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**Hypernuclear production spectroscopy  
with an extended shell model**

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**collaborated with**

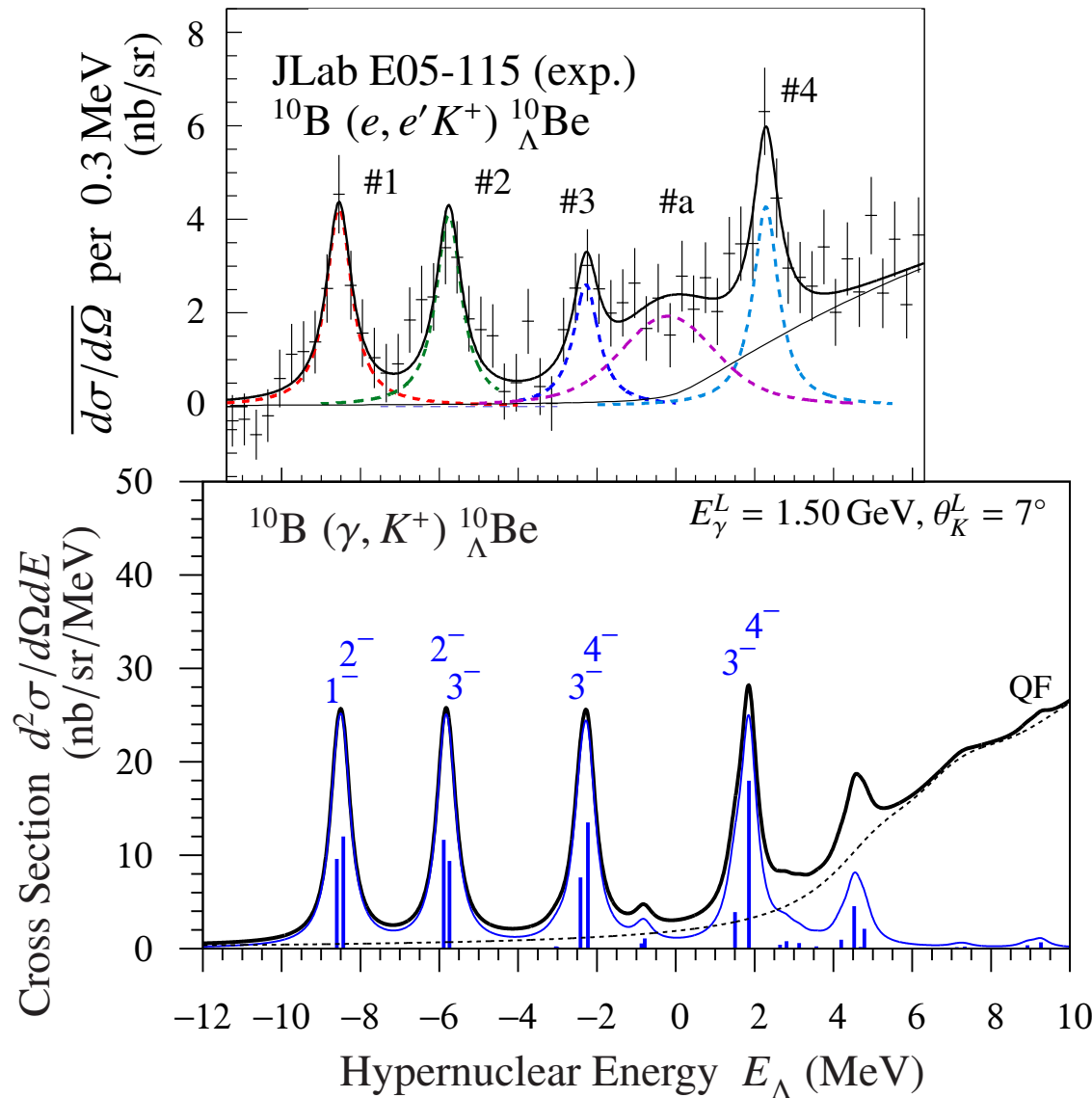
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## Introduction

- In production of hypernuclei,  $(e, e' K^+)$  reaction experiments with high-resolution have been performed at Jlab.
- New projects of  $(K^-, \pi^-)$  and  $(\pi^+, K^+)$  reaction experiments with high-intensity and high-resolution are being at J-PARC.
- Detailed look in Jlab  $(e, e' K^+)$  spectroscopic data requires an extended description with multi-configuration parity-mixing mediated by hyperon.
- We have extended the model space by introducing the new configuration which includes non-normal parity nuclear core-excited states.
- We will show the DWIA cross-sections of  $(K^-, \pi^-)$ ,  $(\pi^+, K^+)$ , and  $(\gamma, K^+)$  reactions.
- We focus on the  $p$ -state  $\Lambda$  hyperon in the  $p$ -shell  $\Lambda$  hypernuclei.

# Recent $(e, e' K^+)$ reaction experiments done at the Jefferson Lab



## Recent experimental result

**T. Gogami *et al.*, PRC93, 034314 (2016)**

## Shell-model prediction

**T. Motoba *et al.*, PTPS117, 123 (1994)**

- Core nucleus calculated with conventional  $p$ -shell model
- $\Lambda$  in  $s$ -orbit

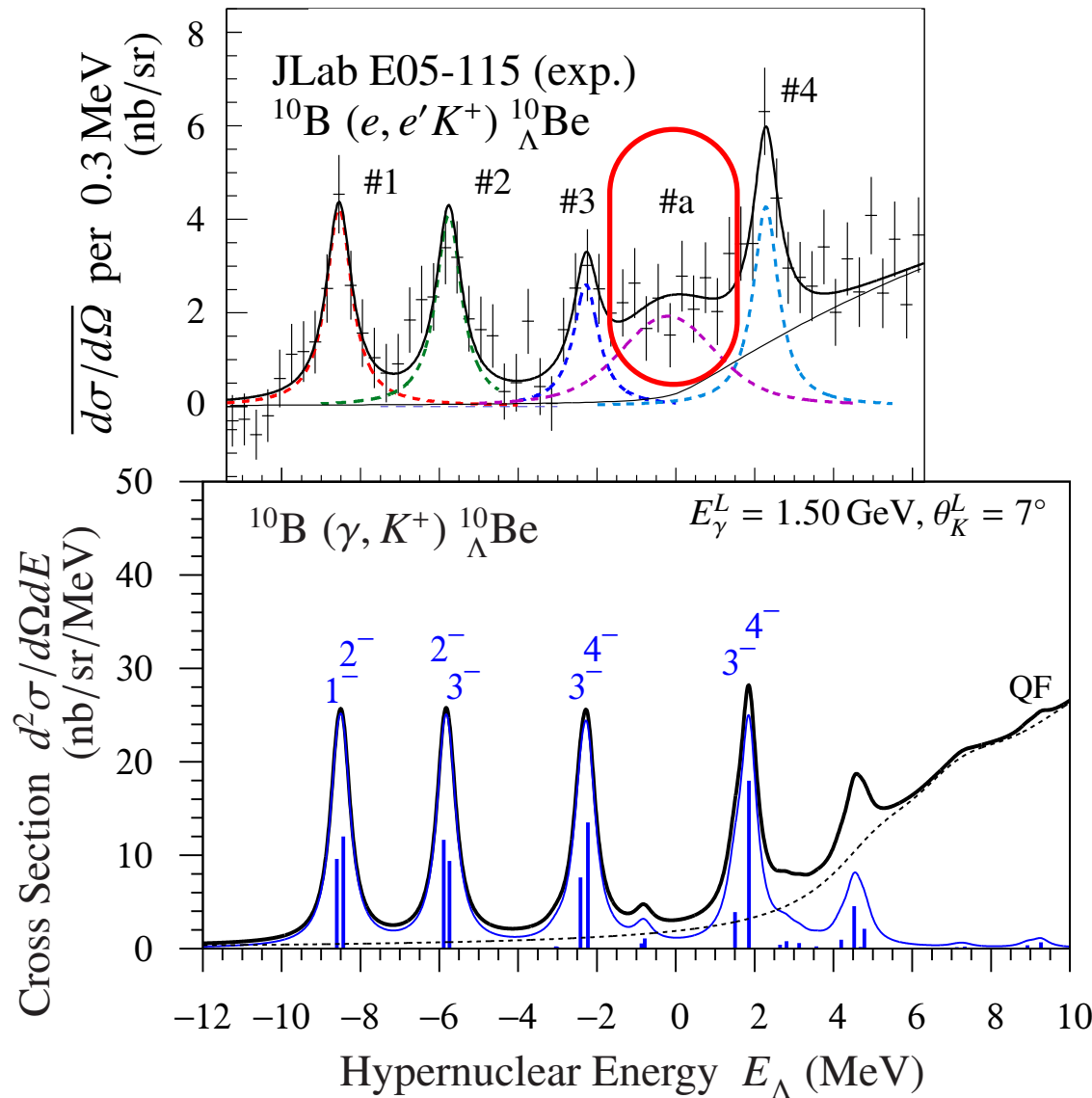
This experiment has confirmed the major peaks (#1, #2, #3, #4) predicted in DWIA by employing the  $\Lambda$  particle in  $s$ -orbit coupled with the nuclear core states confined within the  $p$ -shell configuration.

However, it is interesting to observe extra strengths at  $E_{\Lambda} = 0 \text{ MeV}$  excitation (a).



**The extension of the model space is necessary and interesting challenge in view of the present hypernuclear spectroscopy.**

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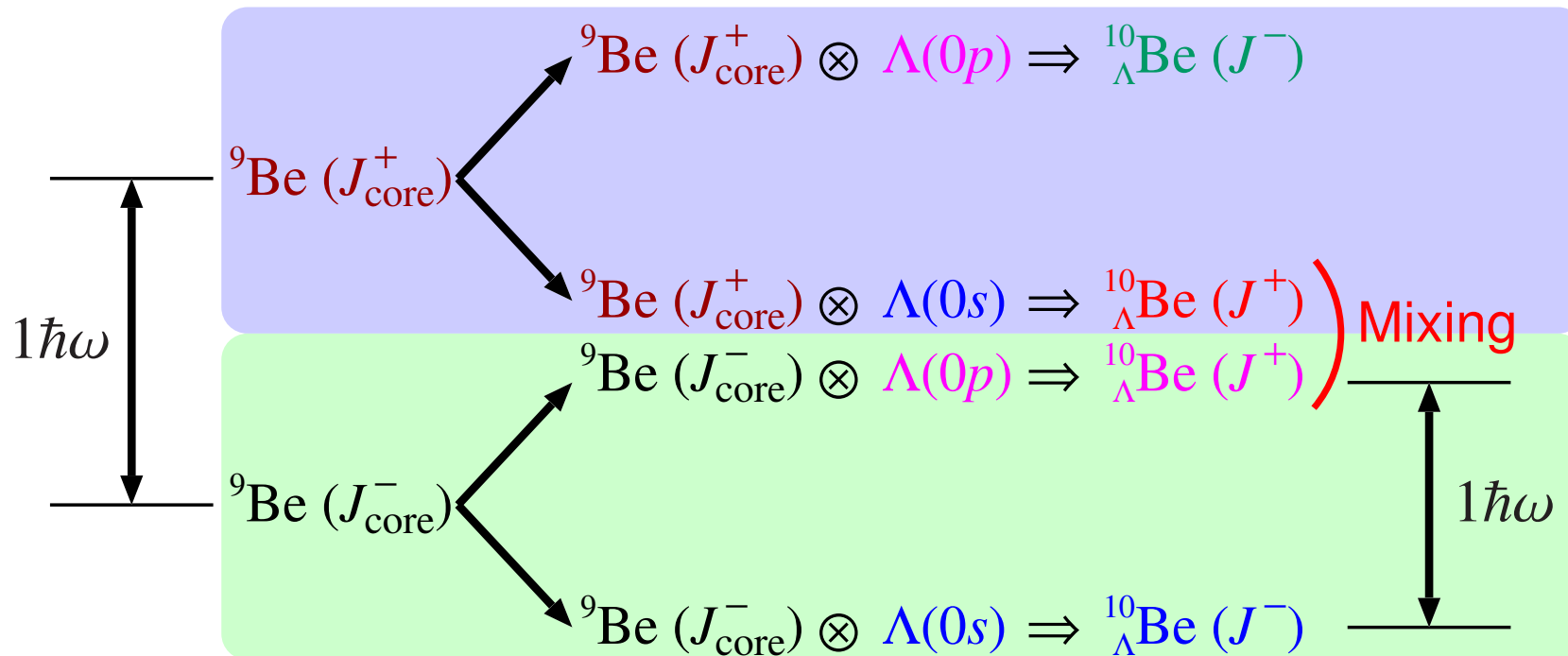
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## Framework of extended shell model ( ${}^{10}_{\Lambda}\text{Be}$ case)

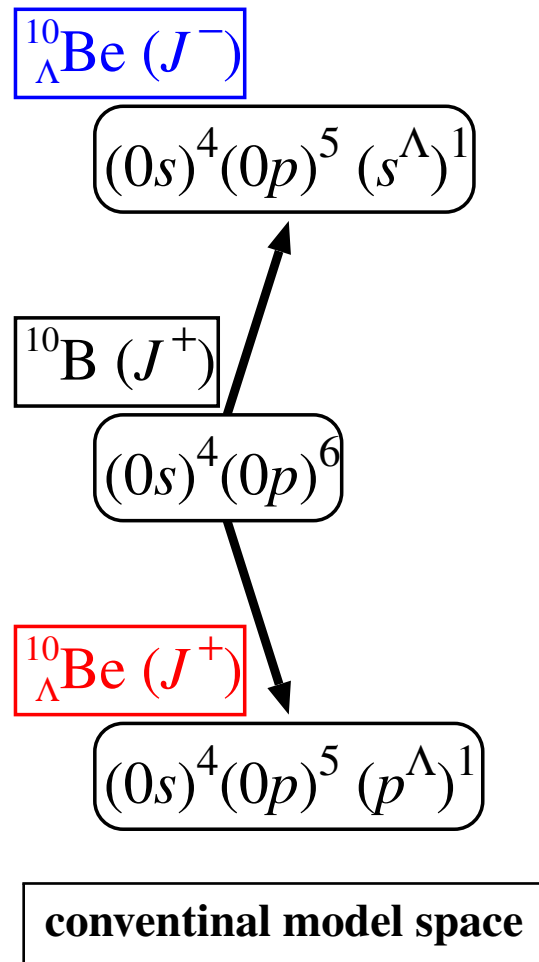


In the conventional shell model, only natural-parity nucleaer-core states ( $J_{\text{core}}^-$ ) are taken into account.  $\Lambda$  particle is in the  $0s$  orbit in  ${}^{10}_{\Lambda}\text{Be}(J^-)$ .

In  ${}^{10}_{\Lambda}\text{Be}(J^+)$ , the energy difference between  $\Lambda(0s)$  and  $\Lambda(0p)$  is  $1\hbar\omega$ , and the energy difference between  ${}^9\text{Be}(J_{\text{core}}^-)$  and  ${}^9\text{Be}(J_{\text{core}}^+)$  is  $1\hbar\omega$ .

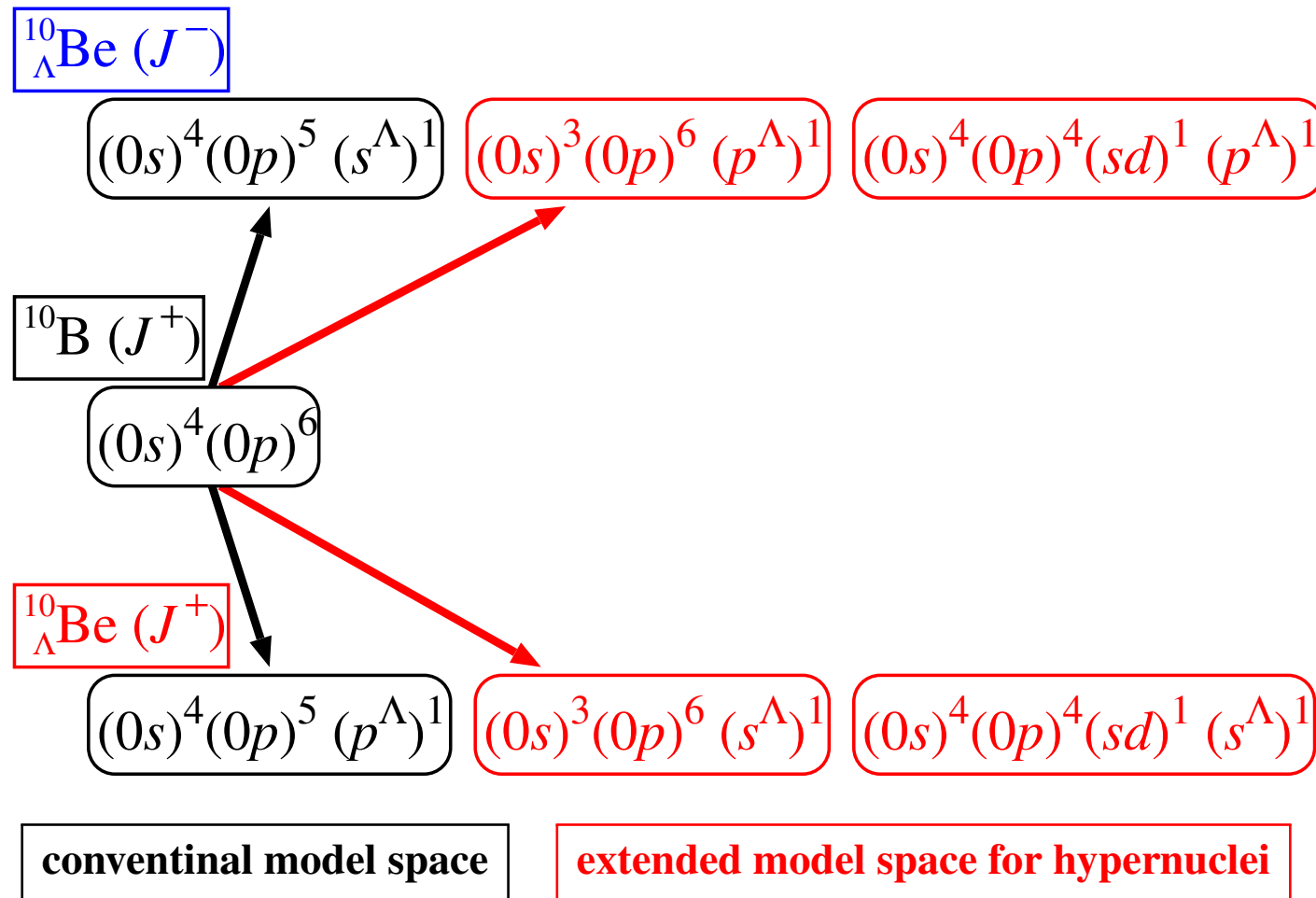
By  $\Lambda N$  interaction, natural-parity nucleaer-core configurations and unnatural-parity nucleaer-core configurations can be mixed.

## Extended model space for target nucleus $^{10}\text{B}$



Extension of model space for target nucleus  $^{10}\text{B}$  up to  $2p-2h$  ( $2\hbar\omega$ ) allows the  $^{10}_{\Lambda}\text{Be}$  production through various configurations.

## Extended model space for target nucleus $^{10}\text{B}$

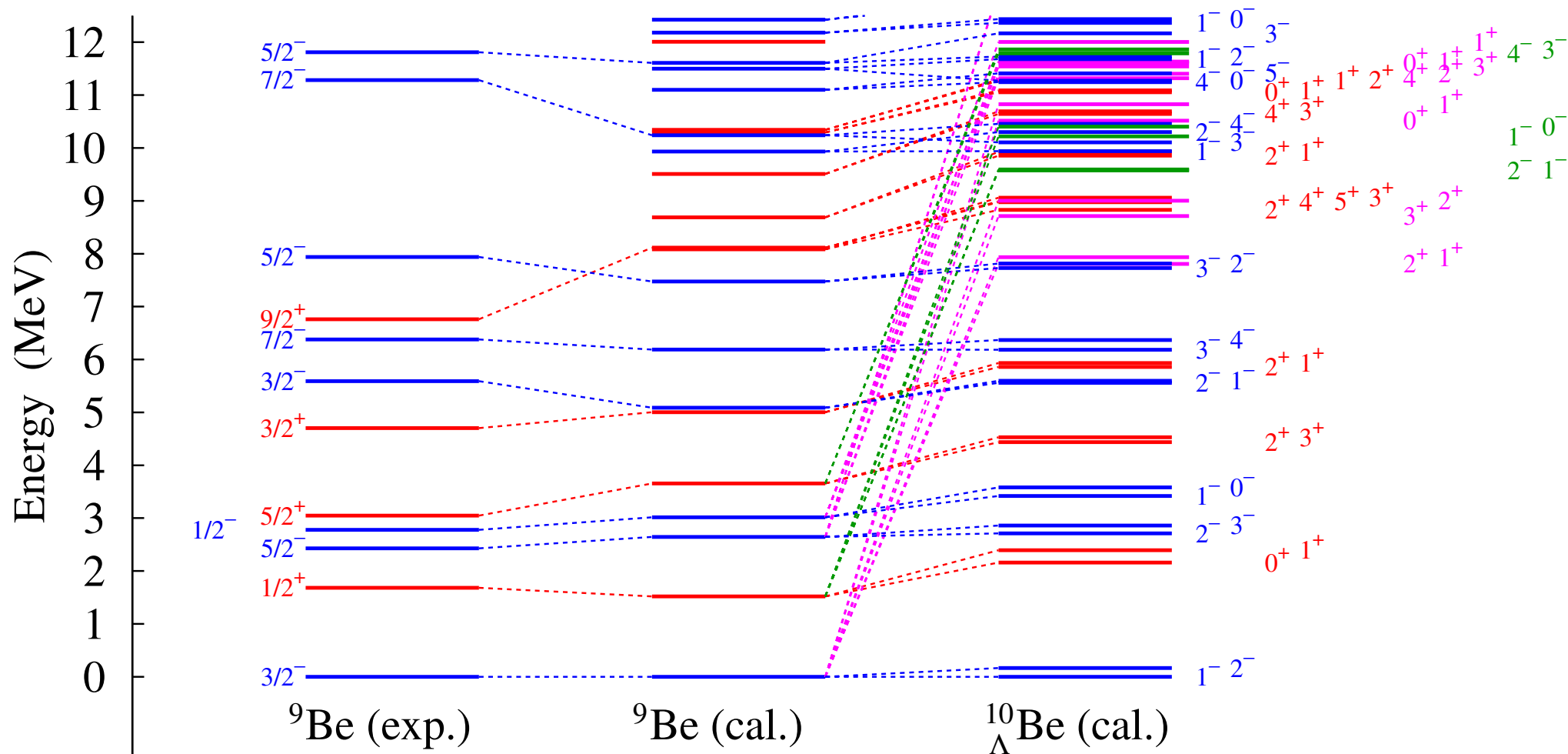


Extension of model space for target nucleus  $^{10}\text{B}$  up to  $2p-2h$  ( $2\hbar\omega$ ) allows the  $^{10}_{\Lambda}\text{Be}$  production through various configurations.





# Results : Energy levels of ${}^9\text{Be}$ and ${}^{10}_{\Lambda}\text{Be}$



dominant configurations

blue

$J^-; {}^9\text{Be}(J_{\text{core}}^-) \otimes \Lambda(0s)$

green

$J^+; {}^9\text{Be}(J_{\text{core}}^+) \otimes \Lambda(0p)$

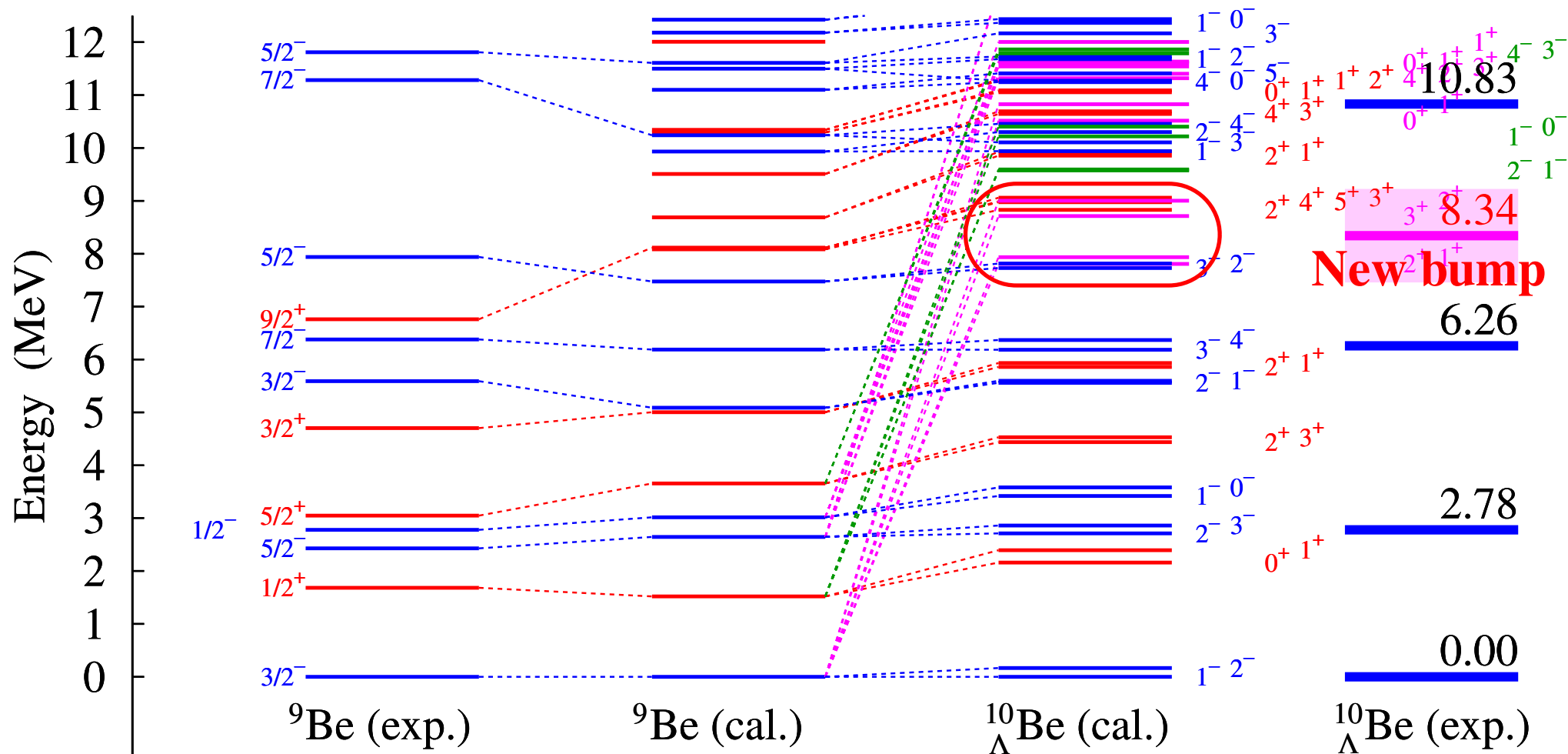
magenta

$J^+; {}^9\text{Be}(J_{\text{core}}^-) \otimes \Lambda(0p)$

red

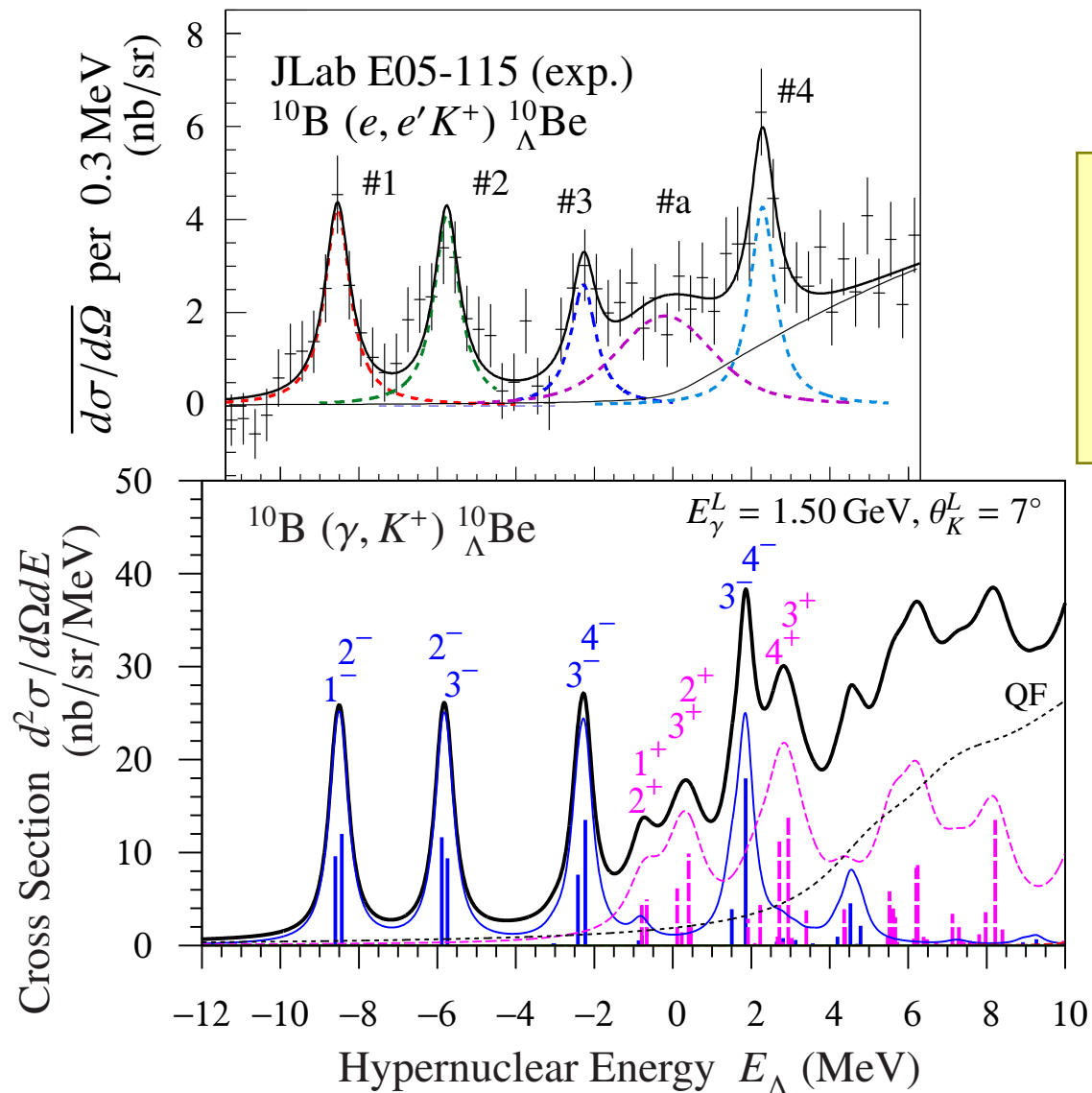
$J^+; {}^9\text{Be}(J_{\text{core}}^+) \otimes \Lambda(0s)$

# Results : Energy levels of ${}^9\text{Be}$ and ${}^{10}_{\Lambda}\text{Be}$



↑↑  
**T. Gogami *et al.*, PRC93, 034314 (2016)**

# Results : Cross sections of the $^{10}\text{B} (\gamma, K^+) ^{10}_{\Lambda}\text{Be}$ reaction (1)



**Recent experimental result**

**T. Gogami *et al.*, PRC93, 034314 (2016)**

**For hypernucleus  $^{10}_{\Lambda}\text{Be}$**

- (1)  $1p-1h$  ( $1\hbar\omega$ ) core excitation**
- (2) Configuration mixing by  $\Lambda N$  int. are taken into account**

**DWIA calculation by using Saclay-Lyon model A**

**Our new calculation reproduces the four major peaks (#1, #2, #3, #4).**

**Our new calculation explains the new bump (a) as a sum of cross sections of some  $J^+$  states.**

# Results : Cross sections of the $^{10}\text{B} (\gamma, K^+) \Lambda^{10}\text{Be}$ reaction (2)

 $E_\gamma = 1.5 \text{ GeV}$ 

EXP = T. Gogami et al, PRC93 (2016)

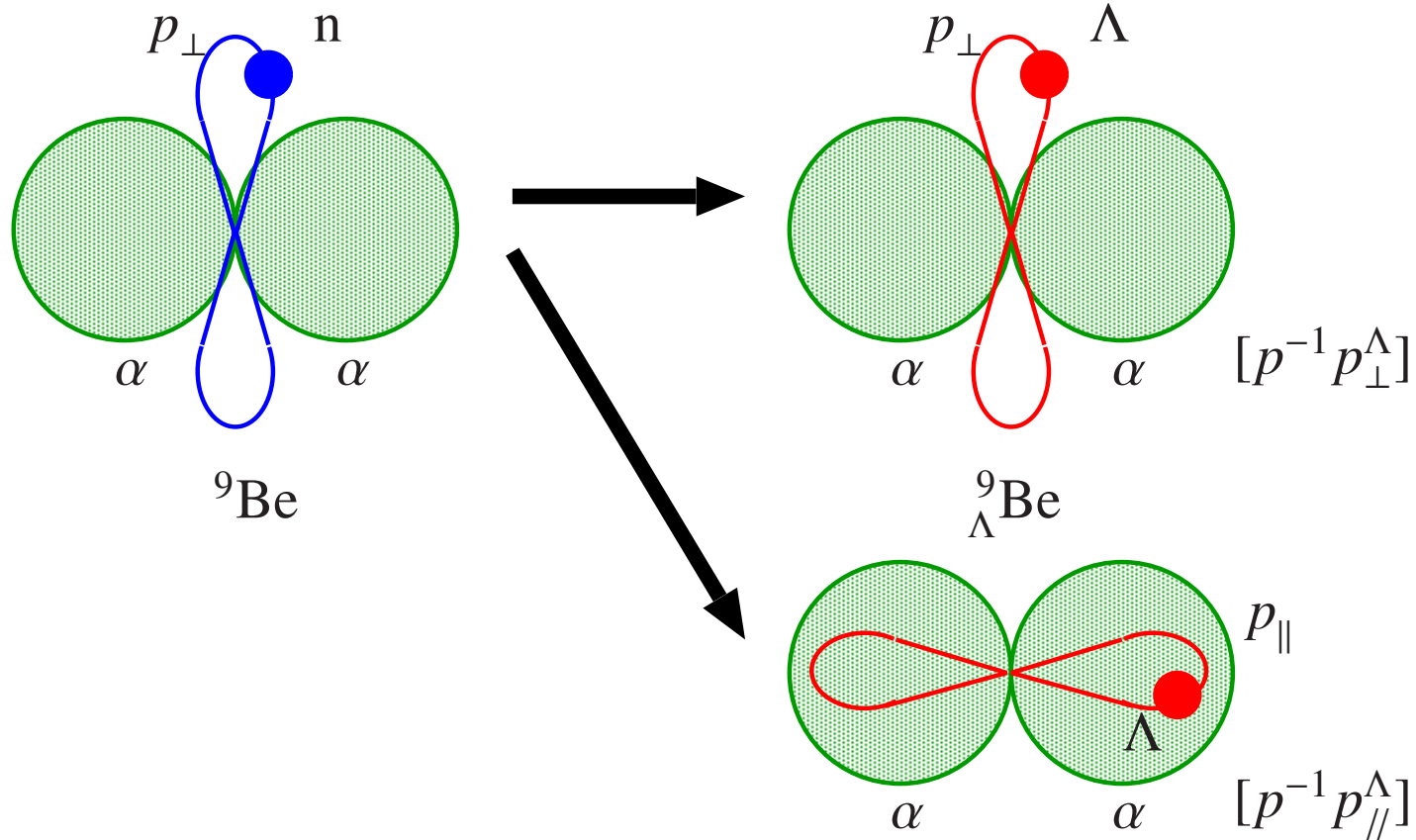
 $\theta = 7 \text{ deg}$ 

$^9\text{Be} (J_i)$			$\Lambda^{10}\text{Be} (J_k) \text{ CAL}$				EXP	Fit I			
$J_i$	$E_i (\text{exp})$ C2S	$E_i (\text{cal})$ C2S	$J_k$	$E_x$ [MeV]	$-B_\Lambda$ [MeV]	$d\sigma/d\Omega$ [nb/sr]	exp peak	$E_x$ [MeV]	$-B_\Lambda$ [MeV]	$d\sigma/d\Omega$ [nb/sr]	
3/2 <sup>-</sup>	0.000	0.000	1 <sup>-</sup>	0.000	-8.600	9.609	#1	0.00	-8.55±0.07	17.0±0.5	
	1.0(rel)	1.0(rel)	2 <sup>-</sup>	0.165	-8.435	12.008					
5/2 <sup>-</sup>	2.429	2.644	2 <sup>-</sup>	2.712	-5.888	11.654	#2	2.78±0.11	-5.76±0.09	16.5±0.5	
	0.958	1.020	3 <sup>-</sup>	2.860	-5.740	9.391					
7/2 <sup>-</sup>	6.380	6.189	3 <sup>-</sup>	6.183	-2.417	7.625	#3	6.26±0.16	-2.28±0.14	10.5±0.3	
	0.668	0.942	4 <sup>-</sup>	6.370	-2.230	13.505					
			2 <sup>+(3)</sup>	7.807	-0.793	4.495	#a	8.34±0.41	-0.20±0.40	23.2±0.7	
			1 <sup>+(3)</sup>	7.935	-0.665	4.968					
			3 <sup>+(2)</sup>	8.712	0.112	6.150					
			2 <sup>+(4)</sup>	8.828	0.228	1.431					
			2 <sup>+(5)</sup>	9.002	0.402	9.893					
			3 <sup>+(3)</sup>	9.059	0.459	2.434					
7/2 <sup>-</sup>	11.283	10.241	3 <sup>-</sup>	10.105	1.505	3.913	#4	10.83±0.10	2.28±0.07	17.2±0.5	
	1.299	1.355	4 <sup>-</sup>	10.455	1.855	17.985					
			1 <sup>+(5)</sup>	10.828	2.228	4.598	29.54 (51.44)				
			4 <sup>+(3)</sup>	11.318	2.718	11.185					
			3 <sup>+(5)</sup>	11.543	2.943	13.759					

## Results : Configurations of $J^+$ states corresponding to the new bump

$J_n^\pi(-B_\Lambda [\text{MeV}])$ XS [nb/sr]	$[J_{\text{core}}^\pi]j^\Lambda$	$[J_{\text{core}}^\pi]j^\Lambda$	$[J_{\text{core}}^\pi]j^\Lambda$
$2_3^+(-0.739)$ 4.49		$[3/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 82.5%	$[5/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 15.8%
$1_3^+(-0.665)$ 4.97		$[3/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 79.5%	$[5/2_1^-]p_{3/2}^\Lambda$ 17.9%
$2_4^+(0.228)$ 1.43	$[5/2_2^+]s_{1/2}^\Lambda$ 87.5%	$[3/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 9.4%	$[5/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 2.4%
$2_5^+(0.402)$ 9.89	$[5/2_2^+]s_{1/2}^\Lambda$ 11.3%	$[3/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 70.9%	$[5/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 10.8%
$3_2^+(0.112)$ 6.15	$[5/2_2^+]s_{1/2}^\Lambda$ 31.6%	$[3/2_1^-]p_{3/2}^\Lambda$ 55.4%	$[5/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 9.7%
$3_3^+(0.459)$ 2.43	$[5/2_2^+]s_{1/2}^\Lambda$ 67.5%	$[3/2_1^-]p_{3/2}^\Lambda$ 27.1%	$[5/2_1^-](p_{3/2}p_{1/2})^\Lambda$ 2.7%

$[p^{-1}p_{\perp}^{\Lambda}]$  and  $[p^{-1}p_{\parallel}^{\Lambda}]$  states of  ${}^9_{\Lambda}\text{Be}$



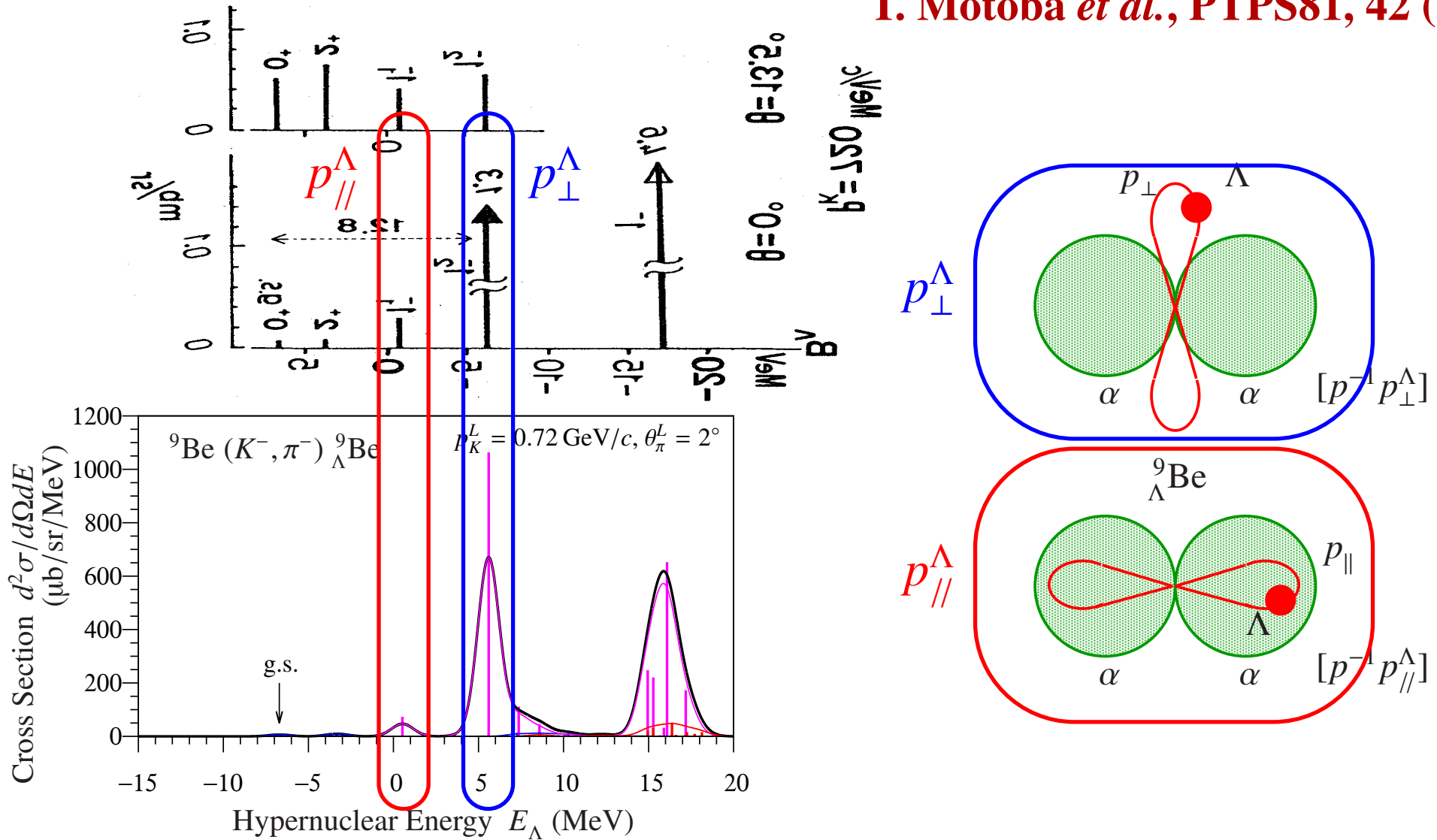
In  ${}^9_{\Lambda}\text{Be}$ , it is well known that the  $p_{\Lambda}$ -state splits into two orbital states expressed by  $p_{\perp}$  and  $p_{\parallel}$ , which is due to the strong coupling with nuclear core deformation having the  $\alpha$ - $\alpha$  structure.

**T. Motoba *et al.*, PTPS81, 42 (1985)**

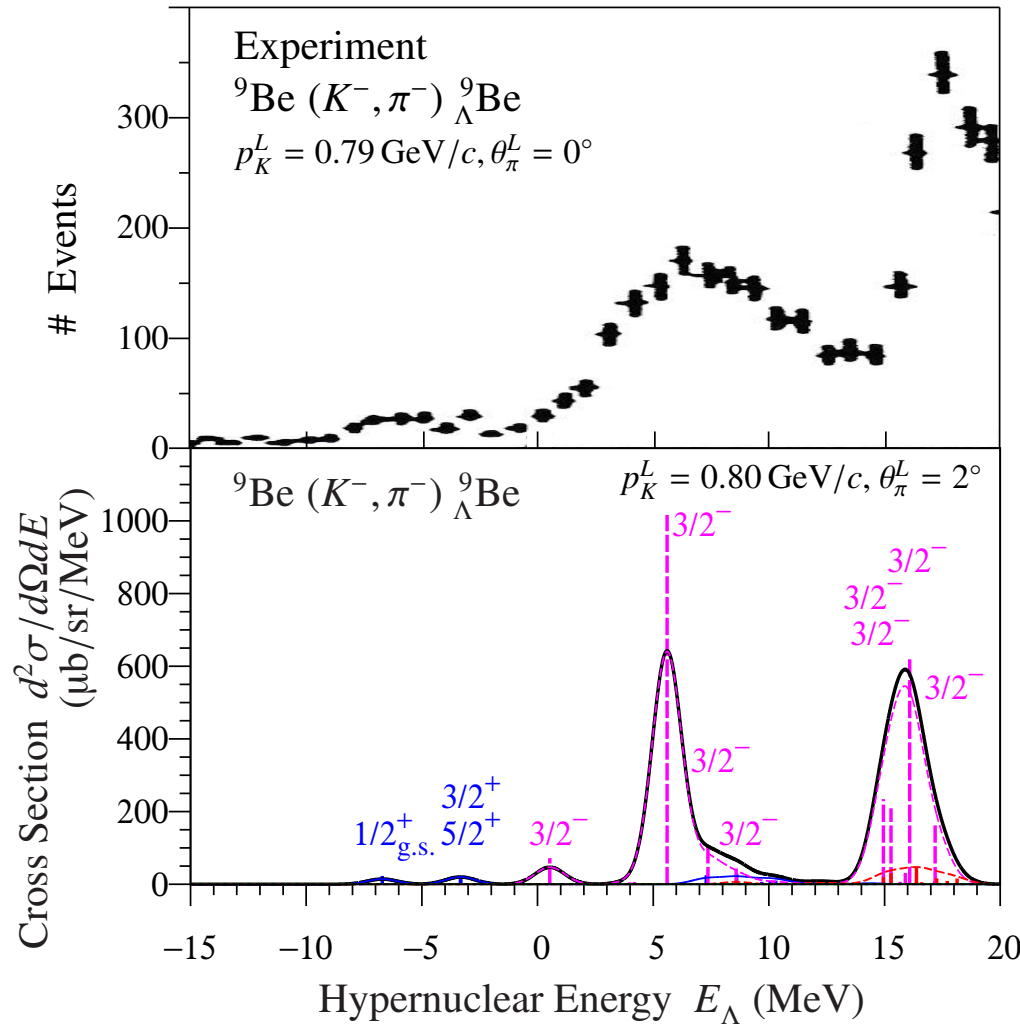
**R. H. Dalitz, A. Gal, PRL36, 362 (1976); AP131, 314 (1981)**

Results : Comparison to the cluster model – Cross section –

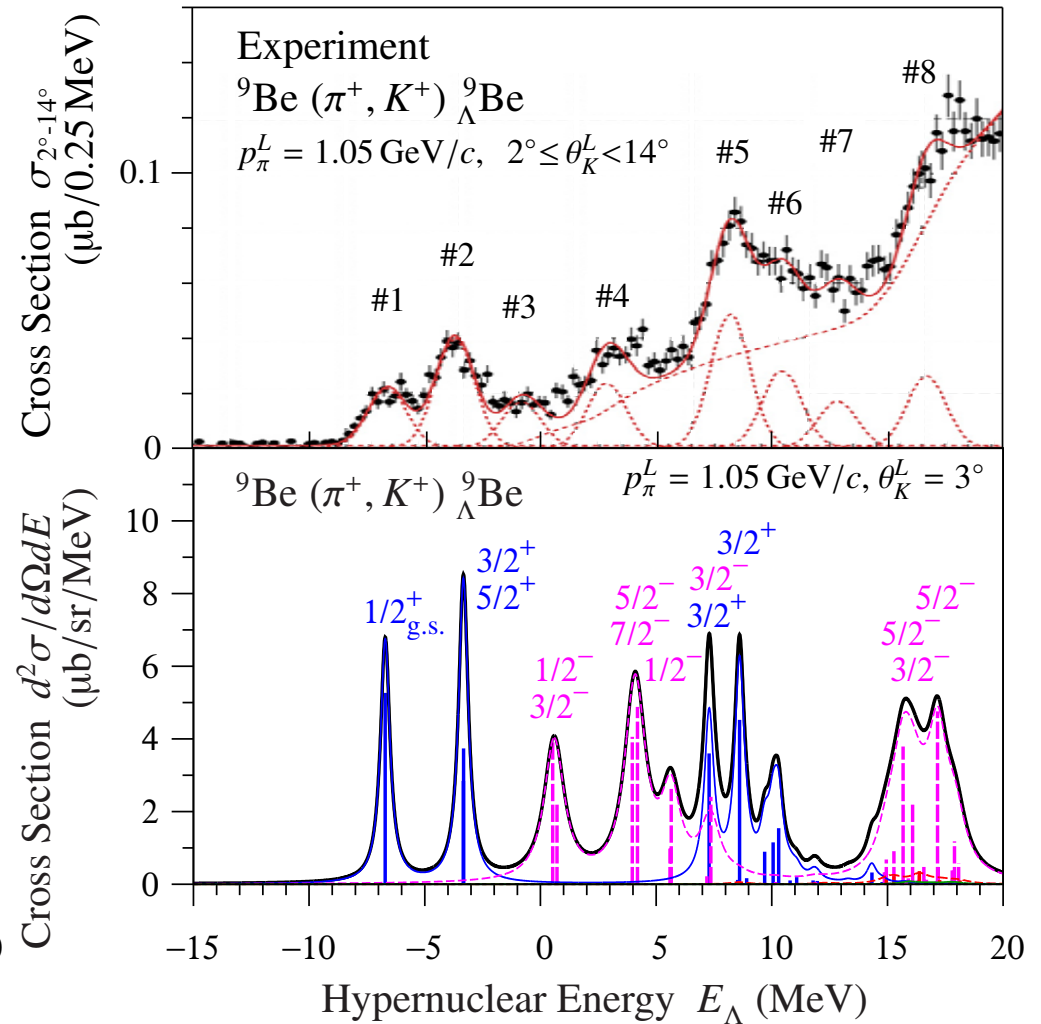
T. Motoba *et al.*, PTPS81, 42 (1985)



# Results : Cross sections of ${}^9\text{Be} (K^-, \pi^-) {}^9_\Lambda\text{Be}$ and ${}^9\text{Be} (\pi^+, K^+) {}^9_\Lambda\text{Be}$ reactions



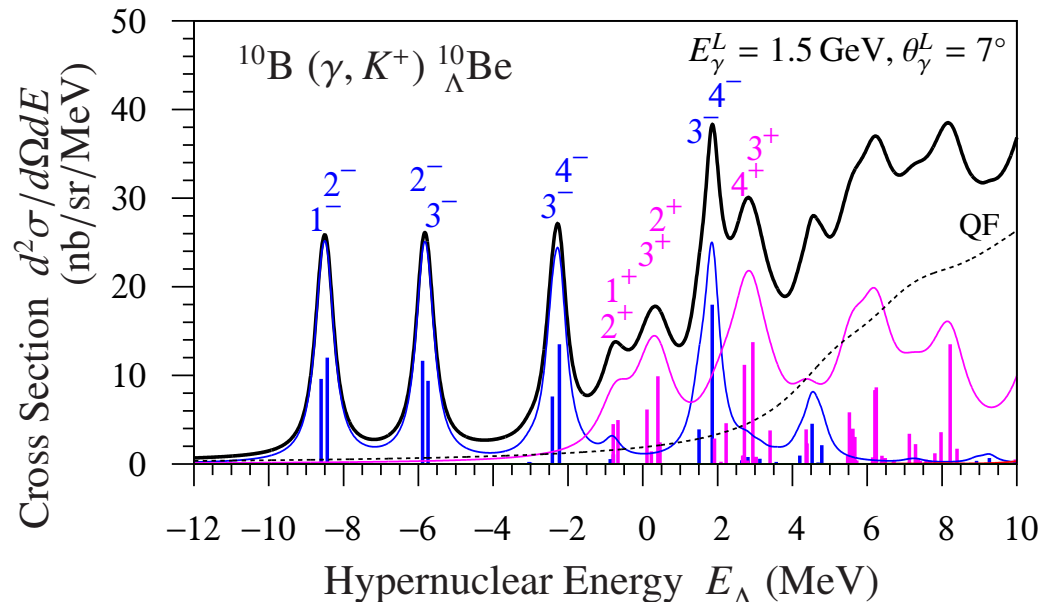
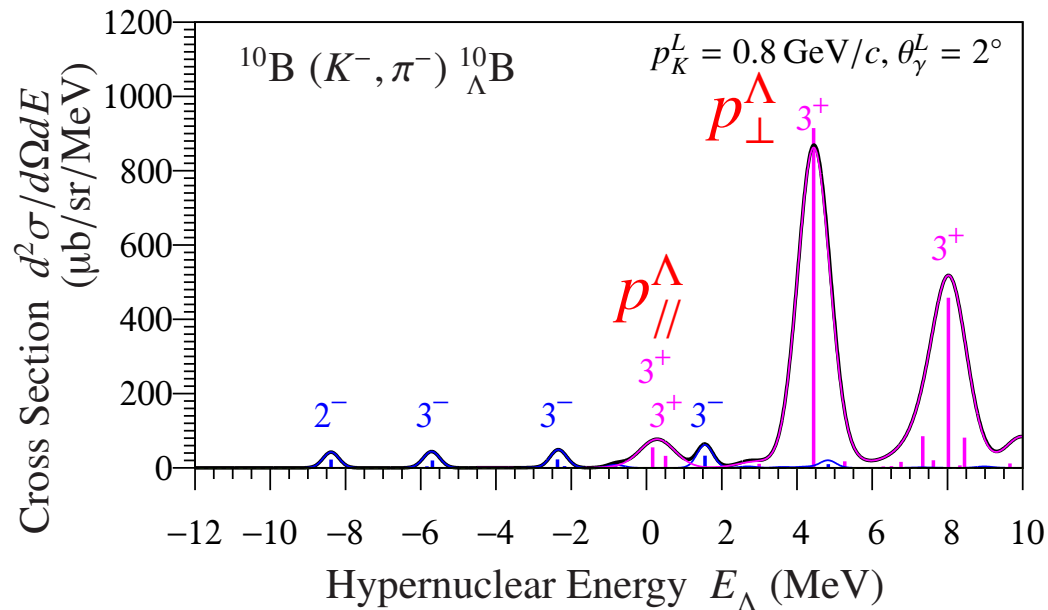
Exp. from **R. Bertini *et al.*,  
 NPA368, 365 (1981)**



Exp. from **O. Hashimoto and H. Tamura,  
 PPNP57, 564 (2006)**



## Results : Cross sections of the $^{10}\text{B} (K^-, \pi^-) ^{10}_{\Lambda}\text{B}$ reaction (1)



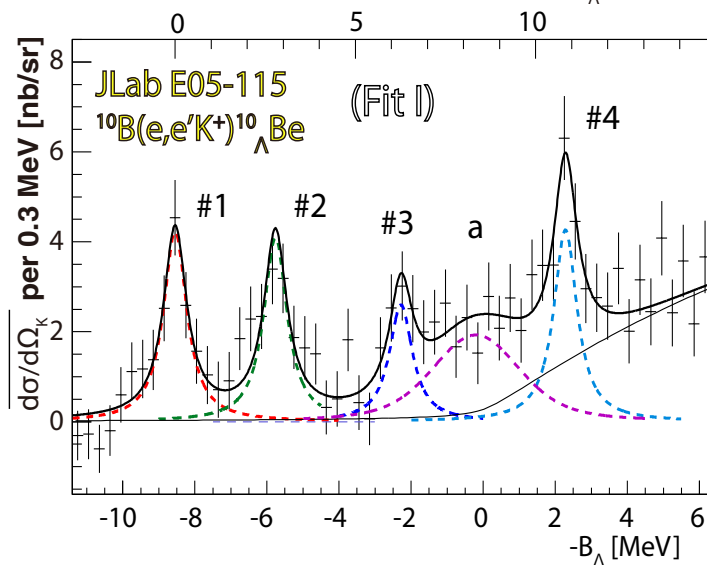
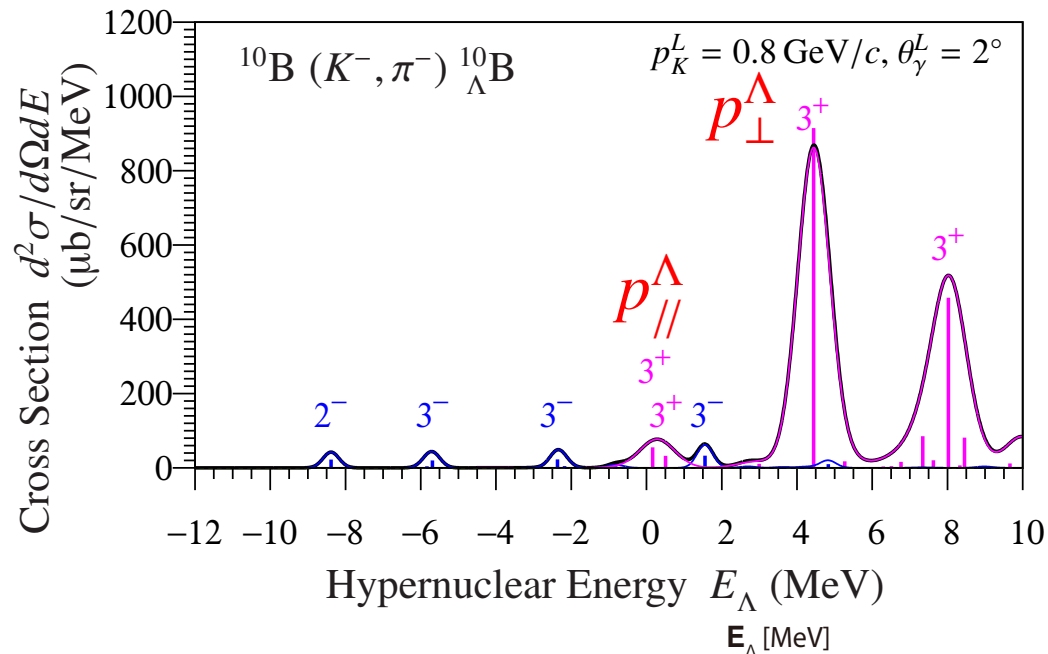
In the  $(K^-, \pi^-)$  reaction, the large peak at  $E_{\Lambda} = 4.4$  MeV is a  **$p$ -substitutional state via the  $p_{3/2}^N \rightarrow p_{3/2}^{\Lambda}$** , which is strongly excited by recoilless reaction.

The small peak at  $E_{\Lambda} = 0$  MeV corresponds to **the new bump** and is explained as **a mixture of  $s^{\Lambda}$  and  $p^{\Lambda}$  states**.

The large peak at  $E_{\Lambda} = 4.4$  MeV in  $^{10}_{\Lambda}\text{Be}$  corresponds to the  **$[p^{-1} p_{\perp}^{\Lambda}]$  state in  $^9_{\Lambda}\text{Be}$  ( $^9\text{Be}$  analog state)**.

The small peak at  $E_{\Lambda} = 0$  MeV in  $^{10}_{\Lambda}\text{Be}$  corresponds to the  **$[p^{-1} p_{\parallel}^{\Lambda}]$  state in  $^9_{\Lambda}\text{Be}$** .

# Results : Cross sections of the $^{10}\text{B} (K^-, \pi^-) ^{10}_{\Lambda}\text{B}$ reaction (2)



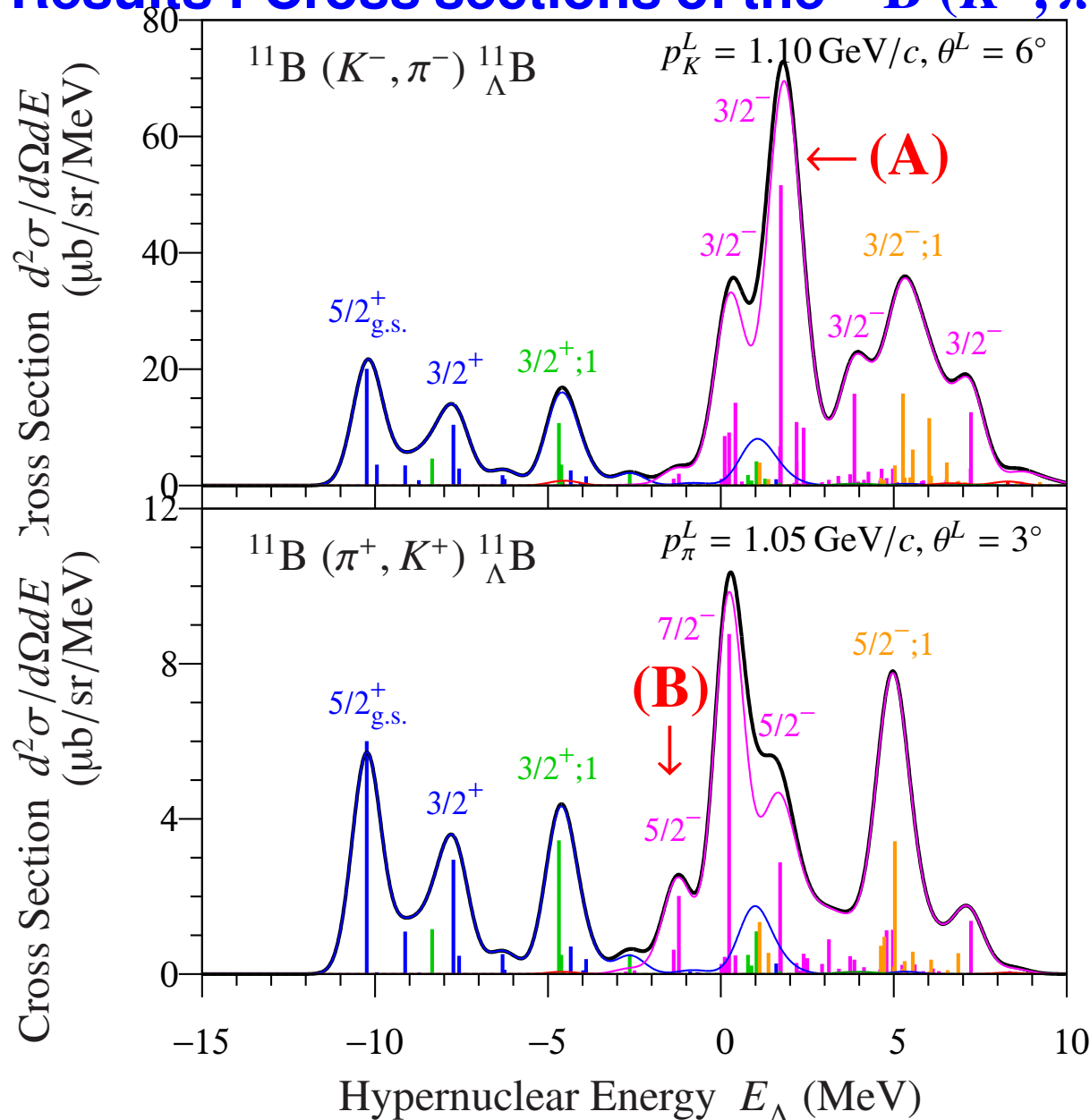
**CONCLUDE:**

**$\alpha\alpha$ -like core deformation causes splitting of  $p^\Lambda$ -states, then low-energy  $p^\Lambda_{//}$  can mix with  $s^\Lambda$ -states.**

$$[{}^9\text{Be}(J^-) \times \Lambda(p_{//})] + [{}^9\text{Be}(J^+) \times \Lambda(s)]$$

**These parity-mixed wave functions at  $E_\Lambda = 0$  MeV can explain the extra peak #a.**

# Results : Cross sections of the $^{11}\text{B} (K^-, \pi^-)$ and $^{11}\text{B} (\pi^+, K^+)$ reactions



FWHM = 1.0 MeV

- $T=0, J^+; J_{\text{core}}^+ \otimes s_\Lambda$
- - -  $T=1, J^+; J_{\text{core}}^+ \otimes s_\Lambda$
- · - ·  $T=0, J^-; J_{\text{core}}^+ \otimes p_\Lambda$
- - -  $T=1, J^-; J_{\text{core}}^+ \otimes p_\Lambda$

**(A)  $3/2^-$**

- $^{10}\text{B}(3_{\text{g.s.}}^+) \otimes p_{3/2}^\Lambda$  51.4%
- $^{10}\text{B}(1_2^+) \otimes p_{1/2}^\Lambda$  23.0%
- $^{10}\text{B}(3_2^+) \otimes p_{3/2}^\Lambda$  9.4%

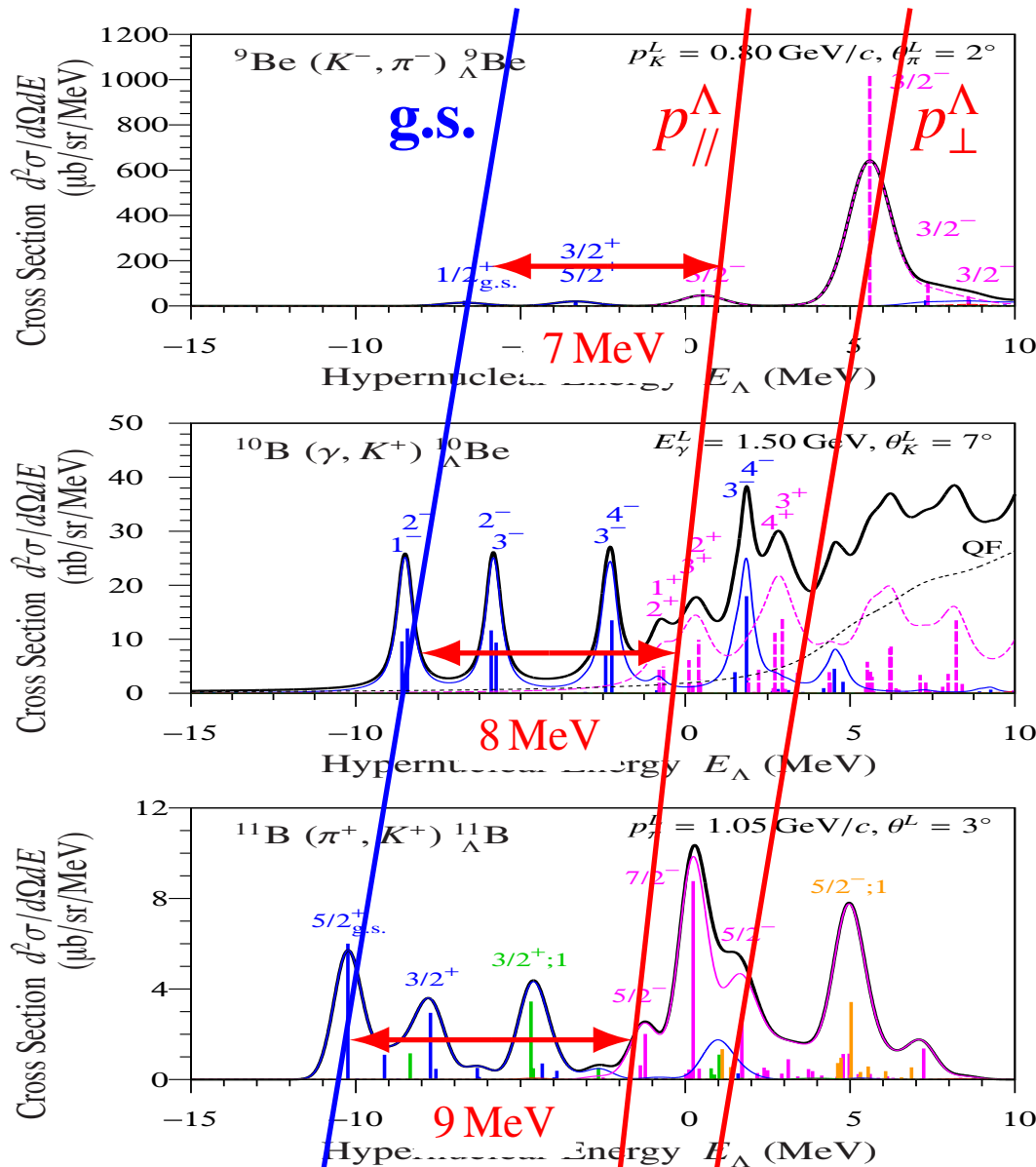
**→  $p_\perp$  state (substitutional)**

**(B)  $5/2^-$**

- $^{10}\text{B}(3_{\text{g.s.}}^+) \otimes p_{3/2}^\Lambda$  56.1%
- $^{10}\text{B}(3_{\text{g.s.}}^+) \otimes p_{1/2}^\Lambda$  35.7%

**→  $p_\parallel$  state**

# Results : Energy of $p_{\parallel}$ -state



The  $p^\Lambda$ -state splits into  $p_\perp$ - and  $p_\parallel$ -states due to the strong coupling with nuclear core deformation.

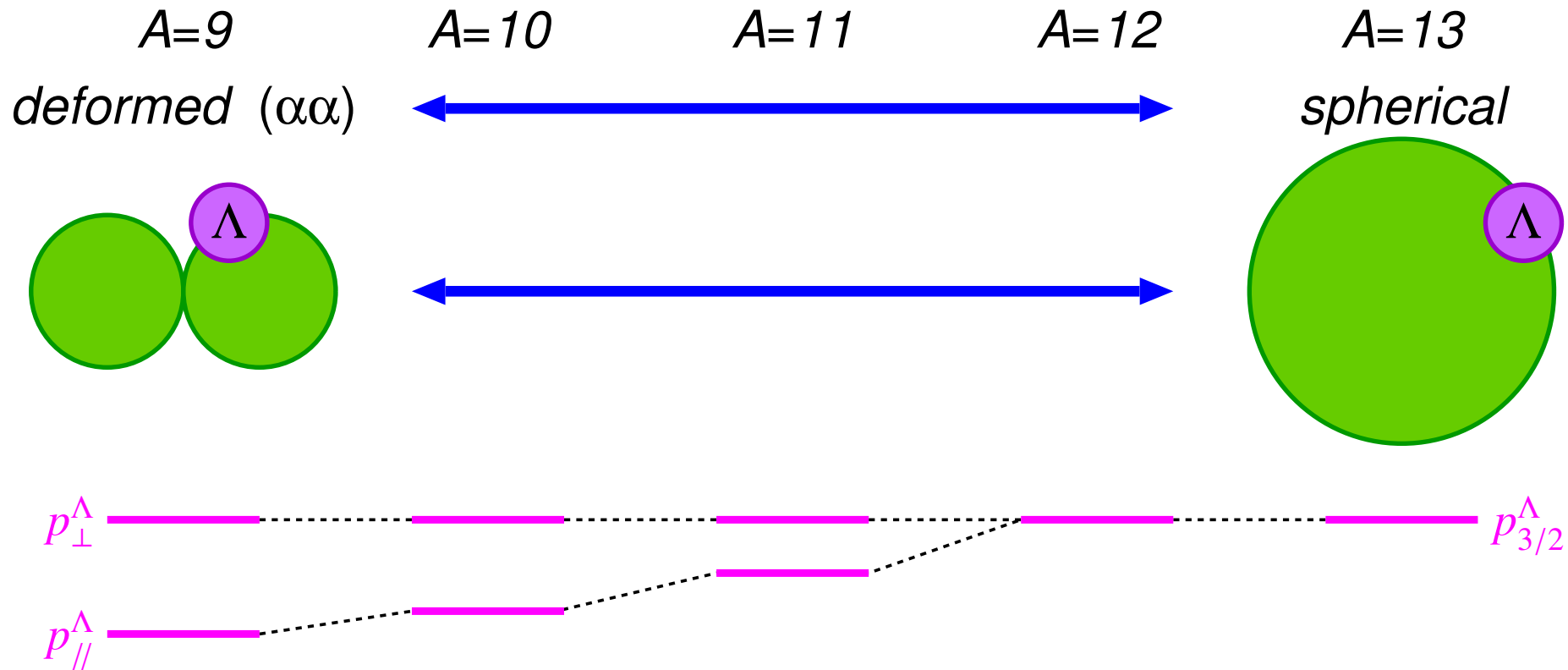
In  ${}^9_\Lambda\text{Be}$ , the energy of  $p_{\parallel}^\Lambda$ -state comes down to  $E_x \approx 7 \text{ MeV}$  from the  $\Lambda$  single-particle energy difference  $\varepsilon_p^\Lambda - \varepsilon_s^\Lambda \approx 11 \text{ MeV}$ .

The bump at  $E_x \approx 8 \text{ MeV}$  in the cross sections of  ${}^{10}_\Lambda\text{Be}$  corresponds to the  $p_{\parallel}^\Lambda$ -state.

In the cross sections of  ${}^{11}_\Lambda\text{B}$ , the small  $5/2^-$  peak at  $E_x \approx 9 \text{ MeV}$  corresponds to the  $p_{\parallel}^\Lambda$ -state.

The energy splitting between  $p_\perp$ - and  $p_\parallel$ -states in  ${}^{11}_\Lambda\text{B}$  is smaller than that in  ${}^9_\Lambda\text{Be}$ , which is due to the small deformation of the nuclear core in  ${}^{11}_\Lambda\text{B}$ .

## $p^\Lambda$ state in the deformed nuclear core



In the spherical nuclear core,  $p^\Lambda$ -state does not split into  $p_\parallel^\Lambda$  and  $p_\perp^\Lambda$ .

The new type wave function should appear in  ${}^{9,10}_\Lambda\text{Be}$  and  ${}^{10,11}_\Lambda\text{B}$  due to the core deformation, but “not” in spherical systems without enough deformation.

## Summary

**We have calculated the DWIA production cross sections for  $p$ -shell hypernuclei by using the extended shell model.**

- **Strong coupling between  $p$ -state  $\Lambda$  and core deformation is realized in  ${}^{9,10}_{\Lambda}\text{Be}$  and  ${}^{10,11}_{\Lambda}\text{B}$ .**
- **In these nuclei,  $p^{\Lambda}$ -state splits into  $p^{\Lambda}_{\parallel}$  and  $p^{\Lambda}_{\perp}$ .**
- **In  ${}^{10}_{\Lambda}\text{Be}$ , the lower  $p^{\Lambda}_{\parallel}$  comes down in energy and  $[{}^9\text{Be}(J^-) \times \Lambda(p_{\parallel})]$  couples easily with  $[{}^9\text{Be}(J^+) \times \Lambda(s)]$ .**
- **Such new type wave functions should appear in  ${}^{9,10}_{\Lambda}\text{Be}$  and  ${}^{10,11}_{\Lambda}\text{B}$  due to the core deformation.**

**The finding of peak #a in  ${}^{10}\text{B} (e, e' K^+) {}^{10}_{\Lambda}\text{Be}$  is a novel evidence for genuine hypernuclear wave function with parity-mixing realized in “deformed” hypernuclei.**