + p + \pi^-$$

In the case of NAGARA event after checking every considerable cases,

$$\Delta B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}He) = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}He) - 2B_{\Lambda}({}^{5}_{\Lambda}He)$$

$$B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}He) = M({}^{4}He) + 2M(\Lambda) - M({}^{6}_{\Lambda\Lambda}He)$$

$$\boxed{\begin{aligned} M({}^{6}_{\Lambda\Lambda}He) &= M({}^{5}_{\Lambda}He) + M(p) + M(\pi^-) \\ &\quad + KE({}^{5}_{\Lambda}He) + KE(p) + KE(\pi^-) \end{aligned}}$$

Range of decay daughter particles → RE relation → Kinetic energy

Range-energy (RE) relation in nuclear emulsion

- ✓ Kinetic energy(KE) of a charged particle is obtained by measuring the range of track

$$\boxed{R = \frac{M}{Z^2}.\lambda(\beta, d) + MZ^{2/3}C_z(\beta/Z)\,,}$$

where,

R = range
Z = charge
M = mass of charged particle in a unit of proton mass
C_z= an empirical function to correct range extension
$\lambda(\beta, d)$ = range of proton at velocity βc and emulsion density(*d*)

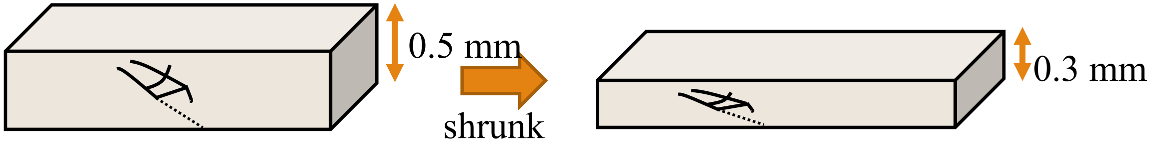
H. H. Heckman et al., Phys. Rev. 117, 544, (1960)
W. H. Barkas, et al., Nuovo Cimento Vol.8 158, 194,195 (1958)

Necessities for range measurement in 3D coordinates

- ✓ Range of charged particles can be obtained by,

$$R = \sqrt{\Delta X^2 + \Delta Y^2 + (\Delta Z)^2}$$

✓ .



Before photographic development

After photographic development

$$R = \sqrt{\Delta X^2 + \Delta Y^2 + (\Delta Z * S)^2}$$

- ✓ Original track range can be obtained by multiplying shrinkage factor(S) in z-axis.

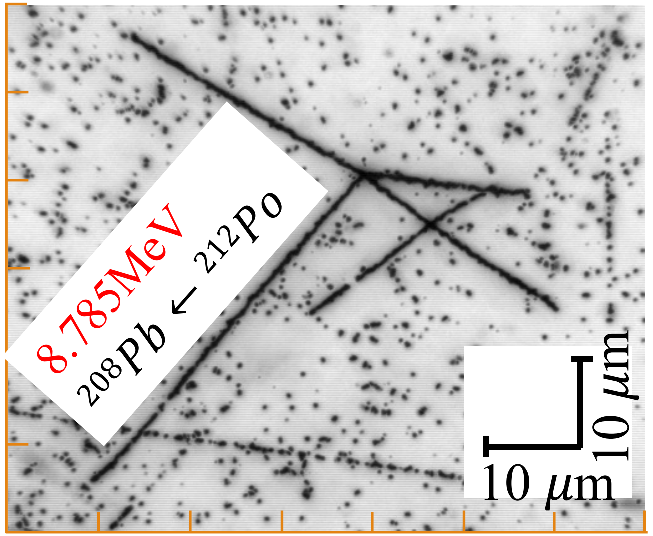
Alpha track from ^{212}Po of Thorium series are used

1. To obtain shrinkage factor
2. To obtain emulsion density

Up to now,

- ✓ Relation between density error and mass measurement error was not fully studied.
- ✓ How many alpha particles are enough to give optimal corresponding mass error?

Natural Isotope (Thorium series)



Superimposed image of alpha tracks from the Thorium series(5 tracks)

3. Objective

- The purpose of this research is to determine sufficient number of alpha tracks for the calibration of density of emulsion layer in order to minimize mass error of double hypernuclei.

4. Analysis method and Results

Calculation Steps to decide enough number of alpha tracks

- To get reasonable energy accuracy from density error, enough number of alpha tracks shall be decided by the following steps:

1. Alpha rage calculation

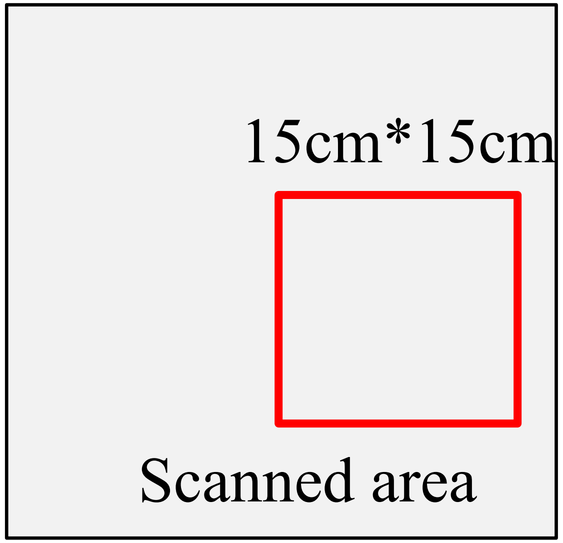
2. Density (d) and density error (d_{err}) calculation

3. Kinetic energy (KE) and kinetic energy error (KE_{err}) calculation
   - KE and KE_{err} calculation using determined d and d_{err}
   - Deciding enough number of alpha tracks by KE_{err} from d_{err}

1. Alpha scanning and selection

Module #030 from E07 experiment

For 3 emulsion plates (PL02, PL03, and PL04)



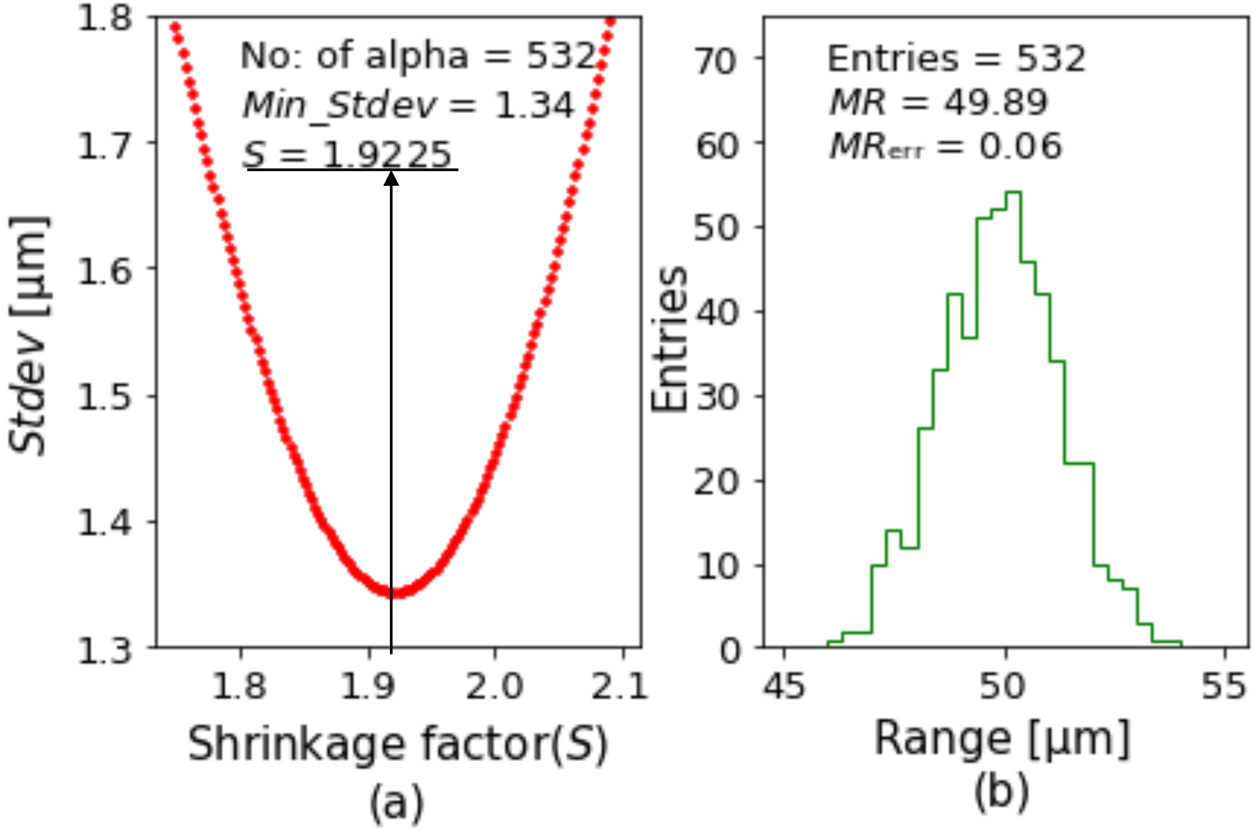
	PL02	PL03	PL04
Alpha candidates by overall scanning	20365	19078	20883
Measured Alpha tracks ^{212}Po → ^{208}Pb	545	532	481

1. Searching for the proper shrinkage factor (S)

$$R = \sqrt{\Delta X^2 + \Delta Y^2 + (\Delta Z * S)^2}$$

- ✓ Alpha range is calibrated changing S from 1.7500 to 2.1000 with an interval of 0.0025.
- ✓ Proper S is taken where alpha mean rage (MR) has minimum standard deviation (Min_stdev).
- ✓ Proper S provide more sharp range distribution.
- ✓ Mean range error (MR_{err}) is calculated by

$$MR_{\mathrm{err}} = \frac{standard\ deviation\ (Stdev)}{\sqrt{N}}$$

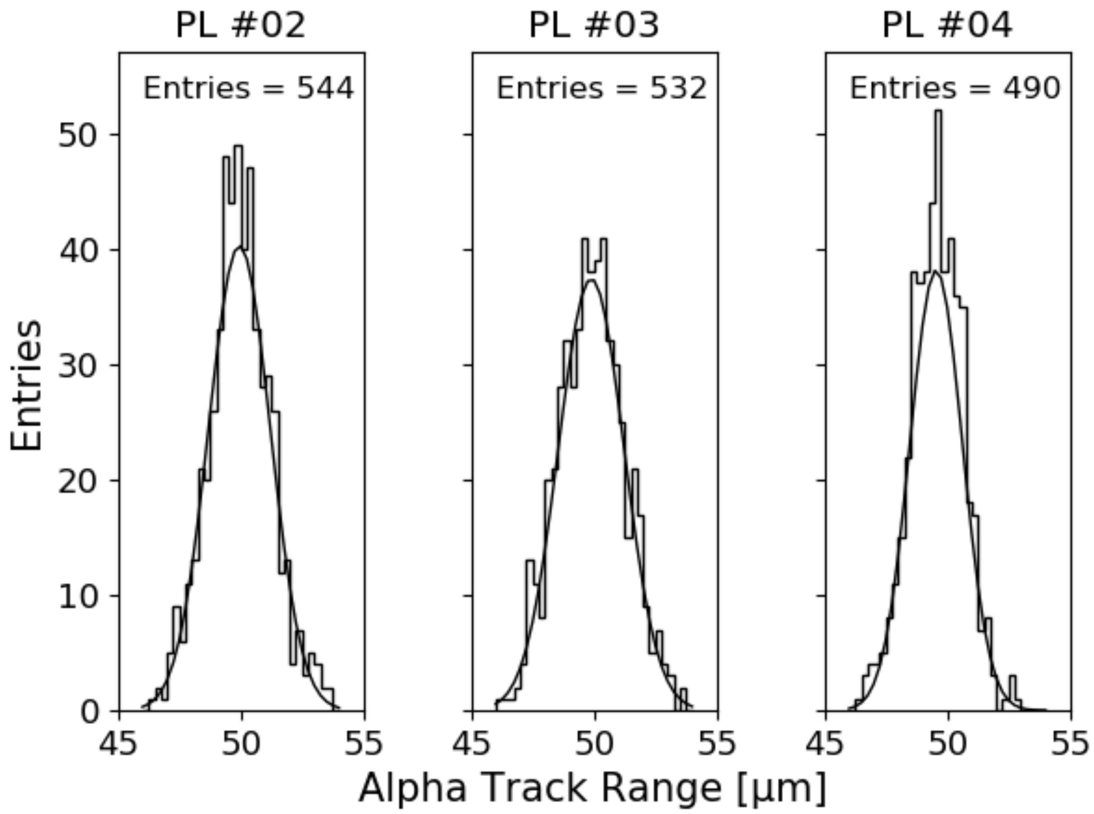


(a) Distribution of stdev for Shrinkage factor (S)
(b) Range distribution using determined S

1. Chi-square test for the distribution of the range to be normal?

Name	**Pl02**	**Pl03**	**Pl04**
χ^2	27.37	21.27	26.29
DOF	29	29	29
p-value	0.552	0.859	0.610

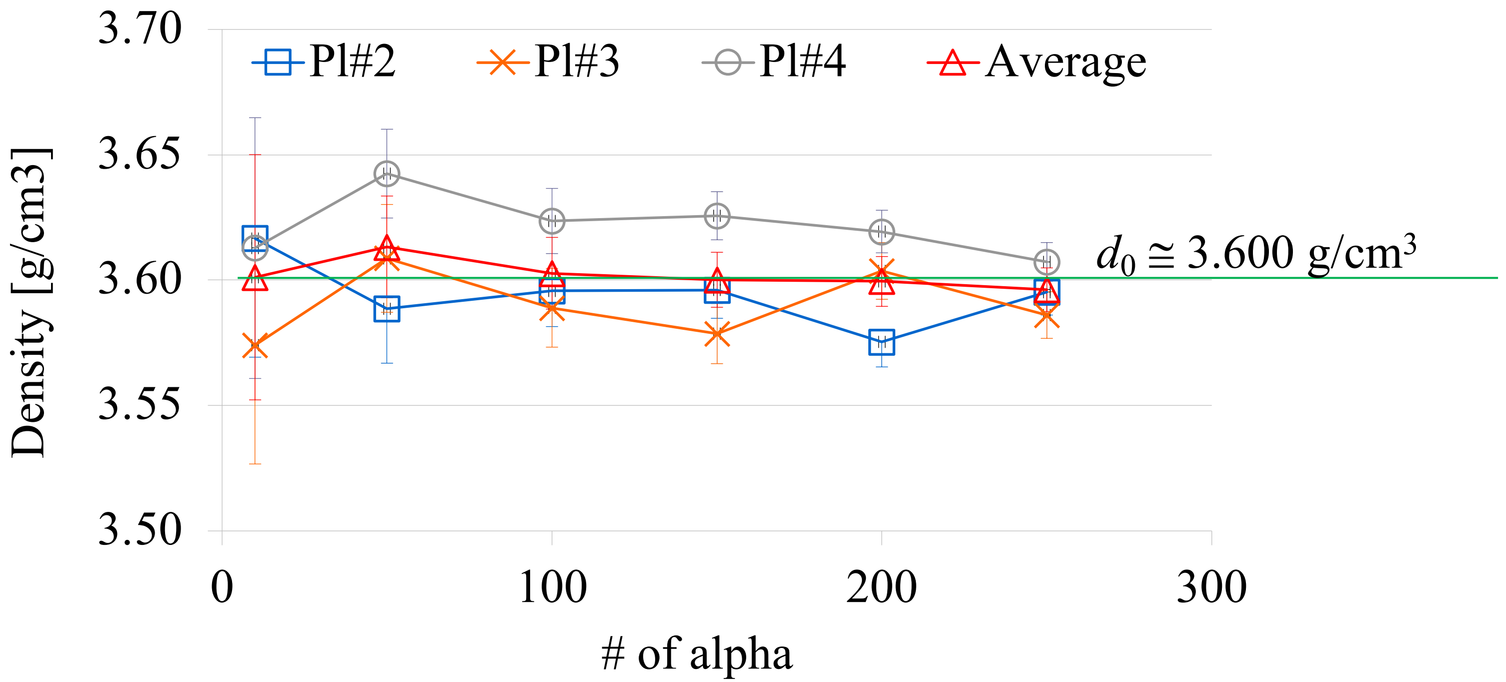
- ✓ p-values for each plate > 0.05.
- ✓ Range distributions each plate are found to be normal distributions.
- ✓ Alpha tracks range in each plate are consistent



Alpha range distribution for three emulsion plates. Employed alpha tracks range were taken between 46 mm and 54 mm.

2. Density (d) and density error (d_{err}) calculation

- ✓ Alpha tracks from each emulsion plates were randomly selected to be six groups (G-10, G-50, G-100, G-150, G-200, and G-250).
- ✓ d and d_{err} were calculated using selected number of alpha tracks groups (G-#).
- ✓ All D ranges from 3.57 to 3.64 g/cm^3. (approximate density, $d_0 \cong 3.600$ g/cm^3)



Density and density error for each group in each emulsion

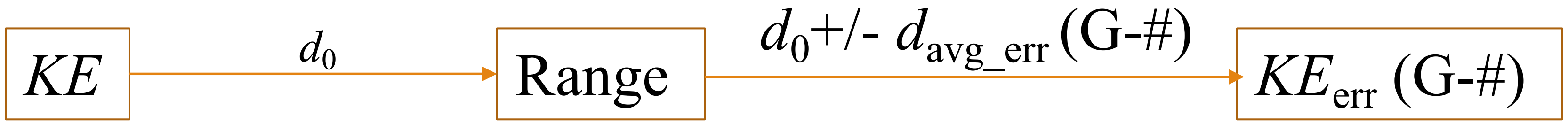
3. KE and KE_{err} calculation using determined d and d_{err}

- ✓ Sufficient number of alpha will be determined using the relation between KE and KE_{err} from d_{err}.

- ✓ To calculate KE_{err} by d_{err}, approximate density (d_0) 3.6 g/cm^3 and average d_{err} (d_{avg_err} (G-#)) for same number of alpha groups were used.

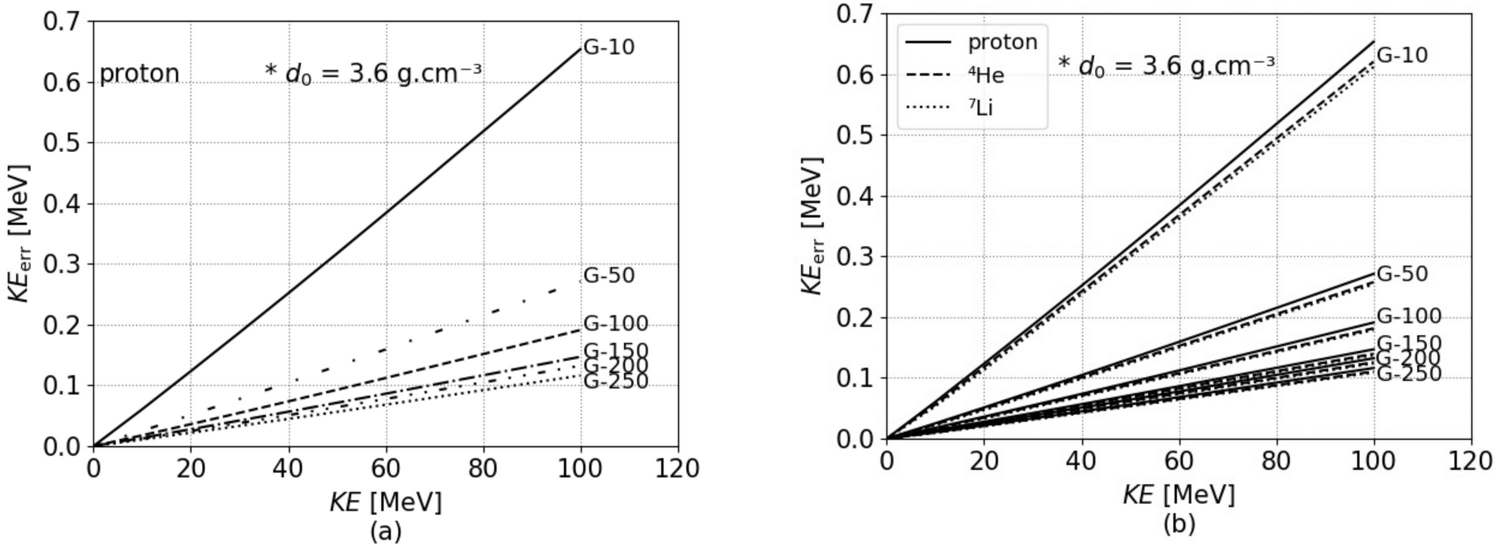
- ✓ KE was varied from 0 to 100 MeV with an interval of 10 MeV to give ranges of the particles: proton (p), helium (^{4}He), lithium (^{7}Li).

- ✓ KE_{err} (G-#) was calculated using obtained range, d_0, and d_{avg_err} (G-#).



KE $\xrightarrow{d_0}$ Range $\xrightarrow{d_0 +/- d_{avg\_err} \text{ (G-\#)}}$ KE_{err} (G-#)

3.3 Determine the enough number of alpha tracks by KEerr from Derr_avg



(a) The relationship between *KE* and KE_{err} for proton, (b) Comparison result for proton, helium (^{4}He), and lithium (^{7}Li).

- ➢ The gaps of KE_{err} in the groups between G-10 and G-150 is large in comparison to between G-150 and G-250.
- ➢ Tendencies of KE_{err} gap for ^{4}He and ^{7}Li are similar to those for the proton.
- ➢ To know variation of KE_{err}, we check KE_{err} by changing density ± 0.10 to d_0. The result gives a difference of ± 2.35 % of KEerr at 100 MeV. It is small enough to be ignored.
- ➢ At least 150 alphas is enough to utilize for density calibration.

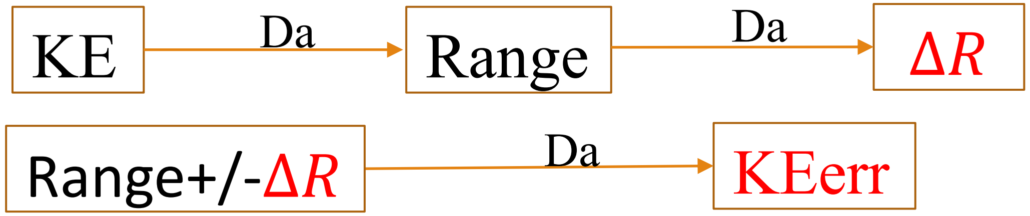
KE_{err} from d_{err} and range straggling (ΔR)

- As ΔR is a statistical error in the analysis of double hypernuclei, the KE_{err} obtained from ΔR was also calculated using following equation.

$$\Delta R\,(KE) = \frac{\sqrt{M}}{Z^2}\,.\,\Delta R_p\left(\frac{KE}{M}\right)$$

where, ΔR_p represents the error of range straggling by the proton

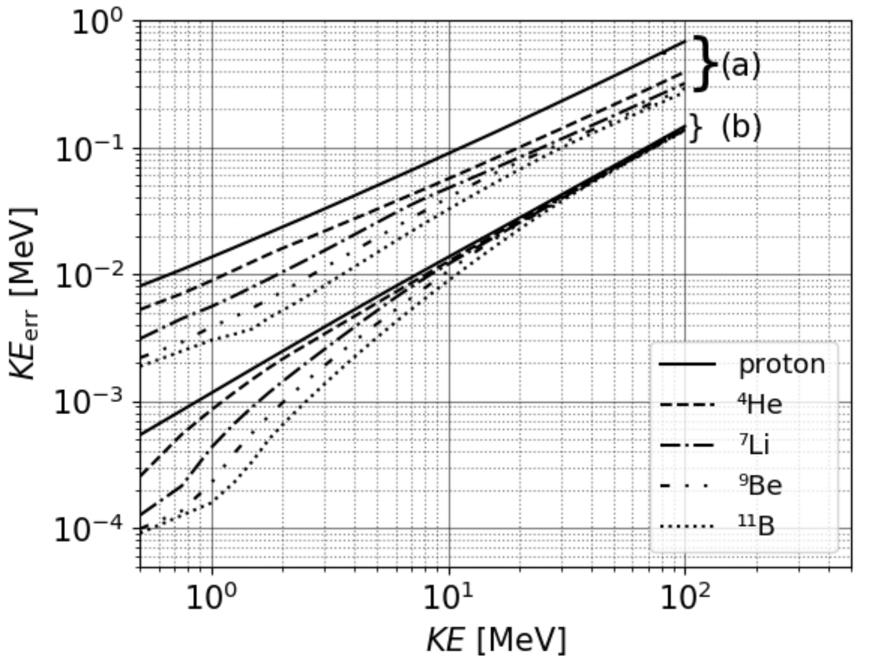
- To calculate ΔR, KE was varied from 0.5 to 100 MeV at intervals of 0.25 MeV.



KE → Da → Range → Da → ΔR

Range+/-ΔR → Da → KEerr

- KE_{err} from $d_{\mathrm{avg\_err}}$ is one order smaller than the KE_{err} from ΔR.



Comparison of KE_{err} value for each KE obtained from (a) ΔR and (b) $d_{\mathrm{err\_avg}}$ for proton, ^{4}He, ^{7}Li, ^{9}Be, and ^{11}B. 150 alpha was used to calibrate KE_{err} from $d_{\mathrm{err\_avg}}$.

4. Summary

- Suitable number of alpha tracks for density calibration was decided to be at least 150.

- The difference of by chaing d_0 with ± 0.10 g/cm^3 is ± 2.35 % of KE_{err} at 100 MeV. It is small enough to be ignored.

- KE_{err} from $d_{\mathrm{avg\_err}}$ is one order of magnitude smaller than the KE_{err} from ΔR.

- This study aimed to provide for the future analysis scheme of double hypernuclei to minimize mass error.