Hypernuclear production spectroscopy with an extended shell model

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collaborated with

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**Introduction**

- In production of hypernuclei, \((e, e' K^+)\) reaction experiments with high-resolution have been performed at Jlab.
- New projects of \((K^-, \pi^-)\) and \((\pi^+, K^+)\) reaction experiments with high-intensity and high-resolution are being at J-PARC.
- Detailed look in Jlab \((e, e' K^+)\) spectroscopic data requires an extended description with multi-configuration parity-mixing mediated by hyperon.
- We have extended the model space by introducing the new configuration which includes non-normal parity nuclear core-excited states.
- We will show the DWIA cross-sections of \((K^-, \pi^-), (\pi^+, K^+),\) and \((\gamma, K^+)\) reactions.
- We focus on the \(p\)-state \(\Lambda\) hyperon in the \(p\)-shell \(\Lambda\) hypernuclei.
Recent experimental result
T. Gogami et al., PRC93, 034314 (2016)

Shell-model prediction
T. Motoba et al., PTPS117, 123 (1994)
- Core nucleus calculated with conventional p-shell model
- Λ in s-orbit

This experiment has confirmed the major peaks (#1, #2, #3, #4) predicted in DWIA by employing the Λ particle in s-orbit coupled with the nuclear core states confined within the p-shell configuration.

However, it is interesting to observe extra strengths at $E_\Lambda = 0$ MeV excitation (a).

The extension of the model space is necessary and interesting challenge in view of the present hypernuclear spectroscopy.
Recent (e, e'K') reaction experiments done at the Jefferson Lab

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However, it is interesting to observe extra strengths at $E_\Lambda = 0$ MeV excitation (a).

The extension of the model space is necessary and interesting challenge in view of the present hypernuclear spectroscopy.
In the conventional shell model, only natural-parity nuclear-core states ($J_{\text{core}}^-$) are taken into account. $\Lambda$ particle is in the 0s orbit in $^{10}_\Lambda$Be($J^-$).

In $^{10}_\Lambda$Be($J^+$), the energy difference between $\Lambda(0s)$ and $\Lambda(0p)$ is $\hbar\omega$, and the energy difference between $^9$Be($J^-_{\text{core}}$) and $^9$Be($J^+_{\text{core}}$) is $\hbar\omega$.

By $\Lambda N$ interaction, natural-parity nuclear-core configurations and unnatural-parity nuclear-core configurations can be mixed.
Extended model space for target nucleus $^{10}\text{B}$

Extension of model space for target nucleus $^{10}\text{B}$ up to $2p-2h$ ($2\hbar\omega$) allows the $^{10}\Lambda\text{Be}$ production through various configurations.
Extended model space for target nucleus $^{10}$B

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Extension of model space for target nucleus $^{10}\text{B}$ up to $2p-2h$ ($2\hbar \omega$) allows the $^{10}_\Lambda\text{Be}$ production through various configurations.
Results: Energy levels of $^9$Be and $^{10}_\Lambda$Be

Energy (MeV)

$^9$Be (exp.) $^9$Be (cal.) $^{10}_\Lambda$Be (cal.)

Dominant configurations:
- Blue: $J^-; ^9$Be($J^-_{\text{core}}$) $\otimes$ $^\Lambda$(0s)
- Green: $J^+; ^9$Be($J^+_{\text{core}}$) $\otimes$ $^\Lambda$(0p)
- Magenta: $J^+; ^9$Be($J^-_{\text{core}}$) $\otimes$ $^\Lambda$(0p)
- Red: $J^+; ^9$Be($J^+_{\text{core}}$) $\otimes$ $^\Lambda$(0s)
Results: Energy levels of $^9$Be and $^{10}_\Lambda$Be

- $^9$Be (exp.)
- $^9$Be (cal.)
- $^{10}_\Lambda$Be (cal.)
- $^{10}_\Lambda$Be (exp.)

New bump

T. Gogami et al., PRC93, 034314 (2016)
Results: Cross sections of the $^{10}\text{B} \ (\gamma, K^+) \ ^{10}_{\Lambda}\text{Be}$ reaction (1)

Recent experimental result
T. Gogami et al., PRC93, 034314 (2016)

For hypernucleus $^{10}_{\Lambda}\text{Be}$
(1) $1p-1h \ (1\hbar\omega)$ core excitation
(2) Configuration mixing by $\Lambda N$ int.
are taken into account

DWIA calculation by using Saclay-Lyon model A

Our new calculation reproduces the four major peaks (#1, #2, #3, #4).

Our new calculation explains the new bump (a) as a sum of cross sections of some $J^+$ states.
### Excitation Energies and Cross Sections (Summary)

**$E_\gamma = 1.5 \text{ GeV}$**

EXP = T. Gogami et al, PRC93 (2016)

<table>
<thead>
<tr>
<th>$^9\text{Be} (J_i)$</th>
<th>$^{10}\Lambda\text{Be} (J_k)$</th>
<th>$E_\gamma = 1.5 \text{ GeV}$</th>
<th>$\theta = 7 \text{ deg}$</th>
<th>EXP</th>
<th>Fit I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J_i$</td>
<td>$E_i (\text{exp})$</td>
<td>$E_i (\text{cal})$</td>
<td>$J_k$</td>
<td>$E_x$</td>
<td>$-B_\Lambda$</td>
</tr>
<tr>
<td></td>
<td>$C2S$</td>
<td>$C2S$</td>
<td>$C2S$</td>
<td>$C2S$</td>
<td>$\text{MeV}$</td>
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<tr>
<td>$3/2^-$</td>
<td>0.000</td>
<td>0.000</td>
<td>1$^{-}$</td>
<td>0.000</td>
<td>-8.600</td>
</tr>
<tr>
<td></td>
<td>1.0(rel)</td>
<td>1.0(rel)</td>
<td>2$^{-}$</td>
<td>0.165</td>
<td>-8.435</td>
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<tr>
<td>$5/2^-$</td>
<td>2.429</td>
<td>2.644</td>
<td>2$^{-}$</td>
<td>2.712</td>
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<td>0.958</td>
<td>1.020</td>
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<tr>
<td></td>
<td>0.668</td>
<td>0.942</td>
<td>4$^{-}$</td>
<td>6.370</td>
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<td></td>
<td>2$^+(3)$</td>
<td>7.807</td>
<td>-0.793</td>
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<td>1$^+(3)$</td>
<td>7.935</td>
<td>-0.665</td>
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<td>3$^+(2)$</td>
<td>8.712</td>
<td>0.112</td>
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<td>2$^+(4)$</td>
<td>8.828</td>
<td>0.228</td>
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<td>2$^+(5)$</td>
<td>9.002</td>
<td>0.402</td>
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<td>3$^+(3)$</td>
<td>9.059</td>
<td>0.459</td>
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<td>1.299</td>
<td>1.355</td>
<td>4$^-$</td>
<td>10.455</td>
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<td>1$^+(5)$</td>
<td>10.828</td>
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<td>4$^+(3)$</td>
<td>11.318</td>
<td>2.718</td>
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<td></td>
<td>3$^+(5)$</td>
<td>11.543</td>
<td>2.943</td>
</tr>
</tbody>
</table>
Results: Configurations of \( J^+ \) states corresponding to the new bump

<table>
<thead>
<tr>
<th>( J_n^+ (-B_{\Lambda}[\text{MeV}]) )</th>
<th>( [J_{\text{core}}^+] j^\Lambda )</th>
<th>( [J_{\text{core}}^+] j^\Lambda )</th>
<th>( [J_{\text{core}}^+] j^\Lambda )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 2^+_3 (-0.739) )</td>
<td>4.49</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">3/2^-_1</a>^\Lambda ) 82.5%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">5/2^-_1</a>^\Lambda ) 15.8%</td>
</tr>
<tr>
<td>( 1^+_3 (-0.665) )</td>
<td>4.97</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">3/2^-_1</a>^\Lambda ) 79.5%</td>
<td>( [5/2^-<em>1]p</em>{3/2}^\Lambda ) 17.9%</td>
</tr>
<tr>
<td>( 2^+_4 (0.228) )</td>
<td>1.43 ( [5/2^+<em>2]s</em>{1/2}^\Lambda ) 87.5%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">3/2^-_1</a>^\Lambda ) 9.4%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">5/2^-_1</a>^\Lambda ) 2.4%</td>
</tr>
<tr>
<td>( 2^+_5 (0.402) )</td>
<td>9.89 ( [5/2^+<em>2]s</em>{1/2}^\Lambda ) 11.3%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">3/2^-_1</a>^\Lambda ) 70.9%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">5/2^-_1</a>^\Lambda ) 10.8%</td>
</tr>
<tr>
<td>( 3^+_2 (0.112) )</td>
<td>6.15 ( [5/2^+<em>2]s</em>{1/2}^\Lambda ) 31.6%</td>
<td>( [3/2^-<em>1]p</em>{3/2}^\Lambda ) 55.4%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">5/2^-_1</a>^\Lambda ) 9.7%</td>
</tr>
<tr>
<td>( 3^+_3 (0.459) )</td>
<td>2.43 ( [5/2^+<em>2]s</em>{1/2}^\Lambda ) 67.5%</td>
<td>( [3/2^-<em>1]p</em>{3/2}^\Lambda ) 27.1%</td>
<td>( <a href="p_%7B3/2%7Dp_%7B1/2%7D">5/2^-_1</a>^\Lambda ) 2.7%</td>
</tr>
</tbody>
</table>
In $^9\text{Be}$, it is well known that the $p_\Lambda$-state splits into two orbital states expressed by $p_\perp$ and $p_\parallel$, which is due to the strong coupling with nuclear core deformation having the $\alpha$-$\alpha$ structure.

T. Motoba et al., PTPS81, 42 (1985)

Results: Comparison to the cluster model – Cross section –

T. Motoba et al., PTPS81, 42 (1985)
Results: Cross sections of $^9$Be ($K^-, \pi^-$) and $^9$Be ($\pi^+, K^+$) reactions

Exp. from R. Bertini et al.,
NPA368, 365 (1981)

Exp. from O. Hashimoto and H. Tamura,
PPNP57, 564 (2006)
In the \((K^-, \pi^-)\) reaction, the large peak at \(E_\Lambda = 4.4\) MeV is a \(p\)-substitutional state via the \(p_{3/2}^N \rightarrow p_{3/2}^{\Lambda}\), which is strongly excited by recoilless reaction.

The small peak at \(E_\Lambda = 0\) MeV corresponds to the new bump and is explained as a mixture of \(s^\Lambda\) and \(p^\Lambda\) states.

The large peak at \(E_\Lambda = 4.4\) MeV in \(^{10}_{\Lambda}\)Be corresponds to the \([p^{-1} p_{\perp}^\Lambda]\) state in \(^9_{\Lambda}\)Be (\(^9\)Be analog state).

The small peak at \(E_\Lambda = 0\) MeV in \(^{10}_{\Lambda}\)Be corresponds to the \([p^{-1} p_{//}^\Lambda]\) state in \(^9_{\Lambda}\)Be.
Results: Cross sections of the $^{10}\text{B} (K^-, \pi^-) ^{10}_{\Lambda}\text{B}$ reaction (2)

CONCLUDE:

$\alpha\alpha$-like core deformation causes splitting of $p^\Lambda$-states, then low-energy $p^\Lambda_{//}$ can mix with $s^\Lambda$-states.

$[{}^9\text{Be}(J^-) \times \Lambda(p_{//})] + [{}^9\text{Be}(J^+) \times \Lambda(s)]$

These parity-mixed wave functions at $E_\Lambda = 0$ MeV can explain the extra peak #a.
Results: Cross sections of the $^{11}$B ($K^-, \pi^-$) and $^{11}$B ($\pi^+, K^+$) reactions

$^{11}$B ($K^-, \pi^-$) $^{\Lambda}$B

- $P_K^L = 1.0$ GeV/c, $\theta_L = 6^\circ$

$^{11}$B ($\pi^+, K^+$) $^{\Lambda}$B

- $P_\pi^L = 1.05$ GeV/c, $\theta_L = 3^\circ$

FWHM = 1.0 MeV

- $T=0, J^+; J^+_\text{core} \otimes s_\Lambda$
- $T=1, J^+; J^+_\text{core} \otimes s_\Lambda$
- $T=0, J^-; J^+_\text{core} \otimes p_\Lambda$
- $T=1, J^-; J^+_\text{core} \otimes p_\Lambda$

(A) $3/2^-$

- $^{10}$B($3^+_{\text{g.s.}}$) $\otimes p_{3/2}^{\Lambda}$ 51.4%
- $^{10}$B($1^+_{2}$) $\otimes p_{1/2}^{\Lambda}$ 23.0%

(B) $5/2^-$

- $^{10}$B($3^+_{\text{g.s.}}$) $\otimes p_{3/2}^{\Lambda}$ 56.1%
- $^{10}$B($3^+_{\text{g.s.}}$) $\otimes p_{1/2}^{\Lambda}$ 35.7%

$p_\perp$ state (substitutional)

$p_\parallel$ state
Results: Energy of $p_{\parallel}$-state

The $p^\Lambda$-state splits into $p_{\perp}$- and $p_{\parallel}$-states due to the strong coupling with nuclear core deformation.

In $^9\Lambda\text{Be}$, the energy of $p_{\parallel}^\Lambda$-state comes down to $E_x \approx 7\text{ MeV}$ from the $\Lambda$ single-particle energy difference $\varepsilon^\Lambda_p - \varepsilon^\Lambda_s \approx 11\text{ MeV}$.

The bump at $E_x \approx 8\text{ MeV}$ in the cross sections of $^{10}\Lambda\text{Be}$ corresponds to the $p_{\parallel}^\Lambda$-state.

In the cross sections of $^{11}\Lambda\text{Be}$, the small $5/2^-$ peak at $E_x \approx 9\text{ MeV}$ corresponds to the $p_{\parallel}^\Lambda$-state.

The energy splitting between $p_{\perp}$- and $p_{\parallel}$-states in $^{11}\Lambda\text{Be}$ is smaller than that in $^9\Lambda\text{Be}$, which is due to the small deformation of the nuclear core in $^{11}\Lambda\text{Be}$. 
$p^\Lambda$ state in the deformed nuclear core

In the spherical nuclear core, $p^\Lambda$-state does not split into $p^\Lambda_{\parallel}$ and $p^\Lambda_{\perp}$.

The new type wave function should appear in $^{9,10}_\Lambda$Be and $^{10,11}_\Lambda$B due to the core deformation, but “not” in spherical systems without enough deformation.
Summary

We have calculated the DWIA production cross sections for $p$-shell hypernuclei by using the extended shell model.

- Strong coupling between $p$-state $\Lambda$ and core deformation is realized in $^{9,10}_\Lambda$Be and $^{10,11}_\Lambda$B.
- In these nuclei, $p^\Lambda$-state splits into $p^\Lambda_\parallel$ and $p^\Lambda_\perp$.
- In $^{10}_\Lambda$Be, the lower $p^\Lambda_\parallel$ comes down in energy and $[^{9}\text{Be}(J^-) \times \Lambda(p_\parallel)]$ couples easily with $[^{9}\text{Be}(J^+) \times \Lambda(s)]$.
- Such new type wave functions should appear in $^{9,10}_\Lambda$Be and $^{10,11}_\Lambda$B due to the core deformation.

The finding of peak #a in $^{10}_\Lambda$B $(e,e'K^+)^{10}_\Lambda$Be is a novel evidence for genuine hypernuclear wave function with parity-mixing realized in “deformed” hypernuclei.