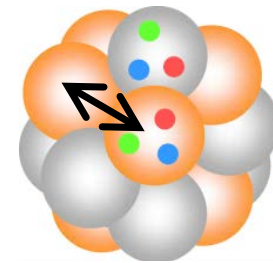
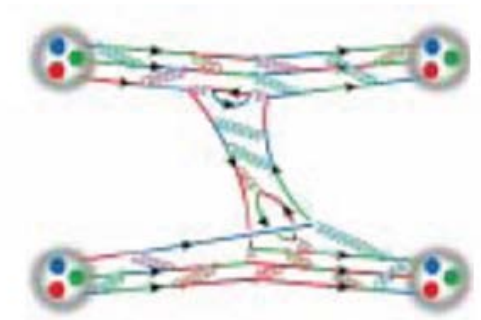
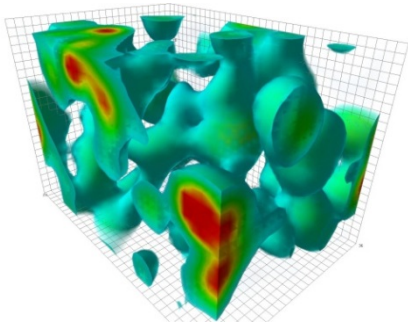
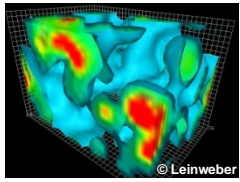


# Lattice QCD study of hadron interactions with strangeness

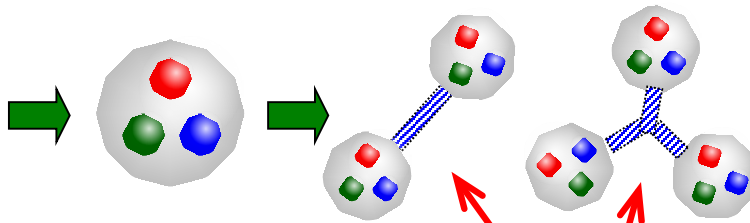
**Takumi Doi**  
(RIKEN iTHEMS)



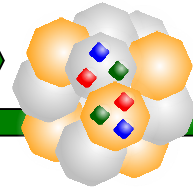
# The Odyssey from Quarks to Universe



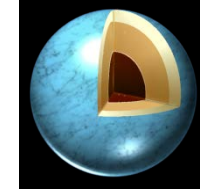
QCD vacuum



Baryons



Nuclei



Neutron Stars / Supernovae  
Nucleosynthesis



QCD

1st-principle  
Lattice QCD

Hadron  
Forces

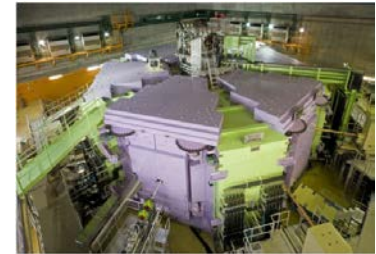
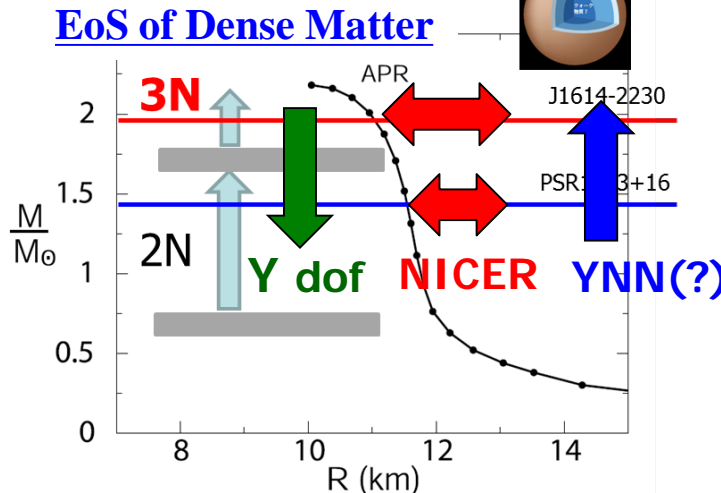
ab-initio nuclear calc.



J-PARC/JLab/MAMI



LHC/RHIC



RIBF/FRIB

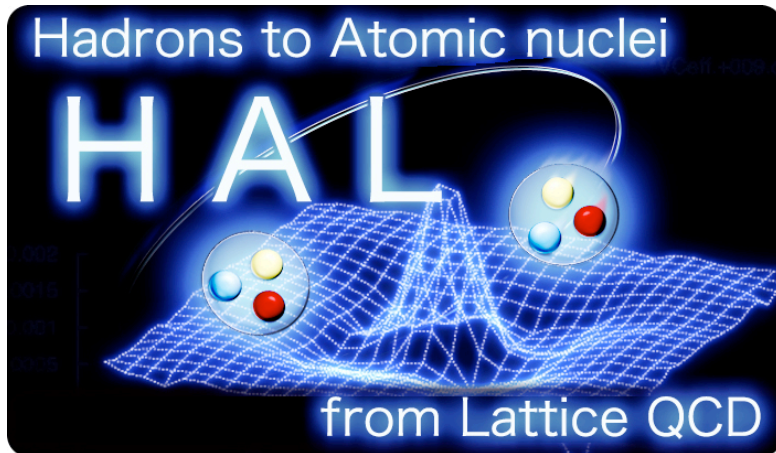


aLIGO/KAGRA



NICER

# Hadrons to Atomic nuclei from Lattice QCD (HAL QCD Collaboration)



**Y. Akahoshi, S. Aoki,**  
**K. Murakami, H. Nemura** (YITP)  
**T. Aoyama** (ISSP)  
**T. Doi, T. Hatsuda, T. Sugiura** (RIKEN)  
**T. M. Doi, Y. Ikeda, N. Ishii, K. Sasaki** (Osaka Univ.)  
**F. Etminan** (Univ. of Birjand)  
**T. Inoue** (Nihon Univ.)  
**Y. Lyu** (Peking Univ.)  
**H. Tong** (Tianjin Normal Univ.)

「20XX年宇宙の旅」  
*from Quarks to Universe*



**+**  
**E. Itou** (RIKEN)  
**I. Kanamori** (RIKEN)  
**K.-I. Ishikawa** (Hiroshima Univ.)

# Luscher's formula: Scatterings on the lattice

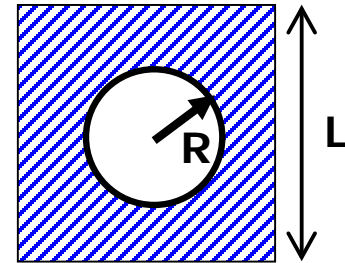
- Consider Schrodinger eq at asymptotic region

$$(\nabla^2 + k^2)\psi_k(\mathbf{r}) = mV_k(\mathbf{r})\psi_k(\mathbf{r})$$

M.Luscher, NPB354(1991)531

$$V_k(\mathbf{r}) = 0 \text{ for } r > R$$

- (periodic) Boundary Condition in finite V  
 → constraint on energies of the system



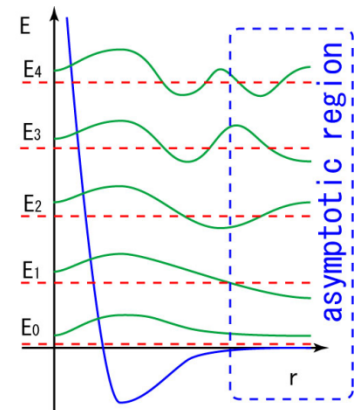
- Quantization condition between Energy E and phase shift (at E)

$$k \cot \delta_{\mathbf{E}} = \frac{2}{\sqrt{\pi}L} Z_{00}(1; (kL/2\pi)^2) \quad E = 2\sqrt{m^2 + k^2}$$

(Lushcer's formula)

- Calculate Finite V spectrum

→ convert them to phase shifts via Luscher's formula



# [HAL QCD method]

- Nambu-Bethe-Salpeter (NBS) wave function

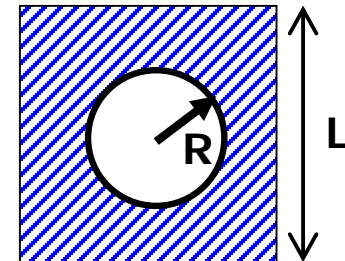
$$\psi(\vec{r}) = \langle 0 | N(\vec{r})N(\vec{0}) | N(\vec{k})N(-\vec{k}); W \rangle$$

$$(\nabla^2 + k^2)\psi(\vec{r}) = 0, \quad r > R$$

- phase shift at asymptotic region

$$\psi(r) \simeq A \frac{\sin(kr - l\pi/2 + \delta(k))}{kr}$$

Extended to multi-particle systems



M.Luscher, NPB354(1991)531

C.-J.Lin et al., NPB619(2001)467

N.Ishizuka, PoS LAT2009 (2009) 119

CP-PACS Coll., PRD71(2005)094504

S. Aoki et al., PRD88(2013)014036

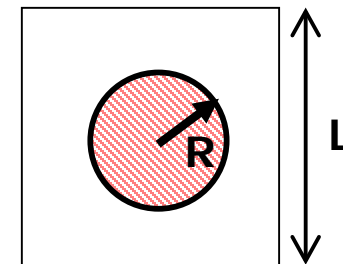
- Consider the wave function at “interacting region”

$$(\nabla^2 + k^2)\psi(\mathbf{r}) = m \int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') \psi(\mathbf{r}'), \quad r < R$$

- $U(\mathbf{r}, \mathbf{r}')$ : faithful to the phase shift by construction

- $U(\mathbf{r}, \mathbf{r}')$ : **E-independent**, while **non-local** in general

- Non-locality  $\rightarrow$  derivative expansion

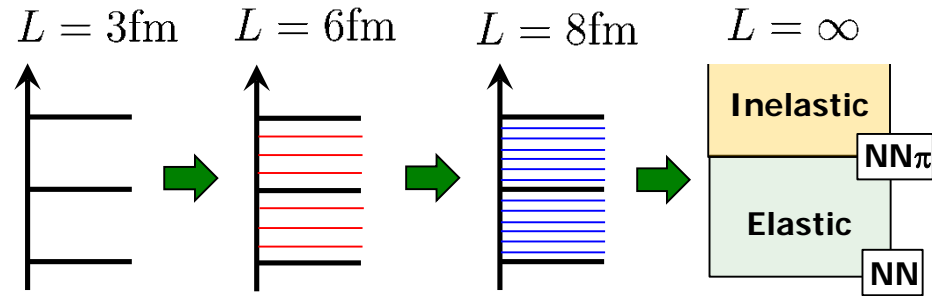


Aoki-Hatsuda-Ishii PTP123(2010)89

# The Challenge in multi-baryons on the lattice

Existence of elastic scatt. states

- (almost) No Excitation Energy
- LQCD method based on G.S. saturation impossible



Signal/Noise issue

$$S/N \sim \exp[-\mathbf{A} \times (\mathbf{m}_N - \mathbf{3}/\mathbf{2}\mathbf{m}_\pi) \times \mathbf{t}]$$

Parisi('84), Lepage('89)

$$L=8\text{fm @ physical point} \quad (E_1 - E_0) \simeq 25\text{MeV} \implies t > 10\text{fm}$$

$$S/N \sim 10^{-32}$$

Direct method (naïve plateau fit @  $t \sim 1\text{fm}$  + Luscher's formula)

→ Does it really reliable? (excited state contaminations)

"Sign Problem"

# [Time-dependent HAL QCD method]

*E-indep of potential  $U(\mathbf{r}, \mathbf{r}')$   $\rightarrow$  (excited) scatt states share the same  $U(\mathbf{r}, \mathbf{r}')$*

$$\int d\mathbf{r}' U(\mathbf{r}, \mathbf{r}') R(\mathbf{r}', t) = \left( -\frac{\partial}{\partial t} + \frac{1}{4m} \frac{\partial^2}{\partial t^2} - H_0 \right) R(\mathbf{r}, t)$$

	Ground state	Excited (elastic)	Excited (inelastic)
Direct method	Signal	Noise	Noise
HAL method	Signal	Signal	Noise

Partial solution of Sign Problem

**HAL method  $\rightarrow$  Exponentially better S/N**

**$\rightarrow$  Coupled channel formalism above inelastic threshold  $\rightarrow$  YN/YY (!)**

## Reliability test of LQCD methods

T. Iritani et al. (HAL) JHEP10(2016)101, PRD96(2017)034521, PRD99(2019)014514, JHE03(2019)007

NN @ heavy quark masses

HAL method (HAL) :	unbound
Direct method (NPL/CalLat/PACS-CS(Yamazaki et al.)):	bound
Semi-improved calc w/ Luscher's formula (Mainz2019) :	unbound
Variational calc w/ Luscher's formula (CalLat2020) :	unbound
Variational calc w/ Luscher's formula (NPL2021) :	(unbound)

- Baryon Forces from LQCD
- Exponentially better S/N
- Coupled channel systems

Ishii-Aoki-Hatsuda (2007)

Ishii et al. (2012)

Aoki et al. (2011,13)

**[Theory]** = HAL QCD method

## Baryon Interactions near the Physical Point

### [Hardware]

= K-computer [10PFlops]

- + FX100 [1PFlops] @ RIKEN
- + HA-PACS [1PFlops] @ Tsukuba

- HPCI Field 5 / Post K Priority Issue 9



### [Software]

= Unified Contraction Algorithm

- Exponential speedup Doi-Endres (2013)

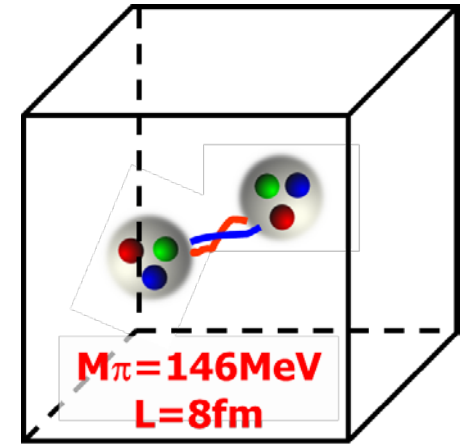
${}^3\text{H}/{}^3\text{He}$	:	$\times 192$
${}^4\text{He}$	:	$\times 20736$
${}^8\text{Be}$	:	$\times 10^{11}$



# Lattice QCD Setup

- **Nf = 2 + 1 gauge configs**
  - clover fermion + Iwasaki gauge w/ stout smearing
  - $V=(8.1\text{fm})^4$ ,  $a=0.085\text{fm}$  ( $1/a = 2.3 \text{ GeV}$ )
  - **$m(\pi) \sim 146 \text{ MeV}$ ,  $m(K) \sim 525 \text{ MeV}$**
  - #traj  $\sim 2000$  generated

PACS Coll., PoS LAT2015, 075



- **Measurement**

- **All of NN/YN/YY** for **central/tensor forces** in  $P=(+)$  (S, D-waves)

## Predictions for Hyperon forces

S=0	S=-1	S=-2	S=-3	S=-4	S=-5	S=-6
NN	N $\Lambda$ , N $\Sigma$	$\Lambda\Lambda$ , $\Lambda\Sigma$ , $\Sigma\Sigma$ , N $\Xi$	$\Lambda\Xi$ , $\Sigma\Xi$ , N $\Omega$	$\Xi\Xi$	$\Xi\Omega$	$\Omega\Omega$

**EXP**  
rich data

**LQCD**  
better S/N

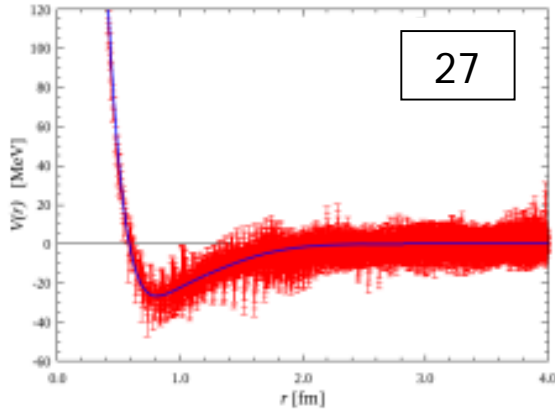
# Birds-eye View: diag-Pot in SU(3) base in S=-2

$$8 \times 8 = 27 + 8s + 1 + 10^* + 10 + 8a$$

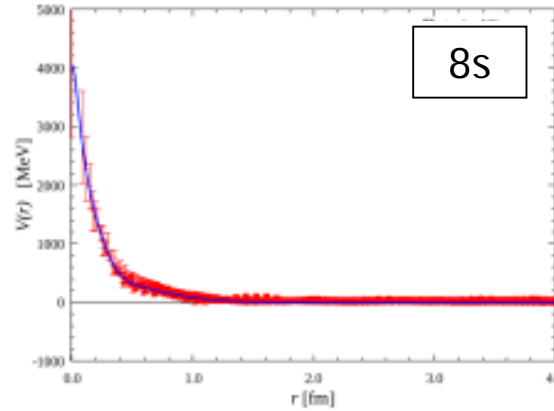
T.Inoue (HAL), AIP Conf. Proc. 2130 (2019) 020002

(off-diag pot relatively small)

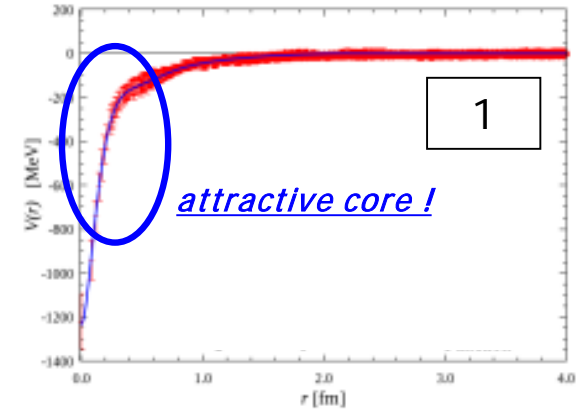
$1S_0$



27

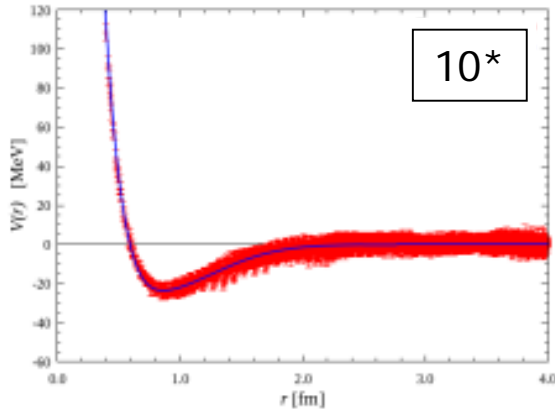


8s

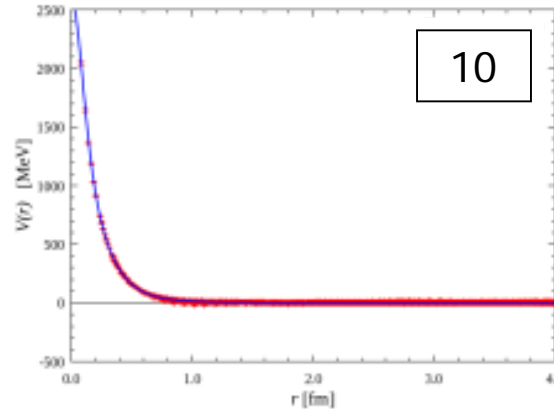


1

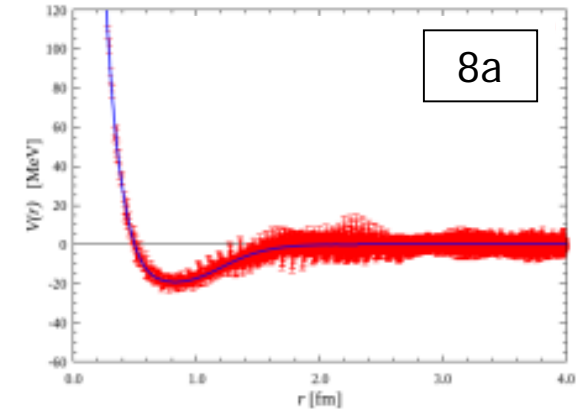
$3S_1-3D_1$



10\*



10



8a

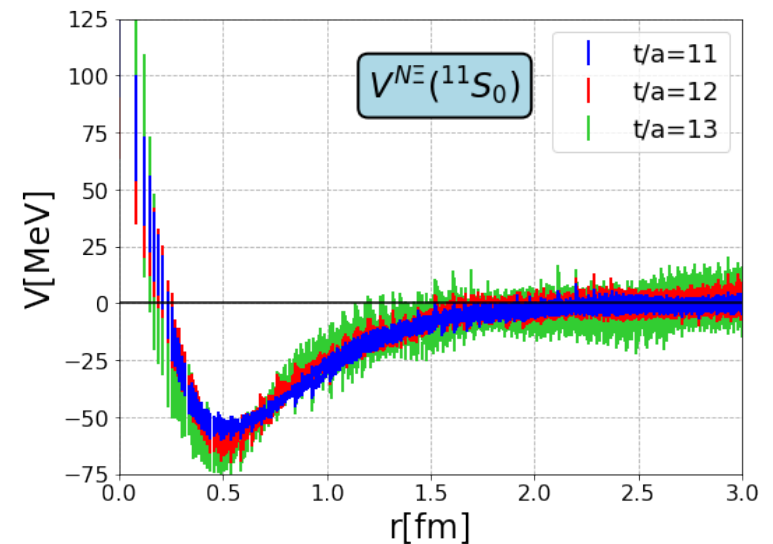
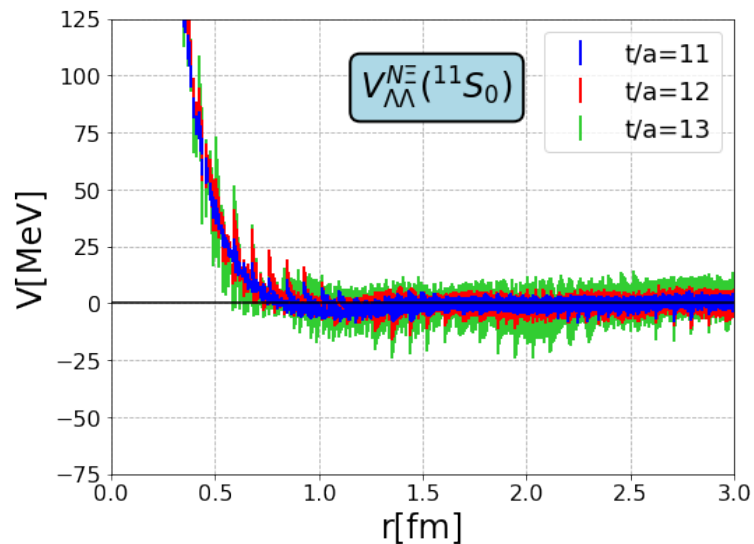
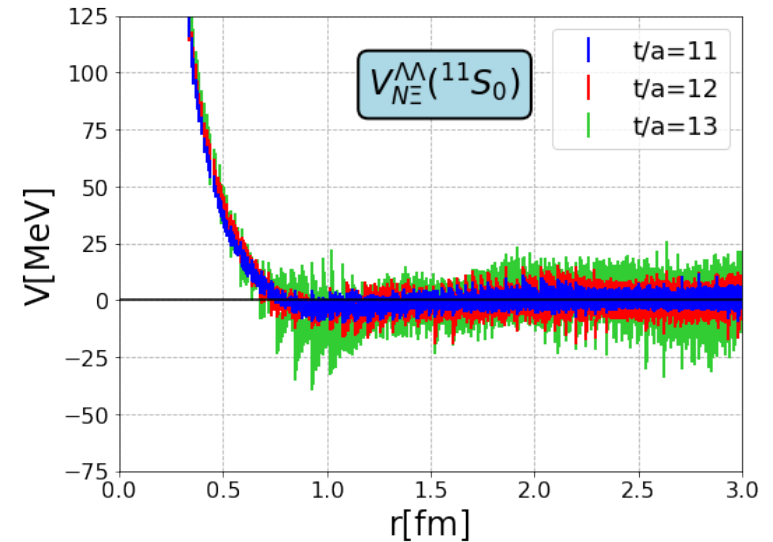
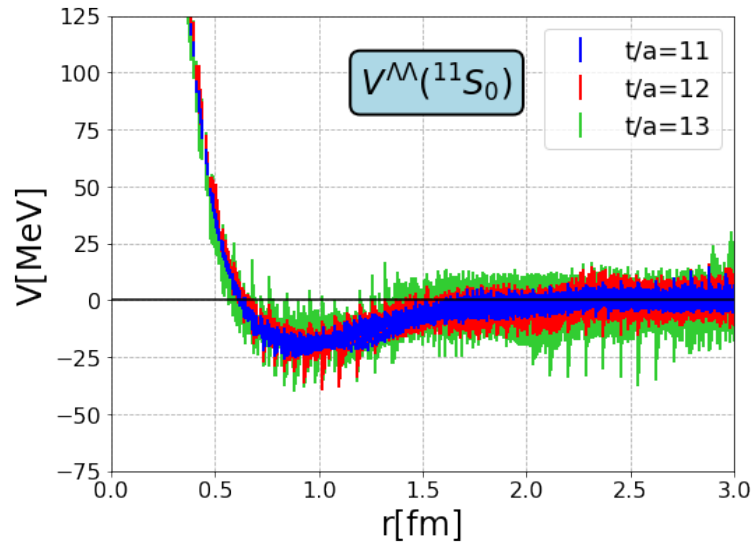
27, 10\*:  
NN-type

8s, 10:  
strong repulsive core

1s: deep attractive pocket  
8a: weak repulsive core

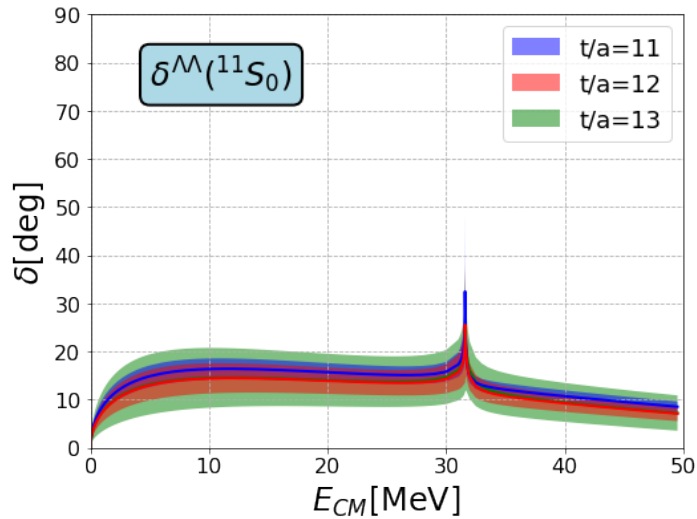
Quark Pauli repulsion + OGE for short range

# $\Lambda\Lambda, N\Xi$ (effective) 2x2 coupled channel analysis



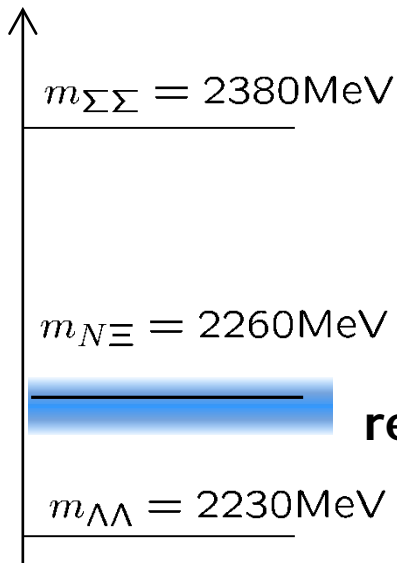
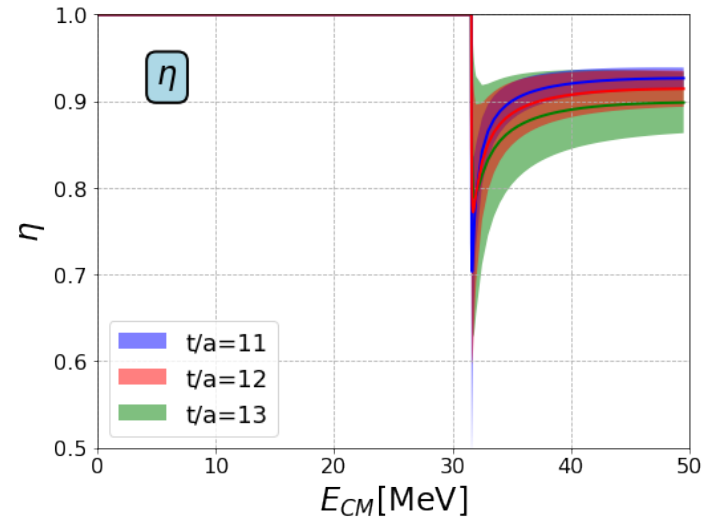
$N\Xi$  ( $^{11}S_0$ ) channel is attractive  
 $N\Xi-\Lambda\Lambda$  coupling is small

# $\Lambda\Lambda, N\Xi$ 2x2 coupled channel analysis



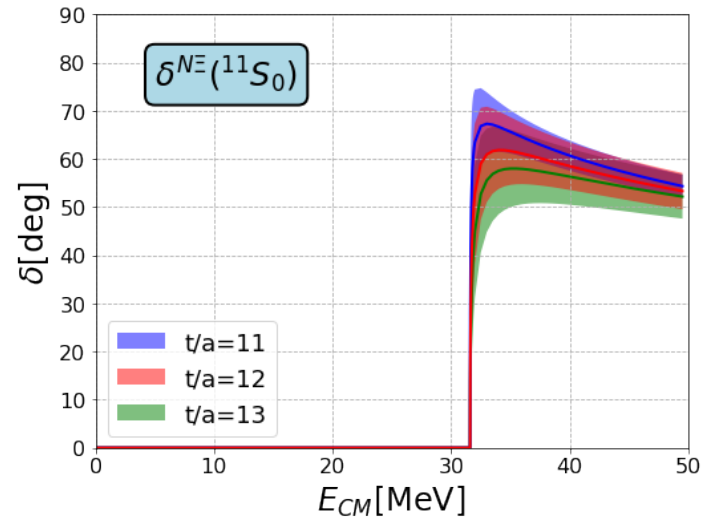
$$a_0 = -0.81(23)(+0.00/-0.13) \text{ [fm]}$$

$$r_{\text{eff}} = 5.47(78)(+0.09/-0.55) \text{ [fm]}$$



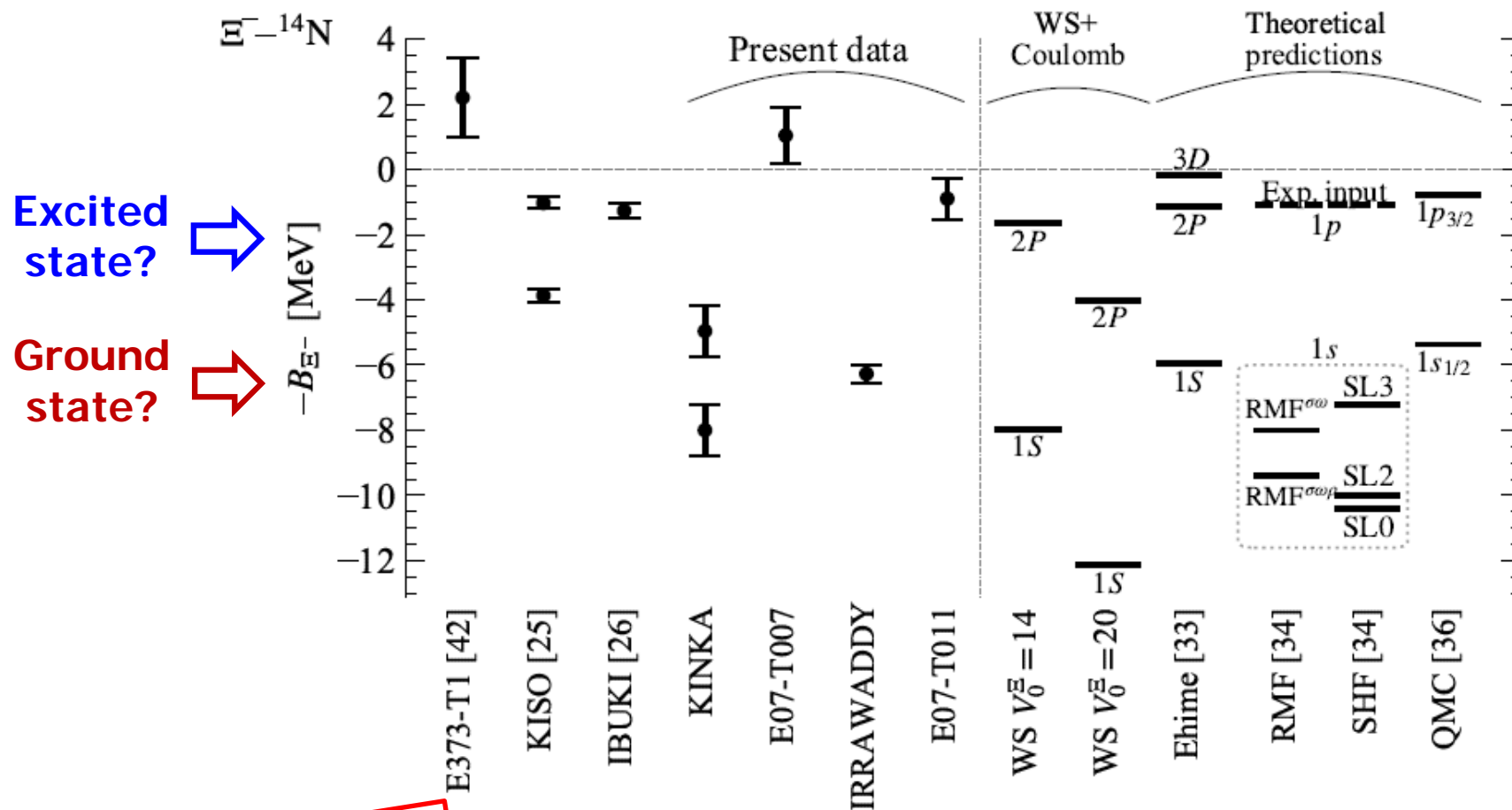
**J-PARC E42 (Ahn)**

**$N\Xi$  ~ unitary limit  
remnant of "H-dibaryon"**



(N.B.  $N\Xi = 1\text{rep } 50\%, 27\text{rep } 30\%$  in  $SU(3)$ )

# Recent experimental progress on $\Xi$ -Hypernuclei



**J-PARC E07, E70,  
more in HEF-EX**

Attractive  $N_{\Xi}$ -int well established  
Small  $N_{\Xi}$ - $\Lambda\Lambda$  coupling indicated

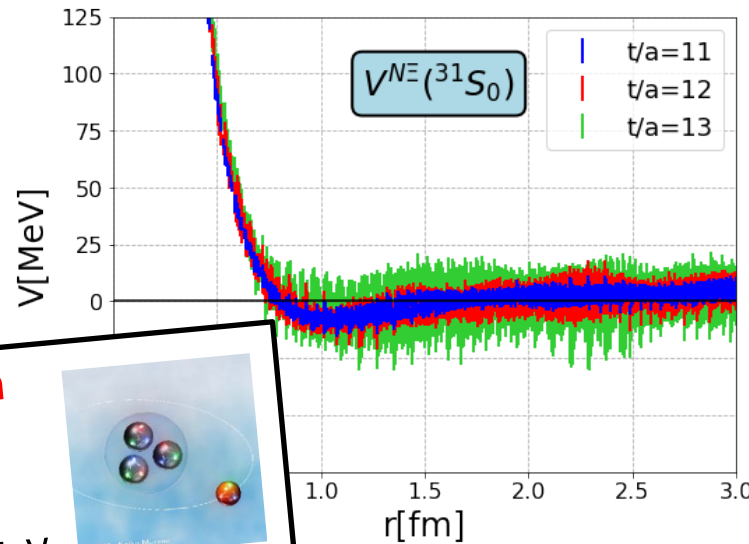
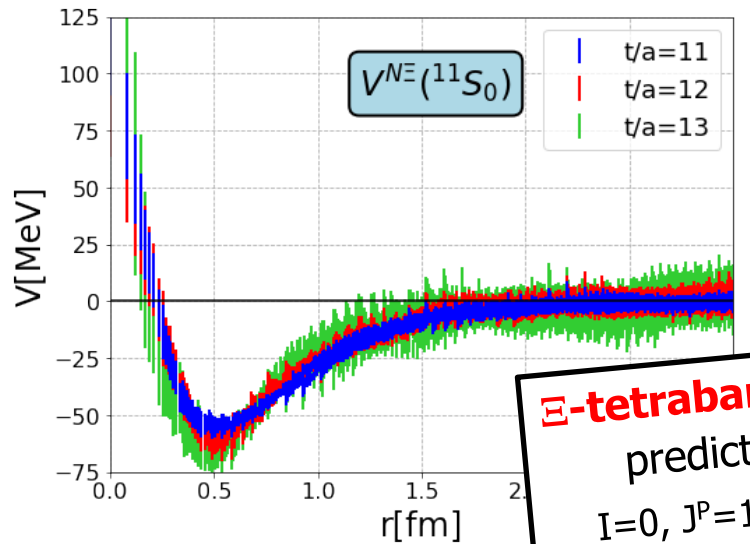
M. Yoshimoto et al.,  
PTEP2021, 073D02

# Spin-Isospin dependence of $N\Xi$ potentials

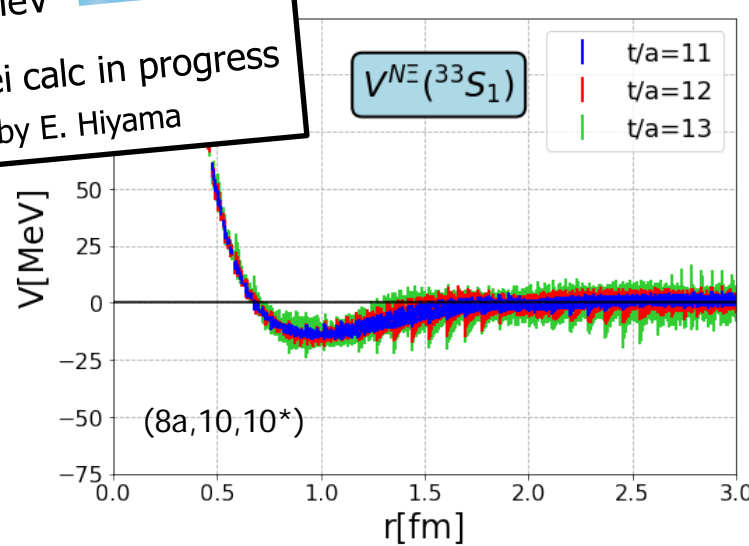
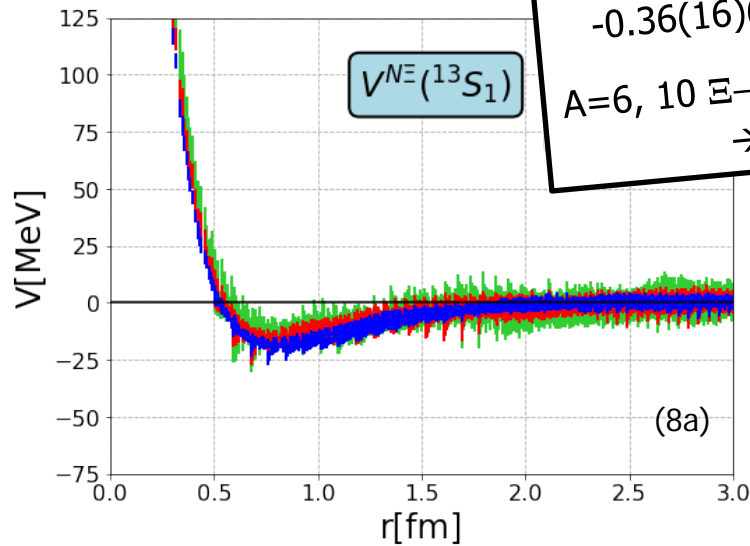
**I=0**

**I=1**

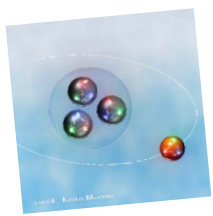
**S=0**



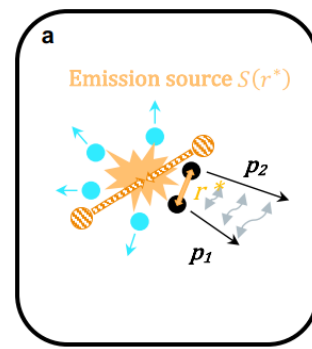
**S=1**



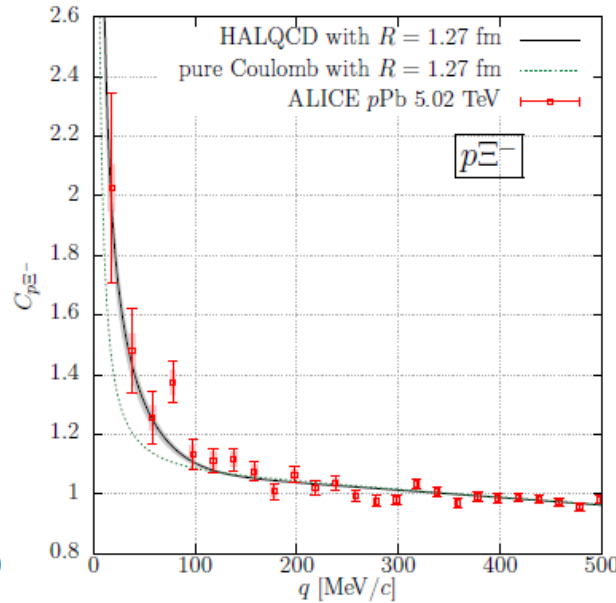
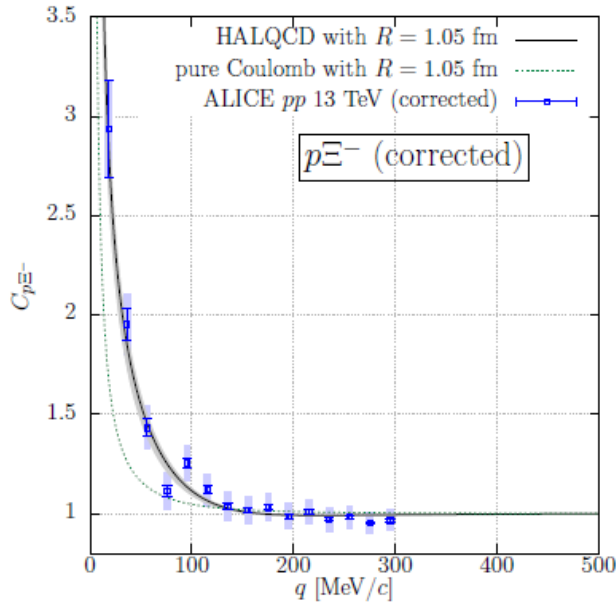
**$\Xi$ -tetrabaryon predicted!**  
 $I=0, J^P=1^+$   
 $-0.36(16)(26)\text{MeV}$   
 $A=6, 10 \Xi$ -nuclei calc in progress  
 $\rightarrow$  talk by E. Hiyama



# Femtoscscopy from nucleus collisions

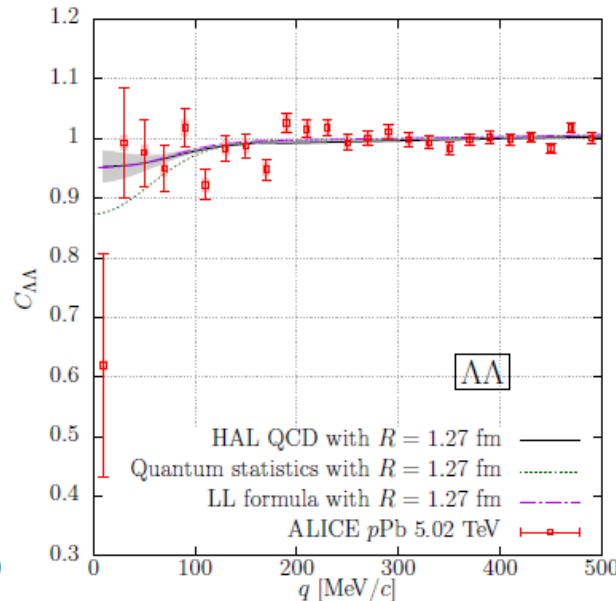
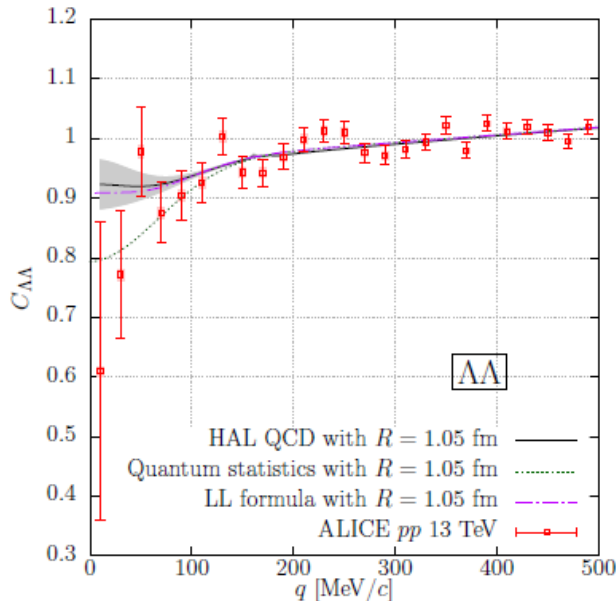


(Fig from ALICE)



←  $p\Xi^-$

LQCD prediction confirmed by experiment!



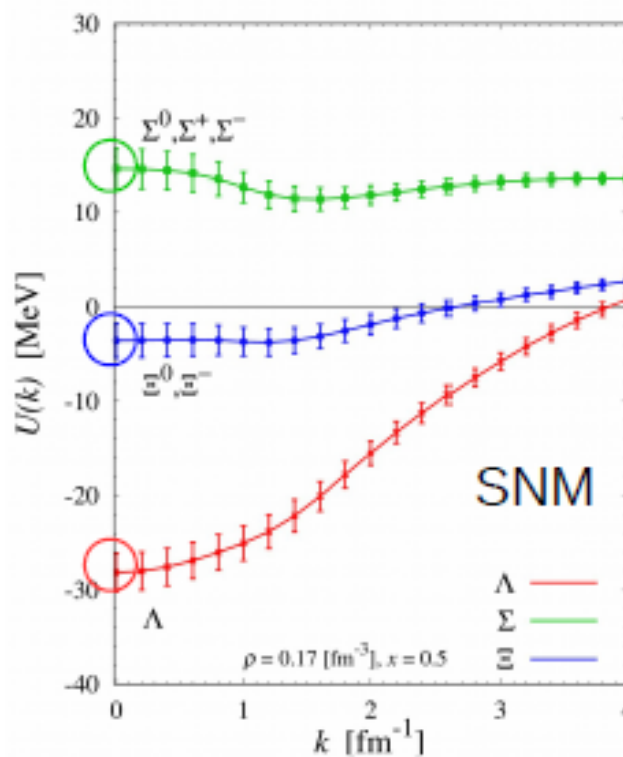
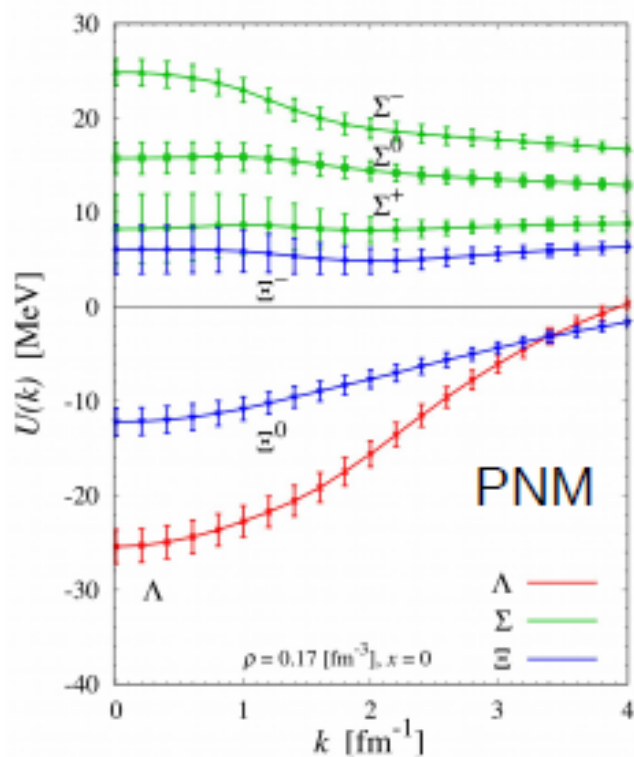
←  $\Lambda\Lambda$

Y. Kamiya et al.,  
PRC105(2022)014915

See also ALICE Coll., PLB797(2019)134822,  
PRL123(2019)112002, Nature 588(2020)232

# “Super-super heavy nuclei”: Dense matter from LQCD

## Hyperon single-particle potential



@ $\rho=0.17$  [fm<sup>-3</sup>]

Preliminary

T. Inoue (HAL Coll.)  
PoS INPC2016, 277

- obtained by using  $YN, YY$  S-wave forces from QCD.
- Results are compatible with experimental suggestion.

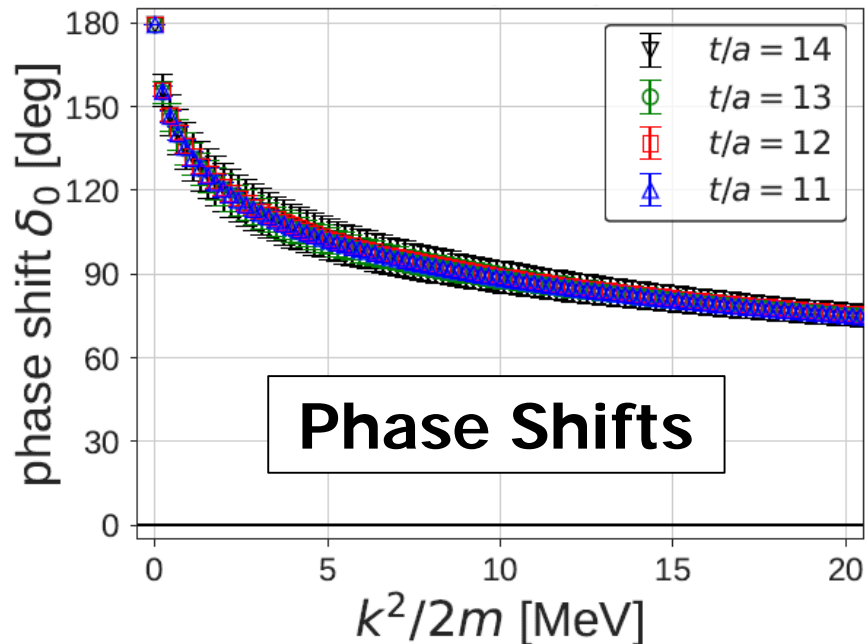
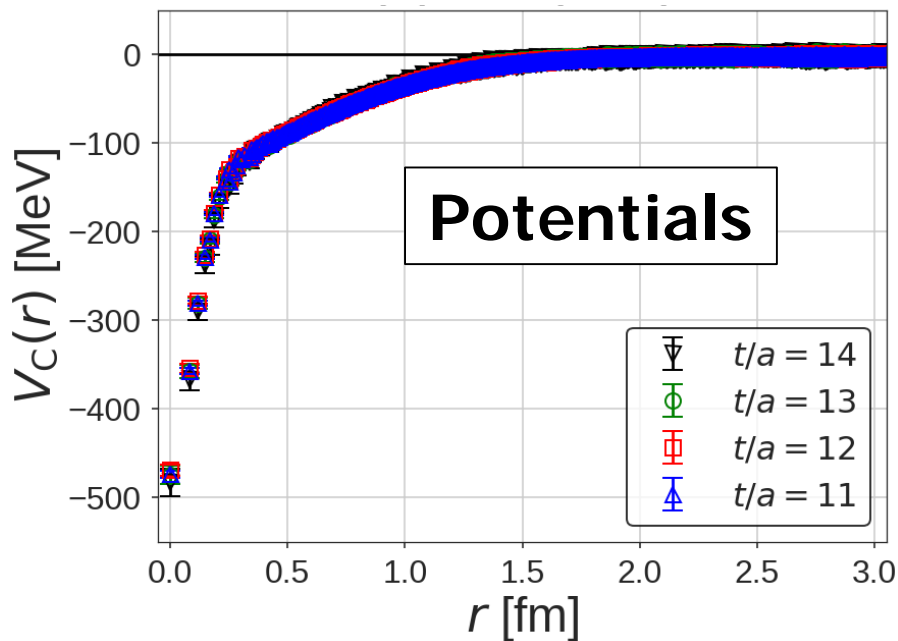
$$U_{\Lambda}^{\text{Exp}}(0) \simeq -30, \quad U_{\Xi}(0)^{\text{Exp}} \simeq -10?, \quad U_{\Sigma}^{\text{Exp}}(0) \geq +20? \quad [\text{MeV}]$$

attraction
attraction small
repulsion

49



# $N\Omega$ system ( ${}^5S_2$ )

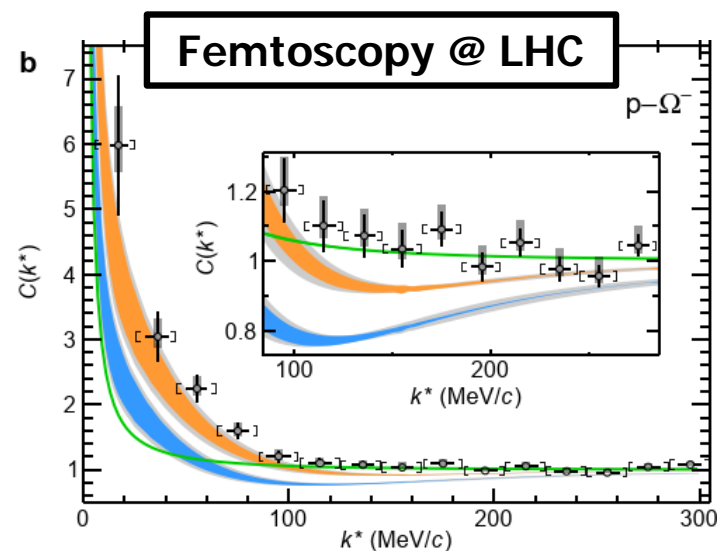


**(Quasi) Bound state**

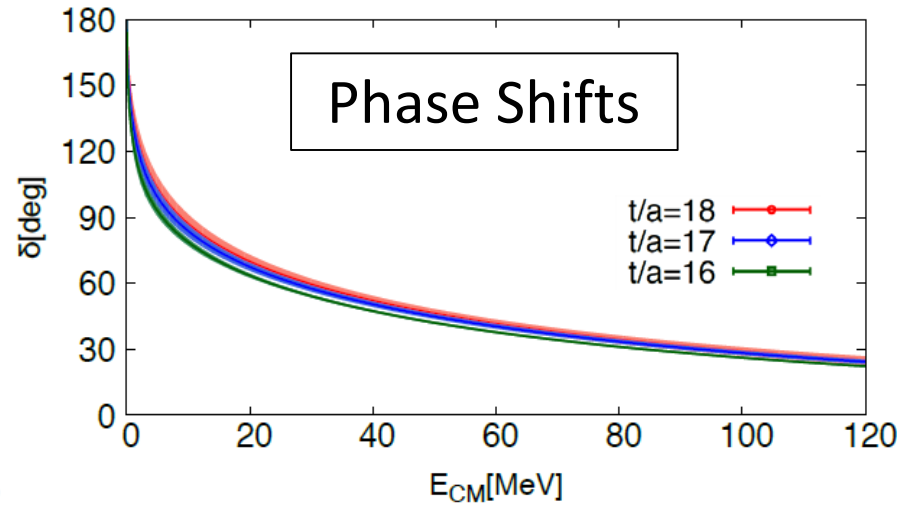
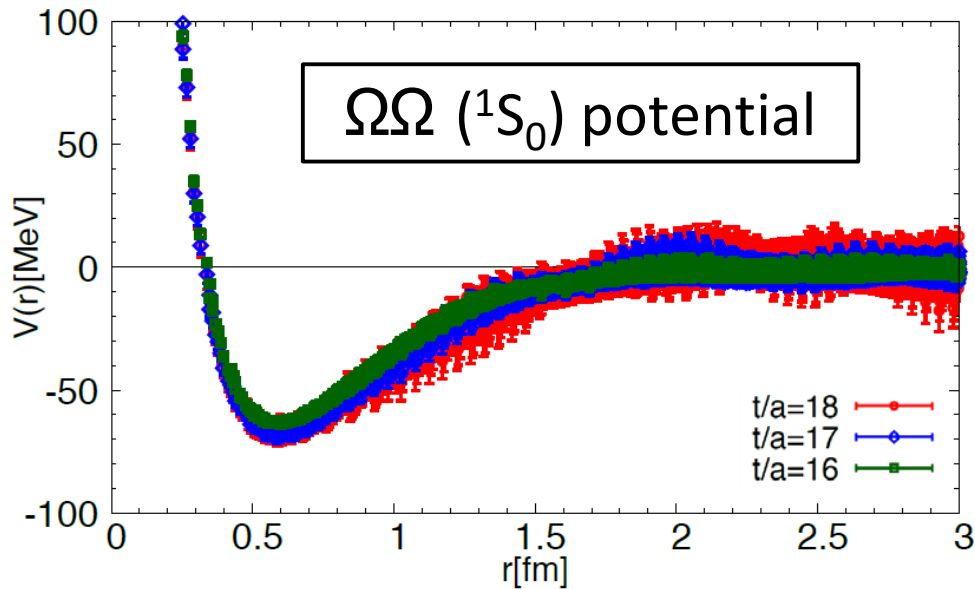
**[~ Unitary limit]**

$$B_{N\Omega} = 1.54(0.30)^{+0.04}_{-0.10} \text{ MeV}$$

$$B_{p\Omega^-} = 2.46(0.34)^{+0.04}_{-0.11} \text{ MeV}$$



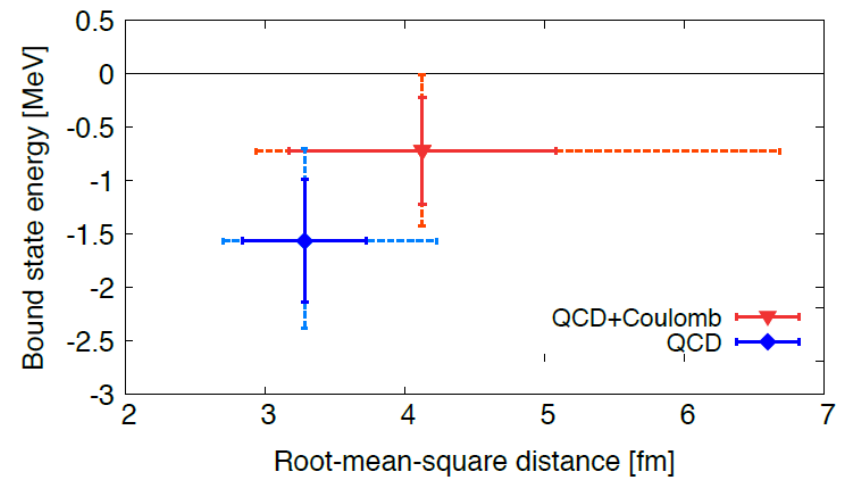
# "Most Strange" Dibaryon : $\Omega\Omega$



**"di-Omega"**

**[~ Unitary limit]**

could be searched in LHC RUN3

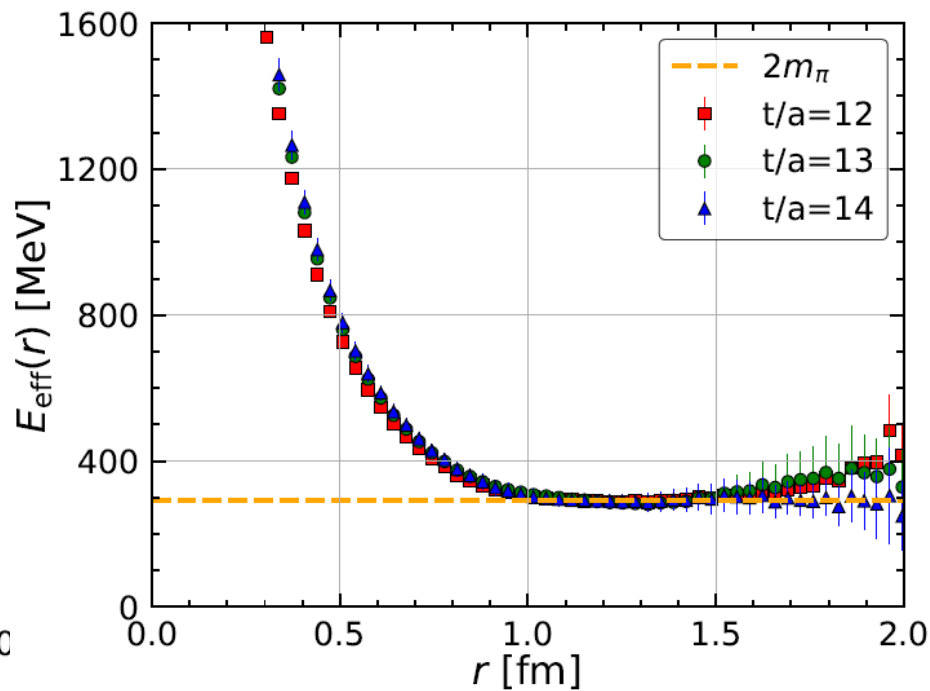
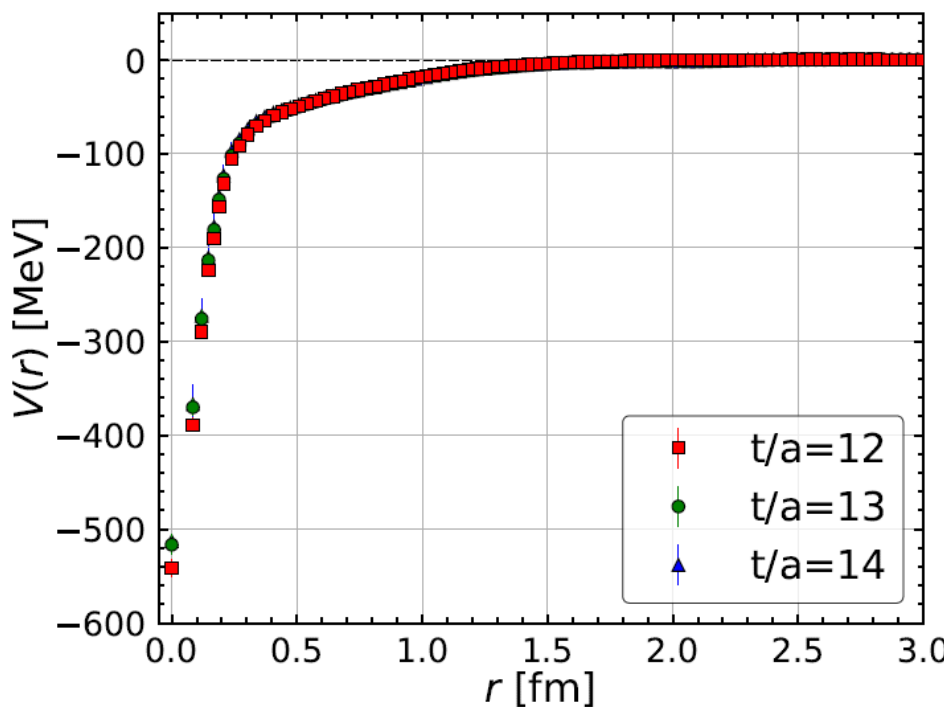


# N $\phi$ system ( $^4S_{3/2}$ )

Potential



Tail structure of potential



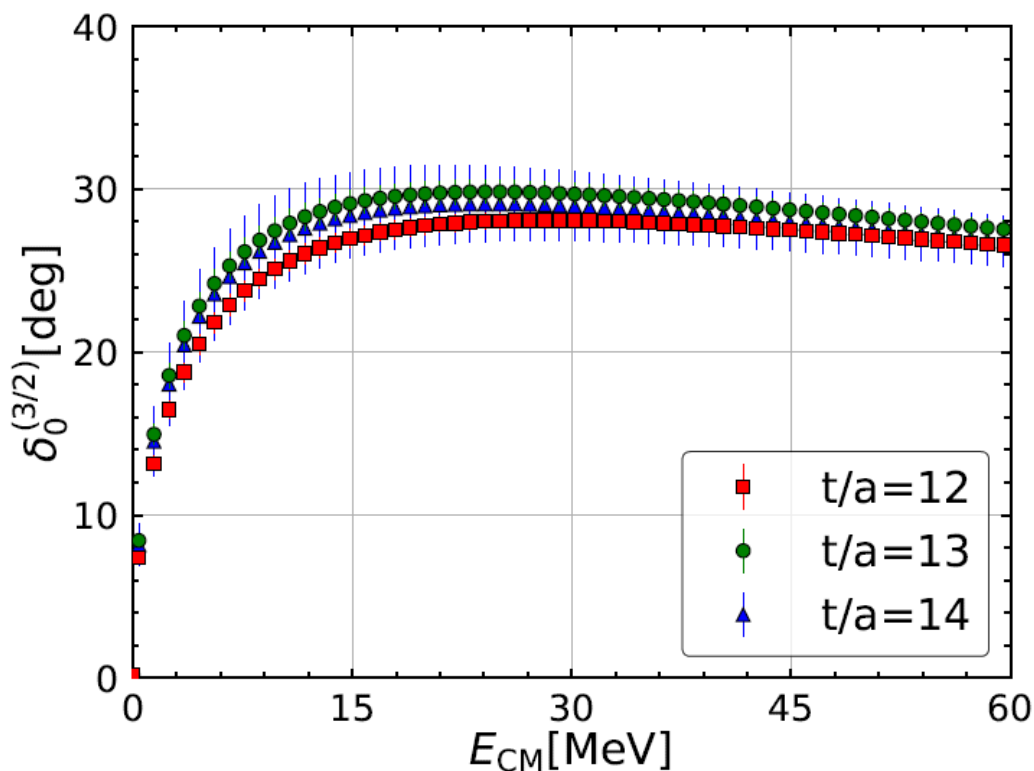
$$V(r) \xrightarrow{r \rightarrow \infty} -\alpha \frac{e^{-2m_\pi r}}{r^2} \implies E_{\text{eff}}(r) = -\frac{\ln[-V(r)r^2/\alpha]}{r} \xrightarrow{r \rightarrow \infty} 2m_\pi$$

Potential is attractive at all distances

Tail is consistent w/ two-pion exchange (TPE) !

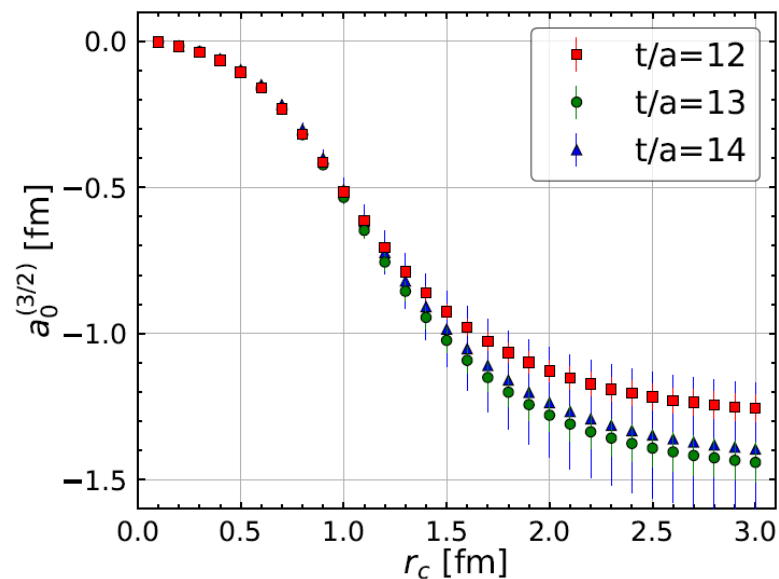
# No system ( ${}^4\text{S}_{3/2}$ )

## Phase shifts



## Scattering length from

$$V(r; r_c) = \theta(r_c - r)V(r)$$



→ Dominated by TPE tail

$$a_0^{(3/2)} = -1.43(23) \begin{pmatrix} +36 \\ -06 \end{pmatrix} \text{ fm}, \quad r_{\text{eff}}^{(3/2)} = 2.36(10) \begin{pmatrix} +02 \\ -48 \end{pmatrix} \text{ fm}$$

# Future prospects: physical point simulation!

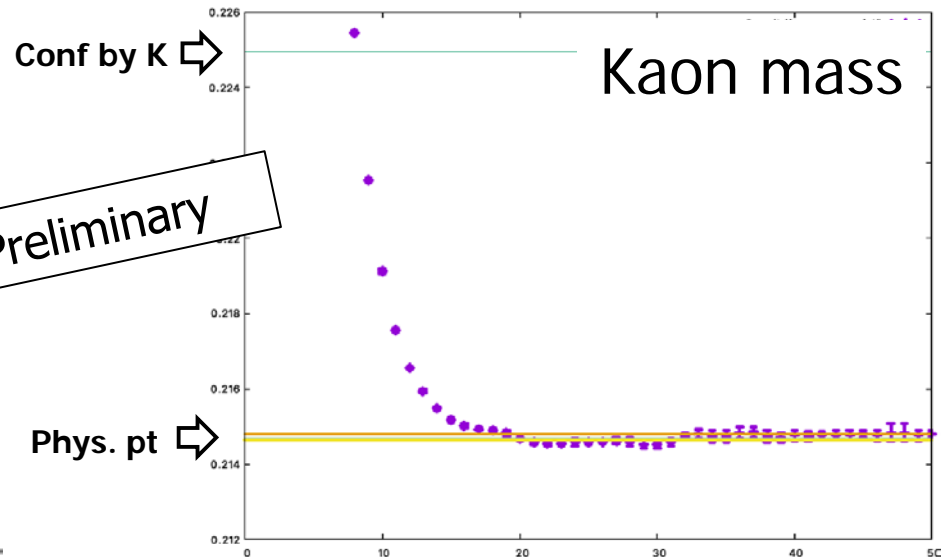
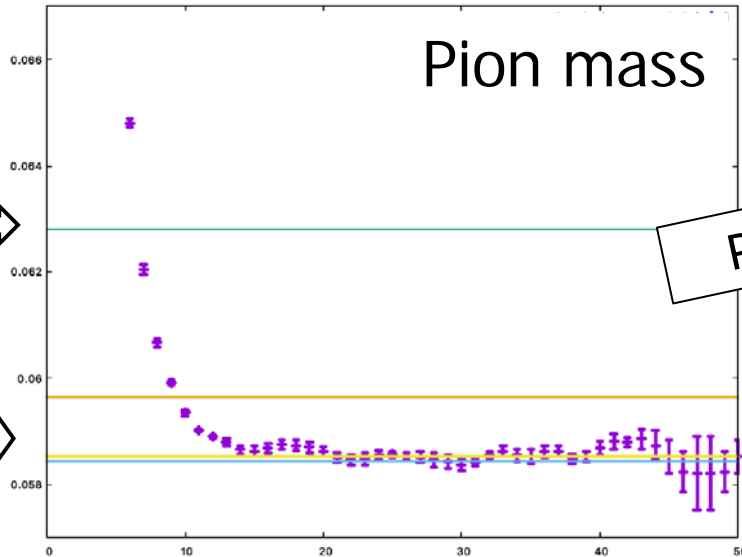
## Fugaku (富岳) supercomputer

Successor of K-computer  
Fastest in the world @ 2020-22



**440 PFlops!**

Our Efficiency =  $\sim 17\%$  (w/ naïve double prec count)



Preliminary

[ E. Ito+ ]

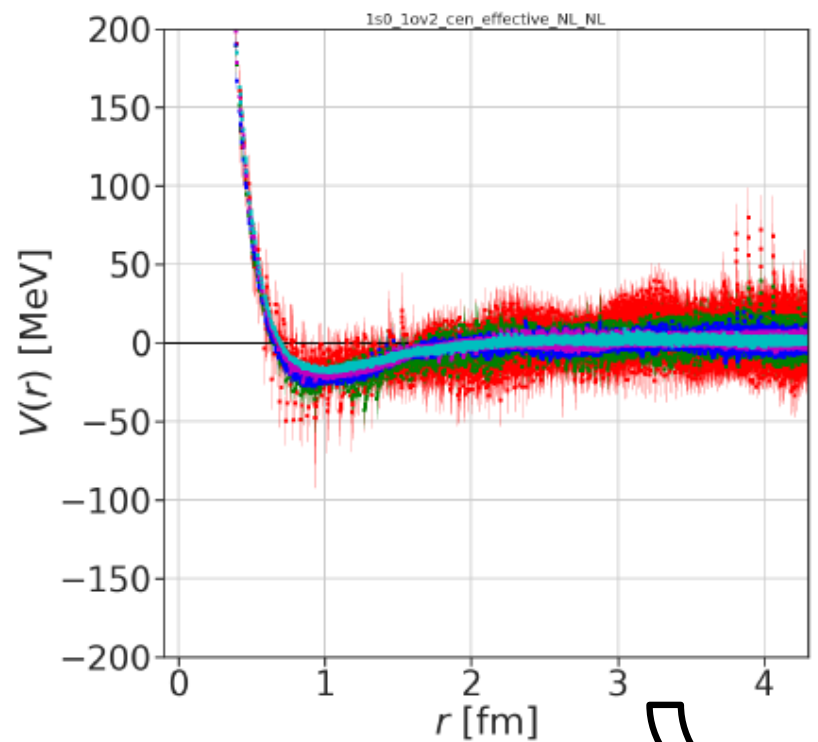
**We are on the physical point!**

# S/N improvement by partial wave decomposition

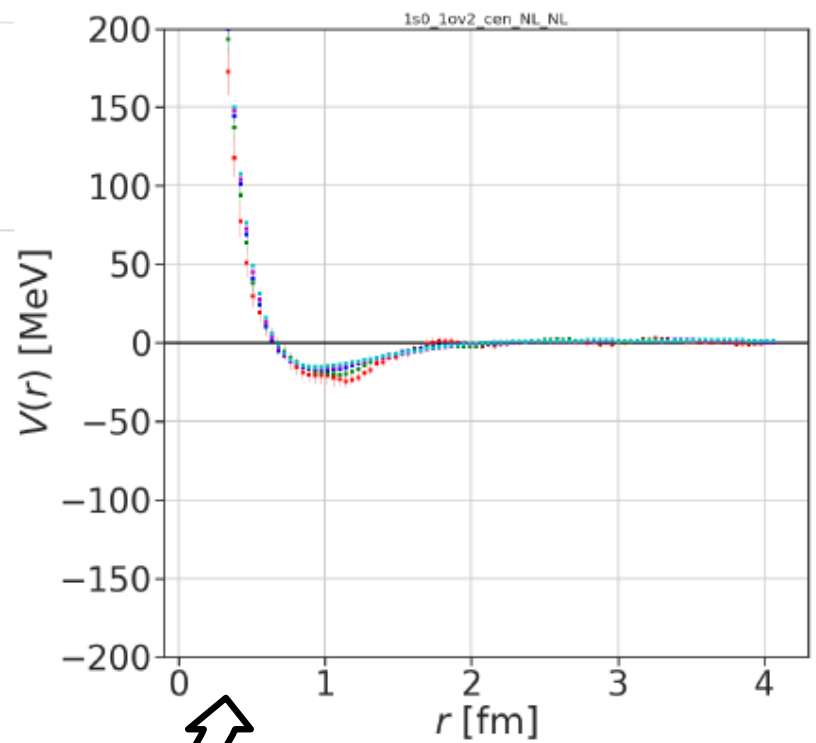
T. Miyamoto et al. (HAL Coll.), PRD101(2020)074514

## Effective $N\Lambda$ ( $^1S_0$ ) central potential

w/o partial wave decomp.



w/ partial wave decomp.



**Significant Improvement!**

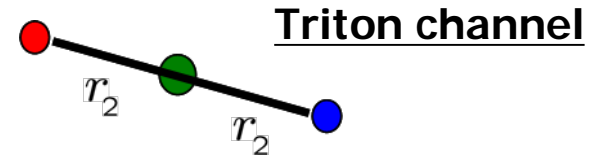
[ T. M. Doi ]

See also H. Nemura, arXiv:2203.07661  
for w/o partial wave decomp

N.B. improvement in phase shifts  
would be much milder

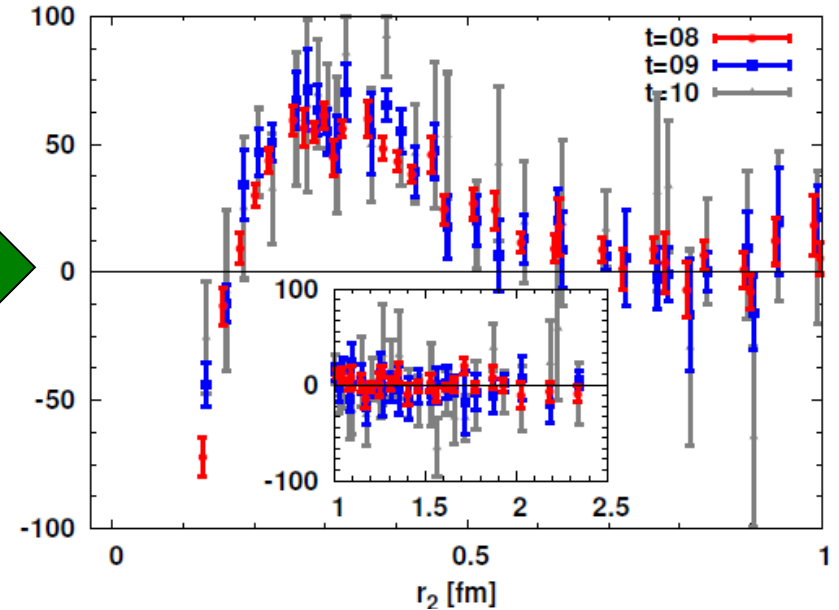
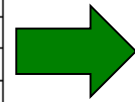
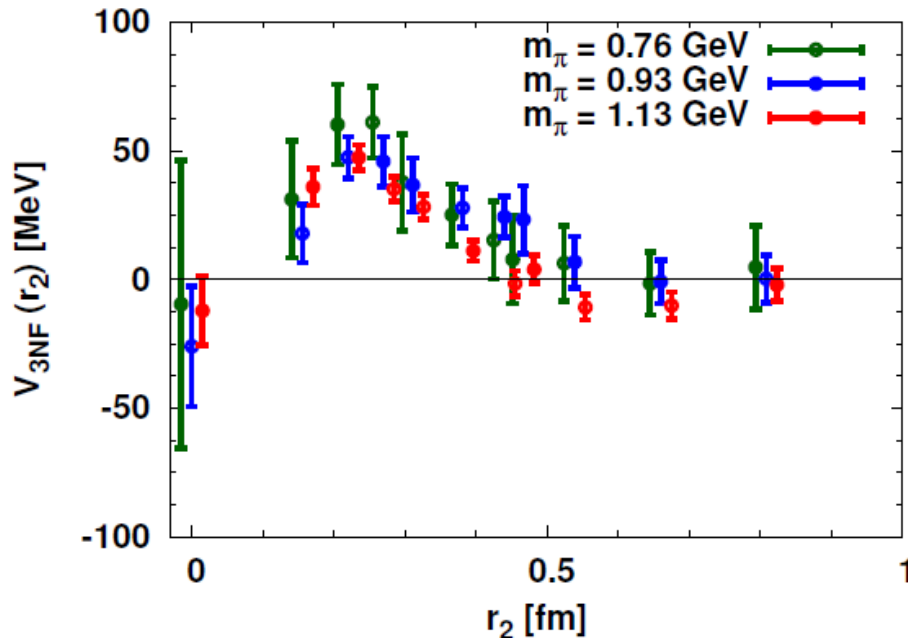
- t=12
- t=11
- t=10
- t=9
- t=8

# 3N-forces (3NF)



Nf=2,  $m_\pi=0.76-1.1$  GeV

Nf=2+1,  $m_\pi=0.51$  GeV



Magnitude of 3NF is similar for all masses  
 Range of 3NF tend to be enlarged for  $m(\pi)=0.5$ GeV

Next challenge: **Calc of P-wave 2BF** : better subtraction of 2BF in 3-body systems  
**YNN** (w/o or w/ P-wave 2BF) : gauge conf generation on Fugaku

# Summary

- Renaissance in particle/nuclear/astro-physics
  - **Observations** of neutron stars (LIGO-Virgo-KAGRA, NICER, ...)
  - **Experiments** of hadrons/nuclei → J-PARC, LHC, Belle II, ...
  - **Theory** by LQCD calc of hadron interactions
- The 1st LQCD for Baryon Interactions near the phys. point
  - Central/Tensor forces for NN/YN/YY in P=(+) channel
  - Dibaryons, Applications to Hypernuclei, EoS
- Prospects
  - **Hadron forces on the physical point** by **Fugaku supercomputer**
    - **Interactions w/ strangeness** are one of the primary targets, **Interactions w/ charm** also coming
  - Future: P=(-) channel, LS-forces, 3-baryon forces, etc., & EoS
  - Resonances & Exotics