$\bar{K}N$ interaction, P-wave terms

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14th International Conference on Hypernuclear and Strange Physics – HYP2022
June 27 – July 1, 2022, Prague

Study of the **meson-baryon interaction** in the **S=-1** sector at moderately high energies.
10 channels involved in this sector: $K^- p, \bar{K}^0 n, \pi^0\Lambda, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^-\Sigma^+, \eta\Lambda, \eta\Sigma^0, K^+\Xi^-, K^0\Xi^0$

1255 MeV — 1810 MeV

$\pi\Lambda$ — $\pi\Sigma$ — $\bar{K}N$ — $\eta\Lambda$ — $\eta\Sigma$ — $K\Xi$

Interaction: **QCD** is a gauge theory which **describes** the **strong interaction** governed by the effects of the color charge of its carriers: quarks and gluons.
Perturbative QCD is inappropriate to treat low energy hadron interactions.

Chiral Perturbation Theory (ChPT) is an effective theory with hadrons as degrees of freedom which respects the symmetries of QCD.

- limited to a moderate range of energies above threshold
- not applicable close to a resonance (singularity in the amplitude)

But it is not so straight forward …

$\bar{K}N$ interaction is dominated by the presence of the $\Lambda(1405)$ resonance, located only 27 MeV below the $\bar{K}N$ threshold.

→ **A nonperturbative resummation is needed!!!**

In 1995, the problem was reformulated in terms of a Unitary extension of ChPT in coupled channels.

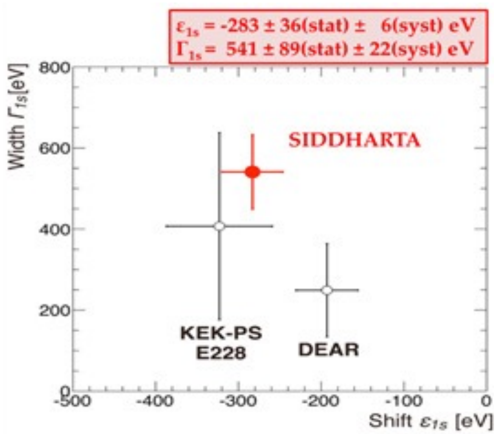
The pioneering work -- *Kaiser, Siegel, Weise,* Nuc. Phys. A 594 (1995) 325.

E. Oset, A. Ramos, Nucl. Phys. A 636, 99 (1998).
J. A. Oller, U. -G. Meissner, Phys. Lett. B 500, 263 (2001).
M. F. M. Lutz, E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002).
B. Borasoy, E. Marco, S. Wetzel, Phys. Rev. C 66, 055208 (2002).
C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003).
D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A 725, 181 (2003).
B. Borasoy, R. Nissler, W. Wiese, Eur. Phys. J. A 25, 79 (2005).
V.K. Magas, E. Oset, A. Ramos, Phys. Rev. Lett. 95, 052301 (2005).
B. Borasoy, U. -G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006).

All of them obtaining in general similar features:
- **$\bar{K}N$ scattering data reproduced very satisfactorily**
- **Two-pole structure of $\Lambda(1405)$**

Introduction: Theoretical Framework and Historical Background

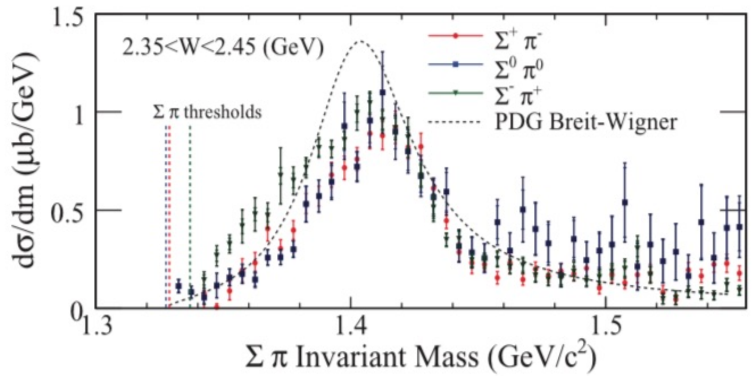
This topic has experienced a renewed interest after recent experimental advances:



M. Bazzi et al.,
Phys. Lett. B 704, 113 (2011).

Energy shift and width of the 1s state in kaonic hydrogen by SIDDHARTA@DAΦNE, 20% precision in the $K^{-}p$ scattering length!!!

Photoproduction $\gamma p \longrightarrow K^{+}\pi\Sigma$ data by the CLAS@Jlab provided detailed line shape results of the $\Lambda(1405)$

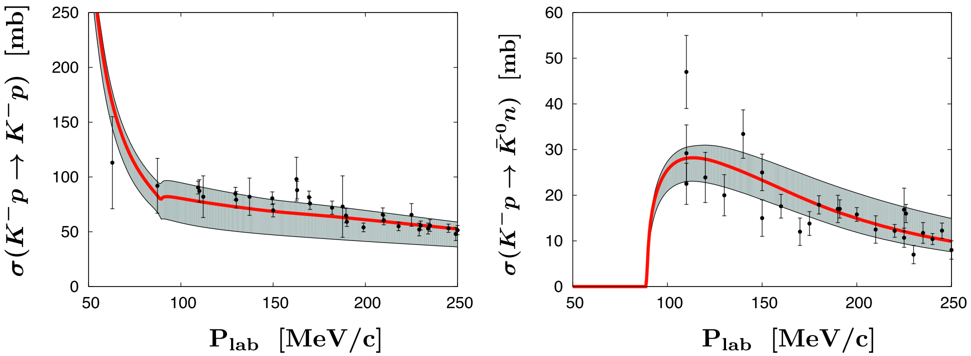


K. Moriya *et al.*, Phys. Rev. C**87**, 035206(2013).

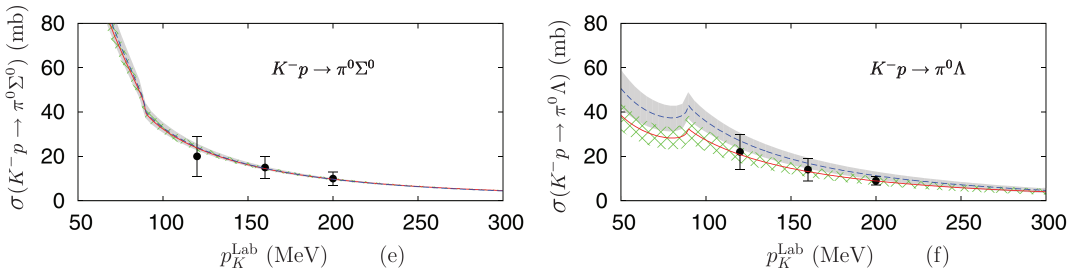
Y. Ikeda, T. Hyodo, W. Wiese, Nucl. Phys. A 881, 98 (2012).
A. Cieply and J. Smejkal, Nucl. Phys. A 881, 115 (2012).
Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).
T. Mizutani, C. Fayard, B. Saghai and K. Tsushima, Phys. Rev. C 87, 035201 (2013).
L. Roca and E. Oset: Phys. Rev. C 87, 055201 (2013), Phys. Rev. C 88, 055206 (2013).
M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).
A. F., V. Magas, A. Ramos, Phys. Rev. C 92, 015206 (2015); Nucl. Phys. A 954, 58 (2016); Phys. Rev. C 99 (2019) 035211.
P.C. Bruns, A. Cieply, Nucl. Phys. A 1019 (2022), 122378.

Introduction: Theoretical Framework and Historical Background

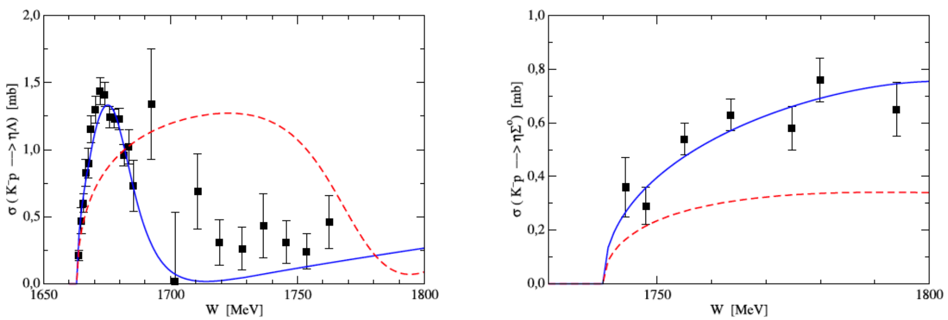
$K^-p \longrightarrow MB\ (S=-1)$ *total cross sections from different groups:*



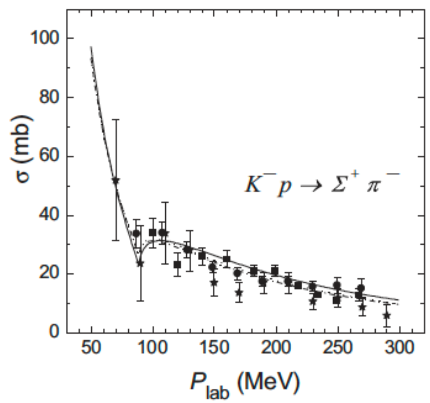
Y. Ikeda, T. Hyodo, W. Wiese, Nucl. Phys. A 881, 98 (2012).



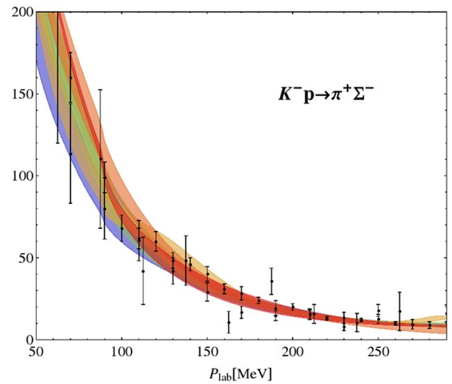
Zhi-Hui Guo, J. A. Oller, Phys. Rev. C 87, 035202 (2013).



P.C. Bruns, A. Cieply, Nucl. Phys. A 1019, (2022) 122378.



N.V. Shevchenko and Révai, Phys. Rev. C 90, 034003 (2014).



M. Mai and U. G. Meissner, Eur. Phys. J. A 51, 30 (2015).

Over the last years, Barcelona group has been working on the NLO contributions of the chiral Lagrangian

1. Paying attention on reactions particularly sensitive to NLO

   e.g. $\bar{K}N \longrightarrow K\Xi$ (it does not proceed through WT at tree level)
   A. F., V. Magas, A. Ramos, Phys. Rev. C 92, 015206 (2015)

2. Analysing the relative relevance between the Born terms and the NLO contributions in the previous type of reactions

   Born and NLO terms play a similar role
   A. Ramos, A. F., V. Magas, Nucl. Phys. A 954, 58 (2016)

3. Studying the constraining effect of isospin filtering reactions especially sensitive to NLO

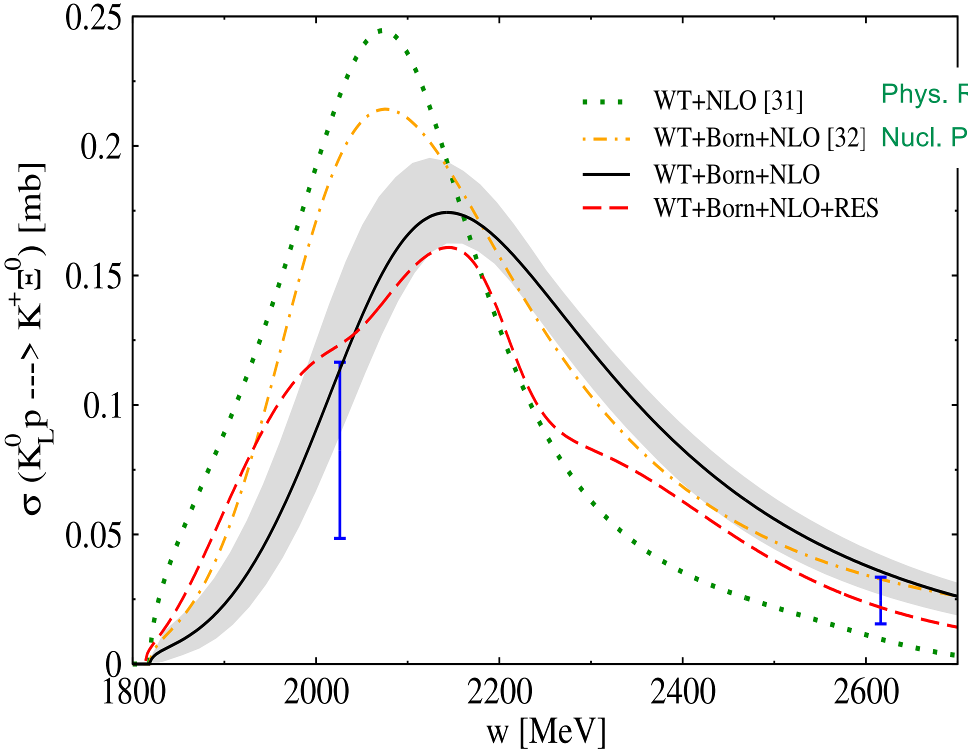
   More reliable values for the NLO LECs
   A. F., V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211

Motivation: Evolution of our chiral model

Prediction for Isospin filtering processes at higher energy:

$K_L^0 p \longrightarrow K^+ \Xi^0$ *reaction (pure* $I = 1$ *process)*

J-Lab proposal for the secondary K_L beam



Phys. Rev. C 92, 015206 (2015)
Nucl. Phys. A 954, 58 (2016)

Motivation: Previous studies including p-wave terms derived from chiral Lagrangians

- Strangeness S=0 sector

  J. Caro Ramon, N. Kaiser, S. Wetzel and W. Weise, Nucl. Phys. A 672, 249 (2000)

LO+NLO terms of the Lagrangian, showing that some processes at higher momenta were governed by p-wave contribution (e.g. $\pi^+ p \rightarrow K^+ \Sigma^+$).

- Strangeness S=+1 sector

  K. Aoki, D. Jido, PTEP 2019, no.1, 013D01 (2019)

Study revisiting the KN scattering by means of Chiral Unitarized approach with an expansion up to NLO terms of the Lagrangian.

- Strangeness S=-1 sector

  D. Jido, E. Oset, A. Ramos, Phys. Rev. C 66, 055203 (2002)

LO Lagr., not much effect of p-wave at low the energy data considered.
(Very useful for theoretical developments!)

D. Sadasivan, M.Mai, M.Döring, Phys. Lett. B 789, 329 (2019)

LO+NLO Lagrangian. A dynamical p-wave pole found! (J^P =1/2+) mimicking the lack of Σ(1385) in J^P=3/2+.

Formalism: Effective Chiral Lagrangian

Lagrangian:

$$\mathcal{L}^{eff}(B,U) = \mathcal{L}^{(1)}_{MB}(B,U) + \mathcal{L}^{(2)}_{MB}(B,U)$$

→ derive an interaction kernel $\mathbf{V}_{ij}$

- **Leading order (LO)**

$$\mathcal{L}^{(1)}_{MB} = \langle \bar{B}(i\gamma_\mu D^\mu - M_0)B\rangle + \frac{1}{2}D\langle \bar{B}\gamma_\mu\gamma_5\{u^\mu, B\}\rangle + \frac{1}{2}F\langle \bar{B}\gamma_\mu\gamma_5[u^\mu, B]\rangle$$

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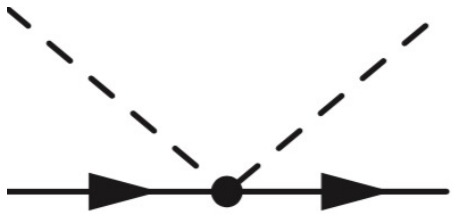
Weinberg-Tomozawa term (WT)



1. Dominant contribution.
2. Interaction mediated by the constant f of the leptonic decay

Formalism: Effective Chiral Lagrangian

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Born terms

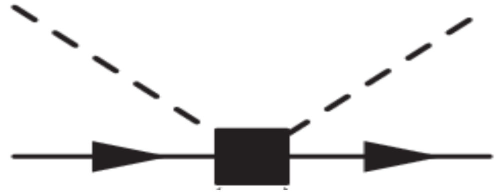
1. Direct diagram (s-channel Born term)

$$V^D_{ij} = V^D_{ij}(D,F)$$

2. Cross diagram (u-channel Born term)

$$V^C_{ij} = V^C_{ij}(D,F)$$

Formalism: Effective Chiral Lagrangian



- **Next to leading order (NLO),** just considering the **contact term**

$$\begin{aligned}
\mathcal{L}_{\phi B}^{(2)} = \; & b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_F\langle \bar{B}[\chi_+, B]\rangle + b_0\langle \bar{B}B\rangle\langle\chi_+\rangle + d_1\langle \bar{B}\{u_\mu,[u^\mu,B]\}\rangle \\
& + d_2\langle \bar{B}[u_\mu,[u^\mu,B]]\rangle + d_3\langle \bar{B}u_\mu\rangle\langle u^\mu B\rangle + d_4\langle \bar{B}B\rangle\langle u^\mu u_\mu\rangle \\
& - \frac{g_1}{8M_N^2}\langle \bar{B}\{u_\mu,[u_\nu,\{D^\mu,D^\nu\}B]\}\rangle - \frac{g_2}{8M_N^2}\langle \bar{B}[u_\mu,[u_\nu,\{D^\mu,D^\nu\}B]]\rangle \\
& - \frac{g_3}{8M_N^2}\langle \bar{B}u_\mu\rangle\langle[u_\nu,\{D^\mu,D^\nu\}B]\rangle - \frac{g_4}{8M_N^2}\langle \bar{B}\{D^\mu,D^\nu\}B\rangle\langle u_\mu u_\nu\rangle \\
& - \frac{h_1}{4}\langle \bar{B}[\gamma^\mu,\gamma^\nu]Bu_\mu u_\nu\rangle - \frac{h_2}{4}\langle \bar{B}[\gamma^\mu,\gamma^\nu]u_\mu[u_\nu,B]\rangle - \frac{h_3}{4}\langle \bar{B}[\gamma^\mu,\gamma^\nu]u_\mu\{u_\nu,B\}\rangle \\
& - \frac{h_4}{4}\langle \bar{B}[\gamma^\mu,\gamma^\nu]u_\mu\rangle\langle u_\nu, B\rangle + h.c.
\end{aligned}$$

New terms taken into account (the g_1–g_4 and h_1–h_4 terms)

- Contributions with g_3 get cancelled
- $b_0, b_D, b_F, d_1, d_2, d_3, d_4, g_1, g_2, g_4, h_1, h_2, h_3, h_4$ are not well established, so they should be treated as parameters of the model!

$$V_{ij}^{WT} = -\frac{N_i N_j}{4f^2} C_{ij} \left\{ (2\sqrt{s} - M_i - M_j)\chi_f^{\dagger s'} \chi_0^s + \frac{2\sqrt{s}+M_i+M_j}{(E_i+M_i)(E_j+M_j)} \chi_f^{\dagger s'} \left[\vec{q}_j \cdot \vec{q}_i + i(\vec{q}_j \times \vec{q}_i)\cdot\vec{\sigma}\right] \chi_0^s \right\}$$

Contributes to S- and P-waves

$$V_{ij}^{NLO} = \frac{N_i N_j}{f^2}\left[D_{ij} - 2L_{ij} {q_j}^{\mu} {q_i}_{\mu} + \frac{1}{2M_N^2} g_{ij} ({p_i}^{\mu} {q_j}_{\mu} {p_i}^{\nu} {q_i}_{\nu} + {p_j}^{\mu} {q_j}_{\mu} {p_j}^{\nu} {q_i}_{\nu})\right]\Bigg(\chi_j^{\dagger s'} \chi_i^s$$
$$-\chi_j^{\dagger s'} \frac{\vec{q}_j\cdot\vec{q}_i + i(\vec{q}_j\times\vec{q}_i)\cdot\vec{\sigma}}{(E_i+M_i)(E_j+M_j)}\chi_i^s\Bigg) + \frac{N_i N_j}{f^2} h_{ij}\Bigg[-\Bigg(\frac{{q_j}_0 {q_i}^2}{E_i+M_i} + \frac{{q_i}_0 {q_j}^2}{E_j+M_j}$$
$$+\frac{{q_j}^2 {q_i}^2}{(E_i+M_i)(E_j+M_j)} + \frac{(\vec{q}_j\cdot\vec{q}_i)^2}{(E_i+M_i)(E_j+M_j)}\Bigg)\chi_j^{\dagger s'}\chi_i^s$$
$$+\left(\frac{{q_i}_0}{E_i+M_i} + \frac{{q_j}_0}{E_j+M_j}\right)\chi_j^{\dagger s'} \vec{q}_j\cdot\vec{q}_i \chi_i^s + \Bigg(\frac{{q_i}_0}{E_i+M_i} + \frac{{q_j}_0}{E_j+M_j} +$$
$$\frac{\vec{q}_j\cdot\vec{q}_i}{(E_i+M_i)(E_j+M_j)} - 1\Bigg) i\chi_j^{\dagger s'}(\vec{q}_j\times\vec{q}_i)\cdot\vec{\sigma}\chi_i^s\Bigg]$$

Contributes to S-, P- and D-waves

Formalism: Effective Chiral Lagrangian

$$V_{ij}^{D} = \frac{N_i N_j}{12 f^2} \sum_k \frac{C_{\bar{i}i,k}^{(\mathrm{Born})} C_{\bar{j}j,k}^{(\mathrm{Born})}}{s - M_k^2} \Big\{ (\sqrt{s} - M_k)(s + M_i M_j - \sqrt{s}(M_i + M_j)) \chi_j^{\dagger s'} \chi_i^{s}$$
$$+ \frac{(s + \sqrt{s}(M_i + M_j) + M_i M_j)(\sqrt{s} + M_k)}{(E_i + M_i)(E_j + M_j)} \chi_j^{\dagger s'} \left[\vec{q}_j \cdot \vec{q}_i + i(\vec{q}_j \times \vec{q}_i) \cdot \vec{\sigma} \right] \chi_i^{s} \Big\}$$

Contributes to S- and P-waves

$$V_{ij}^{C} = -\frac{N_i N_j}{12 f^2} \sum_k \frac{C_{\bar{j}k,i}^{(\mathrm{Born})} C_{\bar{i}k,j}^{(\mathrm{Born})}}{u - M_k^2} \Big\{ \big[u(\sqrt{s} + M_k) + \sqrt{s}(M_j(M_i + M_k) + M_i M_k)$$
$$- M_j (M_i + M_k)(M_i + M_j) - M_i^2 M_k \big] \chi_j^{\dagger s'} \chi_i^{s} + \big[u(\sqrt{s} - M_k) + \sqrt{s}(M_j(M_i + M_k) + M_i M_k)$$
$$+ M_j (M_i + M_k)(M_i + M_j) + M_i^2 M_k \big] \chi_j^{\dagger s'} \frac{\vec{q}_j \cdot \vec{q}_i + i(\vec{q}_j \times \vec{q}_i) \cdot \vec{\sigma}}{(E_i + M_i)(E_j + M_j)} \chi_i^{s} \Big\}$$

Contributes to S-, P-, D- , F-, …

- **The T-matrix in the CM system can be split into spin-nonflip and spin-flip parts:**

$$T_{ij} = \chi_j^{\dagger s'} [f(\sqrt{s},\theta) - i(\vec{\sigma}\cdot\hat{n}) g(\sqrt{s},\theta)] \chi_i^s$$

Where:

$$\hat{n} = \frac{\vec{q}_j \times \vec{q}_i}{|\vec{q}_j \times \vec{q}_i|}$$

$$f(\sqrt{s},\theta) = \sum_{l=0}^{\infty} f_l(\sqrt{s})\, P_l(cos\theta)$$

$$g(\sqrt{s},\theta) = \sum_{l=1}^{\infty} g_l(\sqrt{s})\, sin\theta\, \frac{dP_l(cos\theta)}{d(cos\theta)}$$

Expansion in Legendre polynomials

Amplitudes should be separated in the Bethe-Salpeter equation and redefined with a definite total angular momentum:

$$f_{l+}^{tree}(\sqrt{s}) = \frac{1}{2l+1}\left(f_l(\sqrt{s}) + l\, g_l(\sqrt{s})\right), \quad j = l + \tfrac{1}{2}$$

$$f_{l-}^{tree}(\sqrt{s}) = \frac{1}{2l+1}\left(f_l(\sqrt{s}) - (l+1)\, g_l(\sqrt{s})\right), \quad j = l - \tfrac{1}{2}$$

Finally, **unitarized amplitudes** can be obtained via the Bethe-Salpeter equation

$$f_{l\pm} = \left[1 - f^{tree}_{l\pm} G\right]^{-1} f^{tree}_{l\pm}$$

where *G* is the meson-baryon loop function (dimensional regularitzation)

$$G_l = \frac{2M_l}{(4\pi)^2}\left\{ a_l(\mu) + \ln\frac{M_l^2}{\mu^2} + \frac{m_l^2 - M_l^2 + s}{2s}\ln\frac{m_l^2}{M_l^2} + \frac{q_{\rm cm}}{\sqrt{s}}\ln\left[\frac{(s+2\sqrt{s}q_{\rm cm})^2 - (M_l^2 - m_l^2)^2}{(s-2\sqrt{s}q_{\rm cm})^2 - (M_l^2 - m_l^2)^2}\right]\right\}$$

subtraction constants for the dimensional regularization scale $\mu = 1GeV$ in all k channels.

Total cross section:

$J^P = 1/2-$ $J^P = 3/2+$ $J^P = 1/2+$ $J^P = 5/2-$ $J^P = 3/2-$

$$\sigma_{ij} = \frac{M_i\, M_j\, q_j}{4\,\pi\, s\, q_i}\left[|f_0|^2 + 2|f_{1+}|^2 + |f_{1-}|^2 + 3|f_{2+}|^2 + 2|f_{2-}|^2\right]$$

S-wave P-wave D-wave

Fitting parameters:

- Decay constant f *partially constrained:* $1.12 f_\pi^{exp} \leq f \leq 1.26\, f_\pi^{exp}$, f_π^{exp}=93 *MeV*
- Axial vector couplings *D, F* *we impose:* $g_A = D + F = 1.26$
- 14 coefficients of the NLO lagrangian terms $b_0, b_D, b_F, d_1, d_2, d_3, d_4, g_1, g_2, g_4, h_1, h_2, h_3, h_4$
- 6 subtracting constants (isospín symmetry):

$$a_{K^-p} = a_{\bar{K}^0 n} = a_{\bar{K}N}$$
$$a_{\pi\Lambda}$$
$$a_{\pi^+\Sigma^-} = a_{\pi^-\Sigma^+} = a_{\pi^0\Sigma^0} = a_{\pi\Sigma}$$
$$a_{\eta\Lambda}$$
$$a_{\eta\Sigma}$$
$$a_{K^+\Xi^-} = a_{K^0\Xi^0} = a_{K\Xi}$$

Formalism: Fitting procedure

Observable	Points	Observable	Points
$\sigma_{K^-p\to K^-p}$	23	$\sigma_{K^-p\to \bar{K}^0 n}$	9
$\sigma_{K^-p\to \pi^0\Lambda}$	3	$\sigma_{K^-p\to \pi^0\Sigma^0}$	3
$\sigma_{K^-p\to \pi^-\Sigma^+}$	20	$\sigma_{K^-p\to \pi^+\Sigma^-}$	28
$\sigma_{K^-p\to \eta\Sigma^0}$	9	$\sigma_{K^-p\to \eta\Lambda}$	49
$\sigma_{K^-p\to K^+\Xi^-}$	46	$\sigma_{K^-p\to K^0\Xi^0}$	29
γ	1	ΔE_{1s}	1
R_n	1	Γ_{1s}	1
R_c	1		

Branching ratios:

$$\gamma = \frac{\Gamma(K^-p\to\pi^+\Sigma^-)}{\Gamma(K^-p\to\pi^-\Sigma^+)} = 2.36\pm0.04$$

$$R_n = \frac{\Gamma(K^-p\to\pi^0\Lambda)}{\Gamma(K^-p\to \text{neutral states})} = 0.664\pm0.011$$

$$R_c = \frac{\Gamma(K^-p\to\pi^+\Sigma^-,\pi^-\Sigma^+)}{\Gamma(K^-p\to \text{inelastic channels})} = 0.189\pm0.015$$

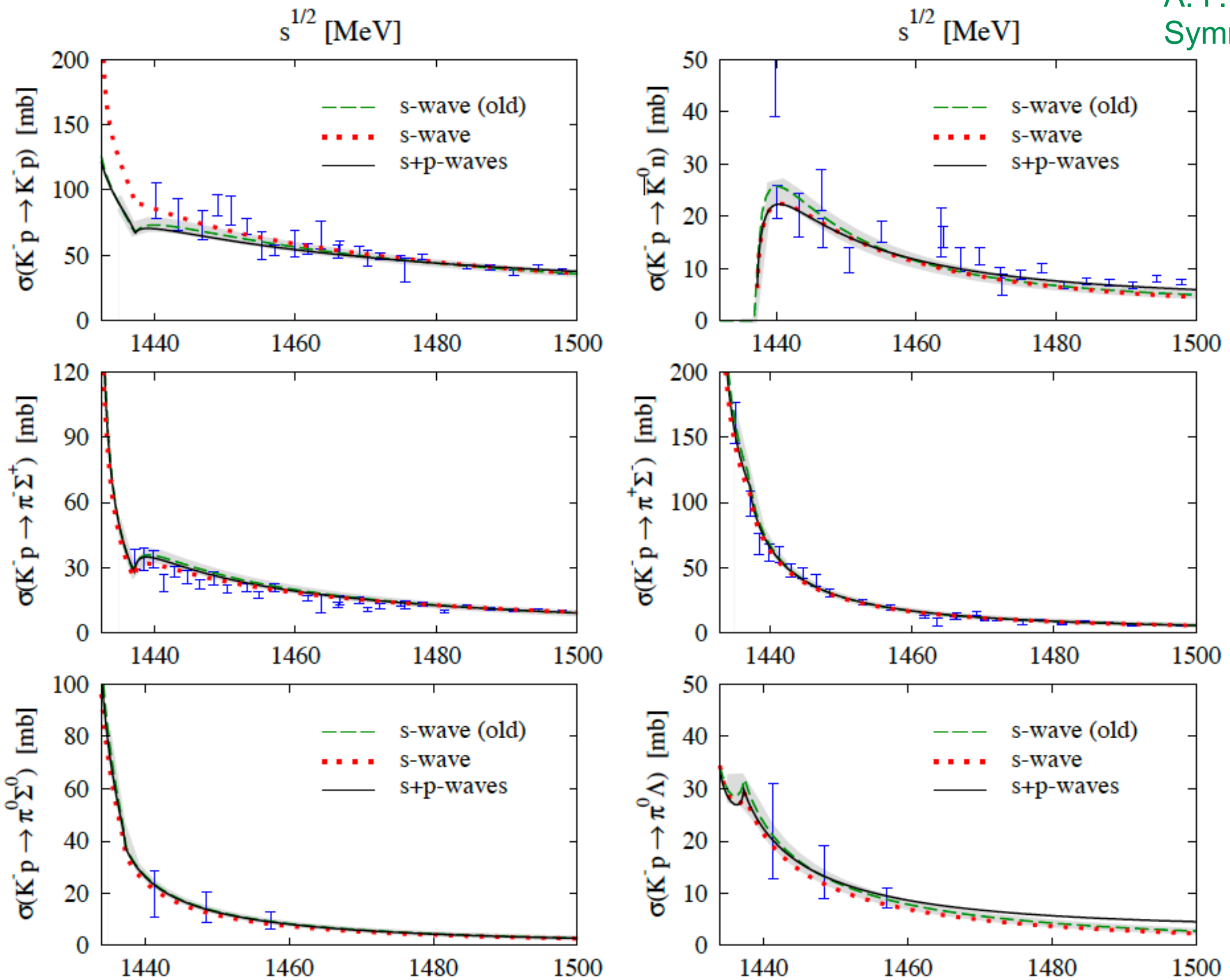
$$\chi^2_{\rm d.o.f} = \frac{\sum_{k=1}^K n_k}{\left(\sum_{k=1}^K n_k - p\right)} \frac{1}{K}\sum_{k=1}^K \frac{\chi^2_k}{n_k}$$

3 models were constructed to tackle the role of P-wave contribution of the scattering amplitude:

- s-wave (old): fit of the s-wave projection from the WT+Born+NLO (without g_i, h_i terms) kernels
  A. F., V. Magas, A. Ramos, Phys. Rev. C 99 (2019) 035211
- s-wave: fit of the s-wave projection from the WT+Born+NLO kernels
- s+p-wave: the s- and the p-wave projections from the WT+Born+NLO kernels are taken into account in the fit.

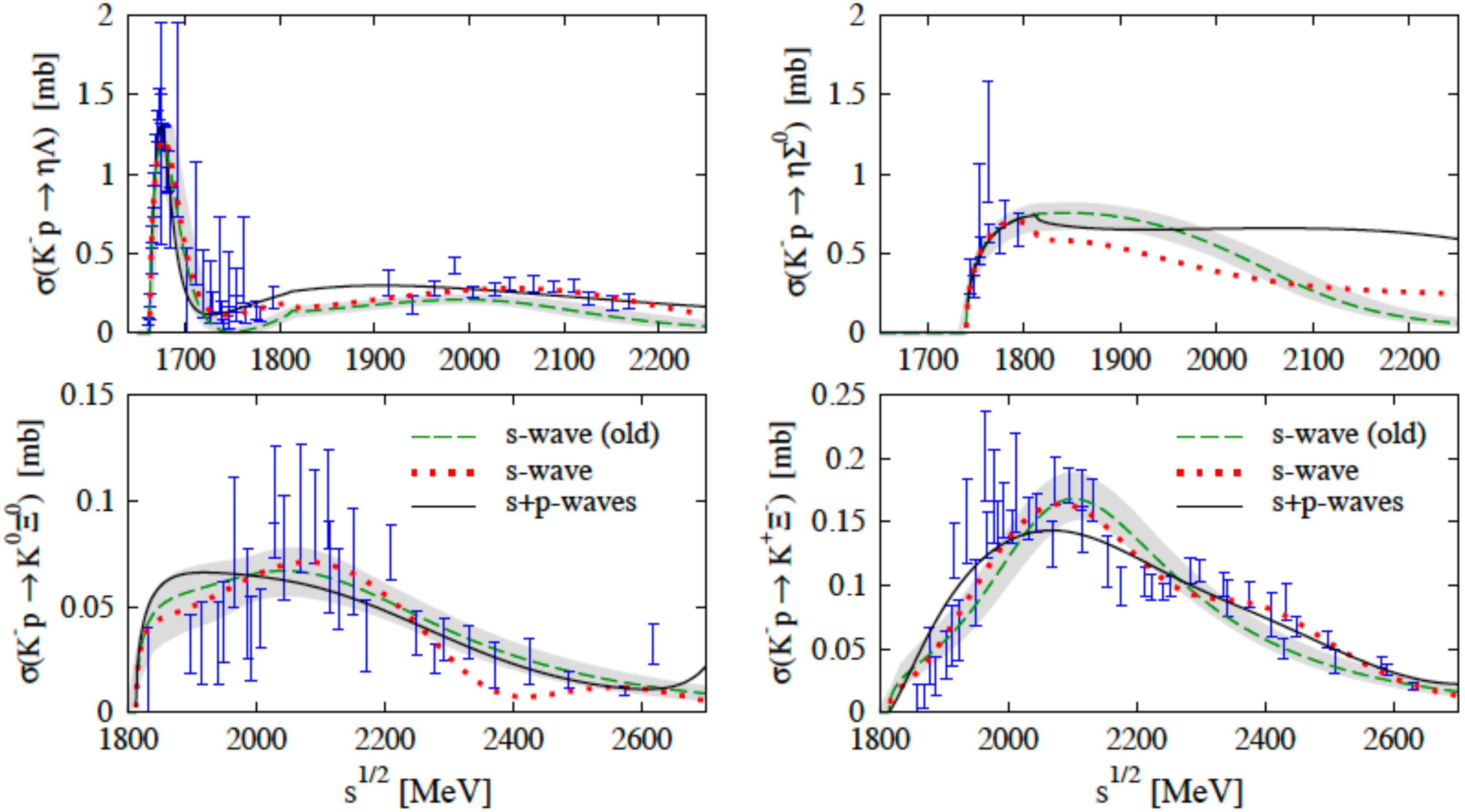
Results: cross sections of the classical processes

A. F., D. Gazda, V. Magas, A. Ramos, Symmetry 13 (2021) 8, 1434



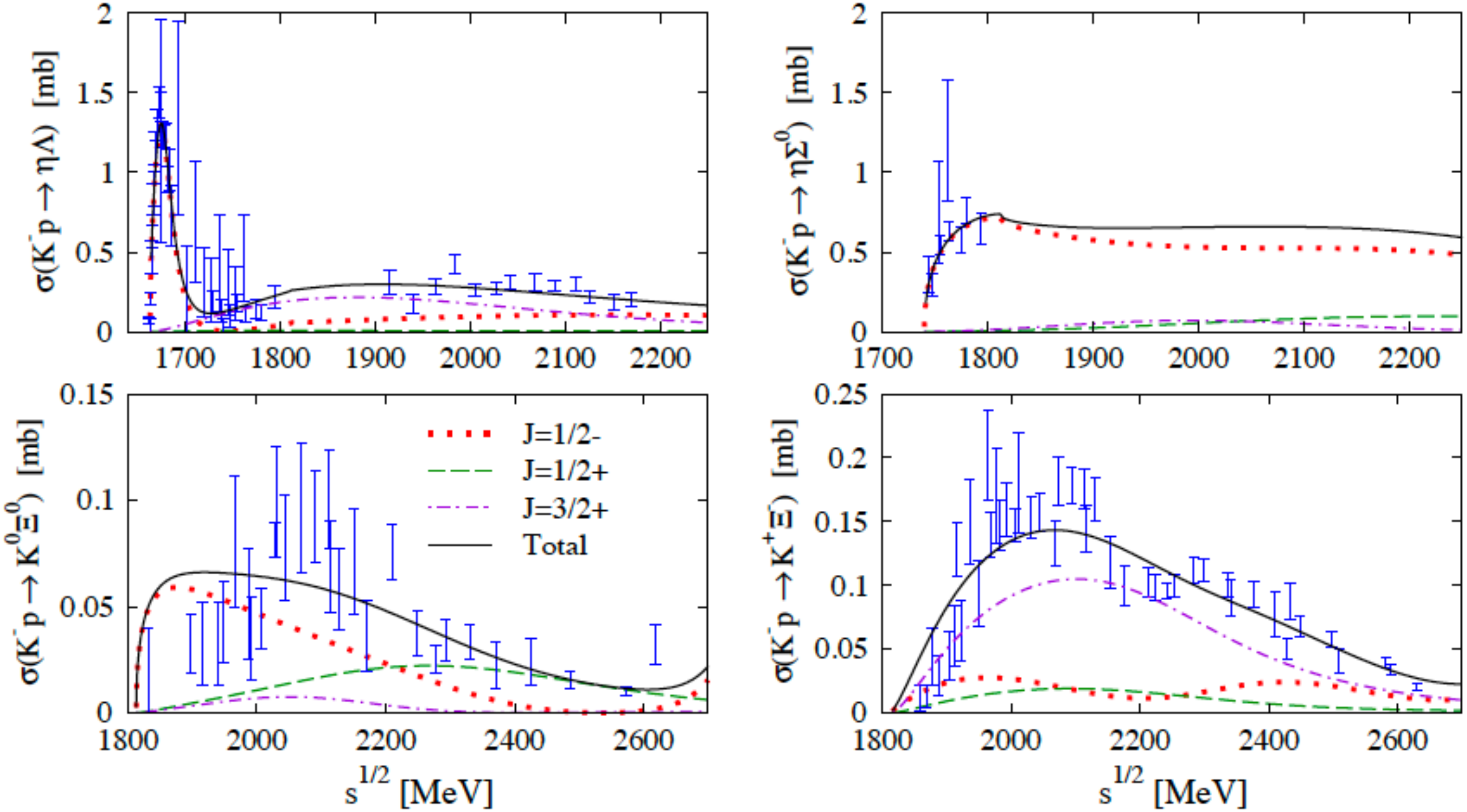
Results: cross sections of higher mass channels

A. F., D. Gazda, V. Magas, A. Ramos, Symmetry 13 (2021) 8, 1434



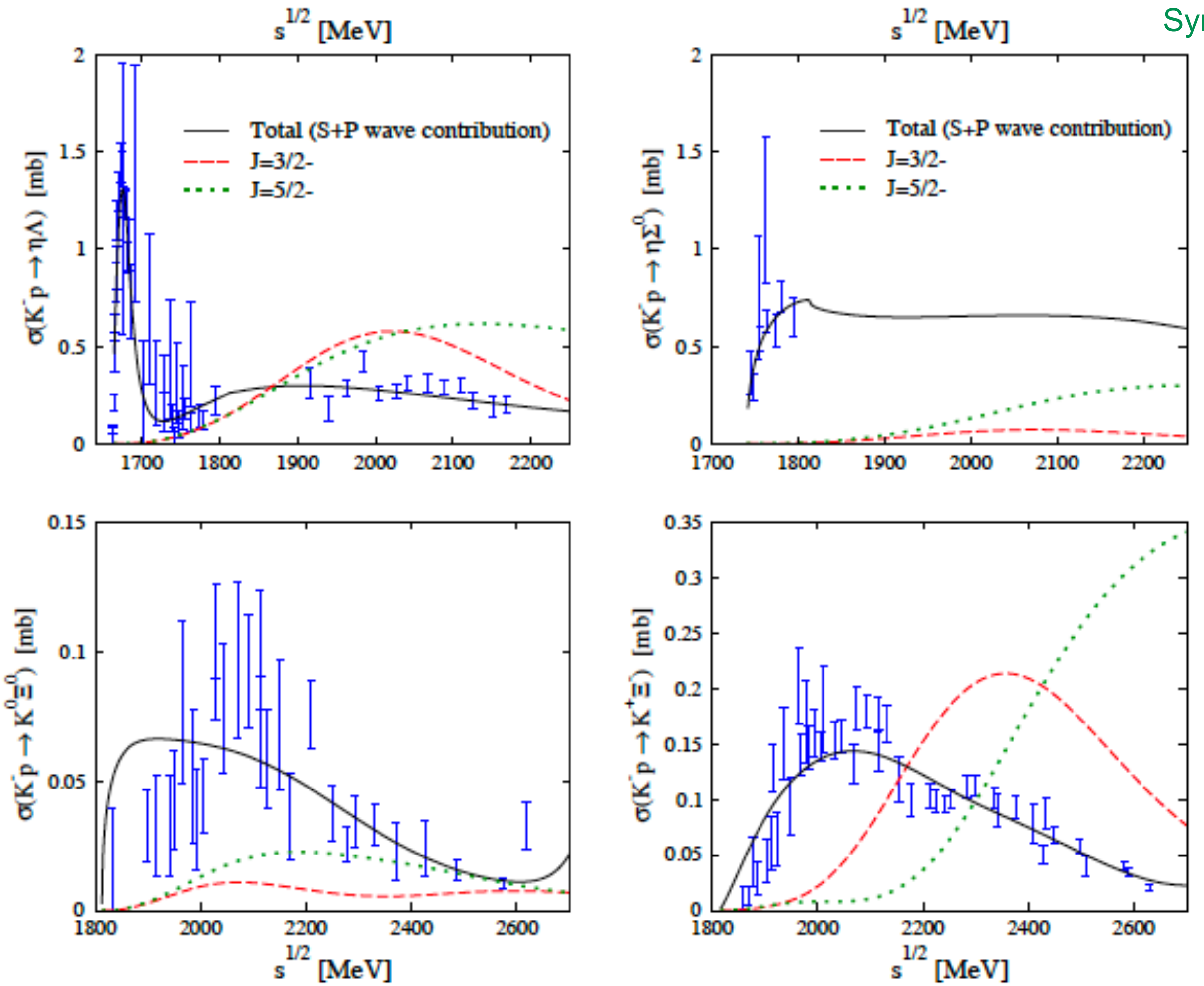
Results: Breakdown into J^P contributions of the cross sections

A. F., D. Gazda, V. Magas, A. Ramos,
Symmetry 13 (2021) 8, 1434



Results: D-wave contributions keeping s+p-wave parametrization

A. F., D. Gazda, V. Magas, A. Ramos, Symmetry 13 (2021) 8, 1434



CONCLUSIONS

- We have shown that a notable fraction of the $K^- p \longrightarrow K^+\Xi^-, K^0\Xi^0, \eta\Lambda, \eta\Sigma^0$ **cross sections** comes from **p-wave contribution** of the chiral Lagrangian. Their incorporation into these production processes is completely needed.

- g_i, h_i terms are also essential given the strong dependence of the p-wave (d-wave) contribution on them.

- Not the end of the story, d-wave contributions have proved to be a non-negligible ingredient in these higher energy processes. The inclusion of all the previous elements requires to consider many more data points (higher energies, differential cross sections, etc…)

WORK IN PROGRESS…

Thank you for your attention

2 fits were performed:

- Unitarized scattering amplitude from Chiral Lagrangian (***WT+Born+NLO***)

$$V_{ij} = V_{ij}^{WT} + V_{ij}^{D} + V_{ij}^{C} + V_{ij}^{NLO} \Longrightarrow T = (1 - VG)^{-1}V \Longrightarrow T_{ij}$$

Only S-wave contribution is taken into account

2 new fits were performed:

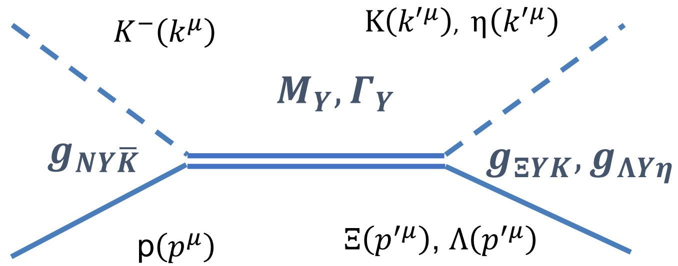
- Unitarized scattering amplitude from Chiral Lagrangian (***WT+Born+NLO***)

$$V_{ij} = V_{ij}^{WT} + V_{ij}^{D} + V_{ij}^{C} + V_{ij}^{NLO} \Longrightarrow T = (1 - VG)^{-1} V \Longrightarrow T_{ij}$$

- Unitarized scattering amplitude from Chiral Lagrangian + resonant contributions (***WT+Born+NLO+RES***)

a) Inclusion of high spin and high mass resonances allows us to study the stability of the NLO parameters.

b) It also simulates the contributions of higher angular momenta of the other channels via rescattering in the energy regime above $K\Xi$ threshold.



Y= $\Lambda(1890), \Sigma(2030), \Sigma(2250)$

Sharov, Korotkikh, Lanskoy, EPJA 47 (2011) 109
Jackson, Oh, Haberzettl and Nakayama,Phys. Rev. C 91, 065208 (2015)
Feijoo, Magas, Ramos, Phys. Rev. C 92, 015206 (2015)

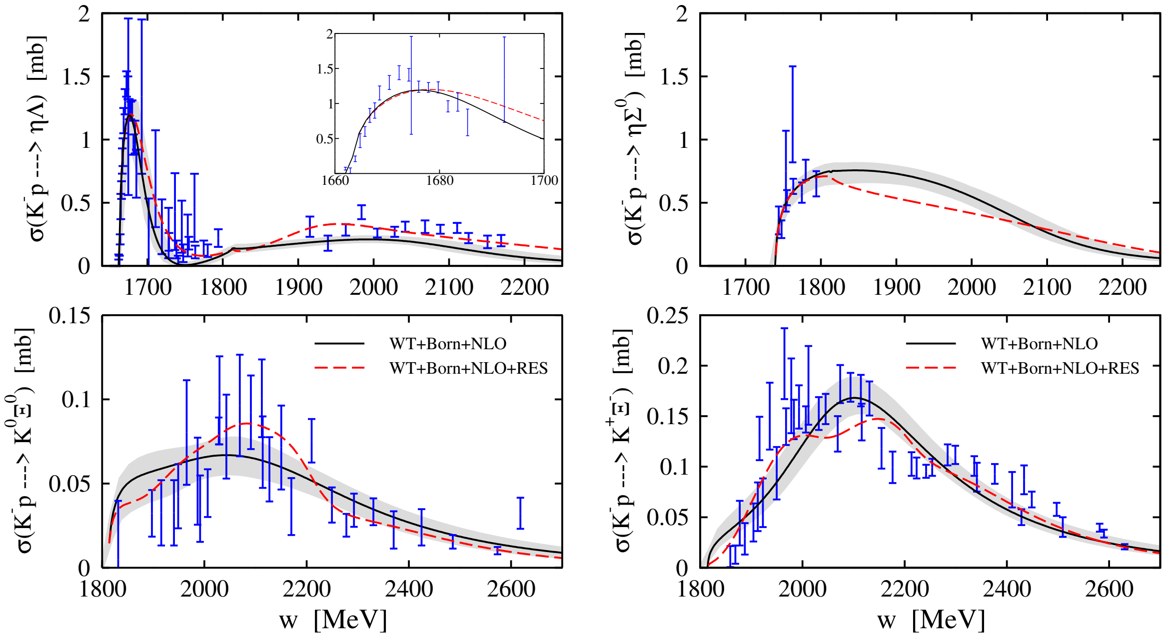
Only for $K^- p \longrightarrow K\Xi$ reactions:

$$T_{ij}^{tot} = T_{ij}^{BS} + \frac{1}{\sqrt{4M_p M_\Xi}} \sum_{J^P} T_{ij}^{J^P}, \; J^P = 3/2^+, 5/2^-, 7/2^+$$

Only for $K^- p \longrightarrow \eta\Lambda$ reaction:

$$T_{ij}^{tot} = T_{ij}^{BS} + \frac{1}{\sqrt{4M_p M_\Lambda}} T_{ij}^{3/2^+}$$

Total cross sections



WT+Born+NLO+RES improves the description of the experimental data

- ***Inclusion of higher partial waves could play similar role***
- ***What about dynamically generating resonances with such contributions?***
- ***A more accurate extension of the $\bar{K}N$ interaction to higher momenta could be useful for $\bar{K}$ bound mesons in nuclei***

Results: values of the parameters involved in the different fits

	s+p-waves	s-wave	s-wave (old)
$a_{\bar{K}N}$ (10^{-3})	1.322 ± 0.643	2.105 ± 0.378	$1.268^{+0.096}_{-0.096}$
$a_{\pi\Lambda}$ (10^{-3})	10.000 ± 19.962	9.999 ± 18.190	$-6.114^{+0.045}_{-0.055}$
$a_{\pi\Sigma}$ (10^{-3})	1.247 ± 1.913	3.413 ± 1.609	$0.684^{+0.429}_{-0.572}$
$a_{\eta\Lambda}$ (10^{-3})	-3.682 ± 6.541	-4.585 ± 1.322	$-0.666^{+0.080}_{-0.140}$
$a_{\eta\Sigma}$ (10^{-3})	5.528 ± 2.571	3.017 ± 0.027	$8.004^{+2.282}_{-0.978}$
$a_{K\Xi}$ (10^{-3})	-2.077 ± 0.931	0.997 ± 0.038	$-2.508^{+0.396}_{-0.297}$
f/f_π	1.110 ± 0.068	1.042 ± 0.003	$1.196^{+0.013}_{-0.007}$
b_0 (GeV^{-1})	0.394 ± 0.087	-0.079 ± 0.005	$0.129^{+0.032}_{-0.032}$
b_D (GeV^{-1})	0.206 ± 0.064	0.112 ± 0.008	$0.120^{+0.010}_{-0.009}$
b_F (GeV^{-1})	0.303 ± 0.058	0.117 ± 0.011	$0.209^{+0.022}_{-0.026}$
d_1 (GeV^{-1})	0.246 ± 0.077	-0.848 ± 0.039	$0.151^{+0.021}_{-0.027}$
d_2 (GeV^{-1})	0.120 ± 0.057	0.634 ± 0.036	$0.126^{+0.012}_{-0.009}$
d_3 (GeV^{-1})	0.270 ± 0.082	0.463 ± 0.047	$0.299^{+0.020}_{-0.024}$
d_4 (GeV^{-1})	0.723 ± 0.085	-0.678 ± 0.032	$0.249^{+0.027}_{-0.033}$
g_1 (GeV^{-1})	0.105 ± 0.114	-1.211 ± 0.049	-
g_2 (GeV^{-1})	-0.024 ± 0.056	0.764 ± 0.041	-
g_4 (GeV^{-1})	0.301 ± 0.097	-1.030 ± 0.036	-
h_1 (GeV^{-1})	0.540 ± 1.070	-0.533 ± 0.373	-
h_2 (GeV^{-1})	0.387 ± 0.483	-1.979 ± 0.229	-
h_3 (GeV^{-1})	0.472 ± 0.821	7.452 ± 0.159	-
h_4 (GeV^{-1})	-0.291 ± 0.832	-2.547 ± 0.319	-
D	0.701 ± 0.102	0.899 ± 0.004	$0.700^{+0.064}_{-0.144}$
F	0.510 ± 0.056	0.510 ± 0.017	$0.510^{+0.060}_{-0.050}$
$\chi^2_{d.o.f.}$	0.86	0.77	1.14

Results: threshold observables and pole content

	γ	R_n	R_c	$a_p(K^-p \to K^-p)$	ΔE_{1s}	Γ_{1s}
s+p-waves	2.36	0.188	0.662	$-0.70 + i\,0.81$	297	532
s-wave	2.40	0.179	0.665	$-0.64 + i\,0.83$	280	560
s-wave (old)	$2.36^{+0.03}_{-0.03}$	$0.188^{+0.010}_{-0.011}$	$0.659^{+0.005}_{-0.002}$	$-0.65^{+0.02}_{-0.08} + i\,0.88^{+0.02}_{-0.05}$	288^{+23}_{-8}	588^{+9}_{-40}
Exp.	2.36 ± 0.04	0.189 ± 0.015	0.664 ± 0.011	$(-0.66 \pm 0.07) + i\,(0.81 \pm 0.15)$	283 ± 36	541 ± 92

s+p-waves

$J^P = \frac{1}{2}^-$	$(I,S) = (0,-1)$			$(I,S) = (1,-1)$	
	$\Lambda(1405)$		$\Lambda(1670)$		Σ^*
M [MeV]	1399.71	1423.30	1674.05	M [MeV]	1590.97
Γ [MeV]	118.50	58.02	31.08	Γ [MeV]	480.14
	$\lvert g_i \rvert$	$\lvert g_i \rvert$	$\lvert g_i \rvert$		$\lvert g_i \rvert$
$\pi\Sigma$	3.45	2.57	0.37	$\pi\Lambda$	1.24
$\bar{K}N$	3.19	3.70	0.40	$\pi\Sigma$	1.36
$\eta\Lambda$	0.66	0.85	1.33	$\bar{K}N$	1.79
$K\Xi$	0.72	0.52	3.65	$\eta\Sigma$	0.37
				$K\Xi$	0.57

s-wave

$J^P = \frac{1}{2}^-$	$(I,S) = (0,-1)$			$(I,S) = (1,-1)$	
	$\Lambda(1405)$		$\Lambda(1670)$		Σ^*
M [MeV]	1364.13	1419.54	1679.16	M [MeV]	–
Γ [MeV]	190.58	39.14	62.36	Γ [MeV]	–
	$\lvert g_i \rvert$	$\lvert g_i \rvert$	$\lvert g_i \rvert$		$\lvert g_i \rvert$
$\pi\Sigma$	3.00	1.59	0.26	$\pi\Lambda$	–
$\bar{K}N$	2.41	3.14	0.64	$\pi\Sigma$	–
$\eta\Lambda$	0.71	1.21	1.78	$\bar{K}N$	–
$K\Xi$	1.86	2.14	4.50	$\eta\Sigma$	–
				$K\Xi$	–

s-wave (old)

$J^P = \frac{1}{2}^-$	$(I,S) = (0,-1)$			$(I,S) = (1,-1)$	
	$\Lambda(1405)$		$\Lambda(1670)$		Σ^*
M [MeV]	1419^{+16}_{-22}	1420^{+15}_{-21}	1675^{+10}_{-11}	M [MeV]	1701^{+16}_{-1}
Γ [MeV]	142^{+48}_{-62}	54^{+36}_{-22}	62^{+8}_{-14}	Γ [MeV]	340^{+4}_{-14}
	$\lvert g_i \rvert$	$\lvert g_i \rvert$	$\lvert g_i \rvert$		$\lvert g_i \rvert$
$\pi\Sigma$	3.40	2.31	0.47	$\pi\Lambda$	1.96
$\bar{K}N$	2.98	3.51	0.59	$\pi\Sigma$	0.47
$\eta\Lambda$	1.10	1.26	1.74	$\bar{K}N$	1.21
$K\Xi$	0.65	0.36	3.71	$\eta\Sigma$	0.36
				$K\Xi$	0.98

Formalism: Fitting procedure

Observable	Points	Observable	Points
$\sigma_{K^-p\to K^-p}$	245	$\sigma_{K^-p\to \bar{K}^0 n}$	317
$\sigma_{K^-p\to \pi^0\Lambda}$	225	$\sigma_{K^-p\to \pi^0\Sigma^0}$	125
$\sigma_{K^-p\to \pi^-\Sigma^+}$	198	$\sigma_{K^-p\to \pi^+\Sigma^-}$	213
$\sigma_{K^-p\to \eta\Sigma^0}$	9	$\sigma_{K^-p\to \eta\Lambda}$	106
$\sigma_{K^-p\to K^+\Xi^-}$	54	$\sigma_{K^-p\to K^0\Xi^0}$	30
γ	1	ΔE_{1s}	1
R_n	1	Γ_{1s}	1
R_c	1		

All available points for this energy range (1527 experimental points):

- A. Baldini et al., *Numerical Data and Func- tional Relationships in Science and Technology, Group I, Vol. 12*, edited by H. Schopper (Springer, Berlin, 1988).
- Adams, Nuc. Phys. B96, 54-56, (1975).
- A. Starostin *et al.* (Crystal Ball Collaboration), Phys. Rev. C **64**, 055205 (2001).
- R. J. Nowak *et al.*, Nucl. Phys. B **139**, 61 (1978).
- D. N. Tovee *et al.*, Nucl. Phys. B **33**, 493 (1971).
- M. Bazzi *et al.*, Phys. Lett. B **704**, 113 (2011).

Total cross section:

$$\sigma_{ij} = \frac{M_i\, M_j\, q_j}{4\,\pi\, s\, q_i}\left[|f_0|^2 + 2|f_{1+}|^2 + |f_{1-}|^2 + 3|f_{2+}|^2 + 2|f_{2-}|^2\right]$$

Branching ratios:

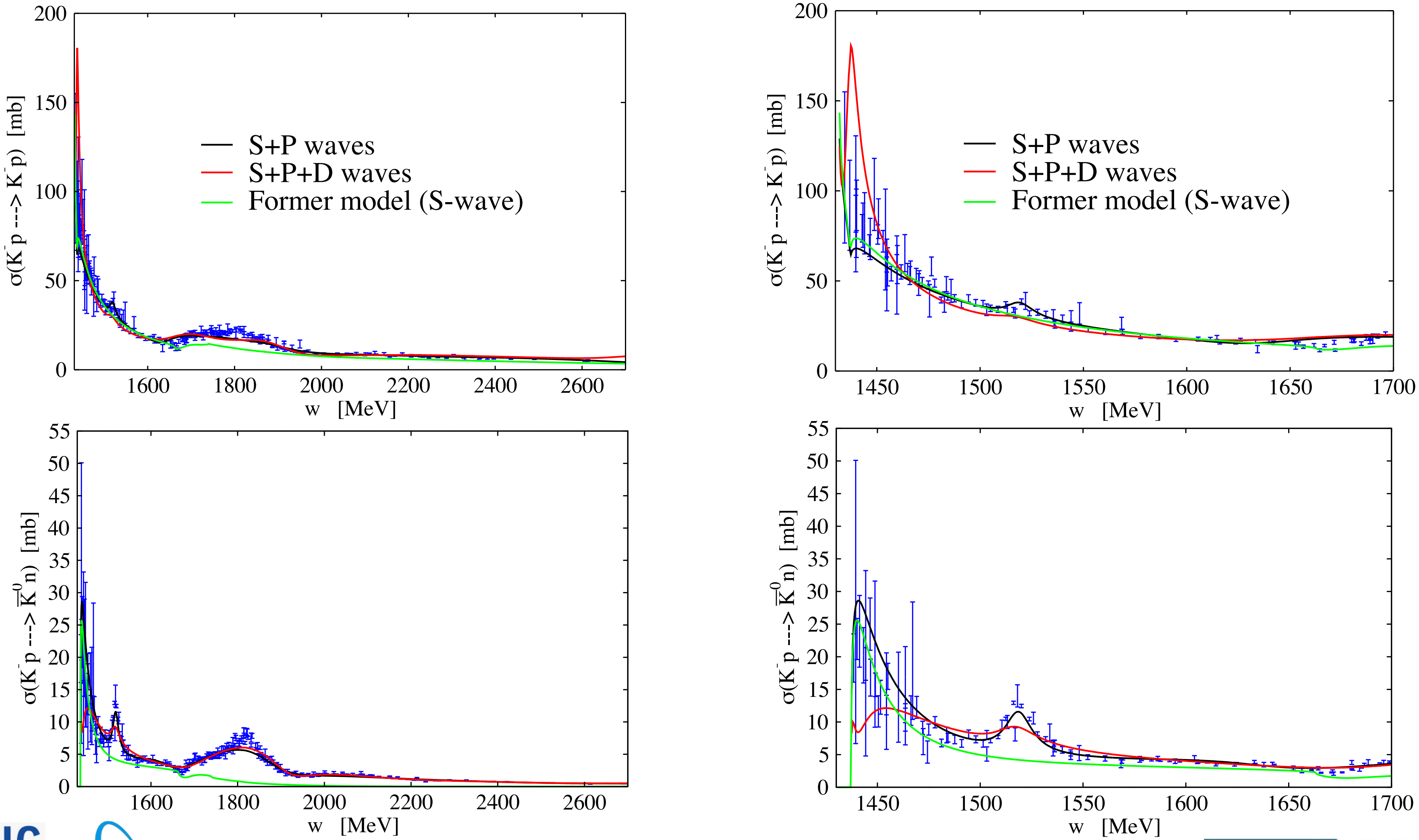
$$\gamma = \frac{\Gamma(K^-p\to\pi^+\Sigma^-)}{\Gamma(K^-p\to\pi^-\Sigma^+)} = 2.36\pm 0.04$$

$$R_n = \frac{\Gamma(K^-p\to\pi^0\Lambda)}{\Gamma(K^-p\to \text{neutral states})} = 0.664\pm 0.011$$

$$R_c = \frac{\Gamma(K^-p\to\pi^+\Sigma^-,\pi^-\Sigma^+)}{\Gamma(K^-p\to \text{inelastic channels})} = 0.189\pm 0.015$$

$$\chi^2_{\text{d.o.f}} = \frac{\sum_{k=1}^K n_k}{\left(\sum_{k=1}^K n_k - p\right)}\frac{1}{K}\sum_{k=1}^{K}\frac{\chi_k^2}{n_k}$$

Preliminary results!!!

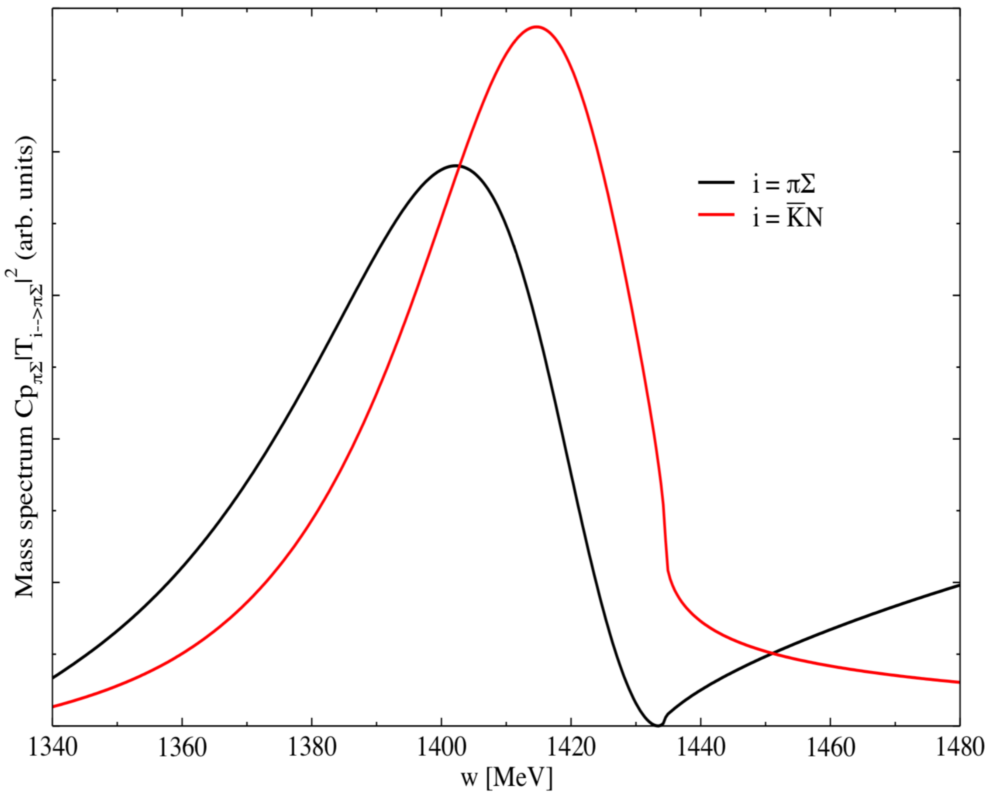


Motivation: Evolution of the model

Pole content (*WT+Born+NLO*)

$0^- \oplus \frac{1}{2}^+$ interaction in $(I,S)=(0,-1)$ sector				
Pole	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\lvert g_{\eta\Lambda}\rvert$	$\lvert g_{K\Xi}\rvert$
$\Lambda(1405)$				
$1419^{+16}_{-22} - i\ 71^{+24}_{-31}$	3.40	2.98	1.10	0.65
$1420^{+15}_{-21} - i\ 27^{+18}_{-11}$	2.31	3.51	1.26	0.36
$\Lambda(1670)$				
$1675^{+10}_{-11} - i\ 31^{+4}_{-7}$	0.47	0.59	1.74	3.71

$0^- \oplus \frac{1}{2}^+$ interaction in $(I,S)=(1,-1)$ sector					
Pole	$\lvert g_{\pi\Lambda}\rvert$	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\lvert g_{\eta\Sigma}\rvert$	$\lvert g_{K\Xi}\rvert$
Σ^*					
$1701^{+16}_{-1} - i\ 170^{+2}_{-7}$	1.96	0.47	1.21	0.36	0.98



Pole content (*WT+Born+NLO*)

$0^- \oplus \frac{1}{2}^+$ interaction in $(I,S)=(0,-1)$ sector				
Pole	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\lvert g_{\eta\Lambda}\rvert$	$\lvert g_{K\Xi}\rvert$
$\Lambda(1405)$				
$1419^{+16}_{-22} - i\ 71^{+24}_{-31}$	3.40	2.98	1.10	0.65
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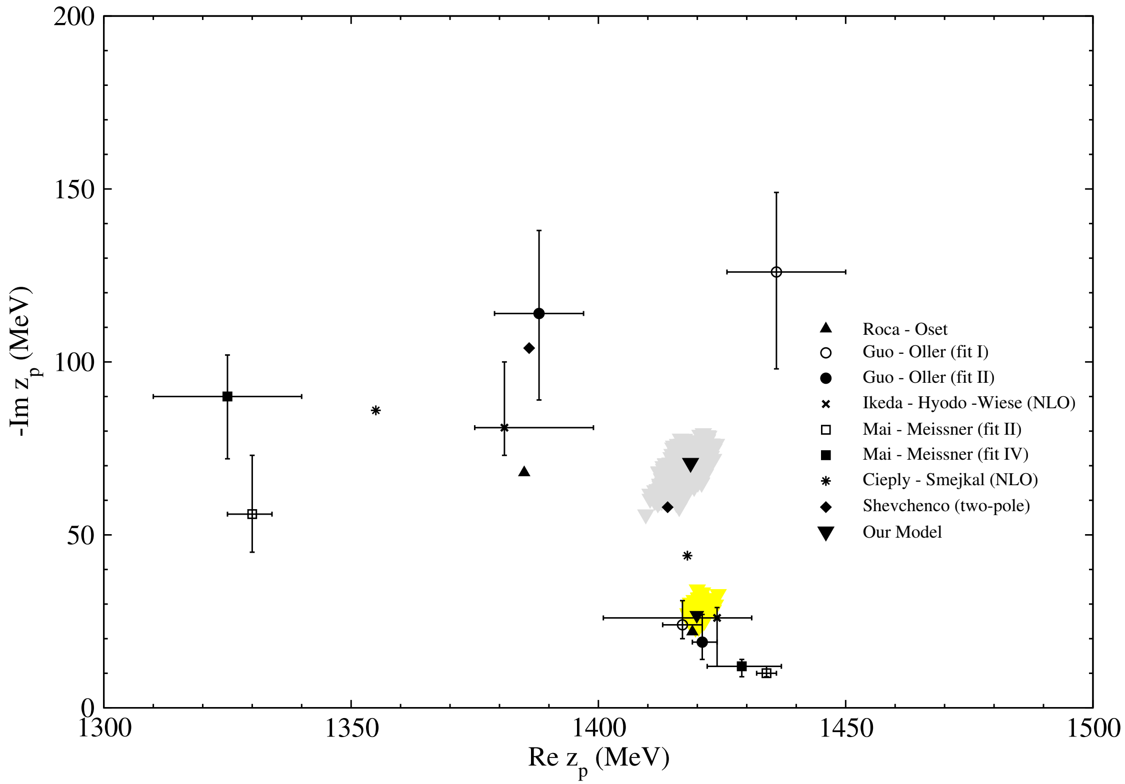
$\Lambda(1670)$				
$1675^{+10}_{-11} - i\ 31^{+4}_{-7}$	0.47	0.59	1.74	3.71

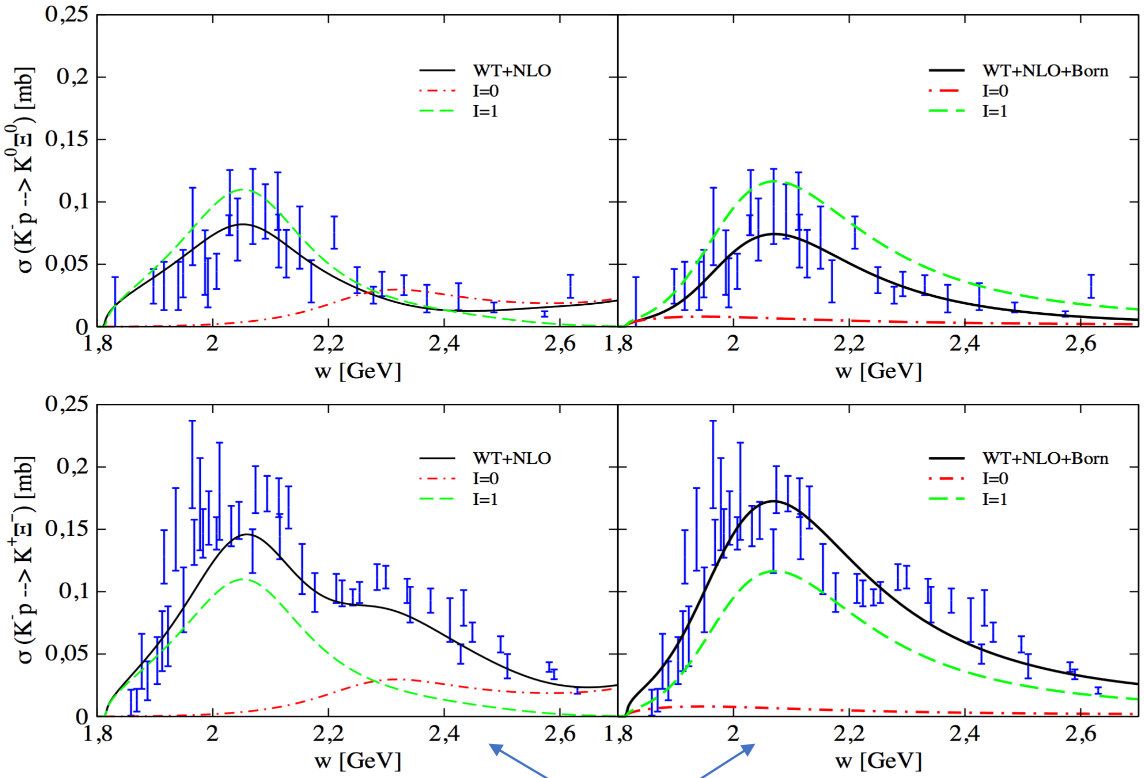
$0^- \oplus \frac{1}{2}^+$ interaction in $(I,S)=(1,-1)$ sector					
Pole	$\lvert g_{\pi\Lambda}\rvert$	$\lvert g_{\pi\Sigma}\rvert$	$\lvert g_{\bar{K}N}\rvert$	$\lvert g_{\eta\Sigma}\rvert$	$\lvert g_{K\Xi}\rvert$
Σ^*					
$1701^{+16}_{-1} - i\ 170^{+2}_{-7}$	1.96	0.47	1.21	0.36	0.98



Threshold observables obtained from our fits and from other recent studies

	γ	R_n	R_c	$a_p(K^-p \to K^-p)$	ΔE_{1s}	Γ_{1s}
Ikeda-Hyodo-Weise (NLO) [23]	2.37	0.19	0.66	$-0.70 + \mathrm{i}\,0.89$	306	591
Guo-Oller (fit I + II) [25]	$2.36^{+0.24}_{-0.23}$	$0.188^{+0.028}_{-0.029}$	$0.661^{+0.012}_{-0.011}$	$(-0.69 \pm 0.16) + \mathrm{i}\,(0.94 \pm 0.11)$	308 ± 56	619 ± 73
Mizutani et al (Model s) [26]	2.40	0.189	0.645	$-0.69 + \mathrm{i}\,0.89$	304	591
Mai-Meissner (fit 4) [29]	$2.38^{+0.09}_{-0.10}$	$0.191^{+0.013}_{-0.017}$	$0.667^{+0.006}_{-0.005}$		288^{+34}_{-32}	572^{+39}_{-38}
Cieply-Smejkal (NLO) [76]	2.37	0.191	0.660	$-0.73 + \mathrm{i}\,0.85$	310	607
Shevchenko (two-pole Model) [77]	2.36			$-0.74 + \mathrm{i}\,0.90$	308	602
WT+Born+NLO	$2.36^{+0.03}_{-0.03}$	$0.188^{+0.010}_{-0.011}$	$0.659^{+0.005}_{-0.002}$	$-0.65^{+0.02}_{-0.08} + \mathrm{i}\,0.88^{+0.02}_{-0.05}$	288^{+23}_{-8}	588^{+9}_{-40}
WT+NLO+Born+RES	2.36	0.189	0.661	$-0.64 + \mathrm{i}\,0.87$	283	587
Exp.	2.36 ± 0.04	0.189 ± 0.015	0.664 ± 0.011	$(-0.66 \pm 0.07) + \mathrm{i}\,(0.81 \pm 0.15)$	283 ± 36	541 ± 92

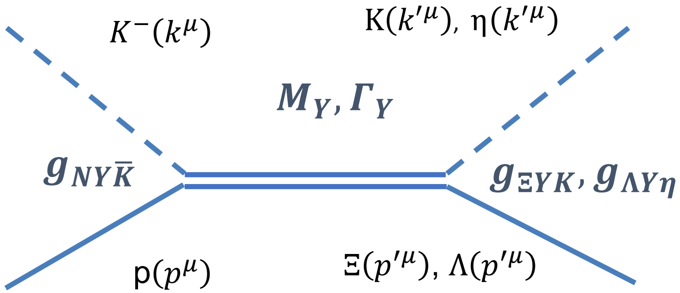
Motivation: Evolution of the model 2



The two models predict very different isospin contributions

➔ Isospin-filter observables are needed!

Isospin filtering processes: Inclusion of Hyperonic resonances



$$\bar{K}N \longrightarrow Y \longrightarrow K\Xi, \eta\Lambda$$

Y= $\Lambda(1890), \Sigma(2030), \Sigma(2250)$

Only for $K^- p \longrightarrow K\Xi$ reactions:

$$T_{ij}^{tot} = T_{ij}^{BS} + \frac{1}{\sqrt{4M_p M_\Xi}} \sum_{J^P} T_{ij}^{J^P}, \quad J^P = 3/2^+, 5/2^-, 7/2^+$$

Only for $K^- p \longrightarrow \eta\Lambda$ reaction:

$$T_{ij}^{tot} = T_{ij}^{BS} + \frac{1}{\sqrt{4M_p M_\Lambda}} T_{ij}^{3/2^+}$$

K. Nakayama, Y. Oh, H. Habertzettl, Phys. Rev. C74, 035205 (2006)

K. Shing Man, Y. Oh, K. Nakayama,, Phys. Rev. C83, 055201 (2011)

$\Lambda(1890)$, $J^P = \frac{3}{2}^+$

$$\mathcal{L}_{BYK}^{3/2^\pm}(q) = \mathrm{i}\frac{g_{BY_{3/2}K}}{m_K}\bar{B}\Gamma^{(\pm)}Y_{3/2}^{\mu}\partial_\mu K + H.c.$$

$$T_{ij}^{3/2^+}(s', s) = F_{3/2}(k, k')\,\bar{u}_j^{s'}(p')\gamma_5 k'_{\beta_1} S_{3/2}(q) k_{\beta_2}\gamma_5 u_i^s(p)$$

$\Sigma(2030)$, $J^P = \frac{7}{2}^+$

$$\mathcal{L}_{BYK}^{7/2^\pm}(q) = -\frac{g_{BY_{7/2}K}}{m_K^3}\bar{B}\Gamma^{(\mp)}Y_{7/2}^{\mu\nu\alpha}\partial_\mu\partial_\nu\partial_\alpha K + H.c.$$

$$T_{ij}^{7/2^+}(s', s) = F_{7/2}(k, k')\,\bar{u}_j^{s'}(p')k'_{\beta_1}k'_{\beta_2}k'_{\beta_3}S_{7/2}(q)k^{\alpha_1}k^{\alpha_2}k^{\alpha_3}u_i^s(p)$$

$\Sigma(2250)$, $J^P = \frac{5}{2}^-$

$$\mathcal{L}_{BYK}^{5/2^\pm}(q) = \mathrm{i}\frac{g_{BY_{5/2}K}}{m_K^2}\bar{B}\Gamma^{(\pm)}Y_{5/2}^{\mu\nu}\partial_\mu\partial_\nu K + H.c.$$

$$T_{ij}^{5/2^-}(s', s) = F_{5/2}(k, k')\,\bar{u}_j^{s'}(p')k'_{\beta_1}k'_{\beta_2}S_{5/2}(q)k^{\alpha_1}k^{\alpha_2}u_i^s(p)$$

$$F_J(k, k') = \frac{g_{BY_J M}\, g_{NY_J\bar{K}}}{m_K^{2J-1}} \exp\left(-\vec{k}^2/\Lambda_J^2\right)\exp\left(-\vec{k'}^2/\Lambda_J^2\right)$$

FORM FACTORS

Sharov, Korotkikh, Lanskoy, EPJA 47 (2011) 109

Isospin filtering processes: Results

Fitting parameters:

	WT+Born+NLO	WT+NLO+Born+RES
$a_{\bar{K}N}$ (10^{-3})	$1.268^{+0.096}_{-0.096}$	1.517 ± 0.208
$a_{\pi\Lambda}$ (10^{-3})	$-6.114^{+0.045}_{-0.055}$	-2.624 ± 13.926
$a_{\pi\Sigma}$ (10^{-3})	$0.684^{+0.429}_{-0.572}$	2.146 ± 1.174
$a_{\eta\Lambda}$ (10^{-3})	$-0.666^{+0.080}_{-0.140}$	0.756 ± 1.215
$a_{\eta\Sigma}$ (10^{-3})	$8.004^{+2.282}_{-0.978}$	10.105 ± 3.660
$a_{K\Xi}$ (10^{-3})	$-2.508^{+0.396}_{-0.297}$	-2.013 ± 0.743
f/f_π	$1.196^{+0.013}_{-0.007}$	1.180 ± 0.028
b_0 (GeV^{-1})	$0.129^{+0.032}_{-0.032}$	-0.071 ± 0.016
b_D (GeV^{-1})	$0.120^{+0.010}_{-0.009}$	0.128 ± 0.015
b_F (GeV^{-1})	$0.209^{+0.022}_{-0.026}$	0.271 ± 0.022
d_1 (GeV^{-1})	$0.151^{+0.021}_{-0.027}$	0.144 ± 0.034
d_2 (GeV^{-1})	$0.126^{+0.012}_{-0.009}$	0.133 ± 0.011
d_3 (GeV^{-1})	$0.299^{+0.020}_{-0.024}$	0.405 ± 0.022
d_4 (GeV^{-1})	$0.249^{+0.027}_{-0.033}$	0.022 ± 0.020
D	$0.700^{+0.064}_{-0.144}$	0.700 ± 0.148
F	$0.510^{+0.060}_{-0.050}$	0.400 ± 0.110
$g_{\Lambda Y_{3/2}\eta} \cdot g_{NY_{3/2}\bar{K}}$	-	8.924 ± 11.790
$g_{\Xi Y_{3/2}K} \cdot g_{NY_{3/2}\bar{K}}$	-	6.200 ± 8.214
$g_{\Xi Y_{5/2}K} \cdot g_{NY_{5/2}\bar{K}}$	-	-3.881 ± 9.585
$g_{\Xi Y_{7/2}K} \cdot g_{NY_{7/2}\bar{K}}$	-	-14.306 ± 14.427
$\Lambda_{3/2}$ (MeV)	-	839.66 ± 406.68
$\Lambda_{5/2}$ (MeV)	-	541.31 ± 290.01
$\Lambda_{7/2}$ (MeV)	-	500.00 ± 426.82
$M_{Y_{3/2}}$ (MeV)	-	1910.00 ± 44.70
$M_{Y_{5/2}}$ (MeV)	-	2210.00 ± 39.07
$M_{Y_{7/2}}$ (MeV)	-	2040.00 ± 14.88
$\Gamma_{3/2}$ (MeV)	-	200.00 ± 120.31
$\Gamma_{5/2}$ (MeV)	-	150.00 ± 52.42
$\Gamma_{7/2}$ (MeV)	-	150.00 ± 43.12
$\chi^2_{\text{d.o.f.}}$	1.14	0.96

Naturally sized values for all

Very homogeneous and accurate values

16% improvement on the goodness of the fit