Four-body Faddeev-type calculation of $\bar{K}NNN$ system

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Interest to antikaon-nucleon systems: quasi-bound state in the K^-pp system
[*Y.Akaishi,T.Yamazaki, Phys.Rev.C65, 044005 (2002)*]
$\rightarrow$ experimental and theoretical efforts with different results

Our contribution: series of dynamically exact calculations of three-body antikaon-nucleon systems

- Quasi-bound states in $\bar{K}NN$, spin 0 (K^-pp) and $\bar{K}\bar{K}N$ systems - binding energy and width
- No quasi-bound state in $\bar{K}NN$, spin 1 (K^-np) - proven
- Near-threshold scattering in $\bar{K}NN$, spin 1 (K^-d)
(Faddeev-type AGS equations with coupled $\bar{K}NN - \pi\Sigma N$ channels)
- 1s level shift and width of kaoninc deuterium (Faddeev-type equations with directly included Coulomb interaction)

[N.V. S., Three-Body Antikaon–Nucleon Systems. Few Body Syst. 58, 6 (2017)]

Next step – four-body calculations of the $\bar{K}NNN$ using Faddeev-type equations.

Four-body equations

The four-body Faddeev-type AGS equations, written for separable potentials [*P. Grassberger, W. Sandhas, Nucl. Phys. B 2, 181-206 (1967)*]

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = (1-\delta_{\sigma\rho})(\bar{G_0}^{-1})_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta}(1-\delta_{\sigma\tau})\bar{T}^{\tau}_{\alpha\gamma}(z)(\bar{G_0})_{\gamma\delta}(z)\bar{U}^{\tau\rho}_{\delta\beta}(z),$$

$$\bar{U}^{\sigma\rho}_{\alpha\beta}(z) = \langle g_\alpha|G_0(z)U^{\sigma\rho}_{\alpha\beta}(z)G_0(z)|g_\beta\rangle,$$

$$\bar{T}^{\tau}_{\alpha\beta}(z) = \langle g_\alpha|G_0(z)U^{\tau}_{\alpha\beta}(z)G_0(z)|g_\beta\rangle,$$

$$(\bar{G_0})_{\alpha\beta}(z) = \delta_{\alpha\beta}\tau_\alpha(z).$$

Operators $\bar{U}^{\sigma\rho}_{\alpha\beta}$ and $\bar{T}^{\tau}_{\alpha\beta}$ contain four-body $U^{\sigma\rho}_{\alpha\beta}(z)$ and three-body $U^{\tau}_{\alpha\beta}(z)$ transition operators of the general form, correspondingly.

$\bar{K}NNN$ system:

- two partitions of $3+1$ type: $|\bar{K}+(NNN)\rangle$, $|N+(\bar{K}NN)\rangle$,
- one of the $2+2$ type: $|(\bar{K}N)+(NN)\rangle$

Four-body equations

Separable form of the "effective three-body potentials" :

$$\bar{T}^{\tau}_{\alpha\beta}(z) = |\bar{g}^{\tau}_{\alpha}\rangle \bar{\tau}^{\tau}_{\alpha\beta}(z) \langle \bar{g}^{\tau}_{\beta}|$$

→ the four-body equations can be rewritten as
[A. Casel, H. Haberzettl, W. Sandhas, Phys. Rev. C 25, 1738 (1982)]

$$\bar{X}^{\sigma\rho}_{\alpha\beta}(z) = \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) + \sum_{\tau,\gamma,\delta} \bar{Z}^{\sigma\tau}_{\alpha\gamma}(z) \bar{\tau}^{\tau}_{\gamma\delta}(z) \bar{X}^{\tau\rho}_{\delta\beta}(z)$$

with new four-body transition $\bar{X}^{\sigma\rho}$ and kernel $\bar{Z}^{\sigma\rho}$ operators

$$\begin{aligned} \bar{X}^{\sigma\rho}_{\alpha\beta}(z) &= \langle \bar{g}^{\sigma}_{\alpha} | \bar{G}_0(z)_{\alpha\alpha} \bar{U}^{\sigma\rho}_{\alpha\beta}(z) \bar{G}_0(z)_{\beta\beta} | \bar{g}^{\rho}_{\beta} \rangle, \\ \bar{Z}^{\sigma\rho}_{\alpha\beta}(z) &= (1 - \delta_{\sigma\rho}) \langle \bar{g}^{\sigma}_{\alpha} | \bar{G}_0(z)_{\alpha\beta} | \bar{g}^{\rho}_{\beta} \rangle. \end{aligned}$$

Full system of equations with:

- 1-term separable $\bar{K}N$ potential, 2-term separable NN potential (input)
- 1-term separabelized 3-body NNN, $\bar{K}NN$ "T-matrices" and "3-body" $\bar{K}N + NN$ "T-matrices"

→ system of 18×18 coupled equations

$\bar{K}N$ and NN potentials: separable by construction, separabelization of $3+1$ and 2+2: Energy Dependent Pole Expansion/Approximation (EDPE/ EDPA) *[S. Sofianos, N.J. McGurk, H. Fiedeldeldey, Nucl. Phys. A 318, 295 (1979)]*

Three-body Faddeev-type AGS equations written in momentum basis:

$$X_{\alpha\beta}(p,p';z) = Z_{\alpha\beta}(p,p';z)+\sum_{\gamma=1}^{3}\int_0^\infty Z_{\alpha\gamma}(p,p'';z)\,\tau_\gamma(p'';z)\,X_{\gamma\beta}(p'',p';z)p''^2dp'',$$

eigenvalues λ_n and eigenfunctions $g_{n\alpha}(p;z)$ of the system – from

$$g_{n\alpha}(p;z) = \frac{1}{\lambda_n}\sum_{\gamma=1}^{3}\int_0^\infty Z_{\alpha\gamma}(p,p';z)\,\tau_\gamma(p';z)\,g_{n\gamma}(p';z)p'^2dp'$$

with normalization condition

$$\sum_{\gamma=1}^{3}\int_0^\infty g_{n\gamma}(p';z)\,\tau_\gamma(p';z)\,g_{n'\gamma}(p';z)p'^2dp' = -\delta_{nn'}.$$

EDPE/EDPA method: solution of the eigenequations for a fixed energy z, usually $z = E_B$. After that energy dependent form-factors

EDPE/EDPA separabelization

$$g_{n\alpha}(p;z) = \frac{1}{\lambda_n} \sum_{\gamma=1}^{3} \int_0^\infty Z_{\alpha\gamma}(p,p';z)\, \tau_\gamma(p';E_B)\, g_{n\gamma}(p';E_B) p'^2 dp'$$

and propagators

$$(\Theta(z))^{-1}_{mn} = \sum_{\gamma=1}^{3} \int_0^\infty g_{m\gamma}(p';z)\, \tau_\gamma(p';E_B)\, g_{n\gamma}(p';E_B) p'^2 dp'$$

$$- \sum_{\gamma=1}^{3} \int_0^\infty g_{m\gamma}(p';z)\, \tau_\gamma(p';z)\, g_{n\gamma}(p';z) p'^2 dp'$$

calculated. The separable version of a three-body amplitude:

$$X_{\alpha\beta}(p,p';z) = \sum_{m,n=1}^{\infty} g_{m\alpha}(p;z)\, \Theta_{mn}(z)\, g_{n\beta}(p';z).$$

In contrast to Hilbert-Schmidt expansion, EDPE method needs only one solution of the eigenvalue equations and calculations of the integrals after that. The method is accurate already with one term (i.e. EDPA), and it converges faster than Hilbert-Schmidt expansion.

Two-term Separable New potential (TSN) of nucleon-nucleon interaction

$$V_{NN}^{\mathrm{TSN}}(k,k') = \sum_{m=1}^{2} g_m(k)\, \lambda_m \; g_m(k')\,,$$

$$g_m(k) = \sum_{n=1}^{3} \frac{\gamma_{mn}}{(\beta_{mn})^2 + k^2}, \quad \text{for } m = 1, 2$$

fitted to Argonne $V18$ potential *[R.B. Wiringa, V.G.J. Stoks, R. Schiavilla, Phys. Rev. C 51, 38 (1995)]* phase shifts

Triplet and singlet scattering lengths a and effective ranges r_{eff}

$$a_{np}^{\mathrm{TSN}} = -5.400\,\mathrm{fm}, \qquad r_{\mathrm{eff},np}^{\mathrm{TSN}} = 1.744\,\mathrm{fm}$$
$$a_{pp}^{\mathrm{TSN}} = 16.325\,\mathrm{fm}, \qquad r_{\mathrm{eff},pp}^{\mathrm{TSN}} = 2.792\,\mathrm{fm},$$

deuteron binding energy $E_{\mathrm{deu}} = 2.2246$ MeV.

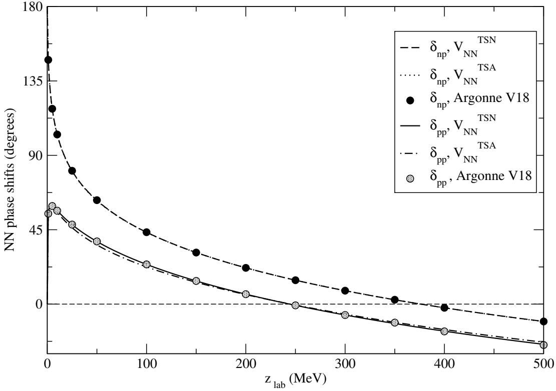

Fig: Phase shifts of np and pp scattering calculated using the new $V_{NN}^{\rm TSN}$ and $V_{NN}^{\rm TSA-B}$ potentials plus phase shifts of Argonne $V18$

By-product result

$K^- np$ subsystem ($\bar{K}NN$, $S = 1$) of the 4-body $K^- ppn$ system:

$I^{(4)} = 0$, $S^{(4)} = 1/2$, orbital momentum $L^{(4)} = 0$

- Previously used in three-body calculations V_{NN}^{TSA} potential *[P.Doleschall]*
  – no quasi-bound state (proven)
- Newly constructed V_{NN}^{TSN} potential
  – quasi-bound state in the $K^- np$ subsystem caused by strong interactions (additionally to the atomic state – kaonic deuterium) was found

Antikaon-nucleon interaction

Three our potentials:

- phenomenological $\bar{K}N - \pi\Sigma$ with one-pole $\Lambda(1405)$ resonance
- phenomenological $\bar{K}N - \pi\Sigma$ with two-pole $\Lambda(1405)$ resonance
- chirally motivated $\bar{K}N - \pi\Sigma - \pi\Lambda$ potential, two-pole $\Lambda(1405)$

reproduce (with the same level of accuracy):

- $1s$ level shift and width of kaonic hydrogen *(SIDDHARTA)*
  direct inclusion of Coulomb interaction, no Deser-type formula used
- Cross-sections of $K^-p \to K^-p$ and $K^-p \to MB$ reactions
- Threshold branching ratios γ, R_c and R_n
- $\Lambda(1405)$ resonance (*one- or two-pole structure*)
  $M^{PDG}_{\Lambda(1405)} = 1405.1^{+1.3}_{-1.0}$ MeV, $\Gamma^{PDG}_{\Lambda(1405)} = 50.5 \pm 2.0$ MeV [*PDG (2016)*]

Subsystems of the $\bar{K}NNN$ system

- $\bar{K}NN\,(I^{(3)} = 1/2, S^{(3)} = 0 \text{ or } 1)$

  Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the K^-pp and K^-np (sub)systems; exact optical $V^{\text{Opt}}_{\bar{K}N}$ and V^{TSN}_{NN}

	B_{K^-pp}	Γ_{K^-pp}	B_{K^-np}	Γ_{K^-np}
$V^{1,\text{SIDD}}_{\bar{K}N}$	53.3	64.7	1.9	68.7
$V^{2,\text{SIDD}}_{\bar{K}N}$	46.7	48.4	5.6	62.7
$V^{\text{Chiral}}_{\bar{K}N}$	29.9	48.2	2.3	45.5

- $NNN\,(I^{(3)} = 1/2, S^{(3)} = 1/2)$ $\qquad B_{NNN} = 9.52$ MeV (V^{TSN}_{NN})
- $\bar{K}N + NN\,(I^{(4)} = 0, S^{(4)} = 1/2)$ – a special system with two non--interacting pairs of particles; 3-body system of equations to be solved

$K^-ppn - \bar{K}^0nnp$ system: $\bar{K}NNN$ with $I^{(4)} = 0, S^{(4)} = 1/2$, $L^{(4)} = 0$

Dependence of the binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^-ppn - \bar{K}^0nnp$ system on three $\bar{K}N$ and three NN interaction models.

	$V_{NN}^{\mathrm{TSA-A}}$		$V_{NN}^{\mathrm{TSA-B}}$		V_{NN}^{TSN}	
	B	Γ	B	Γ	B	Γ
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	52.0	50.4	50.3	49.6	51.2	50.8
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	47.0	39.6	46.4	38.2	46.4	39.9
$V_{\bar{K}N}^{\mathrm{Chiral}}$	32.6	39.7	34.5	50.9	30.5	42.8

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- ppn - \bar{K}^0 nnp$ system, other results

		B	Γ
AY		108.0	20.0
BGL		29.3	32.9
OHHMH,	$V_{\bar{K}N}^{\mathrm{Kyoto-I}}$	45.3	25.5
OHHMH,	$V_{\bar{K}N}^{\mathrm{Kyoto-II}}$	49.7	69.4
ME,	$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	73.5	22.0
ME,	$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	58.5	27.0
ME,	$V_{\bar{K}N}^{\mathrm{IKS\,chiral}}$	41.4	31.5

[AY: Y. Akaishi, T. Yamazaki, Phys. Rev. C 65, 044005 (2002)]
[BGL: N. Barnea, A. Gal, E.Z. Liverts, Phys. Lett. B 712, 132 (2012)]
[S. Ohnishi et.al, Phys. Rev C 95, 065202 (2017)]
[ME: S. Marri, J. Esmaili, Eur. Phys. J. A 55, 43 (2019)]

Binding energy B (MeV) and width Γ (MeV) of the quasi-bound state in the $K^- ppn - \bar{K}^0 nnp$ system ($\bar{K}NNN$ with $I^{(4)} = 0, S^{(4)} = 1/2$, $L^{(4)} = 0$)

	$V_{NN}^{\mathrm{TSA-A}}$		$V_{NN}^{\mathrm{TSA-B}}$		V_{NN}^{TSN}		Other results	
	B	Γ	B	Γ	B	Γ	B	Γ
$V_{\bar{K}N}^{1,\mathrm{SIDD}}$	52.0	50.4	50.3	49.6	51.2	50.8		
$V_{\bar{K}N}^{2,\mathrm{SIDD}}$	47.0	39.6	46.4	38.2	46.4	39.9		
$V_{\bar{K}N}^{\mathrm{Chiral}}$	32.6	39.7	34.5	50.9	30.5	42.8		
AY							108.0	20.0
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- four-body binding energy and width of the $\bar{K}NNN$ system strongly depend on $\bar{K}N$ potential and noticeably - on NN potential, especially together with the chirally motivated antikaon-nucleon interaction model
- quasi-bound K^-ppn state: binding energies $B^{\rm Chiral}_{K^-ppn} \sim 30.5 - 34.5$ MeV obtained with chirally motivated and $B^{\rm SIDD}_{K^-ppn} \sim 46.4 - 52.0$ MeV obtained with phenomenological antikaon-nucleon potentials are close to those for the K^-pp system, calculated with the same $V_{\bar{K}N}$ and V_{NN} potentials
- quasi-bound K^-ppn state: widths of the four-body states $\Gamma_{K^-ppn} \sim 38.2 - 50.9$ MeV are smaller than the three-body widths of K^-pp. Therefore, the neutron added to the K^-pp system slightly influences the binding, but tightens the system