Skyrme Force for Ξ^- Hypernuclei

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- Extended Skyrme-Hartree-Fock (SHF) approach
- Ξ^- Hypernuclei (Data)
- Fit parameters, Results

- PRC 90, 047301 (2014)
- PRC 94, 064319 (2016)
- EPJA 56, 135 (2020)
- PRC 104, L061307 (2021)

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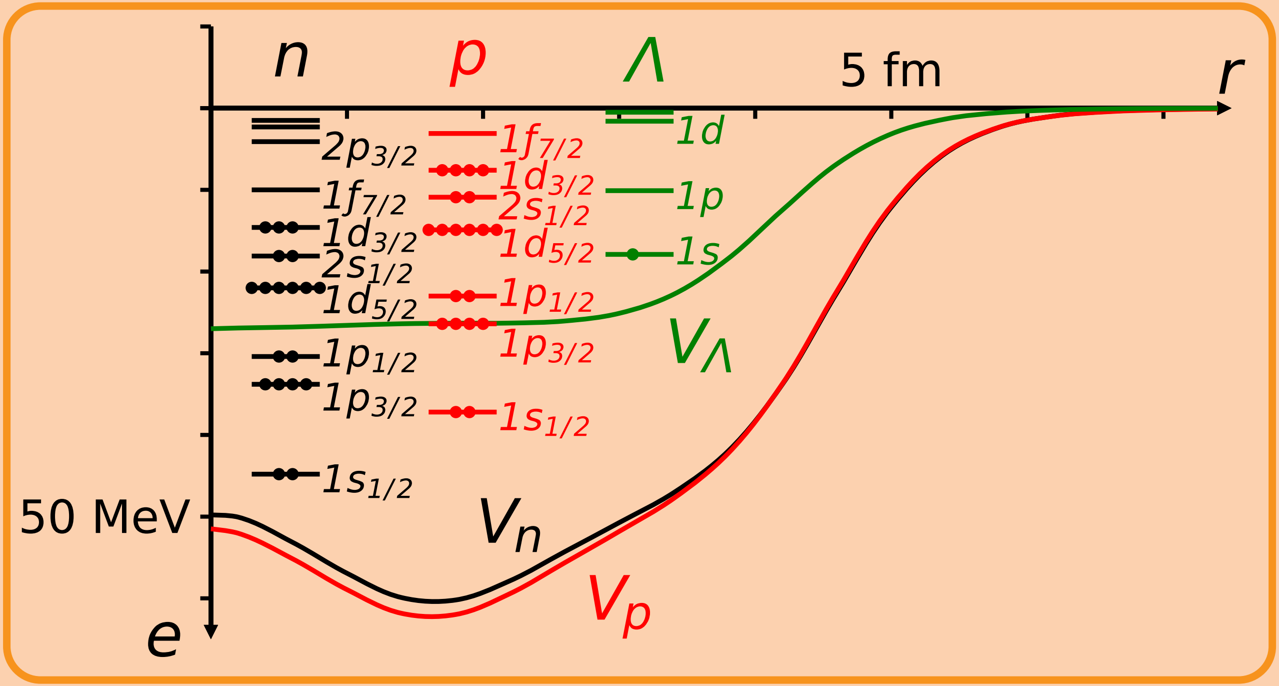
Hypernuclei: Typical Example: $^{40}_{\Lambda}$Ca:



- Theoretical model:
  - Skyrme-Hartree-Fock (SHF) Vautherin & Brink, PRC 5 626 (1972)
  - Standard *NN* force: SIII, SGII, SkI4, **SLy4**, . . .
  - Optimize *N*Λ Skyrme force

Extended SHF Model for Hypernuclei:

M. Rayet, Ann. Phys. 102 226 (1976); Nucl. Phys. A367 381 (1981)
D. Vautherin, PRC 7 296 (1973)

- Total energy of the hypernucleus:

$$E = \int \mathrm{d}^3\boldsymbol{r}\, \epsilon(\boldsymbol{r})$$

Energy density functional:

$$\epsilon = \epsilon_N[\tau_n, \tau_p, \rho_n, \rho_p, \boldsymbol{J}_n, \boldsymbol{J}_p] + \epsilon_\Lambda[\tau_\Lambda, \rho_\Lambda, \rho_N]$$

Local densities:

$$\rho_q = \sum_{i=1}^{N_q} |\phi_q^i|^2\,, \quad \tau_q = \sum_{i=1}^{N_q} |\nabla\phi_q^i|^2\,, \quad \boldsymbol{J}_q = \sum_{i=1}^{N_q} \phi_q^{i*}(\nabla\phi_q^i \times \boldsymbol{\sigma})/i$$

i: occupied states, N_q: number of particles $q = n, p, \Lambda$

2D model: quadrupole constraint: $\beta_2 = \sqrt{\frac{\pi}{5}}\frac{\langle 2z^2 - r^2\rangle}{\langle z^2 + r^2\rangle}$ fixed

- Nucleonic part: standard Skyrme functional:

$$\begin{aligned}\epsilon_N \;=\; & \frac{1}{2m_N}\tau_N + \left[b_0\rho_N^2 - d_0(\rho_n^2+\rho_p^2)\right]/2 \\ & +\; b_1\rho_N\tau_N - d_1(\rho_n\tau_n+\rho_p\tau_p) + d_1(\boldsymbol{J}_n^2+\boldsymbol{J}_p^2)/2 \\ & -\; \left[b_2\rho_N\Delta\rho_N - d_2(\rho_n\Delta\rho_n+\rho_p\Delta\rho_p)\right]/2 \\ & +\; \left[b_3\rho_N^2 - d_3(\rho_n^2+\rho_p^2)\right]\rho_N^\gamma/3 \\ & -\; b_4\rho_N\nabla\cdot\boldsymbol{J}_N - d_4(\rho_n\nabla\cdot\boldsymbol{J}_n+\rho_p\nabla\cdot\boldsymbol{J}_p) + \epsilon_{\text{Coul.}}\end{aligned}$$

- SHF Schrödinger equation:

$$\left[-\nabla\cdot\frac{1}{2m_q^*(\boldsymbol{r})}\nabla + V_q(\boldsymbol{r}) - i\nabla W_q(\boldsymbol{r})\cdot(\nabla\times\boldsymbol{\sigma})\right]\phi_q^i(\boldsymbol{r}) = -e_q^i\phi_q^i(\boldsymbol{r})$$

- SHF mean fields:

$$V_N = V_N^{\text{SHF}} + \frac{\partial\epsilon_\Lambda}{\partial\rho_N} \quad , \quad V_\Lambda = \frac{\partial\epsilon_\Lambda}{\partial\rho_\Lambda} \quad , \quad W_\Lambda = 0$$

Fit to Hypernuclear Data:

- The SHF mean-field approach is not ideal for light nuclei, but one can expect that for $B_{\Xi^-} = E[^{A-1}(Z+1)] - E[^{A}_{\Xi}Z]$ important cancellations regarding the *nuclear* core structure occur. (Results independent of *NN* Skyrme force). We explore this idea pragmatically.

- Data for three Ξ^- hypernuclei:
  $^{12}_{\Xi}$Be (Ξ^-+ ^{11}B) , $^{13}_{\Xi}$B (Ξ^-+ ^{12}C) , $^{15}_{\Xi}$C (Ξ^-+ ^{14}N)
  Most are ambigious and/or inaccurate !

- Fit parameters $a_0, a_1, a_2, a_3, a_4, \alpha, \ldots$ → 'SLX?' ΞN force

Ξ^-(1321) Hypernuclei:

- $\Xi^- + {}^{14}\text{N} \;\rightarrow\; {}^{15}_{\Xi}\text{C} \;\rightarrow\; {}^{10}_{\Lambda}\text{Be} + {}^{5}_{\Lambda}\text{He}$ (KEK-E373-T2 “KISO”)

K. Nakazawa et al. (KEK-E373), PTEP 033D02 (2015)
E. Hiyama & K. Nakazawa, Annu. Rev. Nucl. Part. Sci. 68 131 (2018)

Two possibilities:

1. $B_{\Xi^-} = 3.87 \pm 0.21\,\text{MeV}$ $\left({}^{10}_{\Lambda}\text{Be in ground state: } {}^{15}_{\Xi s}\text{C}\right)$

2. $B_{\Xi^-} = 1.03 \pm 0.18\,\text{MeV}$ $\left({}^{10}_{\Lambda}\text{Be in excited state: } {}^{15}_{\Xi p}\text{C}\right)$

2. confirmed by

S.H. Hayakawa (J-PARC E07), PRL 126 062501 (2021)

$B_{\Xi^-} = 1.27 \pm 0.21\,\text{MeV}$ (J-PARC-E07-T006 “IBUKI”)

S. Aoki et al. (KEK E176), NPA 828 191 (2009)

$B_{\Xi^-} = 1.18 \pm 0.22\,\text{MeV}$ (KEK-E176 #14-03-35)

- Recent candidates for ${}^{15}_{\Xi s}$C:
  (at most one can be the true ground state!)

  M. Yoshimoto et al., PTEP 7 073D02 (2021)

  - $\Xi^- + {}^{14}\mathrm{N} \rightarrow {}^{5}_{\Lambda}\mathrm{He} + {}^{5}_{\Lambda}\mathrm{He} + {}^{4}\mathrm{He} + \mathrm{n}$
    (J-PARC-E07-T010 “IRRAWADDY”)
    $B_{\Xi^-} = 6.27 \pm 0.27\,\mathrm{MeV}$
  - $\Xi^- + {}^{14}\mathrm{N} \rightarrow {}^{9}_{\Lambda}\mathrm{Be} + {}^{5}_{\Lambda}\mathrm{He} + \mathrm{n}$
    (KEK-E373-T3 “KINKA”)
    $B_{\Xi^-} = 8.00 \pm 0.77\,\mathrm{MeV}$ or $B_{\Xi^-} = 4.96 \pm 0.77\,\mathrm{MeV}$

  Only KINKA(8.00) is compatible with all other ${}^{12}_{\Xi}$Be and ${}^{13}_{\Xi}$B event candidates.
  In this analysis we assume therefore
  $B_{\Xi^-}({}^{15}_{\Xi s}\mathrm{C}) = 8.00\,\mathrm{MeV}$
  $B_{\Xi^-}({}^{15}_{\Xi p}\mathrm{C}) = 1.13\,\mathrm{MeV}$

P. Khaustov et al. (AGS E885), PRC 61 054603 (2000)

- $^{12}\mathrm{C}(K^-,K^+)^{12}_{\Xi}\mathrm{Be}$ $(= \Xi^- + {}^{11}\mathrm{B})$

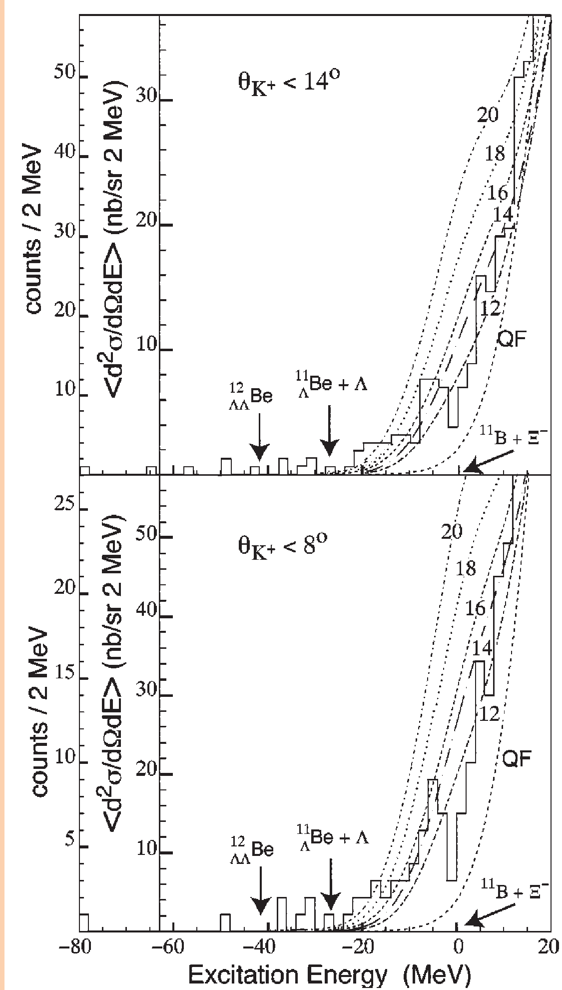
  Ξ^--nucleus Woods-Saxon potential depth $\approx 14\,\mathrm{MeV}$

  ↪ Cluster model: $B_{\Xi^-} \approx 5\,\mathrm{MeV}$

  E. Hiyama et al., PRC 78 054316 (2008)

  ↪ AMD: $B_{\Xi^-} \approx 3-5.5\,\mathrm{MeV}$

  H. Matsumiya et al., PRC 83 024312 (2011)



↪ Direct measurement of $B_{\Xi^-}(^{12}_{\Xi}\mathrm{Be})$ is needed: J-PARC E05 ...

T. Nagae et al. (J-PARC E05) at INPC2016:

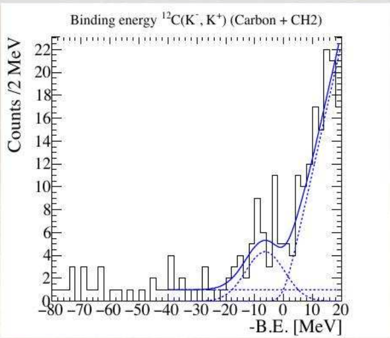
Search for a Ξ bound state in the $^{12}C(K^-, K^+)$ reaction at 1.8 GeV/c

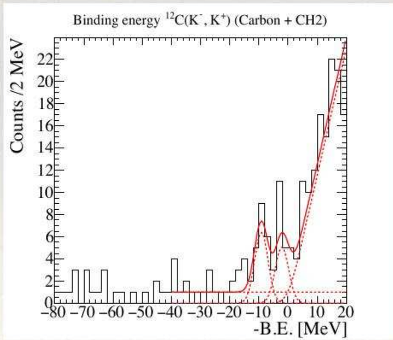
Peak Fittings

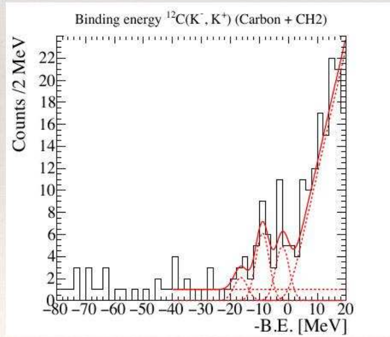
- QFΞ(linear)+Background(Flat)
  +
- One Gaussian (all free)
  - B_{Ξ}=6.3 MeV, ΔB_{FWHM}=15.7 MeV
    too broad !!

- QFΞ(linear)+Background(Flat)
  +
- Two Gaussians
  (fixed width=5.4 MeV)
  - B_{Ξ}=9.1 MeV and 2.1 MeV

- QFΞ(linear)+Background(Flat)
  +
- Three Gaussians
  (fixed width=5.4 MeV)
  - B_{Ξ}=16.4, 8.9, and 2.0 MeV
    too deep ?







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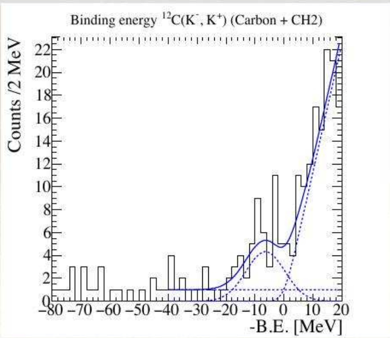

- QFΞ(linear)+Background(Flat)
  +
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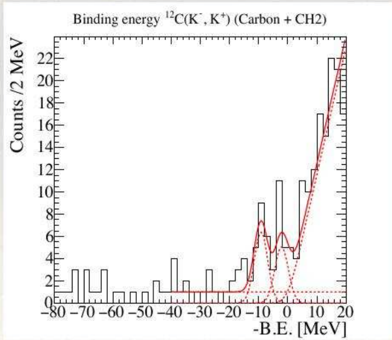
  (fixed width=5.4 MeV)
  - B_Ξ=9.1 MeV and 2.1 MeV



- QFΞ(linear)+Background(Flat)
  +
- Three Gaussians
  (fixed width=5.4 MeV)
  - B_Ξ=16.4, 8.9, and 2.0 MeV
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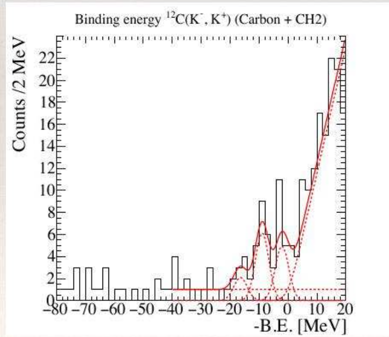


Compatible with Kiso$_p$ **Not** compatible with Kiso$_p$

None is compatible with Kiso$_s$

- KEK-E176 emulsion data with possible formation of ${}^{13}_{\Xi p}$B:

S. Aoki et al. (KEK E176), NPA 828 191 (2009)
M. Yamaguchi & K. Tominaga & Y. Yamamoto & T. Ueda, PTP 105 627 (2001)

- $\Xi^- + {}^{12}\mathrm{C} \rightarrow {}^{4}_{\Lambda}\mathrm{He} + {}^{9}_{\Lambda}\mathrm{Be}$ (KEK-E176 #10-9-6)
  $B_{\Xi^-} = 0.82 \pm 0.17\,\mathrm{MeV}$
- $\Xi^- + {}^{12}\mathrm{C} \rightarrow {}^{4}_{\Lambda}\mathrm{He} + {}^{9}_{\Lambda}\mathrm{Be}^*$ (KEK-E176 #13-11-14)
  $B_{\Xi^-} = 0.82 \pm 0.14\,\mathrm{MeV}$

But no unique interpretation :

Table 13
The Ξ^- binding energies (MeV) in nuclei (^{12}C, ^{14}N or ^{16}O) including excited states of hypernuclei.

Event	${}^{4}_{\Lambda}\mathrm{H} + {}^{9}_{\Lambda}\mathrm{Be}$	${}^{4}_{\Lambda}\mathrm{H} + {}^{11}_{\Lambda}\mathrm{B}$	${}^{4}_{\Lambda}\mathrm{H} + {}^{13}_{\Lambda}\mathrm{C}$
#10-9-6	0.82 ± 0.17 -0.23 ± 0.17 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{9}_{\Lambda}\mathrm{Be}$) ($\chi^2 = 0.4$)	0.79 ± 0.17 -0.26 ± 0.18 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{11}_{\Lambda}\mathrm{B}$) ($\chi^2 = 8.0$)	7.48 ± 0.21 2.6 ± 0.2 (${}^{4}_{\Lambda}\mathrm{H} + {}^{13}_{\Lambda}\mathrm{C}^*$) 6.43 ± 0.21 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{13}_{\Lambda}\mathrm{C}$) 1.5 ± 0.2 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{13}_{\Lambda}\mathrm{C}^*$) ($\chi^2 = 19.5$)
#13-11-14	3.89 ± 0.14 0.82 ± 0.14 (${}^{4}_{\Lambda}\mathrm{H} + {}^{9}_{\Lambda}\mathrm{Be}^*$) 2.84 ± 0.15 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{9}_{\Lambda}\mathrm{Be}$) -0.19 ± 0.15 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{9}_{\Lambda}\mathrm{Be}^*$) ($\chi^2 = 1.3$)	3.55 ± 0.15 2.50 ± 0.15 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{11}_{\Lambda}\mathrm{B}$) ($\chi^2 = 3.5$)	9.90 ± 0.18 5.0 ± 0.2 (${}^{4}_{\Lambda}\mathrm{H} + {}^{13}_{\Lambda}\mathrm{C}^*$) 0.3 ± 0.4 (${}^{4}_{\Lambda}\mathrm{H} + {}^{13}_{\Lambda}\mathrm{C}^*$) 8.85 ± 0.19 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{13}_{\Lambda}\mathrm{C}$) 3.9 ± 0.2 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{13}_{\Lambda}\mathrm{C}^*$) -0.8 ± 0.4 (${}^{4}_{\Lambda}\mathrm{H}^* + {}^{13}_{\Lambda}\mathrm{C}^*$) ($\chi^2 = 11.2$)

Skyrme model for $N\Xi^-$ interaction:

- Disregard Im V_Ξ (decay width $\Xi N \to \Lambda\Lambda$) !

- $$\epsilon_\Xi = \frac{\tau_\Xi}{2m_\Xi} + a_0\rho_\Xi\rho_N + a_3\rho_\Xi{\rho_N}^\alpha - a_2(\rho_\Xi\Delta\rho_N + \rho_N\Delta\rho_\Xi)/2$$
$$+ a_1(\rho_\Xi\tau_N + \rho_N\tau_\Xi) + a_4(\nabla\rho_\Xi\cdot\boldsymbol{J}_N + \nabla\rho_N\cdot\boldsymbol{J}_\Xi)$$

$$\frac{1}{2m^*_\Xi} = \frac{1}{2m_\Xi} + a_1\rho_N + c_1\rho_\Xi$$

- Parameter choice:

  Not enough precise data to fit all parameters

  We fix $a_1 = a_4 = 0$, $\alpha = 2$ or $7/6$

  Trial surface terms $a_2 = 0, 10, \ldots, 40\,\mathrm{MeV\,fm}^5$

  Fit a_0, a_3 to $B_{\Xi^-}({}^{15}_{\Xi s}\mathrm{C}) = 8.00\,\mathrm{MeV}$, $B_{\Xi^-}({}^{15}_{\Xi p}\mathrm{C}) = 1.13\,\mathrm{MeV}$

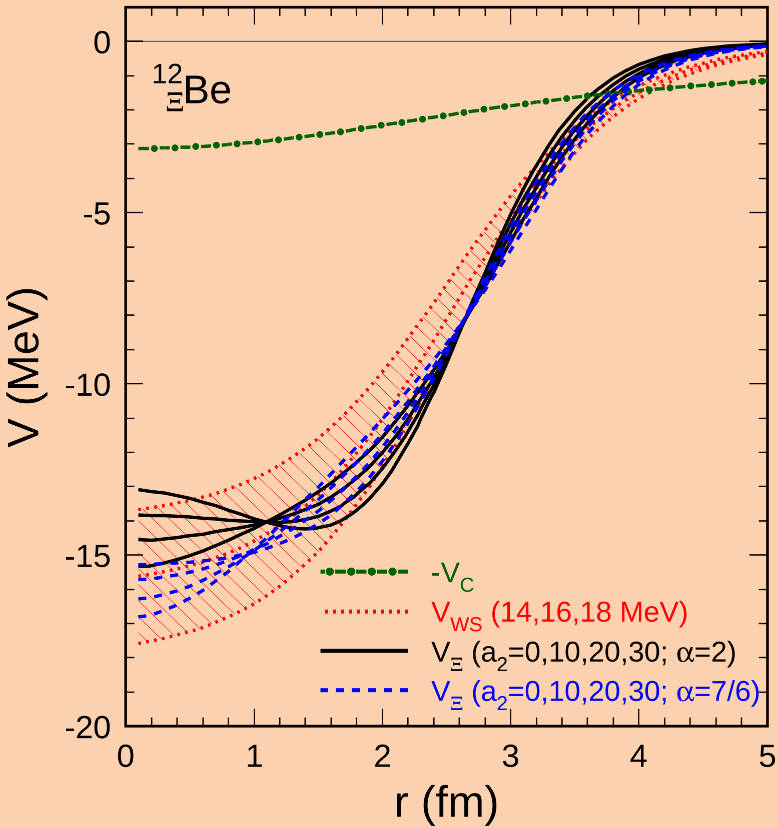
- Results for B_{Ξ^-} (in MeV):

α	a_2	a_0	a_3	${}^{13}_{\Xi s}$B	${}^{13}_{\Xi p}$B	${}^{12}_{\Xi s}$Be
	0	-200.3	704.8	6.62 (6.71)	0.86 (-0.12)	5.83
	10	-198.7	641.3	6.49 (6.65)	0.75 (-0.11)	5.72
2	20	-196.9	576.8	6.37 (6.59)	0.65 (-0.09)	5.61
	30	-194.7	509.3	6.25 (6.53)	0.56 (-0.08)	5.51
	40	-192.5	443.0	6.14 (6.46)	0.47 (-0.07)	5.40
	0	-498.6	551.3	6.44 (6.84)	0.84 (-0.07)	5.87
	10	-470.8	503.0	6.32 (6.76)	0.73 (-0.06)	5.76
7/6	20	-439.4	449.3	6.21 (6.69)	0.63 (-0.06)	5.64
	30	-409.0	397.3	6.10 (6.61)	0.54 (-0.05)	5.53
	40	-378.9	345.5	6.02 (6.54)	0.47 (-0.04)	5.42

${}^{13}_{\Xi}$B values in brackets are for spherical calculations

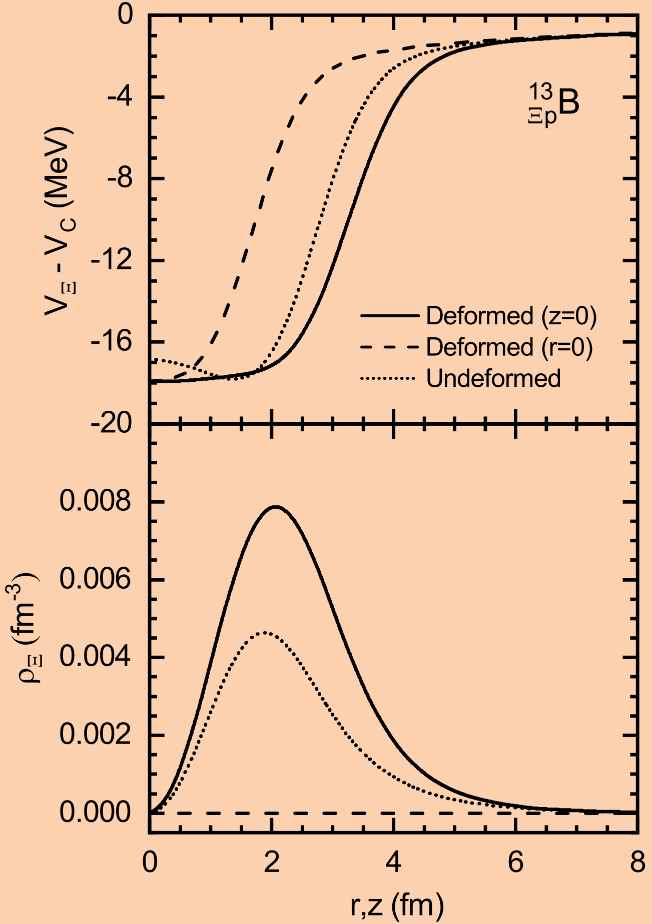
→ Good predictions for ${}^{13}_{\Xi p}$B (deformed) and ${}^{12}_{\Xi s}$Be

- Mean field in $^{12}_{\Xi}$Be:



→ In agreement with derived WS mean field of E885
Shape depends on a_2

- Axially deformed $^{13}_{\Xi}$B:



- Core ^{12}C deformation fixed to exp. value
  $\beta \equiv \frac{\sqrt{5\pi}}{3}\frac{Q_p}{ZR_0^2} \approx -0.58 \pm 0.03$

- Oblately deformed core increases binding of embedded Ξ^- $1p$ state by $\mathcal{O}(1\,\mathrm{MeV})$

Summary:

- Fit all current data for ${}^{15}_{\Xi}$C, ${}^{12}_{\Xi}$Be, and ${}^{13}_{\Xi}$B with a common minimal ΞN Skyrme force.
- The KINKA (8.00 MeV) interpretation of $B_{\Xi^-}({}^{15}_{\Xi s}\mathrm{C})$ is most compatible with both ${}^{12}_{\Xi}$Be and ${}^{13}_{\Xi}$B data.
- Combining KINKA (${}^{15}_{\Xi s}$C) and KISO+IBUKI (${}^{15}_{\Xi p}$C) data, the predicted Ξ^- mean field in ${}^{12}_{\Xi}$Be is very similar to the best-choice WS mean field for BNL-E885 (14–16 MeV).
- ${}^{13}_{\Xi}$B is strongly axially deformed, and $B_{\Xi^-} \approx 0.8$ MeV of some KEK-E176 emulsion data might thus be explained as deformed Ξ^- $1p$ states. Unbound without deformation!

Comparison with Other Works:

- E. Friedman & A. Gal, PLB 820 136555 (2021):

  Optical potential + Shell model

  Fit ${}^{15}_{\Xi}\mathrm{C}$ and ${}^{13}_{\Xi p}\mathrm{B}$, but $V_{\mathrm{WS}} \gtrsim 20\,\mathrm{MeV}$

- J. Hu & Y. Zhang & H. Shen, JPG 49 025104 (2022):

  (Q)RMF

  Fit ${}^{15}_{\Xi p}\mathrm{C}$, $B_{\Xi^-}({}^{15}_{\Xi s}\mathrm{C}) \approx 5.7\,\mathrm{MeV}$, $V_{\mathrm{WS}} \sim 12\,\mathrm{MeV}$, no ${}^{13}_{\Xi p}\mathrm{B}$

- Tanimura & Sagawa & Sun & Hiyama, PRC 105 044324 (2022):

  RMF + residual $\boldsymbol{S}_N \cdot \boldsymbol{S}_\Xi$

  Fit ${}^{15}_{\Xi}\mathrm{C}$, $V_{\mathrm{WS}} \sim (12-14)\,\mathrm{MeV}$, no ${}^{13}_{\Xi p}\mathrm{B}$

↪ Very different approaches due to ambigious data

Outlook:

- Constrain better the SHF ΞN interaction parameters (α, α_1, α_4, ...), include isospin dependence
- Include imaginary parts due to the ΞN–$\Lambda\Lambda$ decay
- Beyond-mean-field HFB etc. for dripline (hyper)nuclei
- Comparison with cluster etc. models
- New unambigious data of KEK-E373 and J-PARC-E07 awaited!