SU(3) Constraints on Hypernuclear Energy Density Functionals

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Agenda

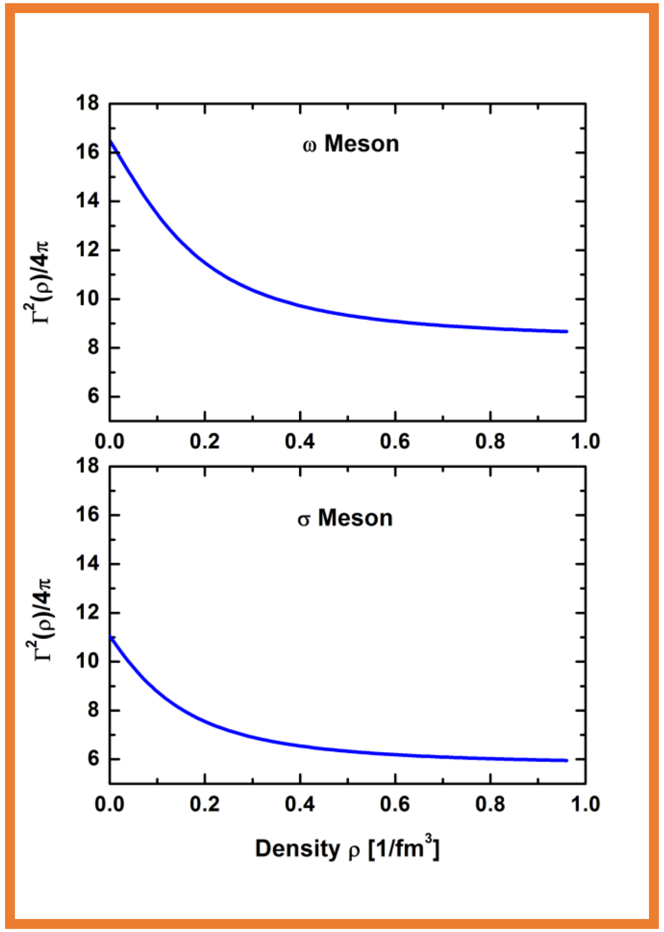
- **Dirac-Brueckner vertex functionals and covariant EDF approach**
- **Hyperon mean-field coupling constants from NN-DBHF vertices**
- **Nucleon and hyperon covariant mean-fields in asymmetric nuclear matter**
- **Λ-Σ mixing induced by the nuclear isovector mean-field**
- **Summary**

Theoretical Background:

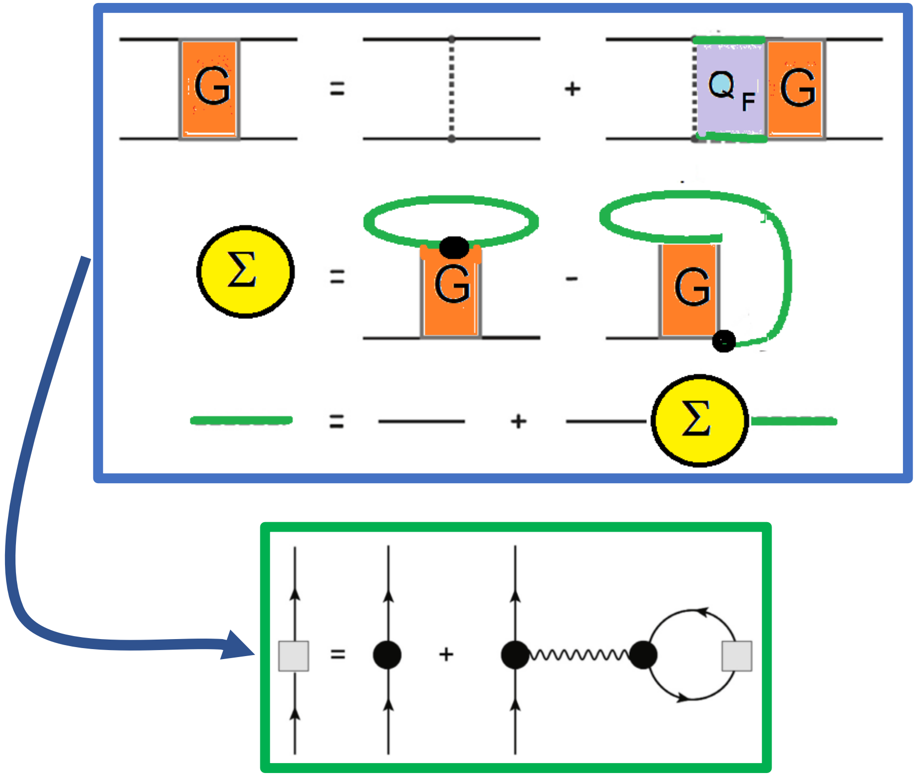
H. Lenske, M. Dhar, ***Lect.Notes Phys. 948 (2018) 161***

H. Lenske, M. Dhar, Th. Gaitanos, Xu Cao, ***Prog.Part.Nucl.Phys.*** **98 (2018) 119**

Nuclear Matter Mean-Field NN-Vertices from GI-DBHF EDF

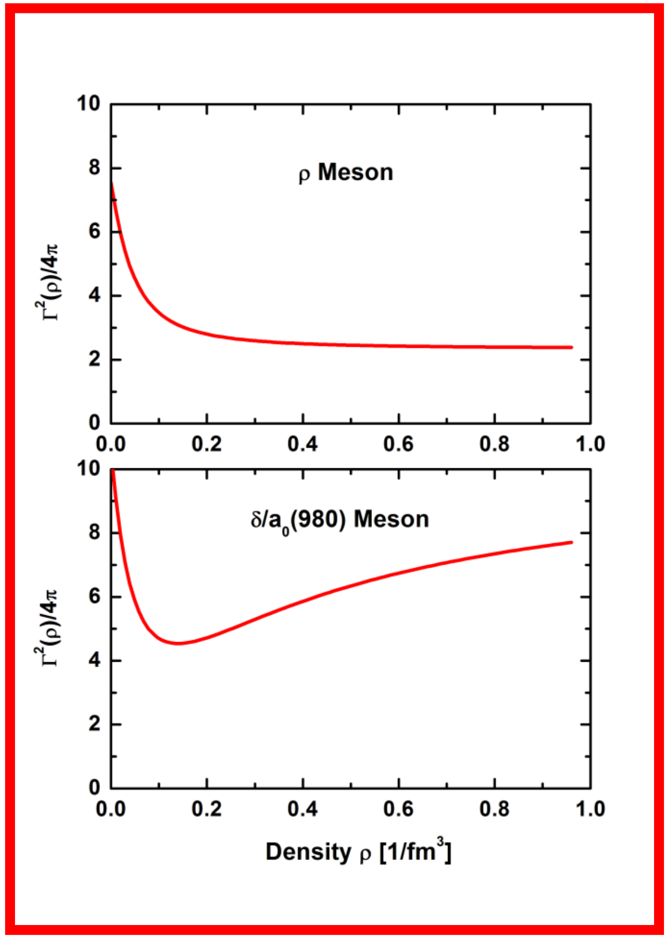


Isoscalar Vertices



$$\Gamma_{\mathcal{B}\mathcal{B}'a}(q_s, k_F) \simeq \frac{1}{1 - \int dq' V_a \mathcal{G}^* Q_F}\Big|_{\mathcal{B}\mathcal{B}'} g_{\mathcal{B}\mathcal{B}'a}. \quad V_a$$

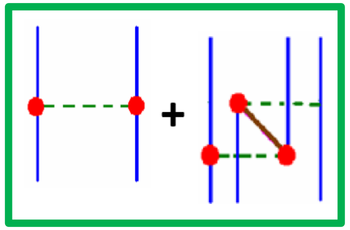
(a=σ,ω,ρ...)

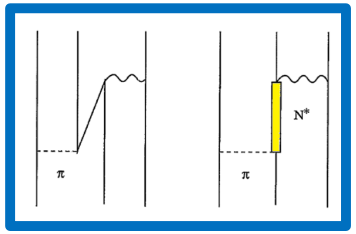


Isovector Vertices

GI-DHBF Theory and EDF-Approaches of that Kind

- **Fully microscopic *ab initio* approach based on free-space covariant NN interactions**
- **Successful as a covariant nuclear EDF for stable and exotic nuclei up to neutron stars**





Saturation Properties of the Nuclear EoS - NN only:

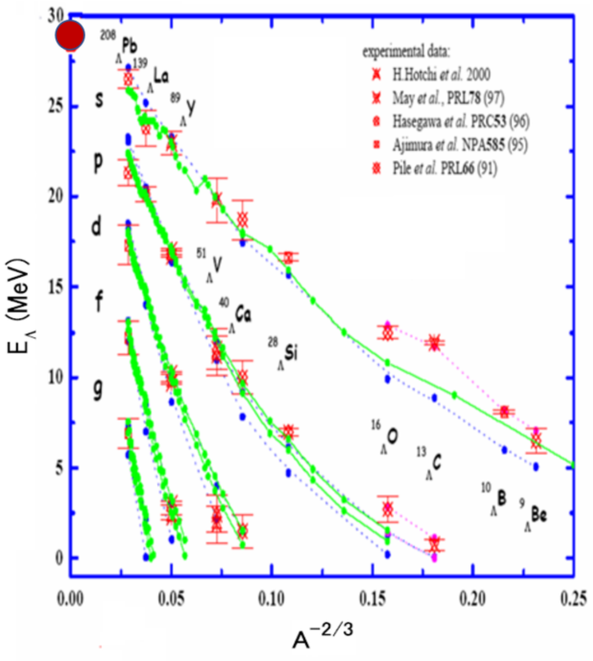
Z/A	rhoSAT	E/A	Compr	Esym	Lsym	Ksym
0.5	0.1800	-15.6033	281.9452	31.1540	88.6270	201.3971
0.4	0.1736	-14.4011	263.0267	30.0276	82.8382	165.8180

Saturation Properties of the Nuclear EoS - NN+NNN:

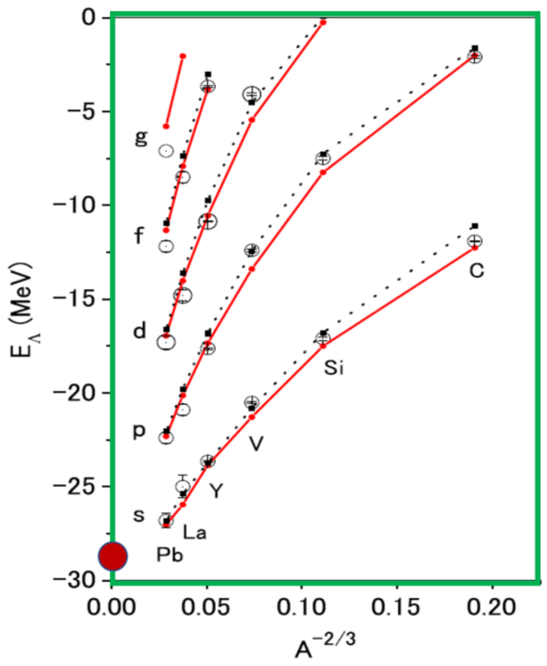
Z/A	rhoSAT	E/A	Compr	Esym	Lsym	Ksym
0.5	0.1600	-16.0000	283.1357	32.0000	90.0000	133.5014
0.4	0.1541	-14.7567	262.7607	31.0696	85.8948	115.0590

GI-DBHF results for symmetric and asymmetric nuclear matter without and with NNN interactions

GI-DHBF Theory and Hypernuclei



Covariant GI-GBHF, H.L. et al. PPNP 98 (2018) 119



ESC-08 G-matrix, Yamamoto et al., PRC 90 (2014) 045805

- **Hypernuclear physics:**
  - **Λ-separation energies are reproduced within exp. error bars**
  - **Empirical „scaling approach“ for hyperon interactions**
  - **Lack of predictive power for Σ and Ξ hyperons**
  - **Hampered by lack of hyperon-nucleon and hyperon-nucleus scattering data**

SU(3) EDF-Approach to Meson-Baryon Octet Interactions

$$\mathcal{L}^{\mathcal{P}}_{int} = -\sqrt{2}\left\{g_D\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_D + g_F\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8\right]_F\right\} - g_S\frac{1}{\sqrt{3}}\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1\right]_S$$

***P*=Pseudoscalar, Vector, and Scalar Meson Exchange**

anti−symmetric $[\bar{B}, B] = \bar{B}B - B\bar{B}$ and symmetric $\{\bar{B}, B\} = \bar{B}B + B\bar{B}$ configurations

$$\left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_D = \mathrm{Tr}\left(\left\{\overline{\mathcal{B}}, \mathcal{B}\right\}\mathcal{P}_8\right) \;,\; \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_F = \mathrm{Tr}\left(\left[\overline{\mathcal{B}}, \mathcal{B}\right]\mathcal{P}_8\right) \;,\; \left[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}\right]_S = \mathrm{Tr}(\overline{\mathcal{B}}\mathcal{B})\mathrm{Tr}(\mathcal{P}_1)$$

SU(3) Octet-Physics:
3 sets of 3 fundamental coupling constants fix the 48 BB'm vertices

Exploit SU(3) relations to construct an Octet BB-EDF from NN-EDF

BBM Vertices under Singlet-Octet Meson Mixing

Guiding Principles

- Interactions inherit SU(3) symmetry by fit to scattering data
- Free space and in-medium Bethe-Salpeter equations conserve the fundamental symmetries
- So do the vertex equations

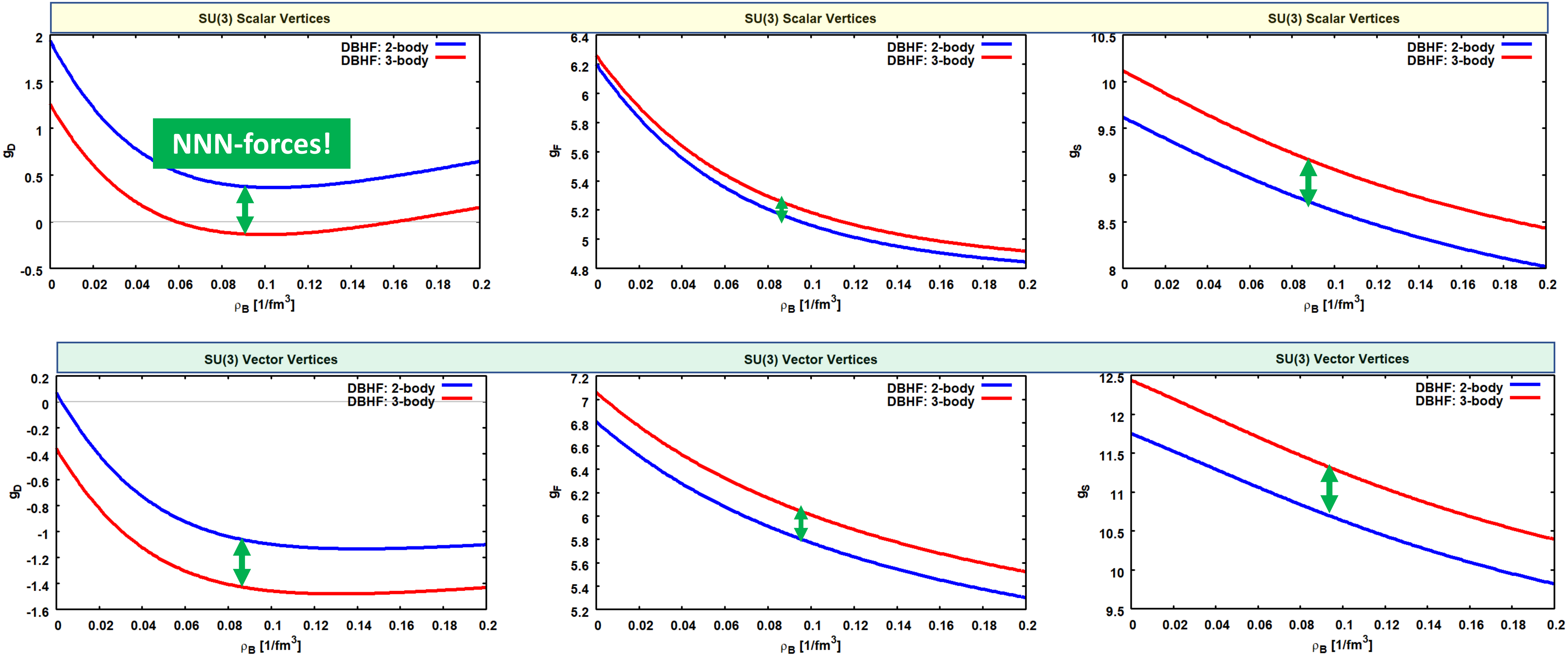
Vertex	Coupling constant
$NN\omega$	$g_N^{\omega}=g_S\cos(\theta)+\sqrt{\frac{3}{2}}g_F\sin(\theta)-\frac{1}{\sqrt{6}}g_D\sin(\theta)$
$NN\phi$	$g_N^{\phi}=g_S\sin(\theta)-\sqrt{\frac{3}{2}}g_F\cos(\theta)+\frac{1}{\sqrt{6}}g_D\cos(\theta)$
$NN\rho$	$g_N^{\rho}=\sqrt{2}(g_F+g_D)$
$\Lambda\Lambda\omega$	$g_\Lambda^{\omega}=g_S\cos(\theta)-\sqrt{\frac{2}{3}}g_D\sin(\theta)$
$\Lambda\Lambda\phi$	$g_\Lambda^{\phi}=g_S\sin(\theta)+\sqrt{\frac{2}{3}}g_D\cos(\theta)$
$\Sigma\Sigma\omega$	$g_\Sigma^{\omega}=g_S\cos(\theta)+\sqrt{\frac{2}{3}}g_D\sin(\theta)$
$\Sigma\Sigma\phi$	$g_\Sigma^{\phi}=g_S\sin(\theta)-\sqrt{\frac{2}{3}}g_D\cos(\theta)$
$\Sigma\Sigma\rho$	$g_\Sigma^{\rho}=\sqrt{2}g_F$
$\Lambda\Sigma\rho$	$g_{\Lambda\Sigma}^{\rho}=\sqrt{\frac{2}{3}}g_D$
$\Xi\Xi\omega$	$g_\Xi^{\omega}=g_S\cos(\theta)-\sqrt{\frac{3}{2}}g_F\sin(\theta)-\frac{1}{\sqrt{6}}g_D\sin(\theta)$
$\Xi\Xi\phi$	$g_\Xi^{\phi}=g_S\sin(\theta)+\sqrt{\frac{3}{2}}g_F\cos(\theta)+\frac{1}{\sqrt{6}}g_D\cos(\theta)$
$\Xi\Xi\rho$	$g_\Xi^{\rho}=\sqrt{2}(g_F-g_D)$

The „SU(3)“ DFT-Program:

- For given θ only 3 physical couplings are needed to fix $\{g_D,g_F,g_S\}$
- Impose $g_{NN\phi}=0$, use ideal mixing: $\theta=35.26\ldots°$, $\tan(\theta)=1/\sqrt{2}$
- use DBHF $g_{NN\omega}(\rho)$ and $g_{NN\rho}(\rho)$ to derive $g_D(\rho)$, $g_F(\rho)$, $g_S(\rho)$ for Dirac-vector interactions
- the full set of BBM vector vertices becomes accessible
- Corresponding approach for the Dirac-scalar couplings

GI-DBHF SU(3) Vertices in Infinite Nuclear Matter

Ideal Mixing: θ=35.26°



g_D g_F g_S

Constructing a „SU(3)“ Covariant DFT

$$\mathcal{L}_{int}^{DF} = -\sqrt{2} \sum_{\mathcal{M}\in\{\mathcal{P},\mathcal{S},\mathcal{V}\}} \left\{ g_D^{*(\mathcal{M})}(\hat{\rho})[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8]_D + g_F^{*(\mathcal{M})} v[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_8]_F - g_S^{*(\mathcal{M})}(\hat{\rho})\frac{1}{\sqrt{6}}[\overline{\mathcal{B}}\mathcal{B}\mathcal{P}_1]_S \right\}.$$

$$\hat{\rho}^2 = j_{B\mu} j_B^{\mu}$$

Field Equations:

$$\left(\partial_\mu \partial^\mu + m_{\mathcal{M}}^2\right) \Phi_{\mathcal{M}}^s = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho})\rho^{BB's}, \qquad \left(\partial_\mu \partial^\mu + m_{\mathcal{M}}^2\right) V_{\mathcal{M}}^\lambda = \sum_{BB'} g_{BB'\mathcal{M}}^*(\hat{\rho})\rho^{BB'\lambda}$$

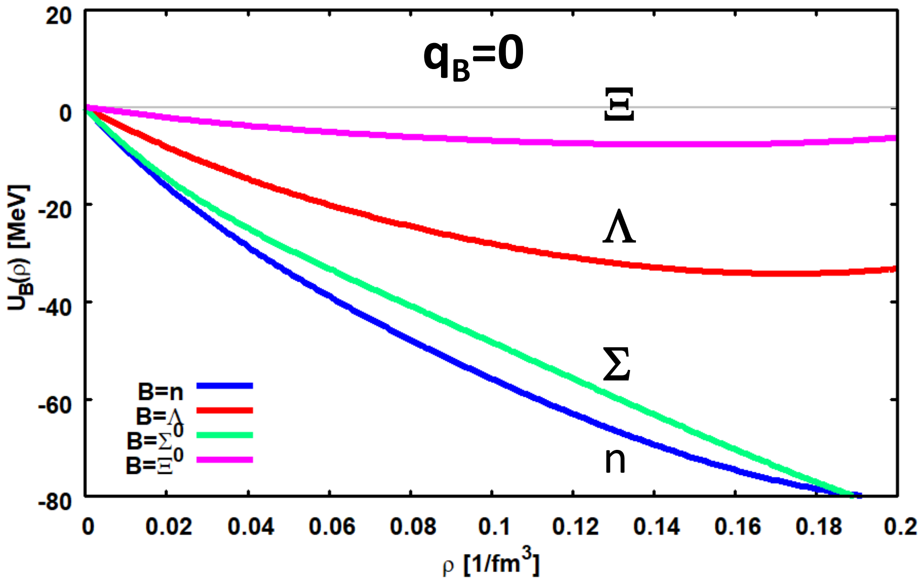
$$\left(\gamma_\mu \left(p^\mu - \Sigma_{\mathcal{B}}^\mu(\hat{\rho})\right) - M_{\mathcal{B}} + \Sigma_{\mathcal{B}}^{(s)}(\hat{\rho})\right) \Psi_{\mathcal{B}} = 0\,.$$

Baryon Self-Energies and Rearrangement

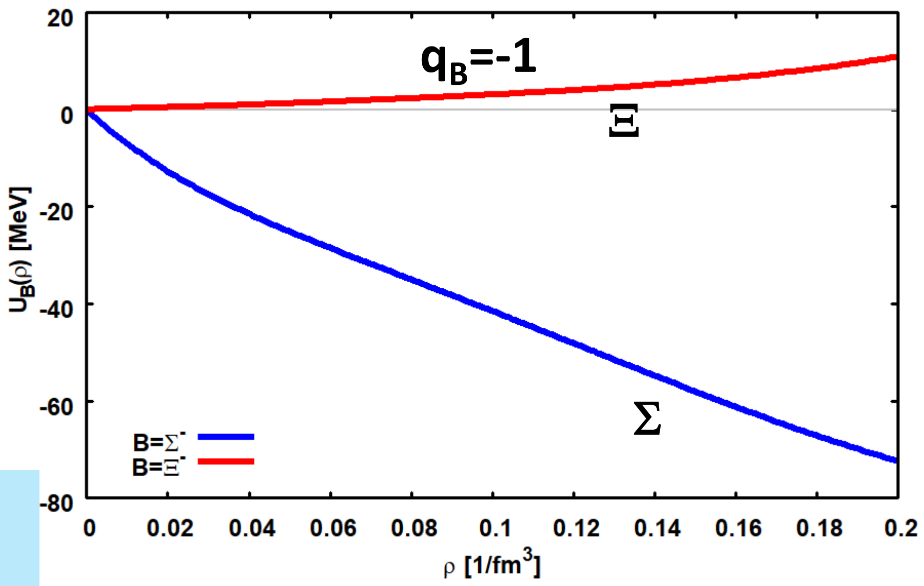
$$\Sigma_B^{(s)}(\hat{\rho}) = \sum_{\mathcal{M}\in S} \Phi_{\mathcal{M}}(\hat{\rho}) g_{BB\mathcal{M}}^*(\hat{\rho}). \qquad \Sigma_B^{(d)\mu}(\hat{\rho}) = \sum_{\mathcal{M}\in V} V_{\mathcal{M}}^\mu(\hat{\rho}) g_{BB\mathcal{M}}^*(\hat{\rho})$$

$$\Sigma_B^{(r)\mu}(\hat{\rho}) = \sum_{B'B''\mathcal{M}} \frac{\partial g_{B'B''M}^*(\hat{\rho})}{\partial j_{B\mu}} \frac{\delta}{\delta g_{B'B'f'M}^*} \mathcal{L}_{int}^{DF},$$

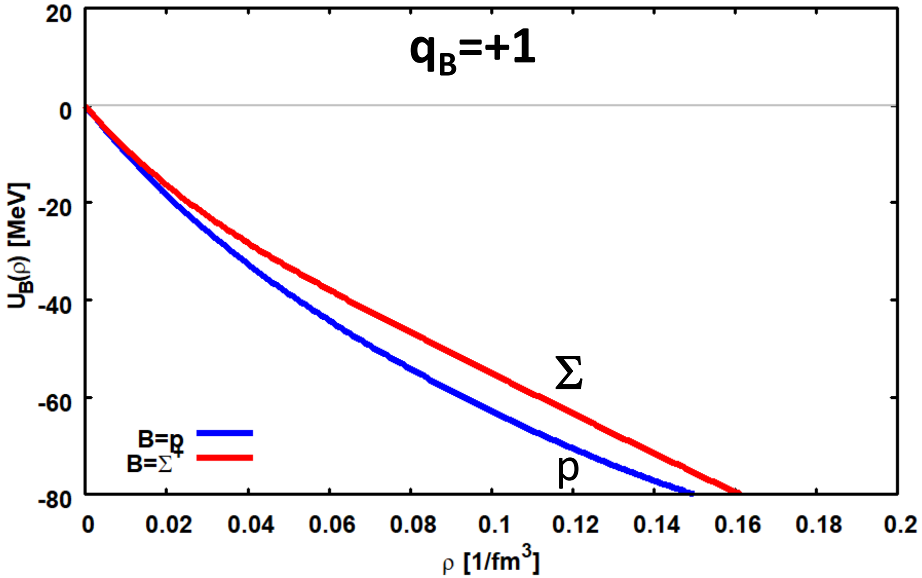
Covariant „SU(3)“ Baryon Mean-Field Self-Energies



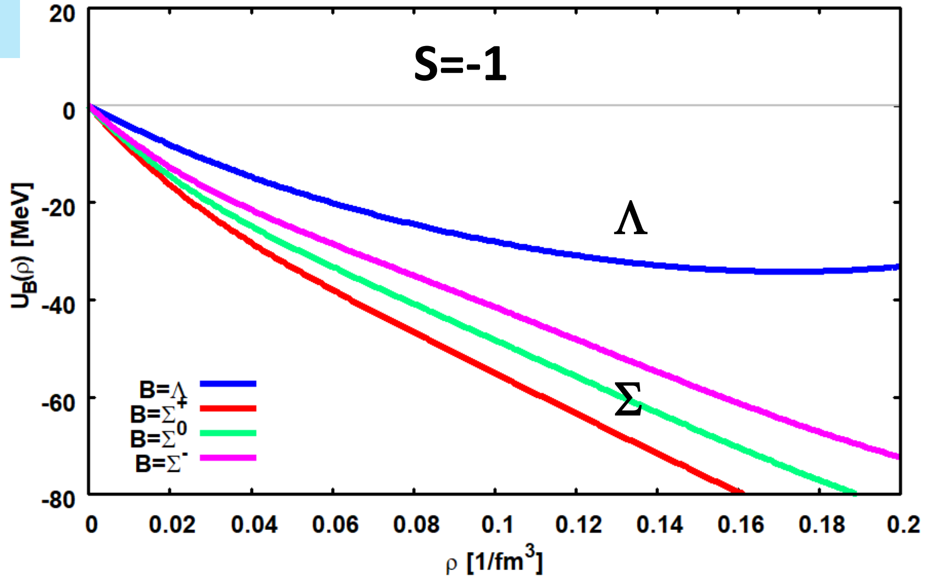
$U_B = U^S_B + U^V_B$



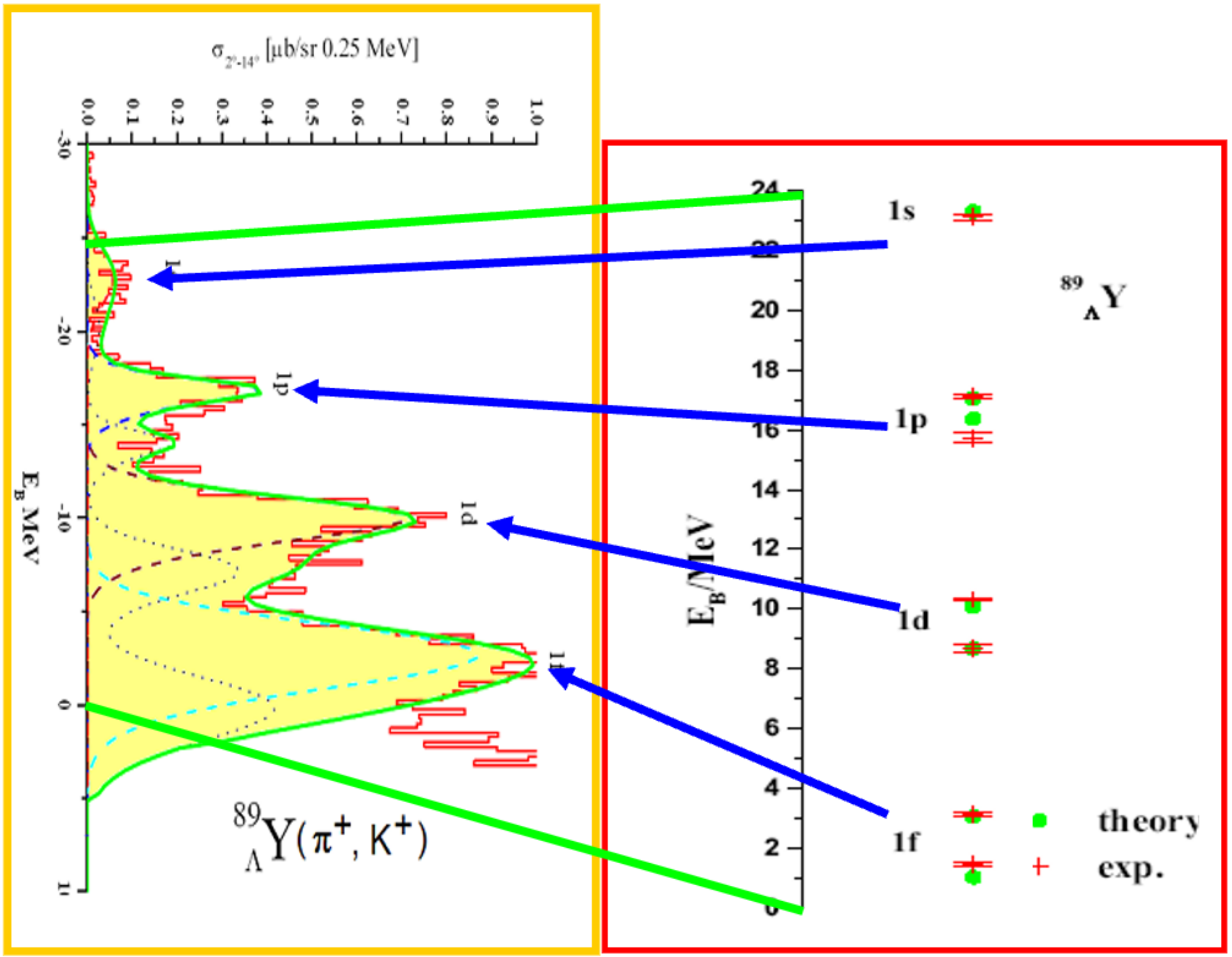
Z/A=0.4
as for example in
^{10}Be, ^{48}Ca, ^{124}Sn, ^{208}Pb



Similarity:
$\{\Sigma^+, \Sigma^0\} \sim \{p, n\}$



Application to a Finite Nucleus: Spectrum of ^{89}Y



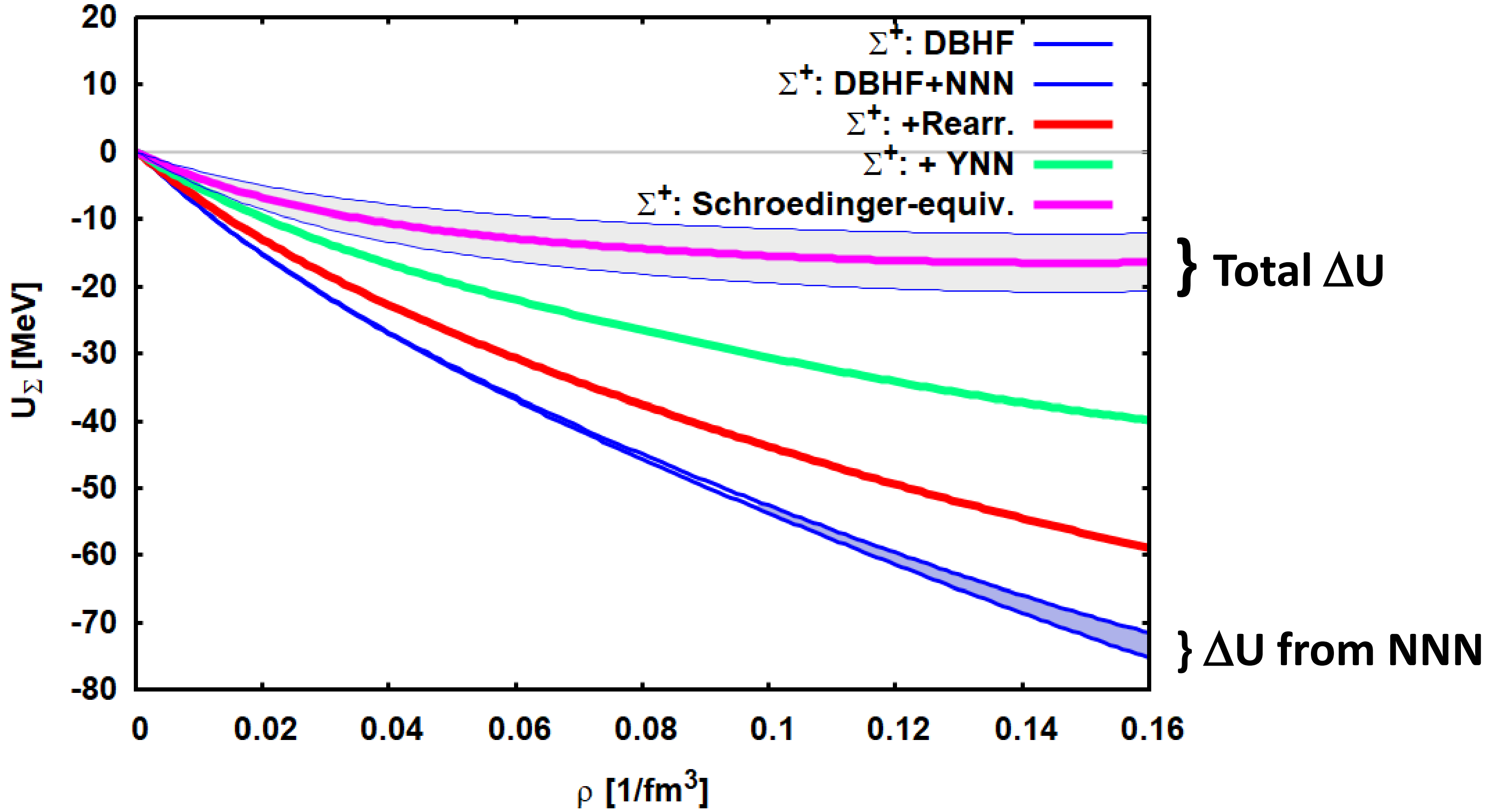
Structure Calculations:
H.L. et al., PPNP 98 (2018) 119

- ^{89}Y = Λ + ^{88}Y(4-,g.s.)
- Λ–Core Spin-Spin Interactions
- Covariant Tensor Interaction

Reaction Calculation:
S. Bender, R. Shyam, H.L., Nucl.Phys.A 839 (2010) 51

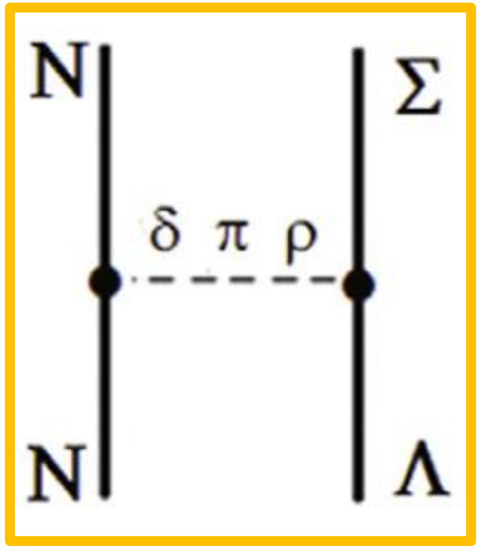
Data: Hotchi et al., Phys. Rev. C64 (2001) 044302

The Emergence of the „Schroedinger-equivalent“ EDF-Potential



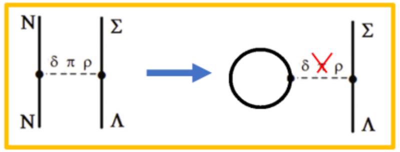
$$U_{Schroed} = \frac{M_Y^*}{M_Y} U_{RMF} + \delta U(k_F)$$ (see G. Adaman, H.L. et al., *Eur.Phys.J.A* 57 (2021) 3, 89

Λ-Σ Mixing by Isovector Interactions



$$L = -g_{\Lambda\Sigma\delta}\overline{\Psi}_\Sigma\vec{\tau}\Psi_\Lambda \bullet \vec{\phi} + g_{\Lambda\Sigma\rho}\overline{\Psi}_\Sigma\vec{\tau}\gamma_\mu\Psi_\Lambda \bullet \vec{V}^\mu + i\frac{1}{m_\pi}g_{\Lambda\Sigma\pi}\overline{\Psi}_\Sigma\vec{\tau}\gamma_5\gamma_\mu\Psi_\Lambda \bullet \partial^\mu\vec{\varphi}$$

Λ-Σ Mixing Induced by the Static Isovector Mean-Field

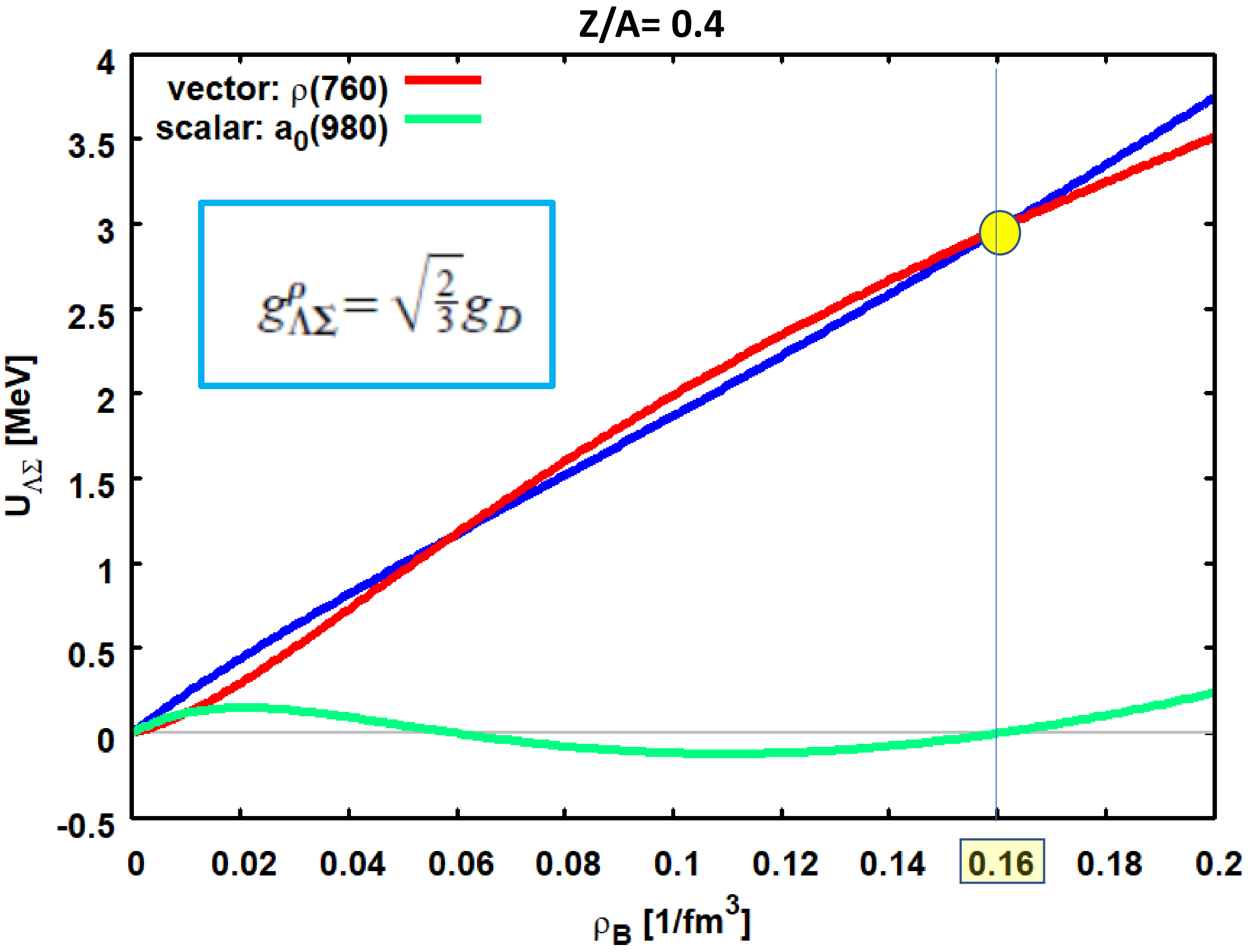


$$U_{\Lambda\Sigma}(\rho_B) = U_\delta^{(NN)}(\rho_B)\left(\frac{g_{\Lambda\Sigma\delta}}{g_{NN\delta}}\right) + U_\rho^{(NN)}(\rho_B)\left(\frac{g_{\Lambda\Sigma\rho}}{g_{NN\rho}}\right)$$

Mean-Field Induced Mixing

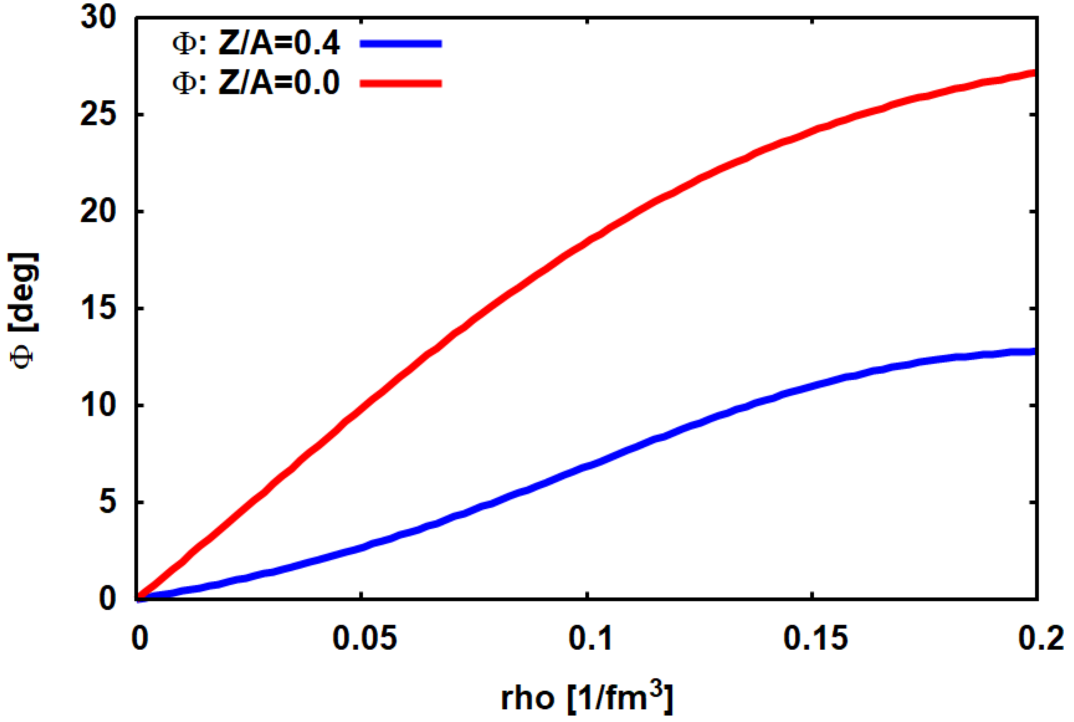
$$\begin{pmatrix} H_{\Lambda A} - E & U_{\Lambda\Sigma} \\ U_{\Lambda\Sigma}^\dagger & H_{\Lambda A} + m_{\Sigma\Lambda} - E \end{pmatrix} \begin{pmatrix} \left[\phi_\Lambda \otimes |A\rangle\right]_{I_A N_A} \\ \left[\phi_\Sigma \otimes |A\rangle\right]_{I_A N_A} \end{pmatrix} = 0$$

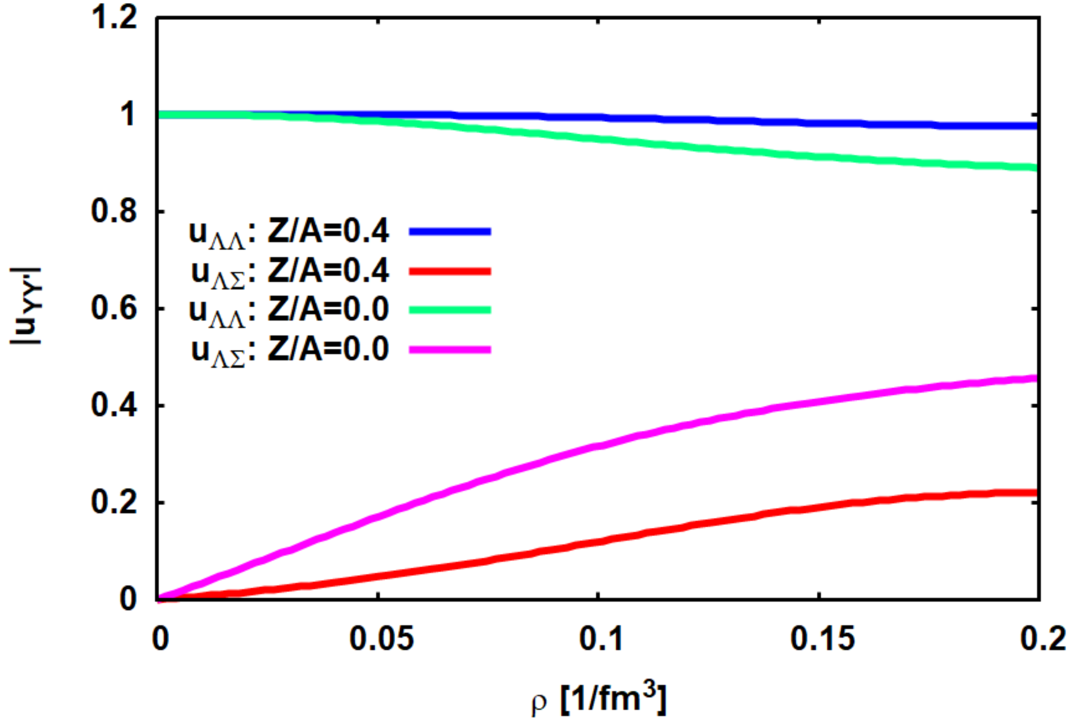
Λ–Σ Mean-Field Mixing SU(3) Potential in Asymmetric Nuclear Matter



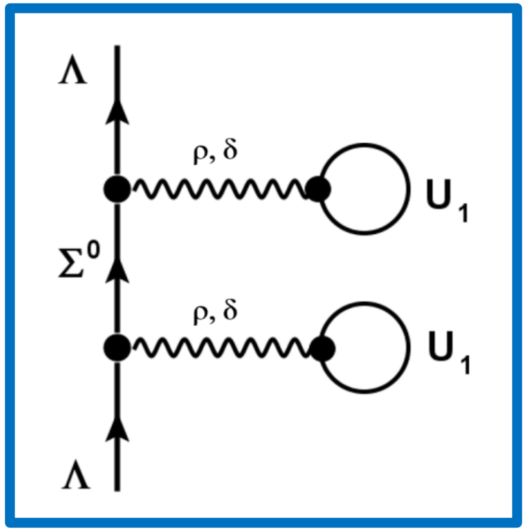
In-Medium Λ–Σ Mixing by the Isovector Mean-field

$$|\Lambda^*(\rho)\rangle = \cos(\Phi)|\Lambda\rangle - \sin(\Phi)|\Sigma\rangle \qquad |\Sigma^*(\rho)\rangle = \cos(\Phi)|\Sigma\rangle + \sin(\Phi)|\Lambda\rangle$$





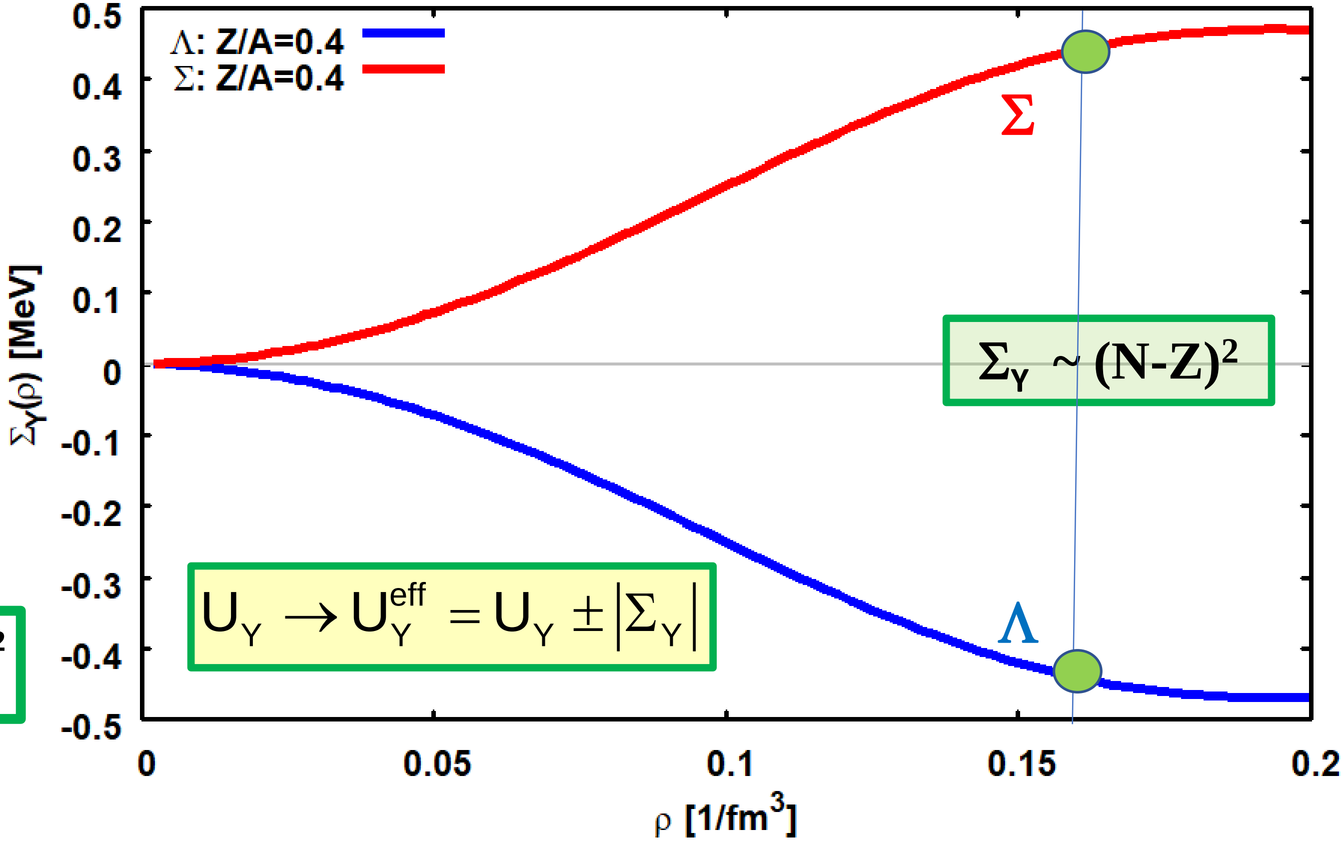
Λ and Σ Self-Energies Induced by the Isovector Mean-Field



$$\Sigma_\Lambda(\rho,\omega) = U_{\Lambda\Sigma}(\rho) G_\Sigma(\rho,\omega) U_{\Sigma\Lambda}(\rho) \left[\tau_{A3}\, T_{Y3}\right]^2$$

Static Limit:

$$G_\Sigma(\rho,\omega) \approx \frac{1}{M^*_\Sigma(\rho) - M^*_\Lambda(\rho)}$$



- Depth of effective Λ potential is increased
- Depth of effective Σ potential is decreased

Summary and Outlook

- **In-medium BB-interaction by covariant GI-DBHF theory**
- **SU(3) relations connecting NN, NY, and YY interactions**
- **„Quark Scaling“ in Y=0,-1 (S=-1,-2) hypercharge multiplets**
- **$\Lambda - \Sigma$ mixing induced by the isovector mean-field and 2nd order self-energies**
- **to come: hypernuclear spectra and magnetic moments, hypermatter, neutron stars, dynamical $\Lambda - \Sigma$ mixing, …**

Background:
***H.L., M. Dhar, Lect.Notes Phys.* 948 (2018) 161**
***H.L., M. Dhar, Th. Gaitanos, Xu Cao, Prog.Part.Nucl.Phys.* 98 (2018) 119**

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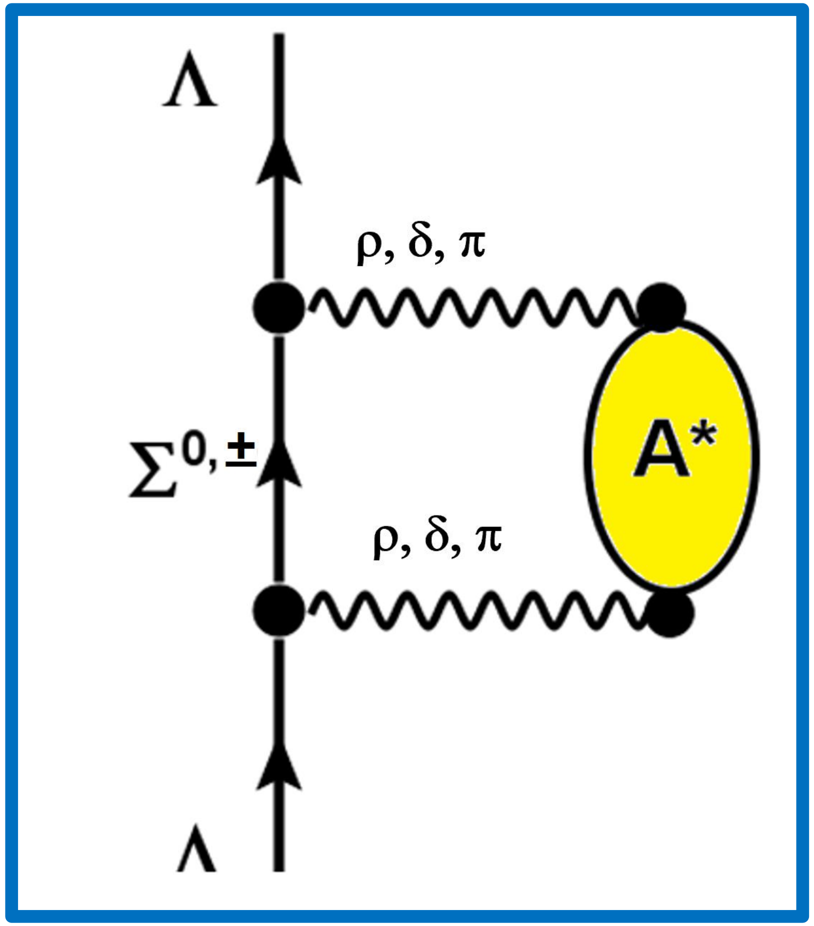
Isovector Λ-Σ Mixing and Induced Dynamical Λ Self-Energy



GDR, Fermi- and Gamow-Teller Isovector Modes

C. Dover, H. Fesbach, A. Gal, Phys.Rev.C 51 (1995) 541

→Influence on magnetic moments of Λ-hypernuclei