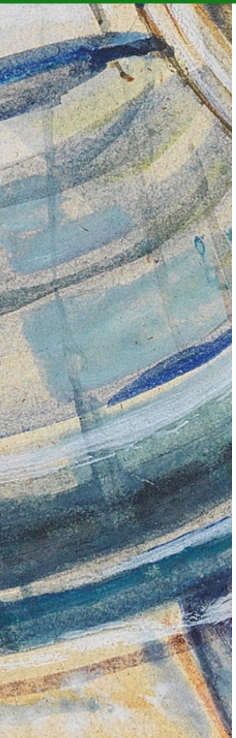
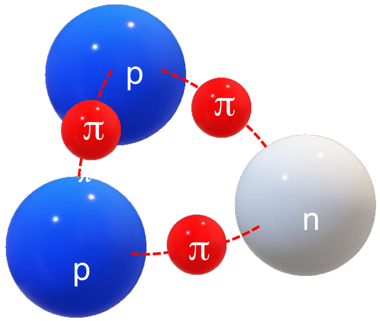
Evgeny Epelbaum, Ruhr University Bochum

HYP2022, June 27 — July 1, 2022
Prague, Czech Republic

Precise 3N interactions from chiral EFT: Where do we stand?



What has been learned about 3-body interactions for (better understood) non-strange systems?



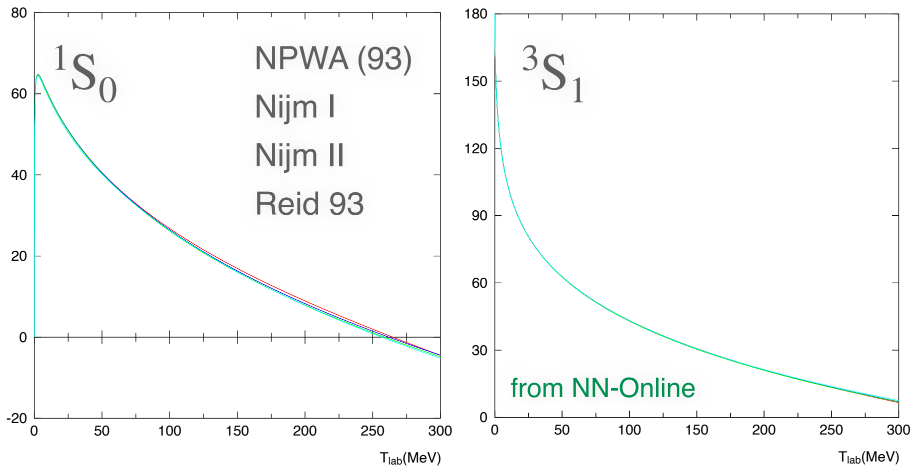
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
erc European Research Council

Nuclear forces

Since the 90-es, we know that:

— 2N force is easy to parametrize:
2 (isospin) × 6 spin-momentum operators

— after removing inconsistent data (~10% pp and ~30% np…), the rest of the data base can be described with $\chi^2/\text{datum} \sim 1$.



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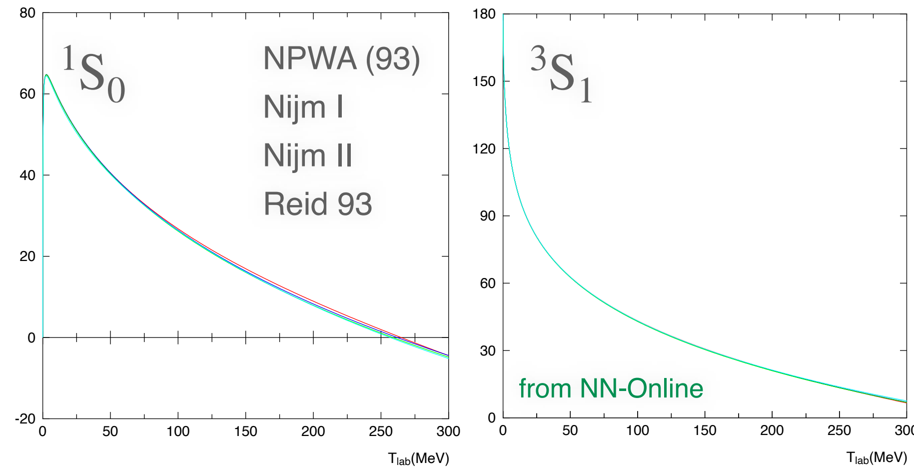
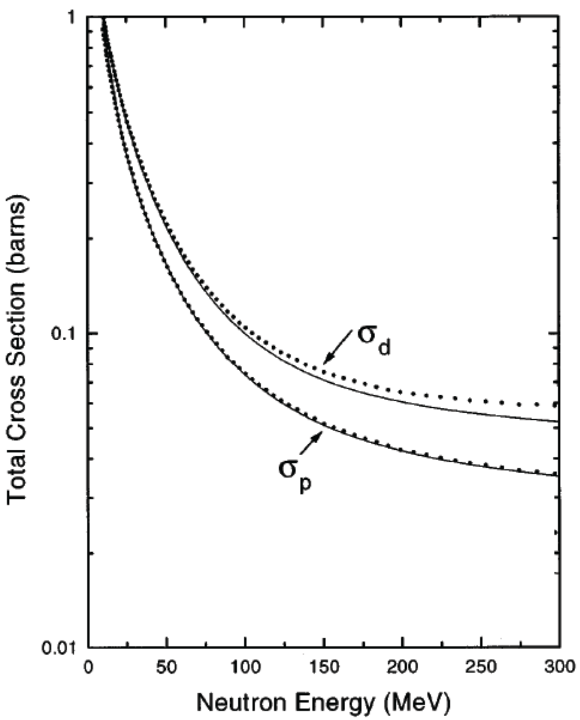
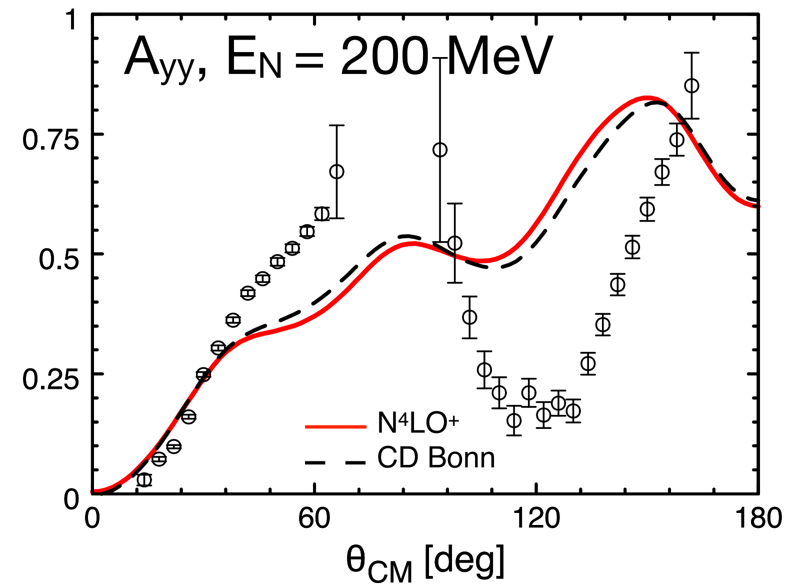




While the NN forces seem under control, large deviations show up for Nd scattering signaling the missing 3N forces



The 3NF challenge

The most general structure of a local, isospin-symmetric 3NF

Krebs, Gasparyan, EE ’13; Phillips, Schat ’13; EE, Gasparyan, Krebs, Schat ’15

Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
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$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1$
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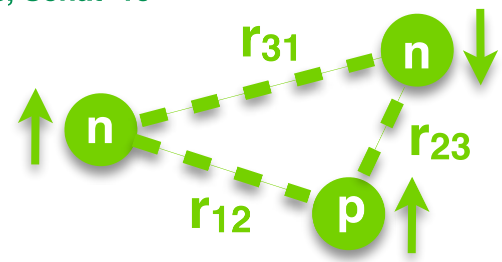

80 operators generated by all possible permutations of **20 structures**:

$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i \, F_i(q_1, q_2, q_3) + \text{permutations}$$

Nonlocal: 320 (!) operators

Topolnicki ’17

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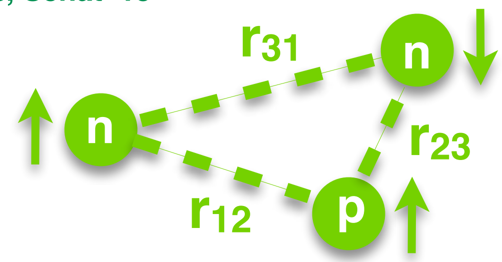

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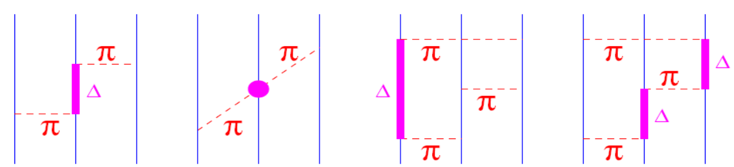
Topolnicki '17

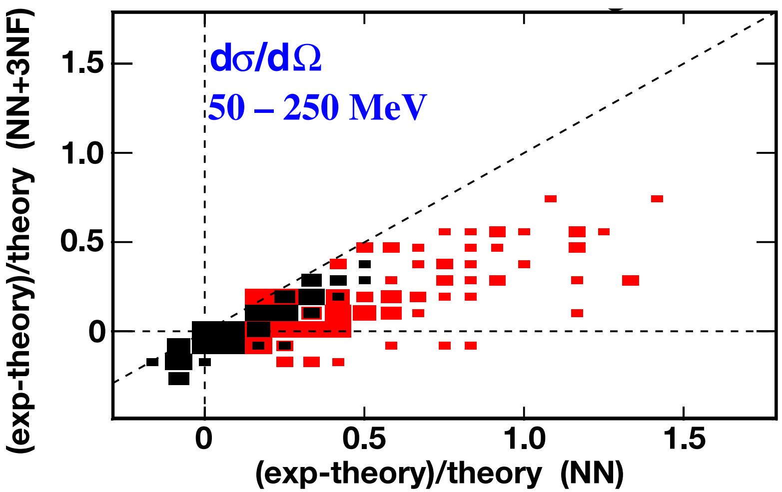
Phenomenological models:

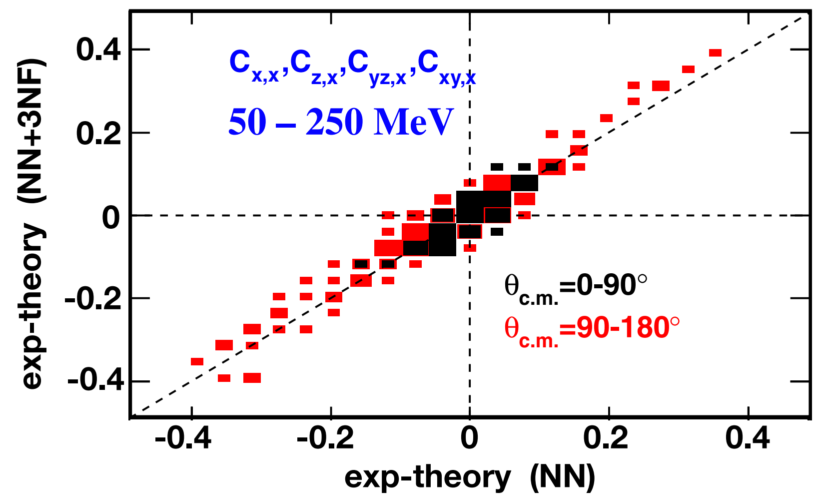
Fujita-Miyazawa, Tucson-Melbourne, Brasil, Urbana IX, Illinois, ...



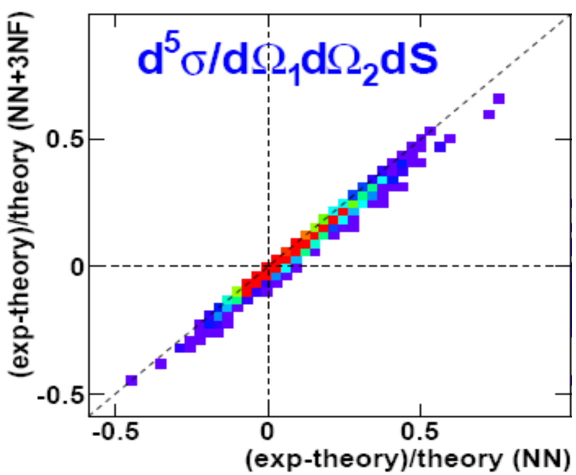
The 3NF challenge

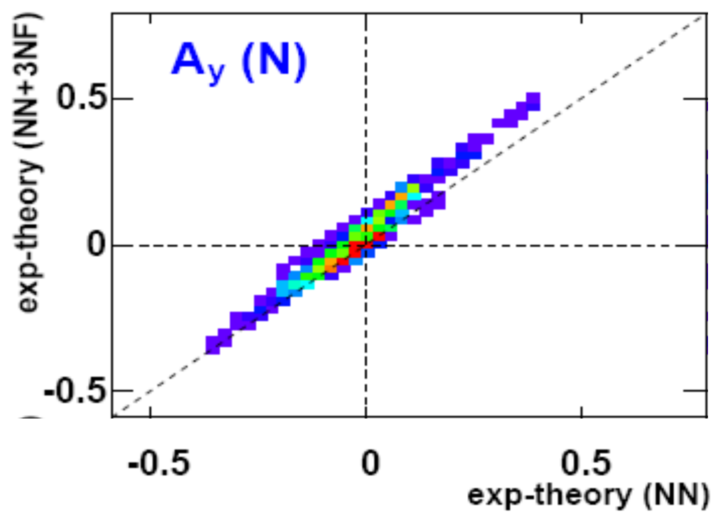
Elastic nucleon-deuteron scattering





Deuteron breakup reaction





from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (2012) 016301

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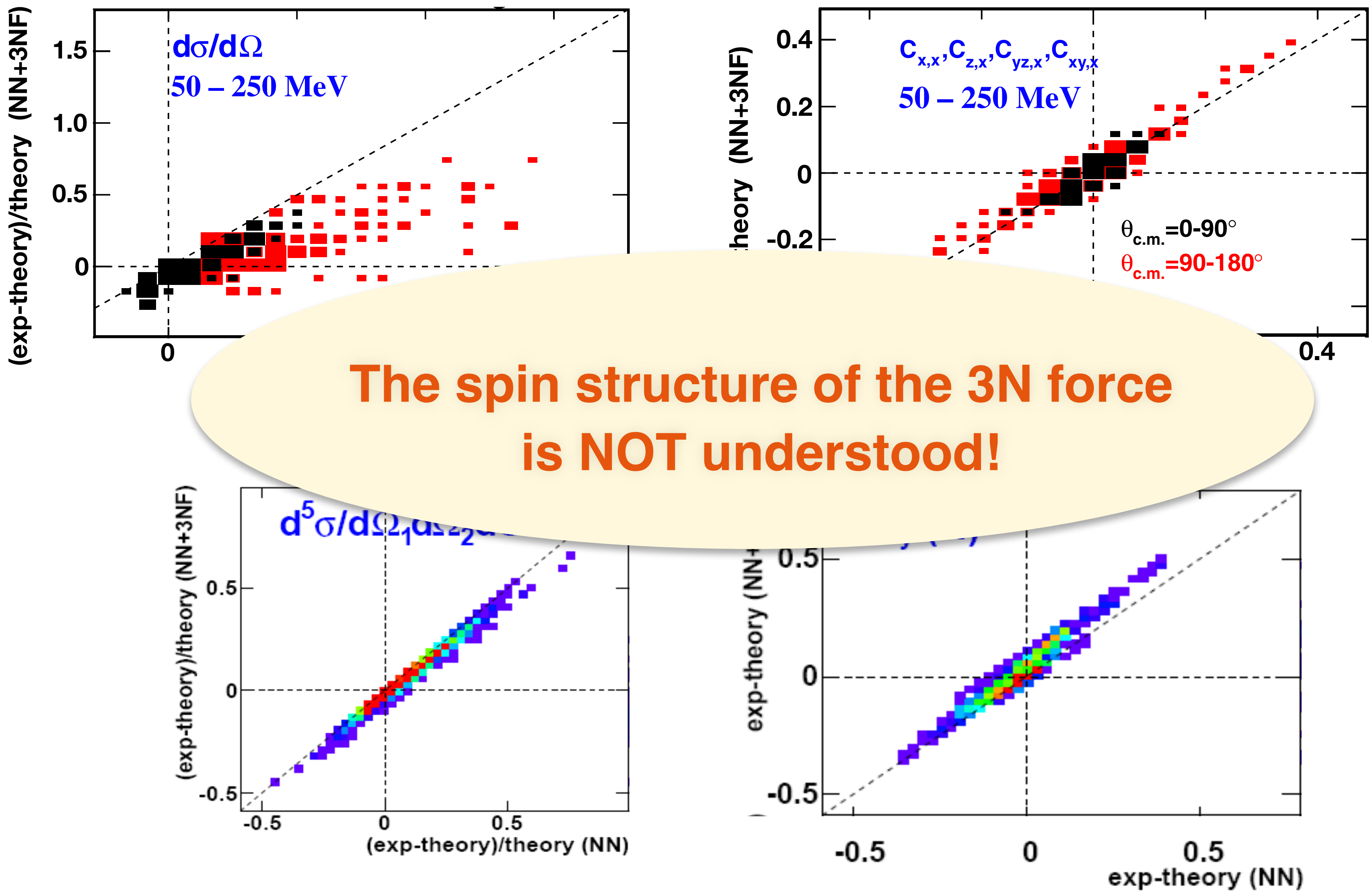


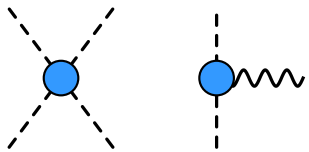
The spin structure of the 3N force is NOT understood!

from: Kalantar-Nayestanaki, EE, Messchendorp, Nogga, Rept. Prog. Phys. 75 (2012) 016301

Chiral Effective Field Theory

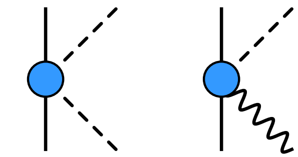
GB dynamics

Weinberg, Gasser, Leutwyler, ...



πN dynamics

Bernard-Kaiser-Meißner et al.



Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi}{\text{breakdown scale } \Lambda_b} \sim \frac{1}{4} \cdots \frac{1}{3}$$

Effective Lagrangian:

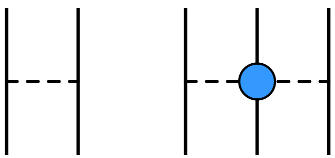
$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots ,$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

Nuclear forces

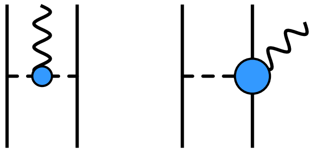
Weinberg, van Kolck, Kaiser, EGM, ...



Combined with ab-initio few-body methods,
provide first-principle approach to nuclear systems

Nuclear currents

Park et al, Bochum-Bonn, JLab-Pisa



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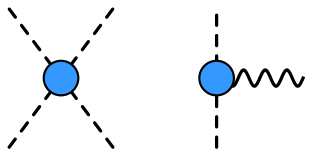

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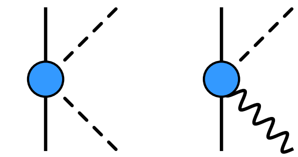

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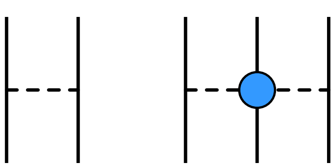

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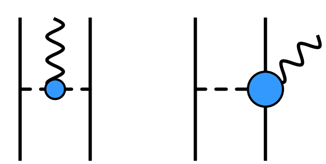

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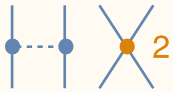
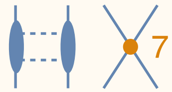
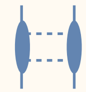
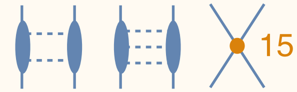
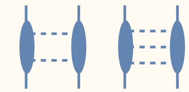
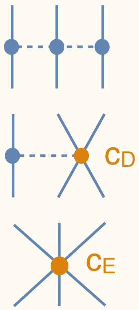
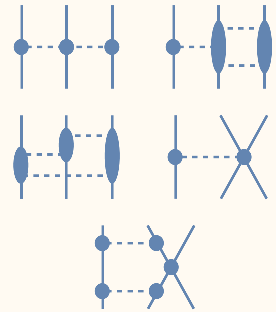
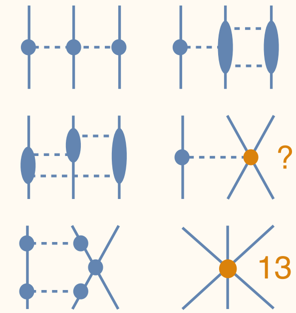
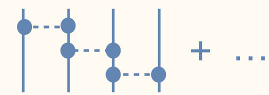
Goal: chiral EFT as a precision tool !

Chiral EFT expansion of the Hamiltonian

	LO (Q^0)	NLO (Q^2)	N^{2}LO (Q^3)	N^{3}LO (Q^4)	N^{4}LO (Q^5)
2NF					
3NF	–	–			
4NF	–	–	–		–

ACCURACY …but also complexity and the number of LECs…

- πN LECs taken from the Roy-Steiner analysis ⇒ long-range topologies are pure predictions
- Loop diagrams calculated using dimensional regularization
- Finite-cutoff EFT [Lepage '97]: NN scattering amplitude is rigorously proven to be renormalizable in the EFT sense Gasparyan, EE, PRC 105 (2022) 024001. Alternative (Λ-independent): talk by Xiu-Lei Ren…

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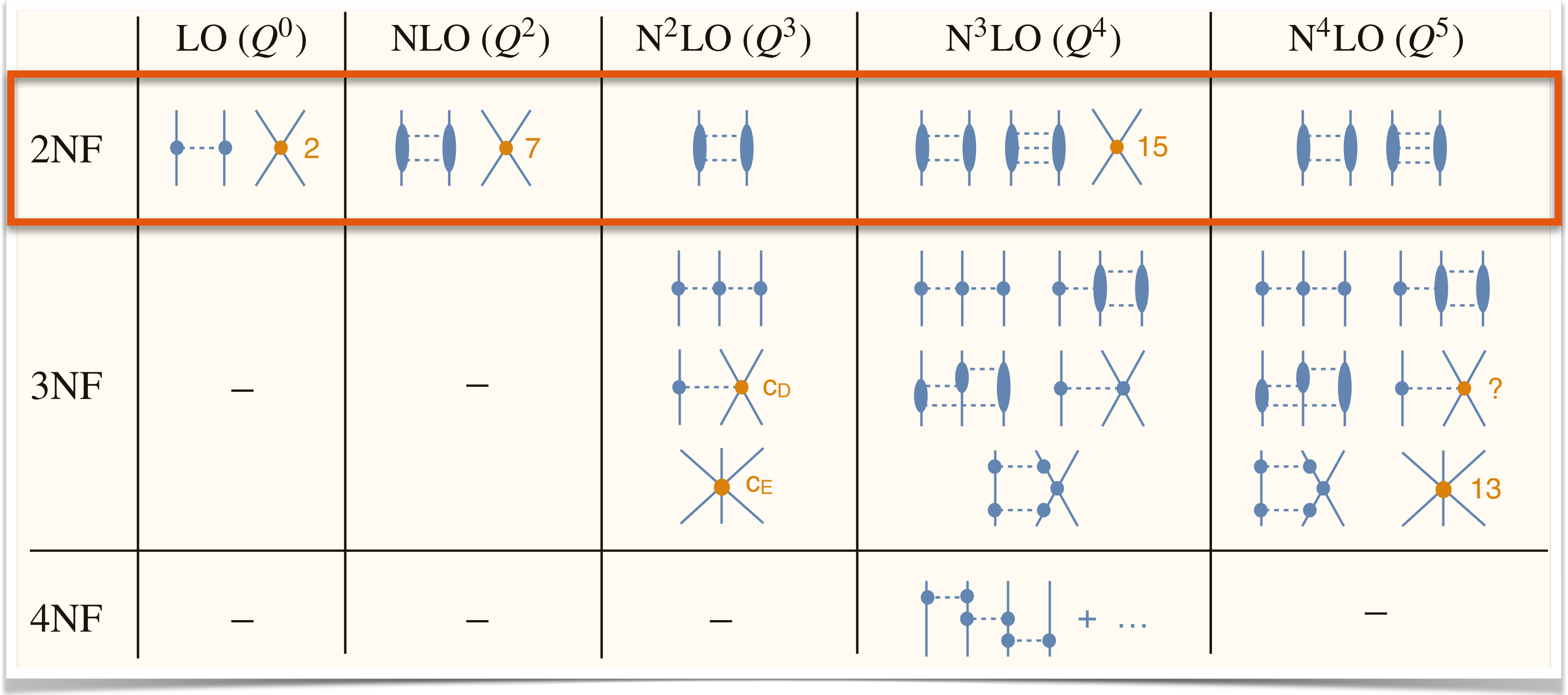

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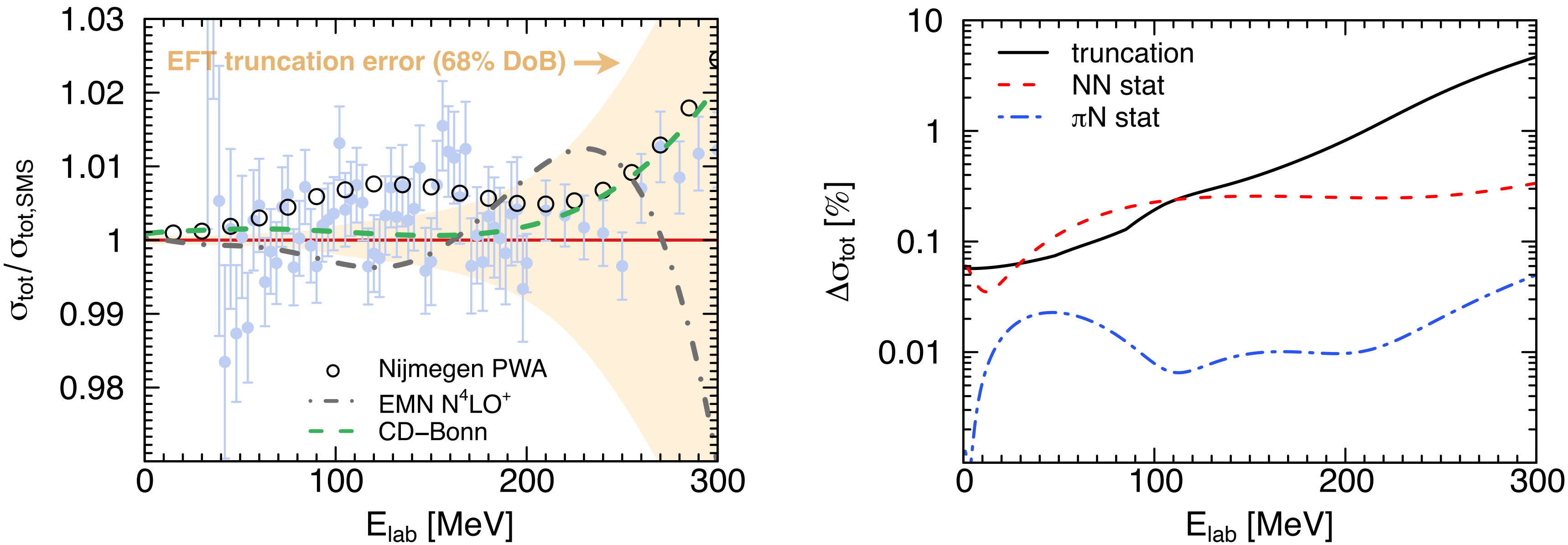
Chiral EFT for NN scattering

- First PWA of NN data up to π-production threshold in χEFT Reinert, Krebs, EE, PRL 126 (2021) 092501

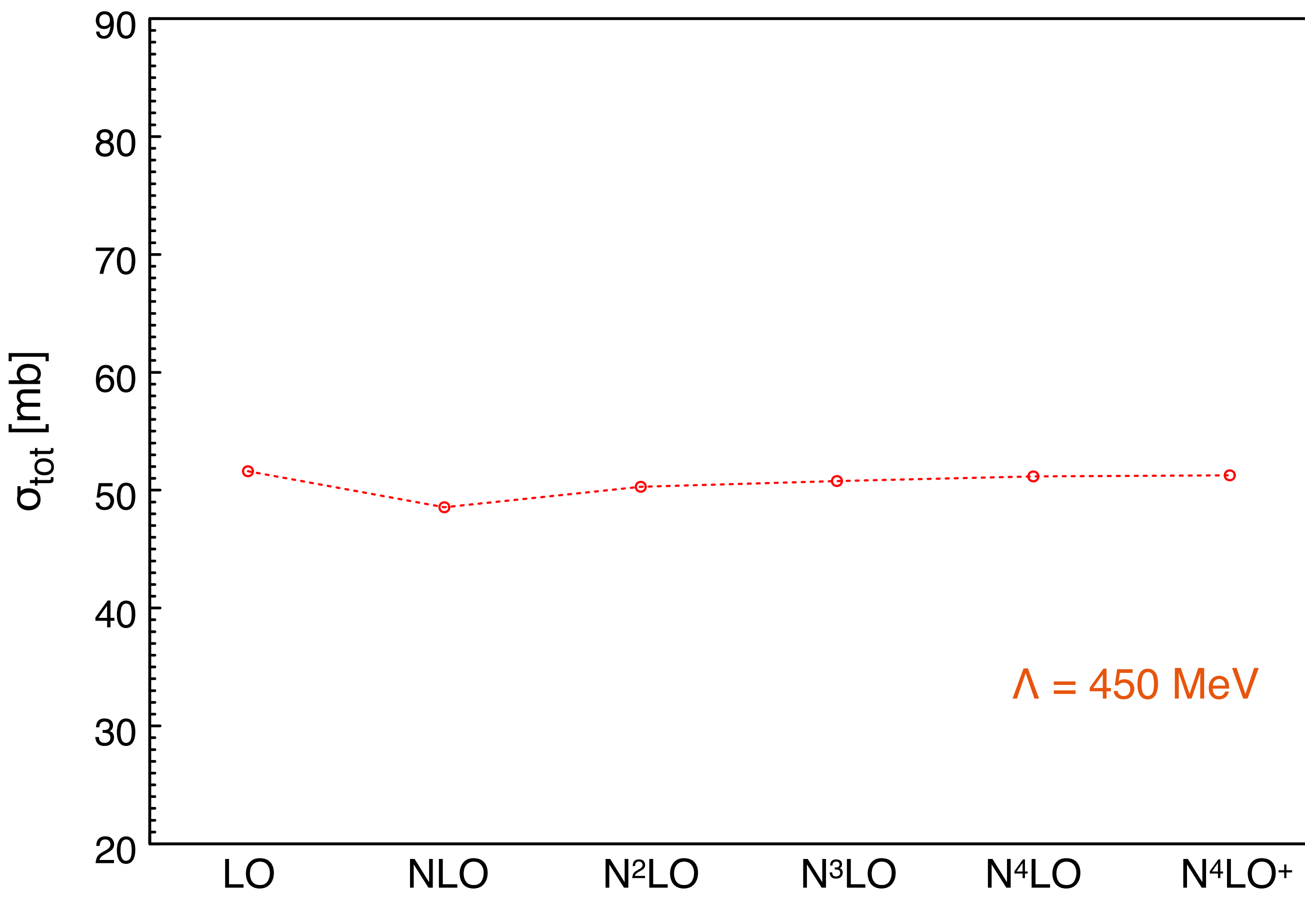
- Statistically satisfactory description of our database of mutually consistent scattering data (2124 pp and 2935 np data below E_{lab} = 290 MeV):

high-precision „realistic“ potentials				*Idaho χEFT*		*Bochum SMS χEFT*	
Nijm I	Nijm II	Reid93	CD Bonn	$N^4LO^+_{450}$	$N^4LO^+_{500}$	$\mathbf{N^4LO^+_{450}}$	$\mathbf{N^4LO^+_{500}}$
1.061	1.070	1.078	1.042	2.019	1.203	**1.013**	**1.015**

- Results for the np total cross section and the error budget:



Uncertainty quantification



Neutron-proton total cross section at 150 MeV [Λ = 450 MeV]

$$\sigma_{\rm tot} = 51.4_{\rm LO} - 3.0_{\rm NLO} + 1.7_{\rm N^2LO} + 0.5_{\rm N^3LO} + 0.4_{\rm N^4LO} + 0.1_{\rm N^4LO+}$$

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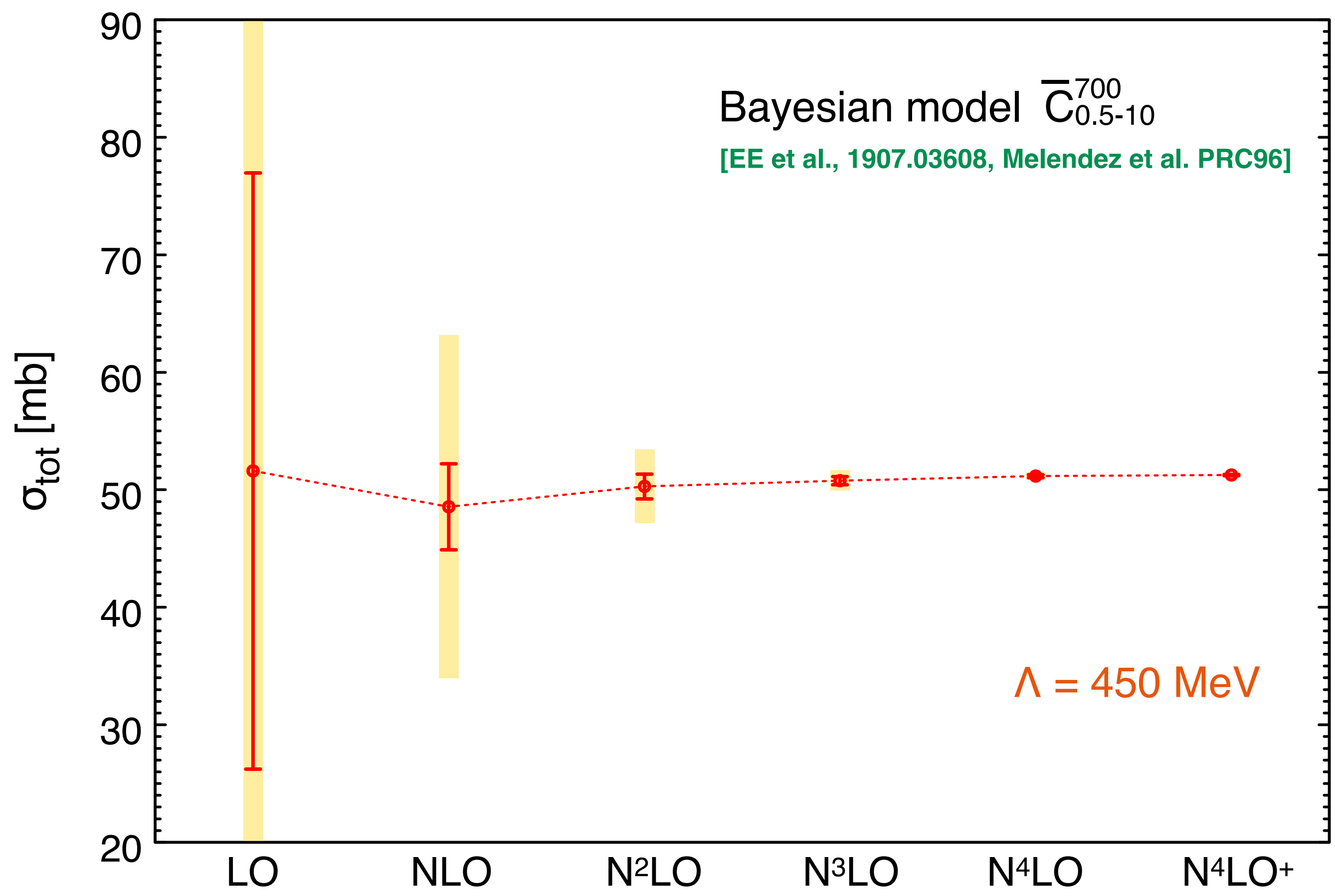

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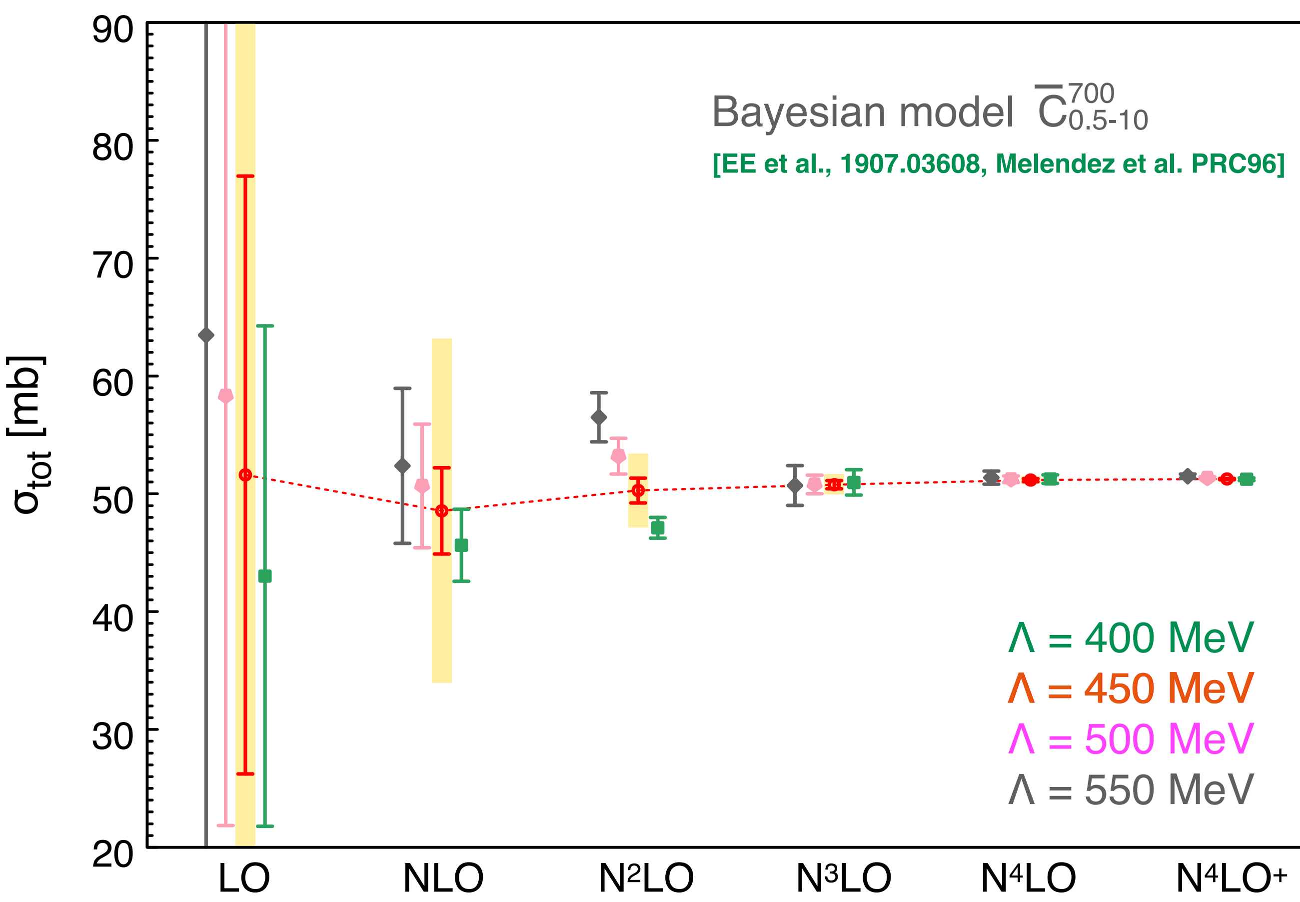
$$= 51.10(12)(12)(19)(6) \text{ mb}$$ to be compared with $$\sigma_{\rm tot}^{\rm exp.} = 51.02 \pm 0.30 \text{ mb}$$

Lisowski et al. ’82

Uncertainty quantification



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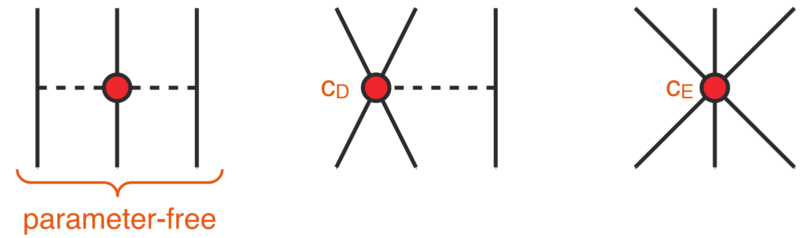
Lisowski et al. ’82

Inclusion of the 3NF at N²LO

EE et al. [LENPIC], PRC99 (2019); Maris et al. [LENPIC], PRC103 (2021); e-Print: 2206.13303

The leading 3NF depends on 2 LECs that need to be determined from few-N data:

- ^{3}H binding energy yields $c_E = f(c_D)$
- c_D is determined from Nd scattering



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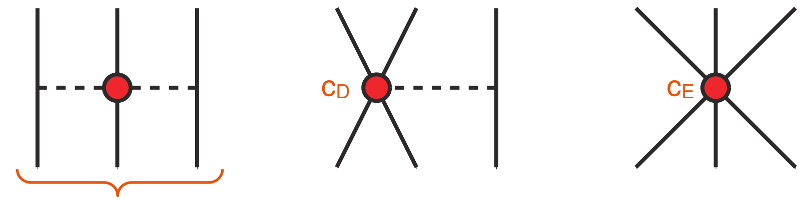
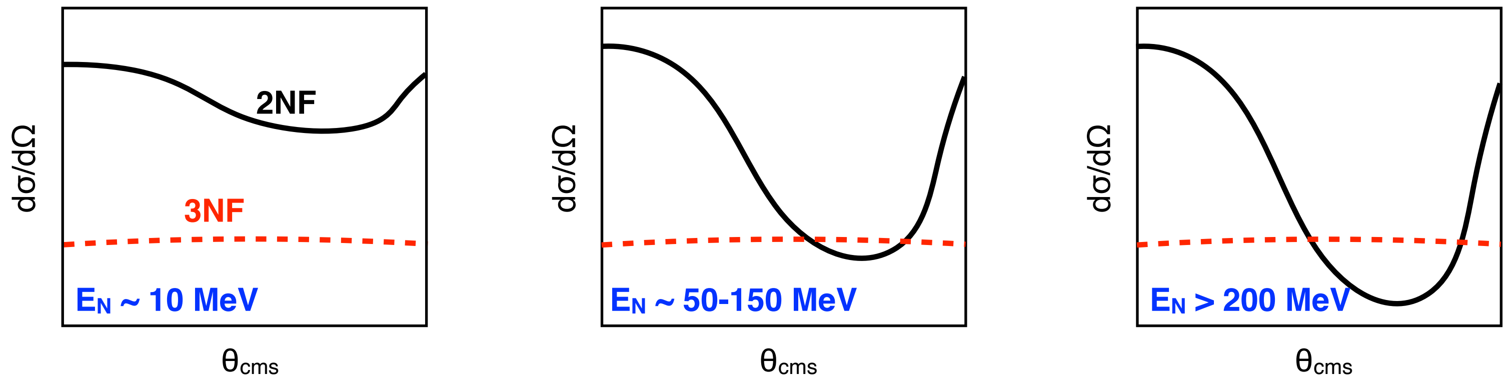


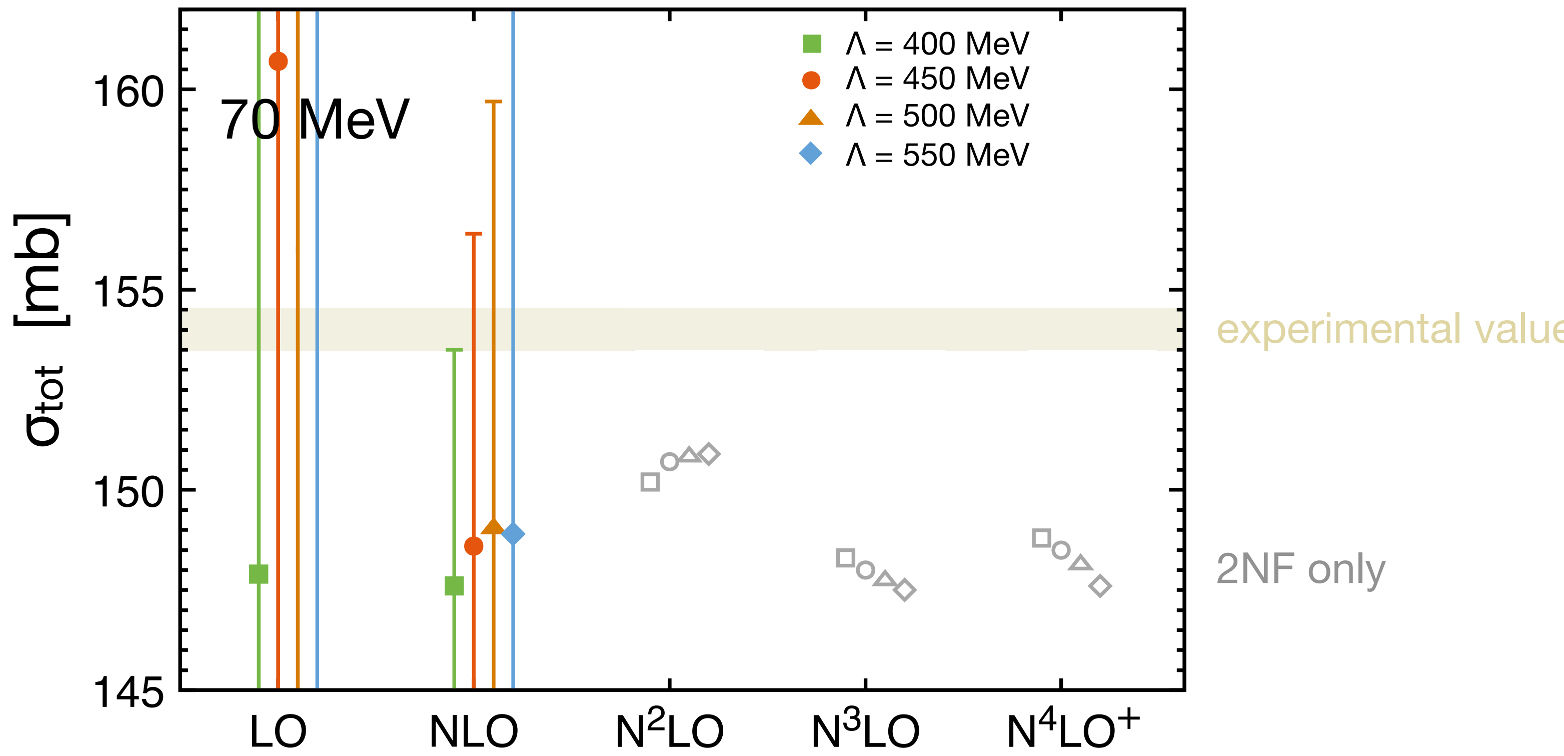
Differential cross section of Nd elastic scattering at intermediate and higher energies is known to be sensitive to the 3NF:



⇒ use precise Nd cross section data at 70 MeV from RIKEN [Kimiko Sekiguchi et al. '02] to fix c_D

Predictions for Nd total cross section

Maris et al. [LENPIC], e-Print: 2206.13303



— 2NF only underestimates the data; adding the 3NF improves the agreement with exp.

— 3NF contributions of natural size (W. counting)

— small residual cutoff dependence

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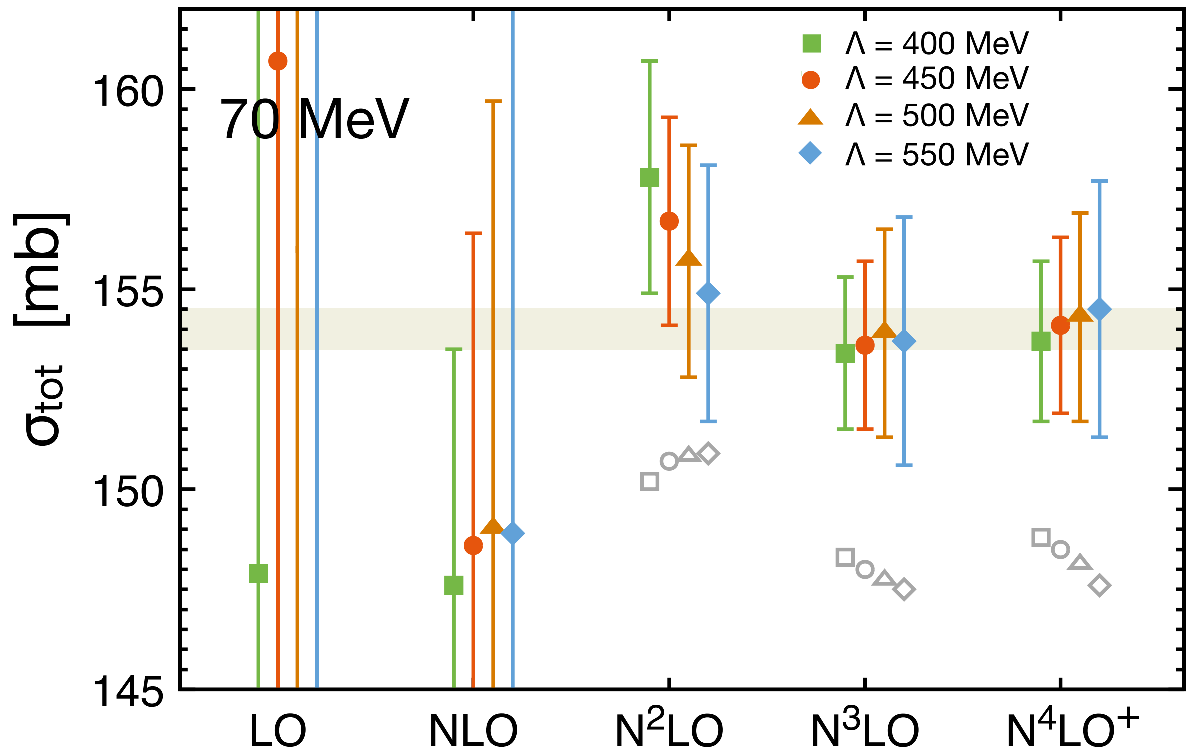

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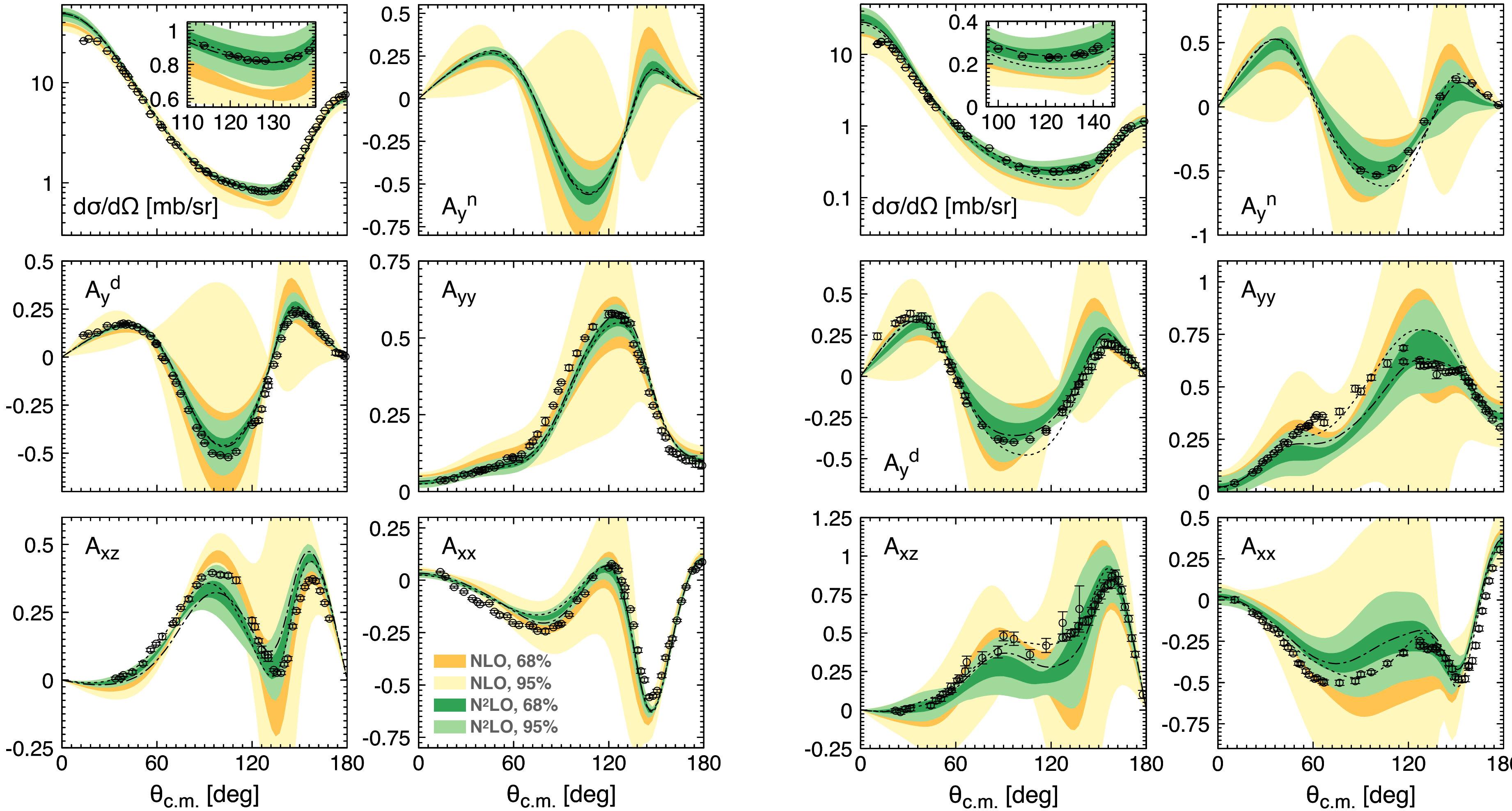
LENPIC: Low Energy Nuclear Physics International Collaboration

Predictions for elastic Nd scattering

Maris et al. [LENPIC], PRC103 (2021)

Elastic Nd scattering at 70 MeV

Elastic Nd scattering at 135 MeV

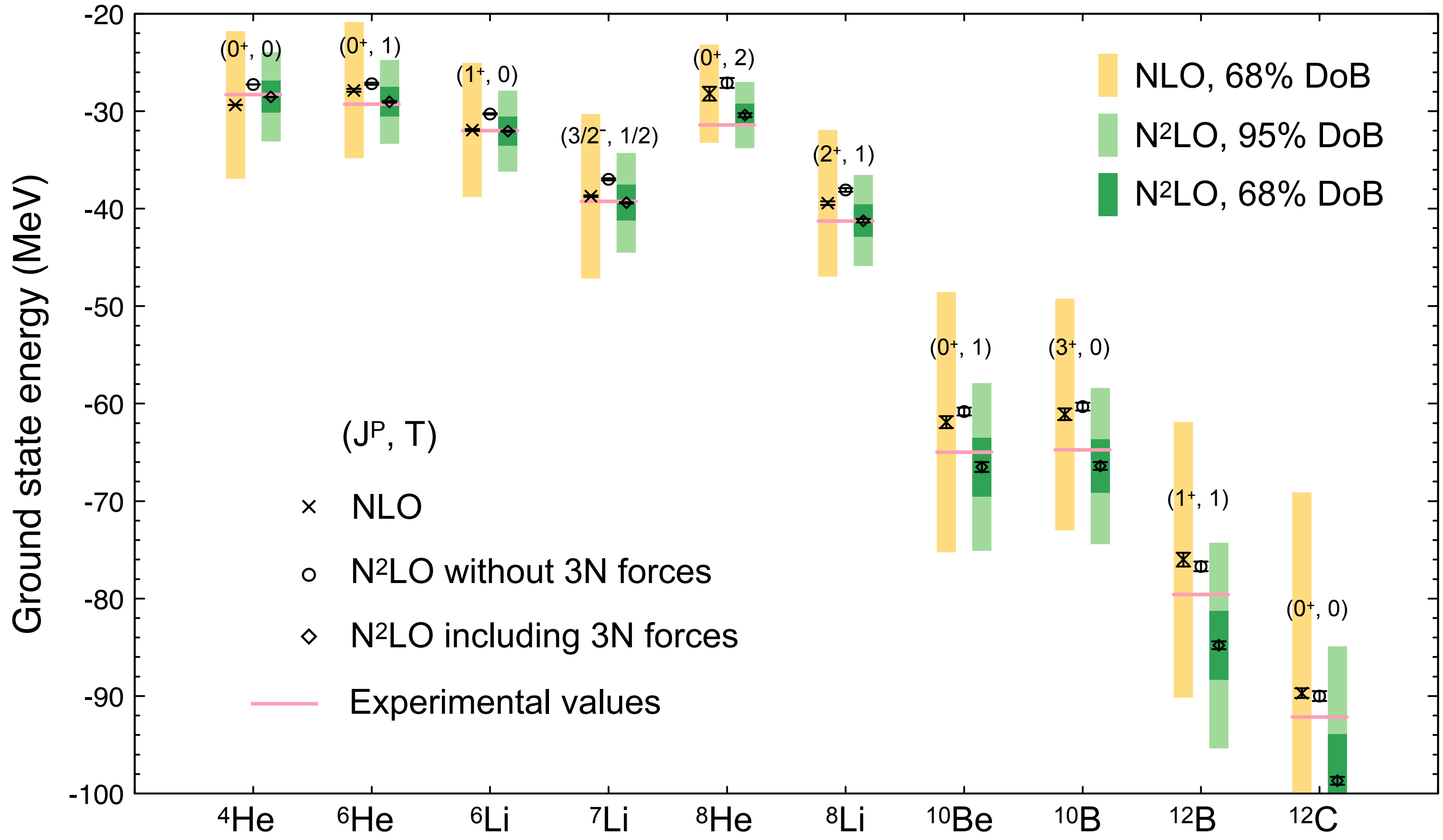


Experimental data are proton-deuteron data from Sekiguchi et al., PRC65 (2002).

LENPIC: Low Energy Nuclear Physics International Collaboration

Predictions for p-shell nuclei (NCSM)

Maris et al. [LENPIC], PRC103 (2021)



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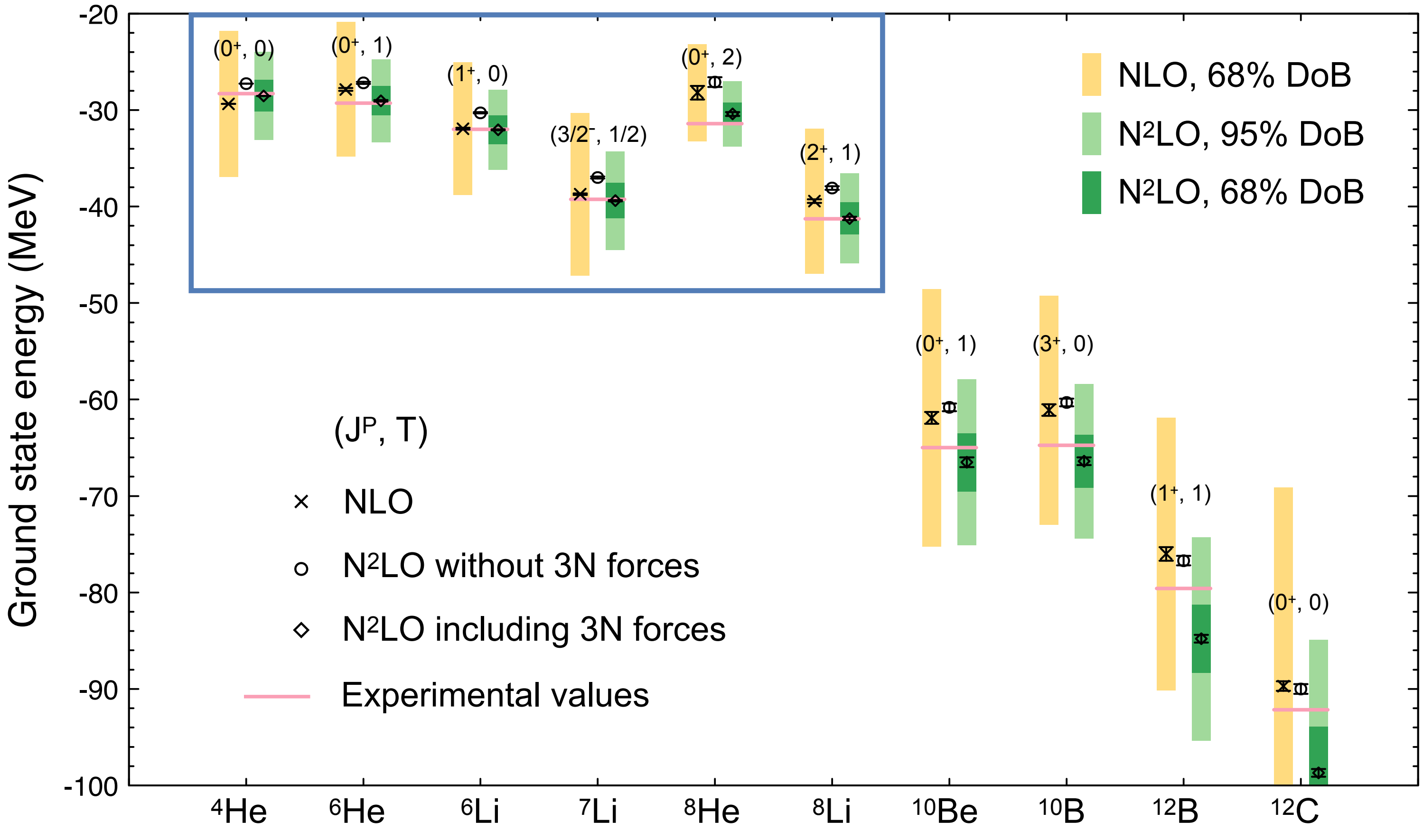

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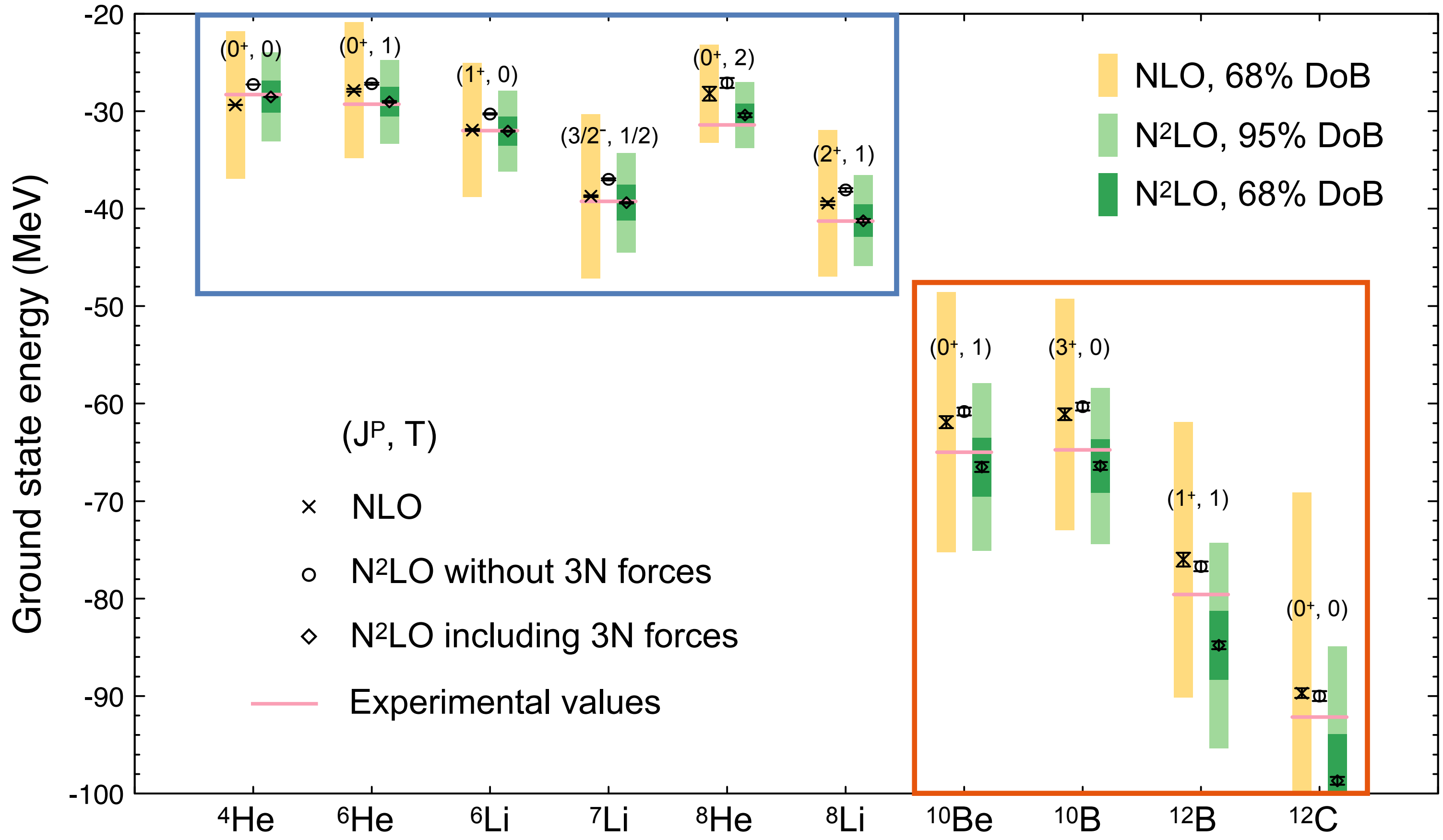


— Excellent agreement with the data

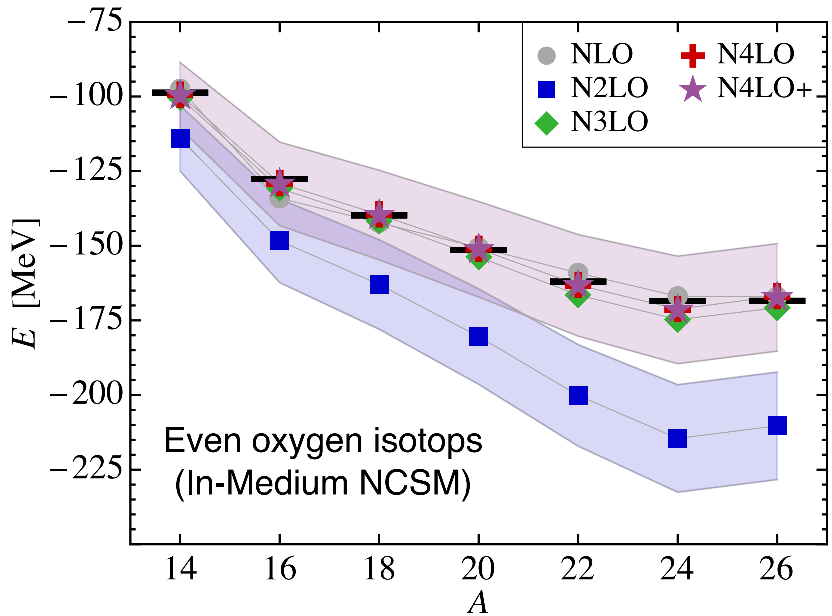
— Overbinding starting from A ~ 10 that increases with A. Deficiency in the 2NF or 3NF?

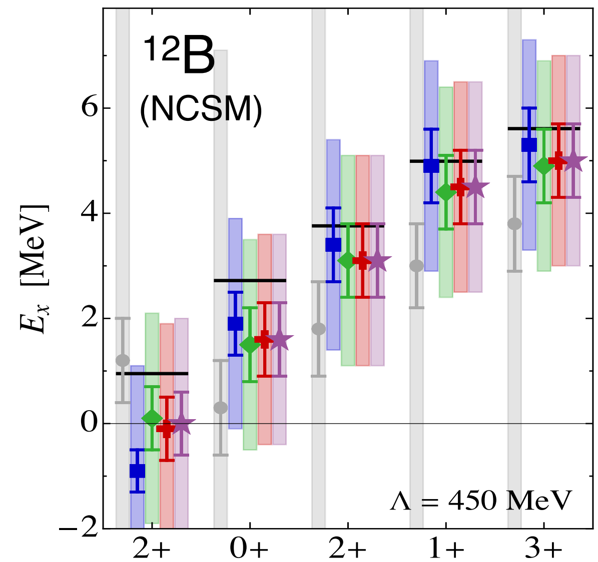
LENPIC: Low Energy Nuclear Physics International Collaboration

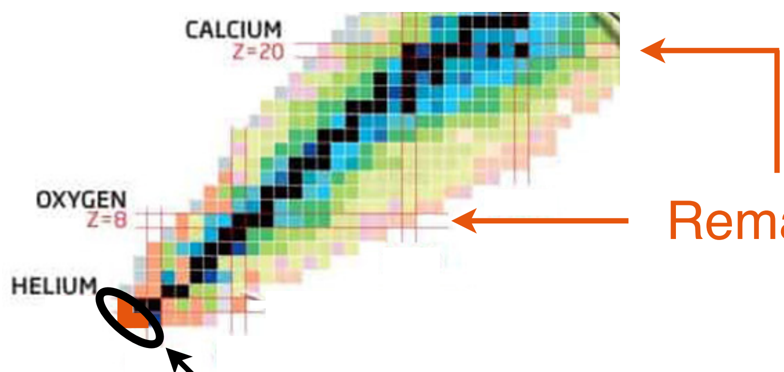
Predictions for ground & excited states

Maris et al. [LENPIC], e-Print: 2206.13303

Repeat the calculations by including higher-order corrections to the 2N force:



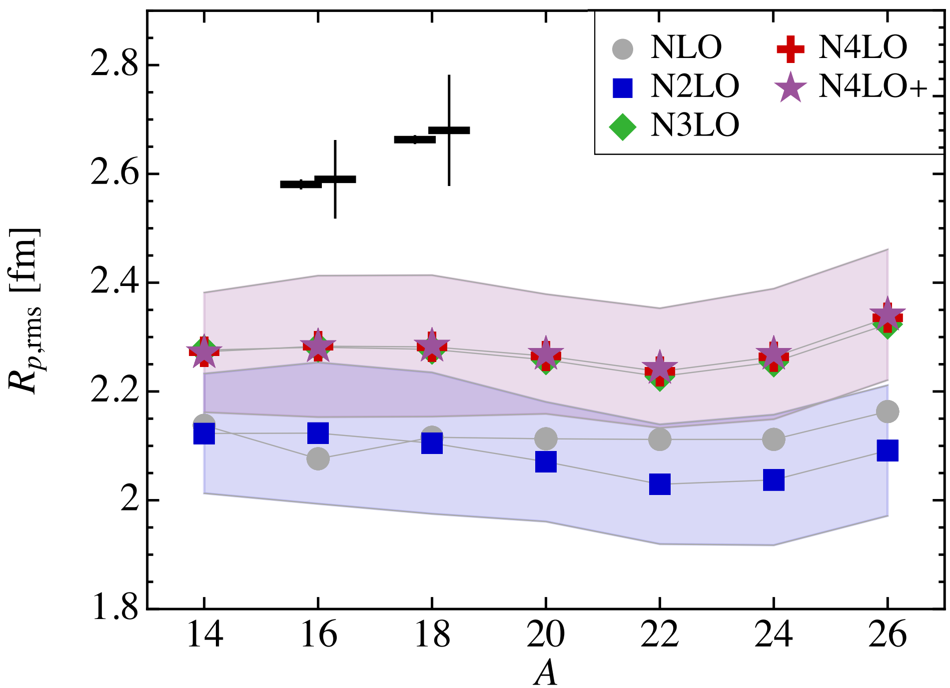




Remarkable predictive power!

Hamiltonian fixed **completely** from A = 2, 3 nuclei

Charge radii: A smoking gun?



A similar underprediction of the radii was observed using the EM 2NF@N³LO with 3NF@N²LO

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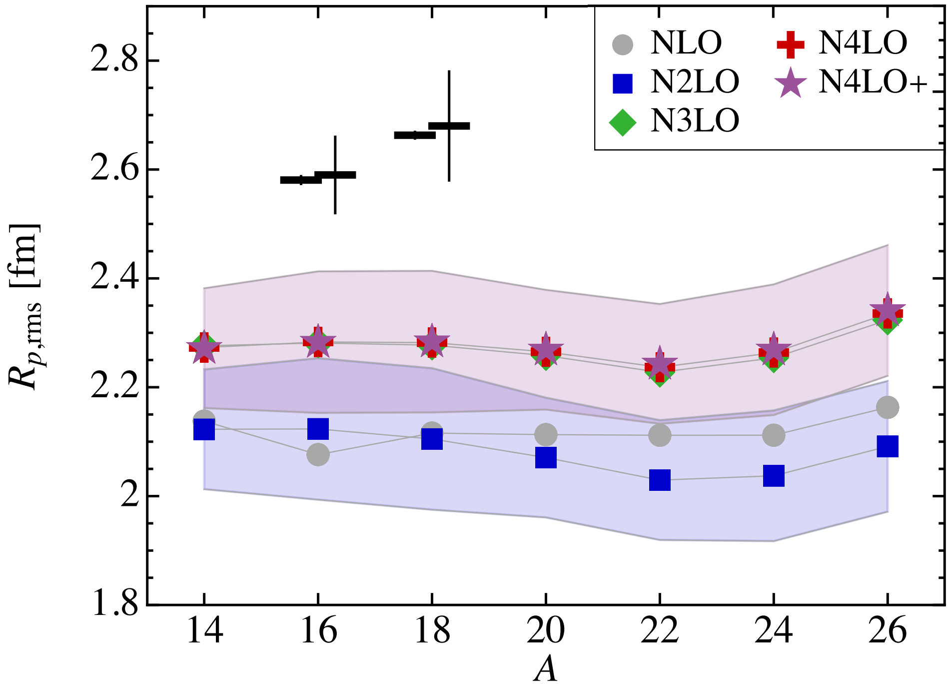

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Recent/ongoing N⁴LO studies of the A = 2,3,4 isoscalar radii including MECs:

A = 2 (²H) Filin et al., PRL 124 (2020) 082501; PRC 103 (2021) 024313

$r_{\mathrm{str}} = 1.9729^{+0.0015}_{-0.0012}$ fm, $\quad Q_{\mathrm{d}} = 0.2854^{+0.0038}_{-0.0017}$ fm^2

A = 3 (³H, ³He) $r = 1.9065(26)$ fm, to be compared with $r^{\mathrm{exp}} = 1.903(29)$ fm
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A = 4 (⁴He) $r = 1.6766(34)$ fm, to be compared with $r^{\mathrm{exp}} = 1.67824(83)$ fm
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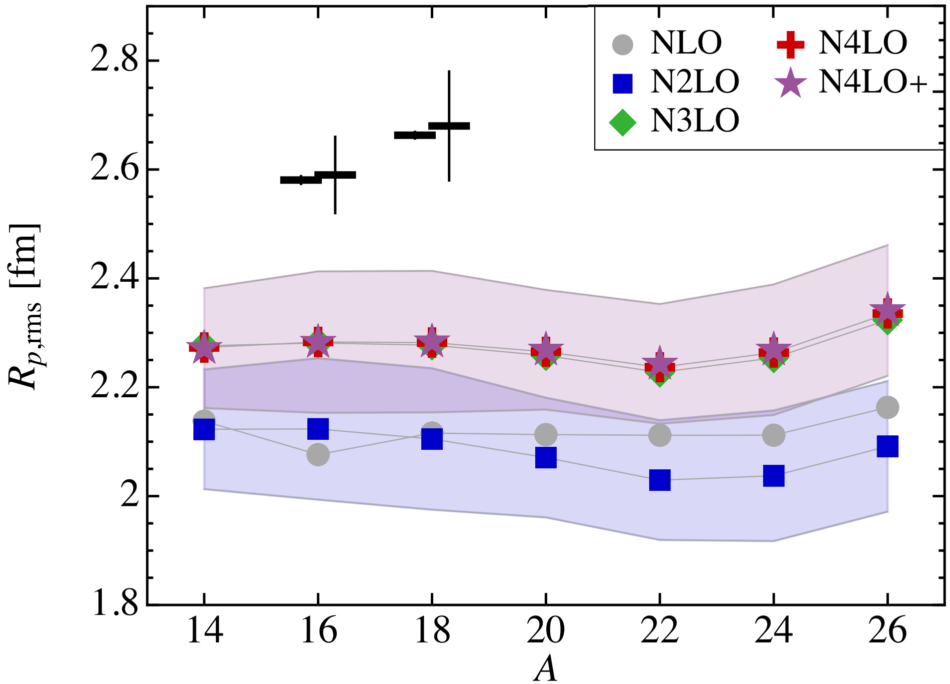

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A = 4 (^{4}He) $r = 1.6766(34)$ fm, to be compared with $r^{\rm exp} = 1.67824(83)$ fm — $\delta r_{\rm MEC} \simeq 3\,\%$

(preliminary) Krauth '21

The 3-body force beyond N^2LO

Conceptual challenge: 3NFs have been worked out using **dimensional regularization**. Is it **consistent** to employ an additional cutoff regulator when solving the Faddeev equation?

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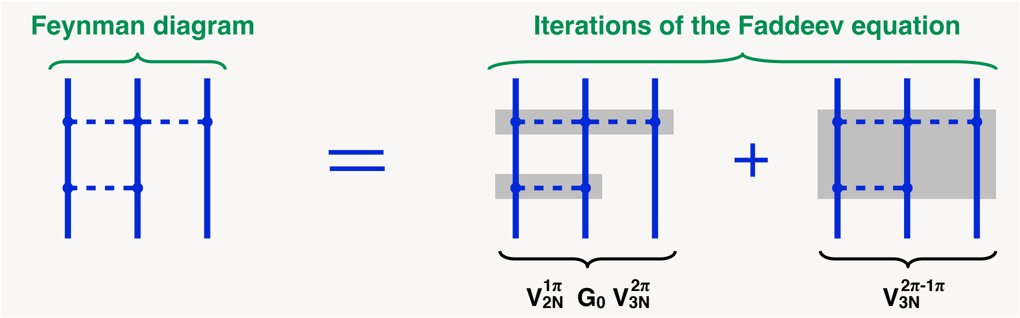

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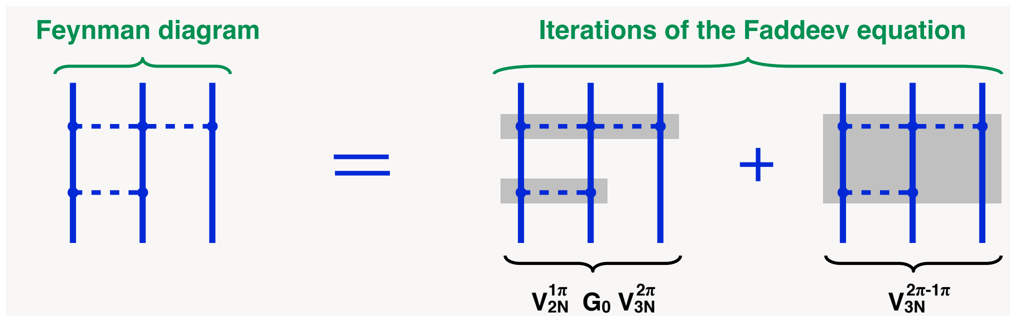
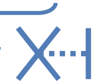


- Using DimReg everywhere: l.h.s. = r.h.s. $\Rightarrow$ consistent
- Calculate the iterative diagram on the r.h.s. using cutoff regularization:

$$V^{1\pi}_{2N,\,\Lambda}\, G_0\, V^{2\pi}_{3N,\,\Lambda} = -\Lambda \frac{g_A^4}{96\sqrt{2\pi^3}F_\pi^6}\left[\underbrace{\tau_1\cdot\tau_3\,(\vec{q}_3\cdot\vec{\sigma}_1)}_{\text{absorbable into } c_D} - \underbrace{\frac{4}{3}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)(\vec{q}_2\cdot\vec{\sigma}_3)}_{\text{violates chiral symmetry...}}\right]\frac{\vec{q}_3\cdot\vec{\sigma}_3}{q_3^3+M_\pi^2} + \ldots$$

absorbable into c_D: 

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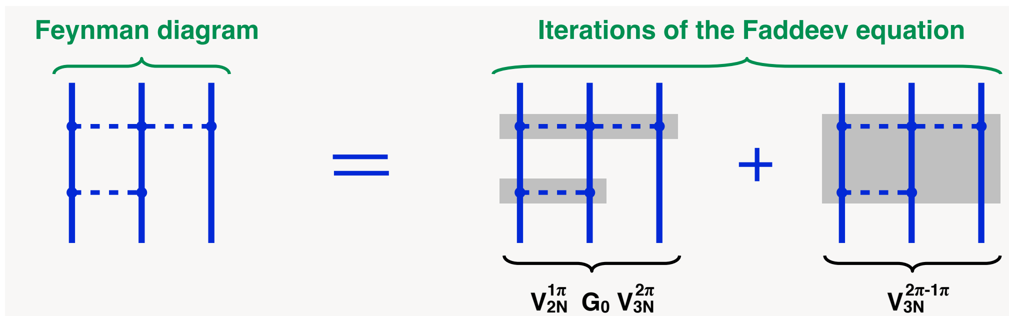

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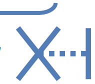
absorbable into c_D:  *violates chiral symmetry…*

⇒ all expressions for the 3NF (and exchange currents) beyond tree level, i.e. N^2LO must be re-calculated using cutoff regularization that maintains chiral symmetry.

— e.g., the higher-derivative [Slavnov '71] or gradient flow regularization [Lüscher '10]

— a new path-integral approach to derive nuclear forces and currents [Krebs, EE, in preparation]

Summary and conclusions

- The main obstacle toward precision nuclear theory is the uncertainty in the 3N force
- The leading 3NF improves the description of the data, but the N^2LO accuracy is still limited…
- Based on our experience in the 2N system, a precise description of Nd scattering data below π-production threshold will likely require going to N^4LO
- Two major challenges towards N^4LO PWA of Nd scattering data:
  - — derivation of the consistently regularized 3NF beyond N^2LO [Krebs, EE, in progress]
  - — development of efficient emulators for 3N scattering to fix LECs [Witala et al., 2203.08499]
- More precision data for Nd elastic scattering [New experiment at RIKEN RIBF by Kimiko Sekiguchi et al.]

Generalization to **hyper-nuclear interactions** requires addressing additional challenges:

— reacher structure of SU(3) vs SU(2) $\Rightarrow$ many more LECs…

— slower convergence of χEFT in the SU(3) sector (M_K vs M_π)

— poor (but improving!) data situation… Help from lattice QCD heavily needed!