# Lecture 3/4

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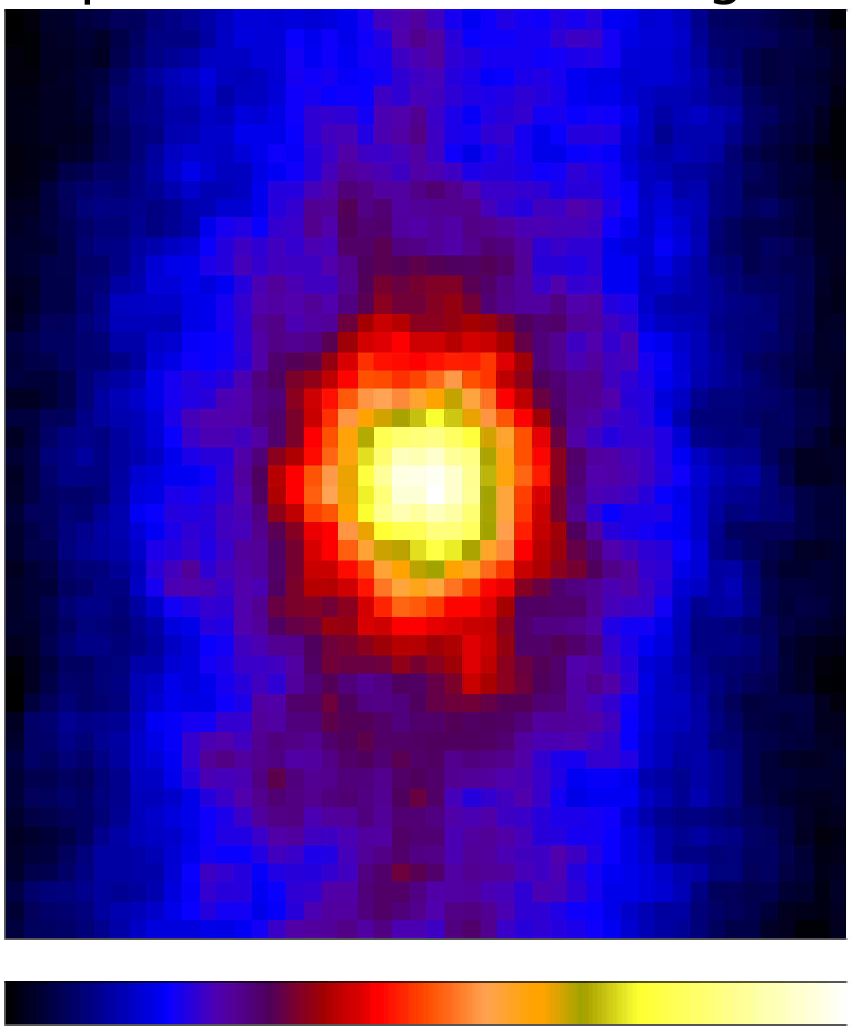
*Neutrino Flavour Oscillations*

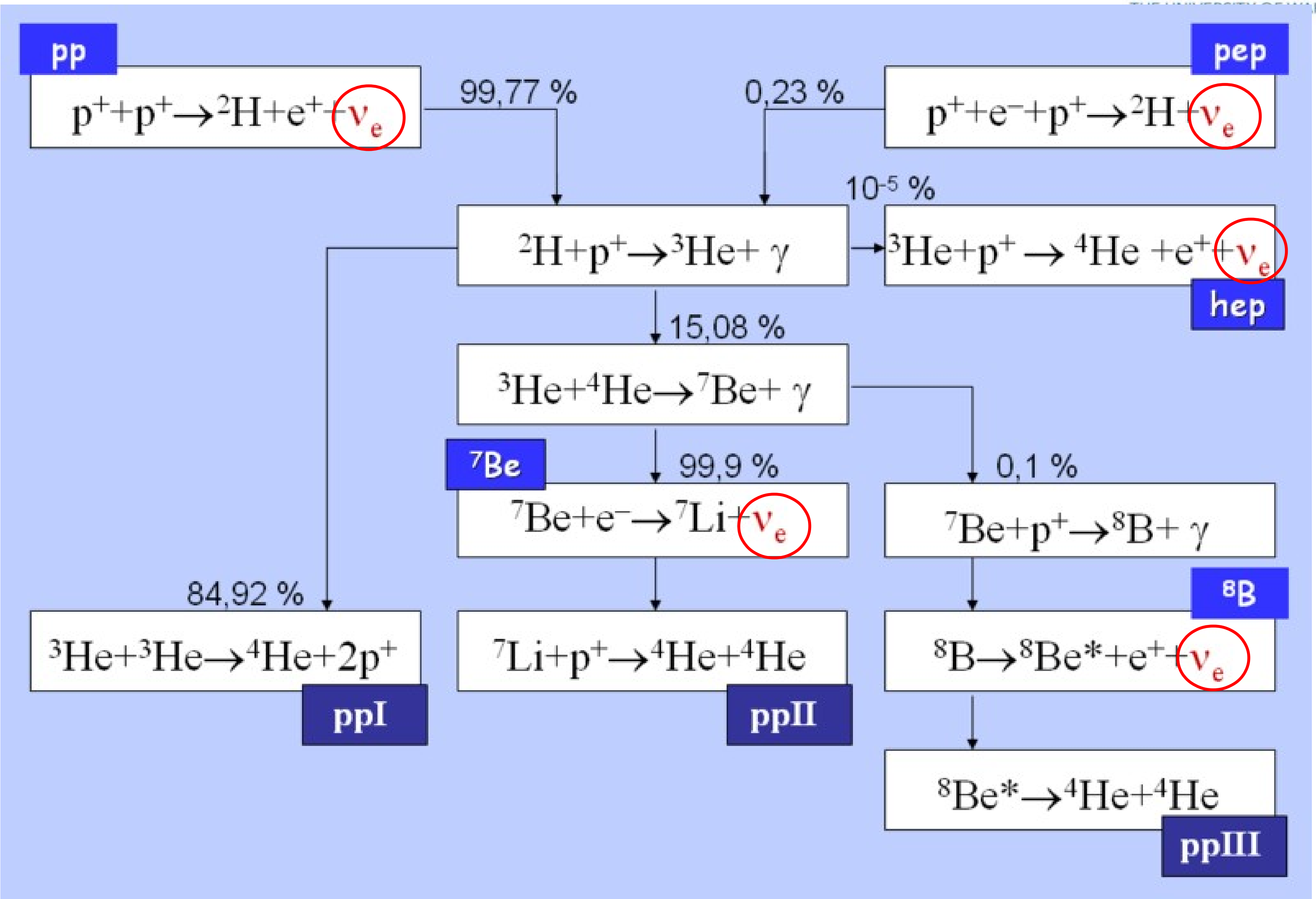
# Solar Neutrinos

SuperK : Solar neutrino-gram

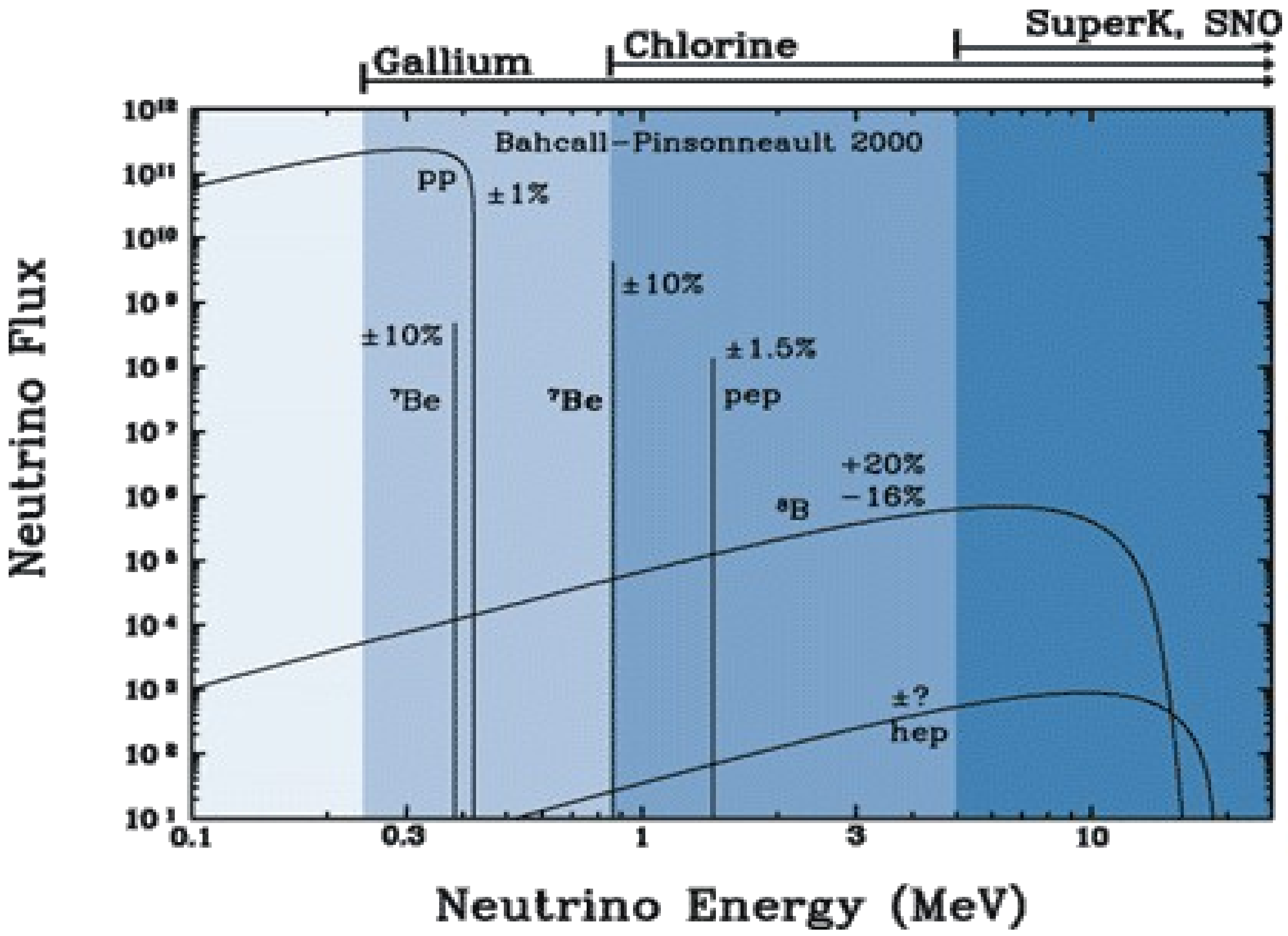


- Light from the solar core takes a million years to reach the surface
- Fusion processes generate electron neutrinos which take 2s to leave
- Solar neutrinos are a direct probe of the solar core
- Roughly 4.0 x $10^{10}$ solar $\nu_e$ per $cm^2$ per second on earth

# Solar neutrino – pp Cycle



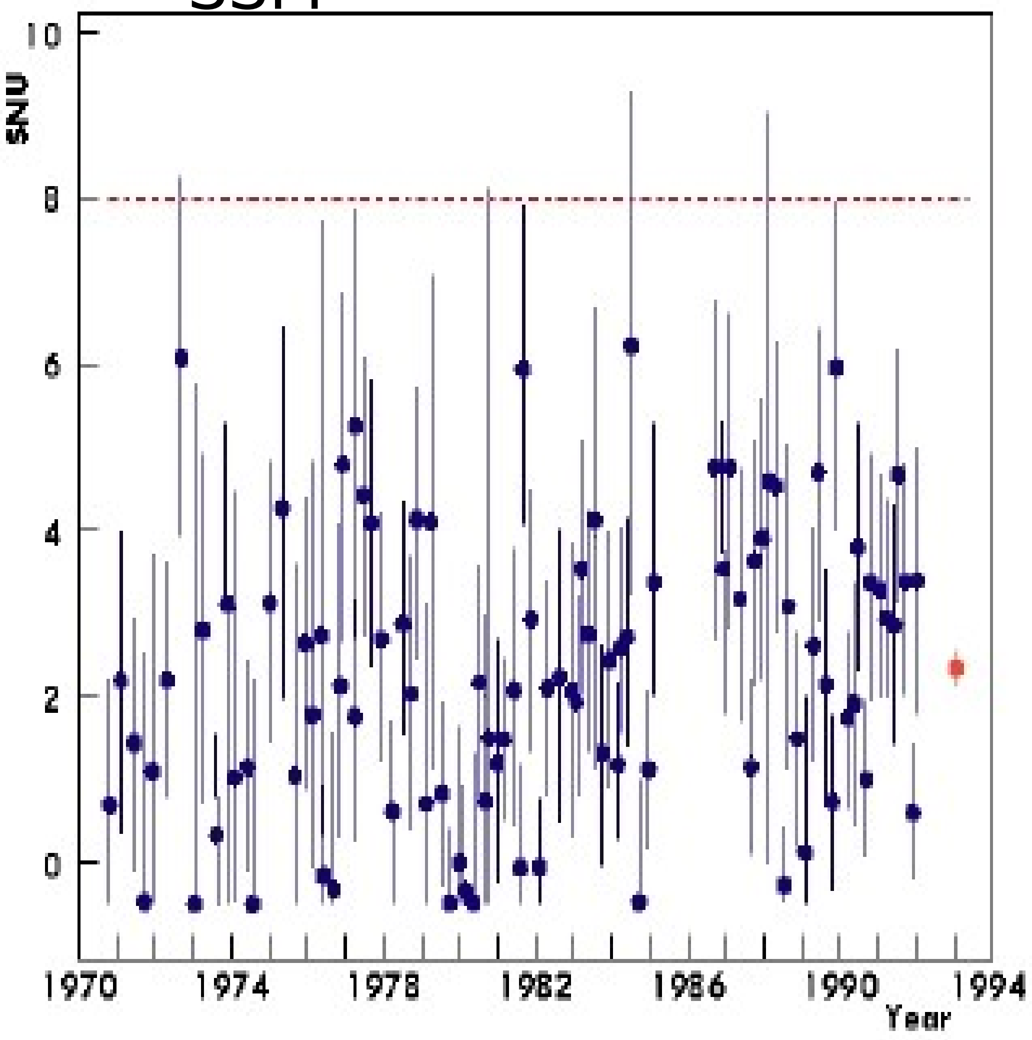
# Solar Neutrino Flux




As predicted by Bahcall's Solar model

# The Solar Neutrino Problem - Homestake

SSM



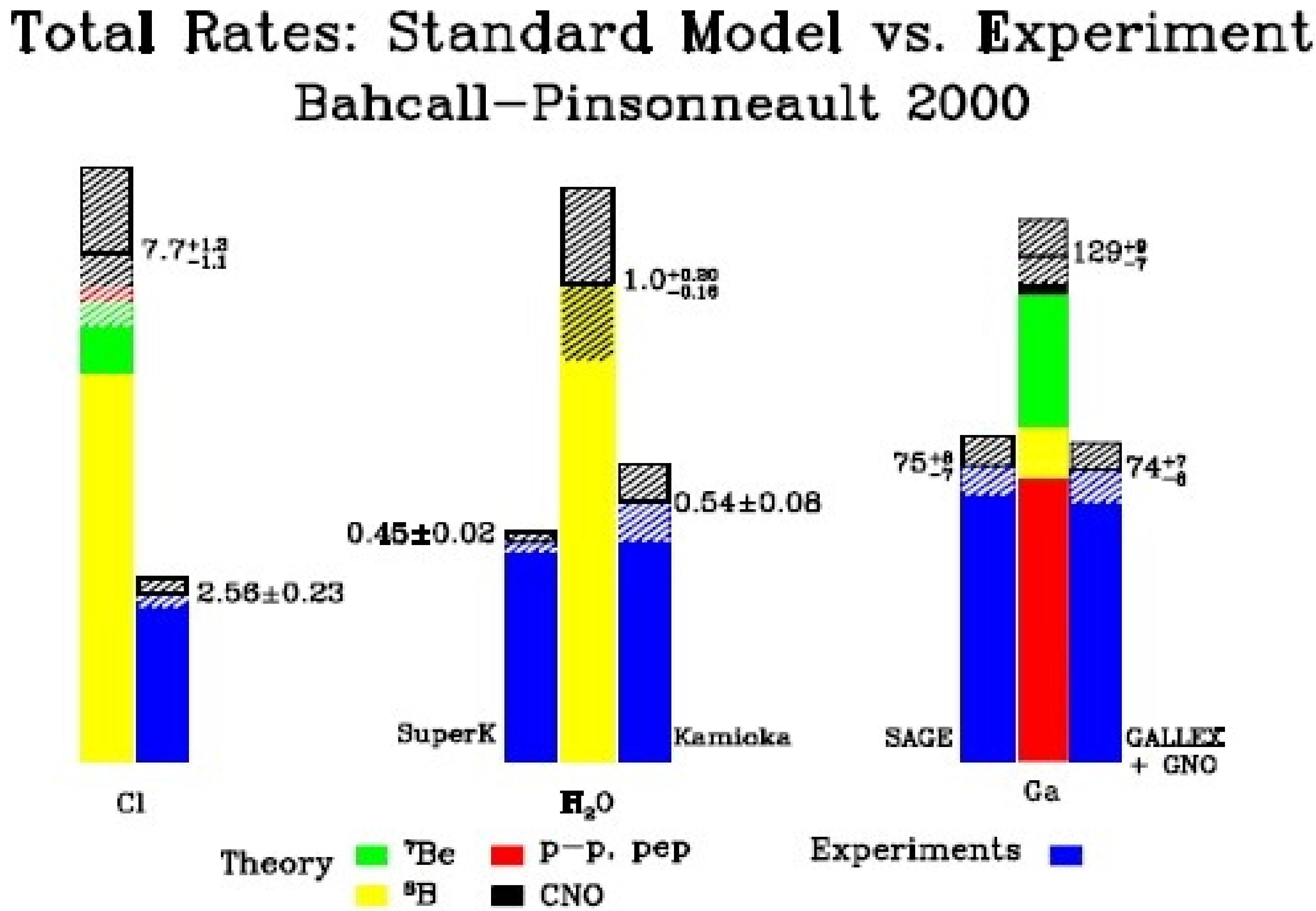

Homestake sensitive to $^{8}B$ and $^{7}Be$ *electron neutrinos*

$E_\nu$ > 800 keV

Observe 1/3 of the expected number of solar neutrinos

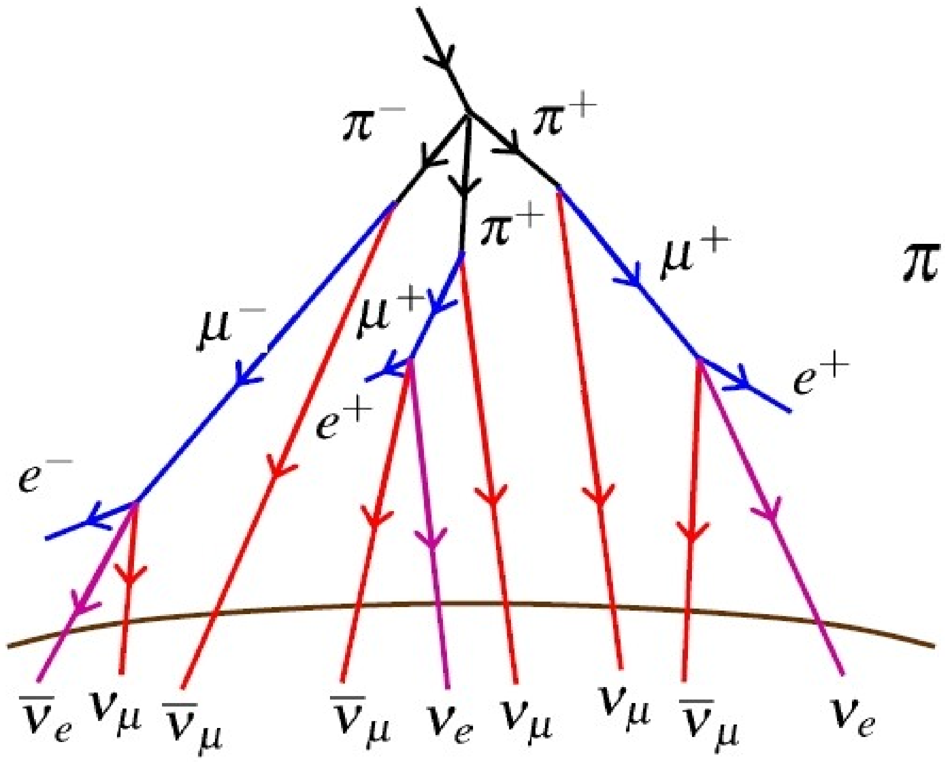
1 SNU = 1 interaction per $10^{36}$ atoms per second

# Experimental summary



# Atmospheric neutrinos

High energy cosmic rays interact in the upper atmosphere producing showers of mesons (mostly pions)




Neutrinos produced by

$$\pi^+ \rightarrow \mu^+ \nu_\mu \qquad \pi^- \rightarrow \mu^- \bar{\nu}_\mu$$

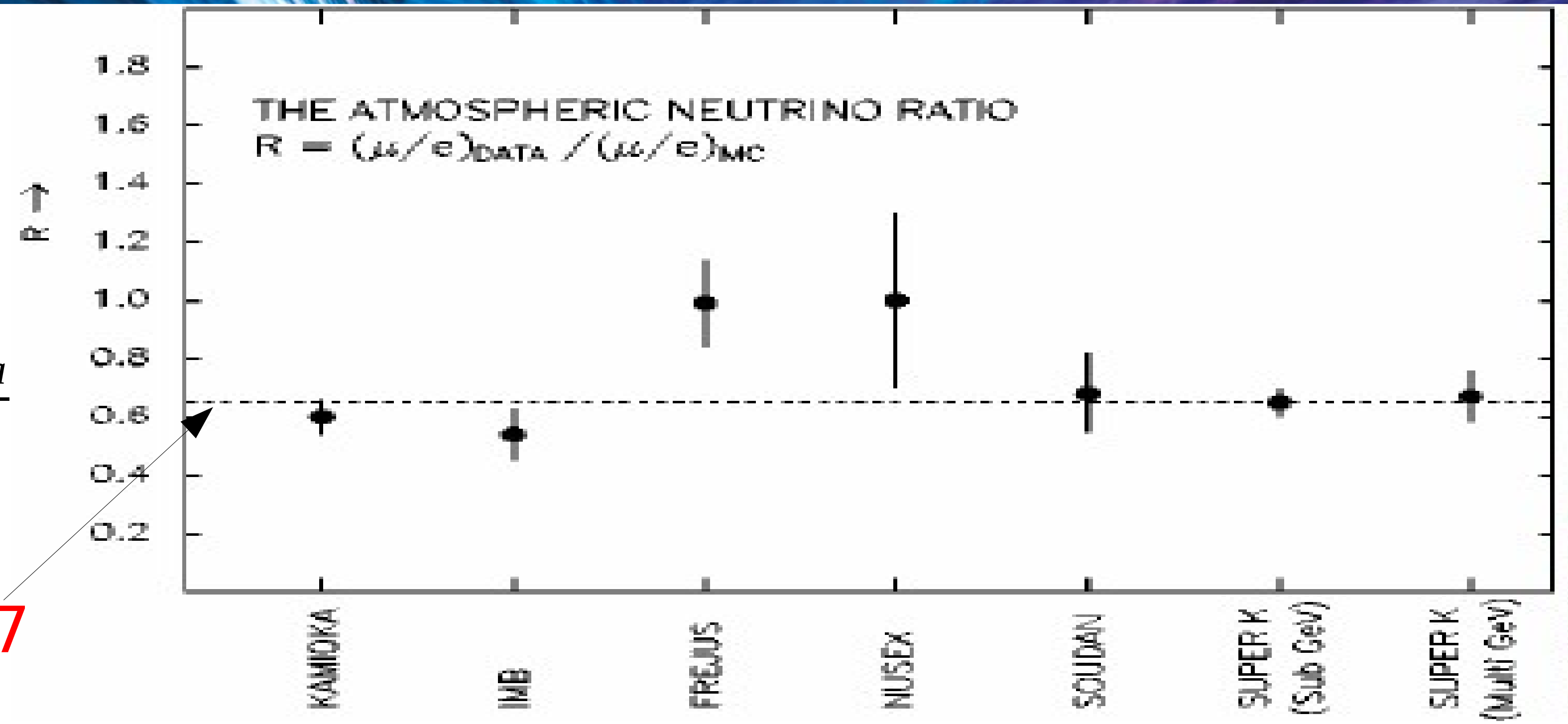
$$\hookrightarrow e^+ \nu_e \bar{\nu}_\mu \qquad \hookrightarrow e^- \bar{\nu}_e \nu_\mu$$

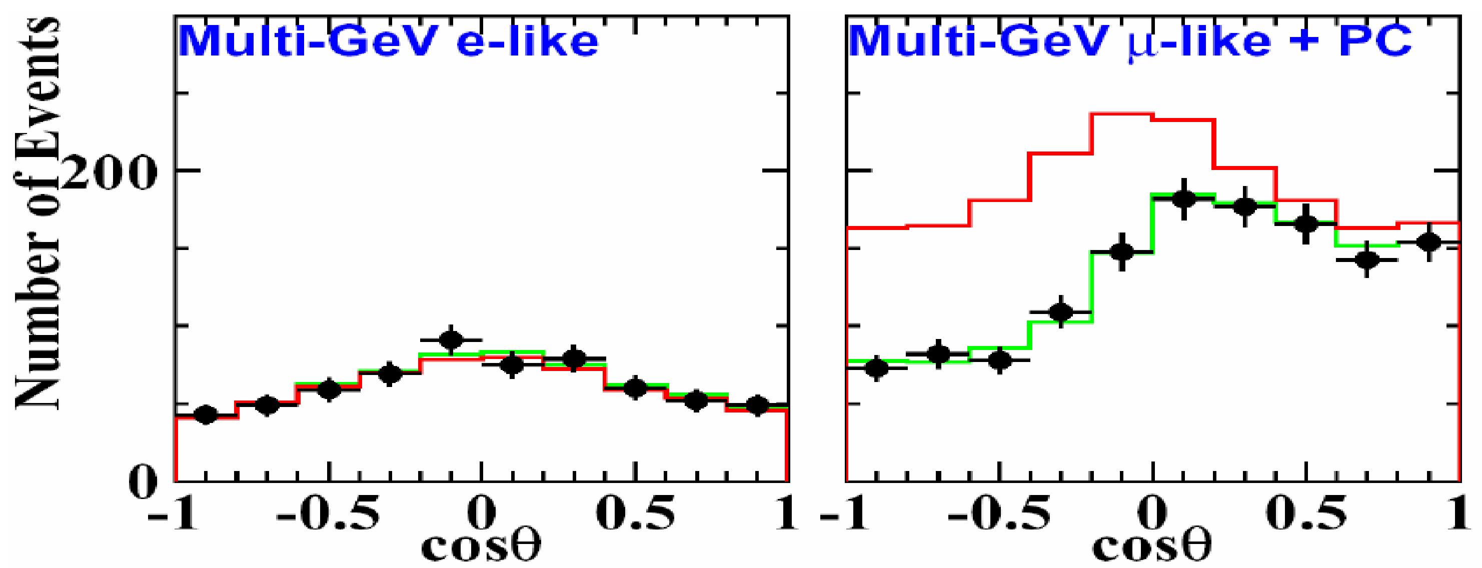
Expect $\frac{N\left(\nu_\mu + \overline{\nu_\mu}\right)}{N\left(\nu_e + \overline{\nu_e}\right)} \approx 2$

At higher energies, the muons can reach the ground before decaying so ratio increases

$$R = \frac{(\mu / e)_{Data}}{(\mu / e)_{MC}}$$

R ~ 0.6 - 0.7




The Atmospheric Neutrino Anomaly

# *Neutrino Flavour Oscillations*

# Mixing

CKM Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

$$d' = d\cos\theta_c + s\sin\theta_c$$

$$s' = -d\sin\theta_c + s\cos\theta_c$$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are solutions of the Dirac equation)

Weak states → 

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 0.97 & 0.23 & 0.003 \\ 0.23 & 0.97 & 0.04 \\ 0.008 & 0.04 & 0.99 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

← Mass states

CKM Matrix

# Mixing

CKM Mechanism

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L , \begin{pmatrix} c \\ s' \end{pmatrix}_L$$

$$d' = d\cos\theta_c + s\sin\theta_c$$

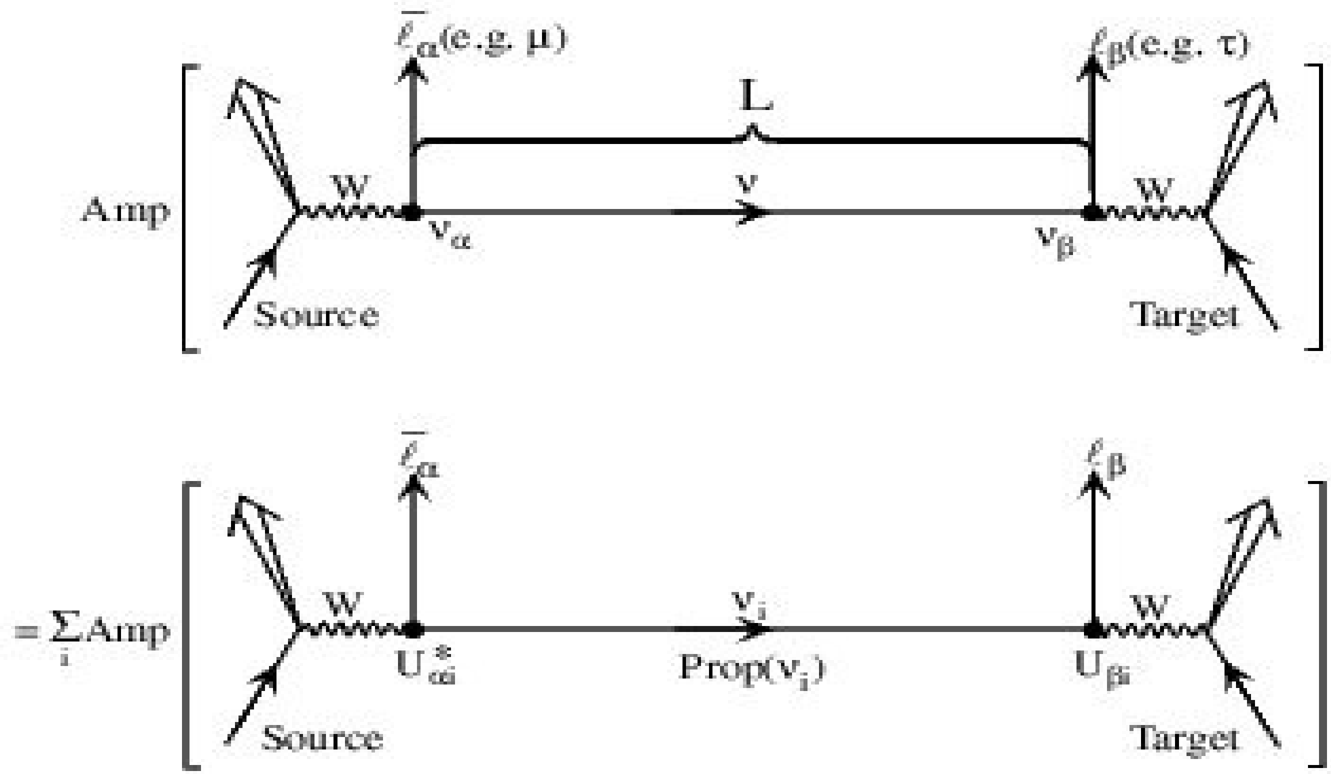
$$s' = -d\sin\theta_c + s\cos\theta_c$$

In the quark sector, the flavour eigenstates (those states which couple to the W/Z) are not identical to the mass eigenstates (those states which are solutions of the Dirac equation)

Weak states → $$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$ ← Mass states

Unitary mixing matrix → $U$

# Neutrino Oscillations




$$Amp(\nu_\alpha \rightarrow \nu_\beta) \propto \sum_i U^*_{\alpha i} \text{Prop}(\nu_i) U_{\beta i}$$

If we can't resolve the individual mass states then the amplitude involves a <u>coherent</u> superposition of $\nu_i$ states

# Bruno Pontercorvo

- Italian nuclear physicist
- Early assisstant of Fermi

- Spent most of his career obsessed with neutrinos
  - **1945** : Proposed detection of neutrinos via radiochemical method used 20 years later by Davis (Nobel)
  - **1957** : Proposed the idea of neutrino flavour oscillations
  - **1958** : Proposed that neutrinos came in different types. Proved by Lederman, Steinberger and Schwartz in 1962 (Nobel)
  - **1968** : Proposed neutrino flavour oscillations as solution of the solar neutrino problem. Later verified by McDonald and Kajita (Nobel)

- Defected to the USSR in 1950. Most of his ideas were locked behind the Iron Curtain for decades.

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$
$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

- If $\Delta m_{ij}^2 = 0$ then neutrinos don't oscillate
- Oscillation depends on $|\Delta m^2|$ - absolute masses, or mass patterns cannot be determined.
- If there is no mixing (If $U_{\alpha i} = 0$) neutrinos don't oscillate
- One can detect flavour change in 2 ways : start with $\nu_\alpha$ and look for $\nu_\beta$ (appearance) or start with $\nu_\alpha$ and see if any disappears (disappearance)
- Flavour change oscillates with L/E. L and E are chosen by the experimenter to maximise sensitivity to a given $\Delta m^2$
- Flavour change doesn't alter total neutrino flux – it just redistributes it amongst different flavours (unitarity)

# Two flavour oscillations

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \Rightarrow U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

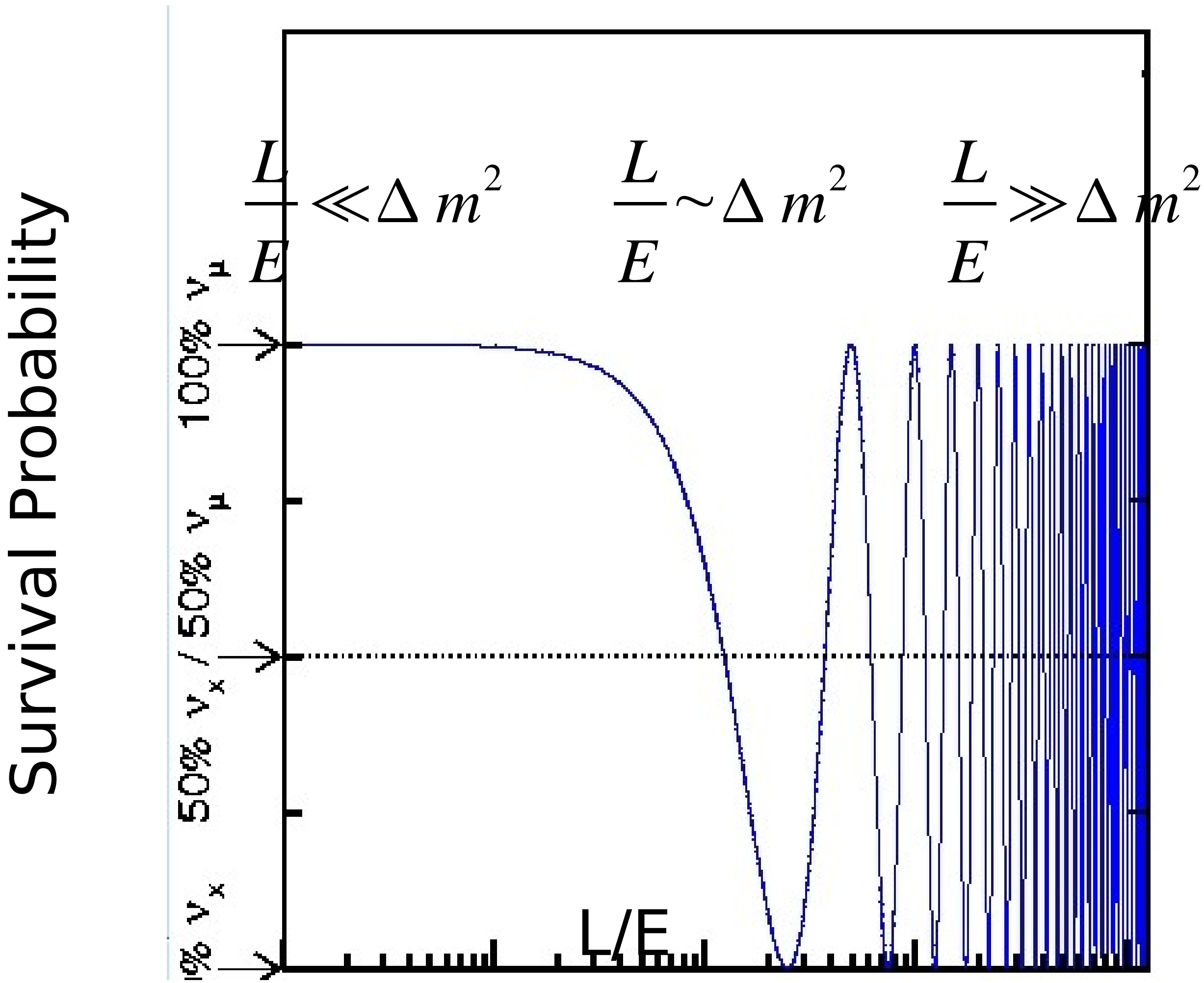
$P(\nu_\alpha \rightarrow \nu_\beta)$ : Appearance Probability

$P(\nu_\alpha \rightarrow \nu_\alpha)$ : Survival Probability

$$P(\nu_\alpha \rightarrow \nu_\beta) = -4(U_{\alpha 1} U_{\beta 1} U_{\alpha 2} U_{\beta 2}) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$

$$. = \sin^2(2\theta) \sin^2(1.27 \Delta m^2 (eV^2) \frac{L(km)}{E(GeV)})$$

(changing to useful units)




$$P(\nu_\alpha(0) \rightarrow \nu_\alpha(x)) = 1 - \sin^2(2\theta)\sin^2(1.27\,\Delta m^2 \frac{(L/km)}{(E/GeV)})$$

# Three Flavour Oscillation



The three flavour case is more complicated, but no different

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \Leftrightarrow U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}$$

U is the Pontecorvo-Maskawa-Nakayama-Sakata (PMNS) matrix

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2(\Delta m_{ij}^2 \frac{L}{4E})$$
$$+ 2\sum_{i>j} \Im(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin(\Delta m_{ij}^2 \frac{L}{2E})$$

# PMNS* matrix

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

2 independent $\Delta m^2$

$$Prob(\nu_\alpha \to \nu_\beta)=\delta_{\alpha\beta}-4\sum_{i>j}\Re(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2(\Delta m^2_{ij}\frac{L}{4E})$$

$$+2\sum_{i>j}\Im(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin(\Delta m^2_{ij}\frac{L}{2E})$$

*Pontercorvo-Maki-Nakagama-Sakata

# PMNS matrix

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Three angles

$$Prob(\nu_\alpha \rightarrow \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

# PMNS matrix

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

CP violating phase

$$Prob(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E})$$

$$+2\sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E})$$

# PMNS matrix

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$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Extra Majorana phases

The extra Majorana matrix does not affect flavour oscillation processes.....so is usually dropped. However it will affect the interpretation of neutrinoless double beta decay results

# Explaining the solar data

# Testing the oscillation hypothesis

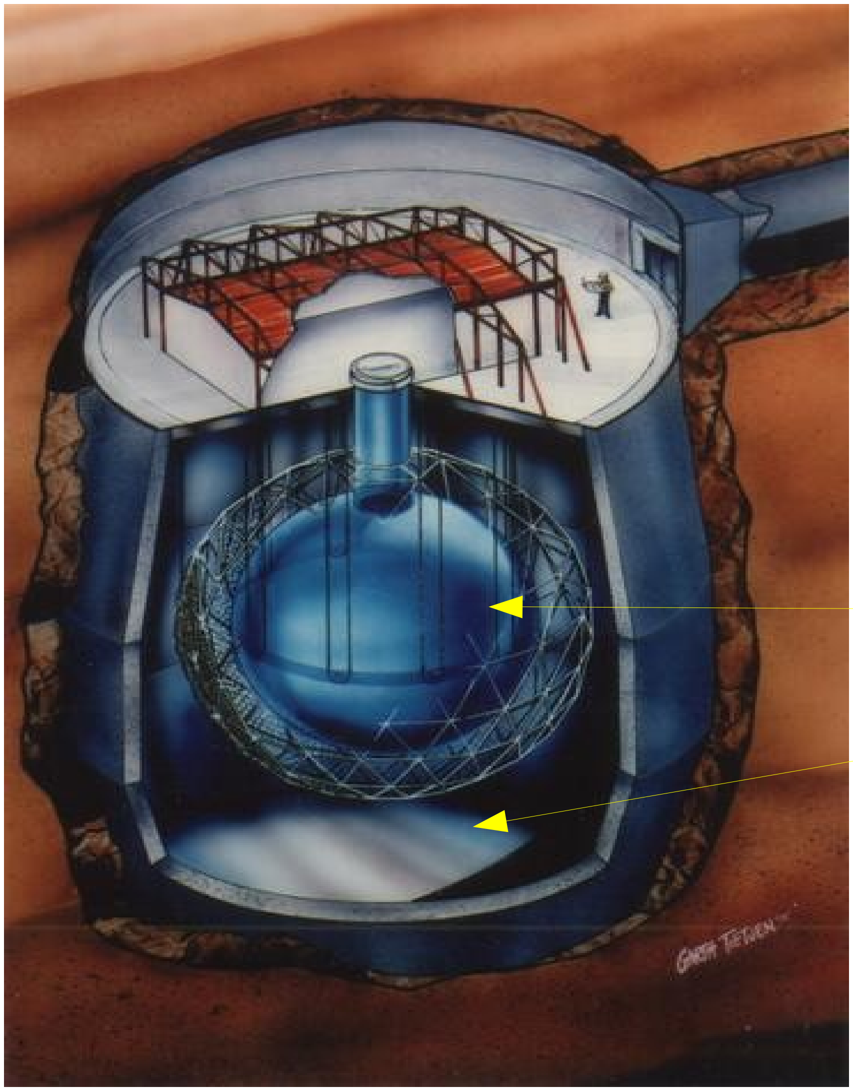
## Solar neutrino problem

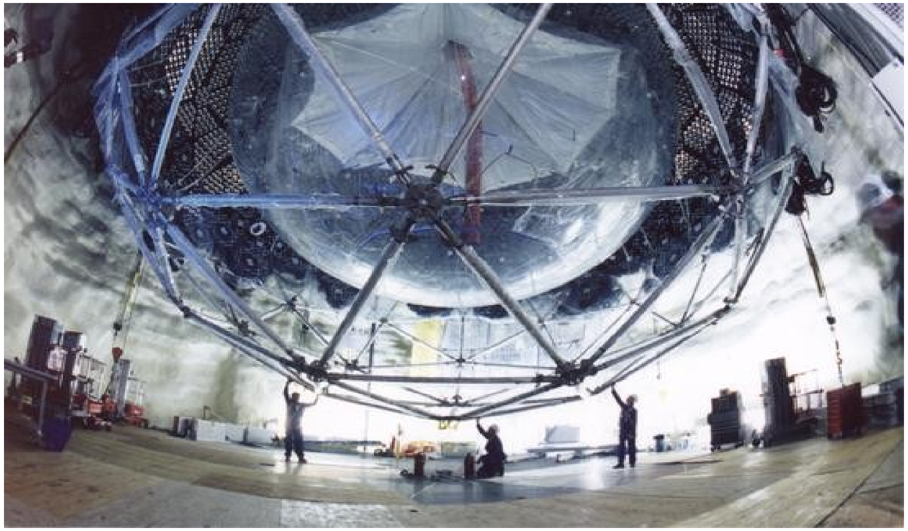
$\nu_e$ from sun would change to $\nu_\mu$ or $\nu_\tau$ . However these have too little energy to interact via the charged current, and all the detectors are only sensitive to charge current interactions.

Non-$\nu_e$ component would effectively disappear, reducing the apparent $\nu_e$ flux.

**Proof : Neutral current event rate shouldn't change.**

# Sudbury Neutrino Observatory





1000 tonnes of $D_2 0$

6500 tons of $H_2 0$

Viewed by 10,000 PMTS

In a salt mine 2km underground in Sudbury, Canada

# SNO

CC $\nu_e + d \rightarrow p + p + e^-$

Produces Cherenkov Light Cone in $D_2O$

$\nu_e$

- Q = 1.445 MeV
- good measurement of $\nu_e$ energy spectrum
- some directional info $\propto (1 - 1/3 \cos\theta)$
- $\nu_e$ only

NC $\nu_x + d \rightarrow p + n + \nu_x$

n captures on deuteron $^2H(n, \gamma)^3H$

Observe 6.25 MeV $\gamma$

$\nu_e+\nu_\mu+\nu_\tau$

- Q = 2.22 MeV
- measures total $^8B$ $\nu$ flux from the Sun
- equal cross section for all $\nu$ types

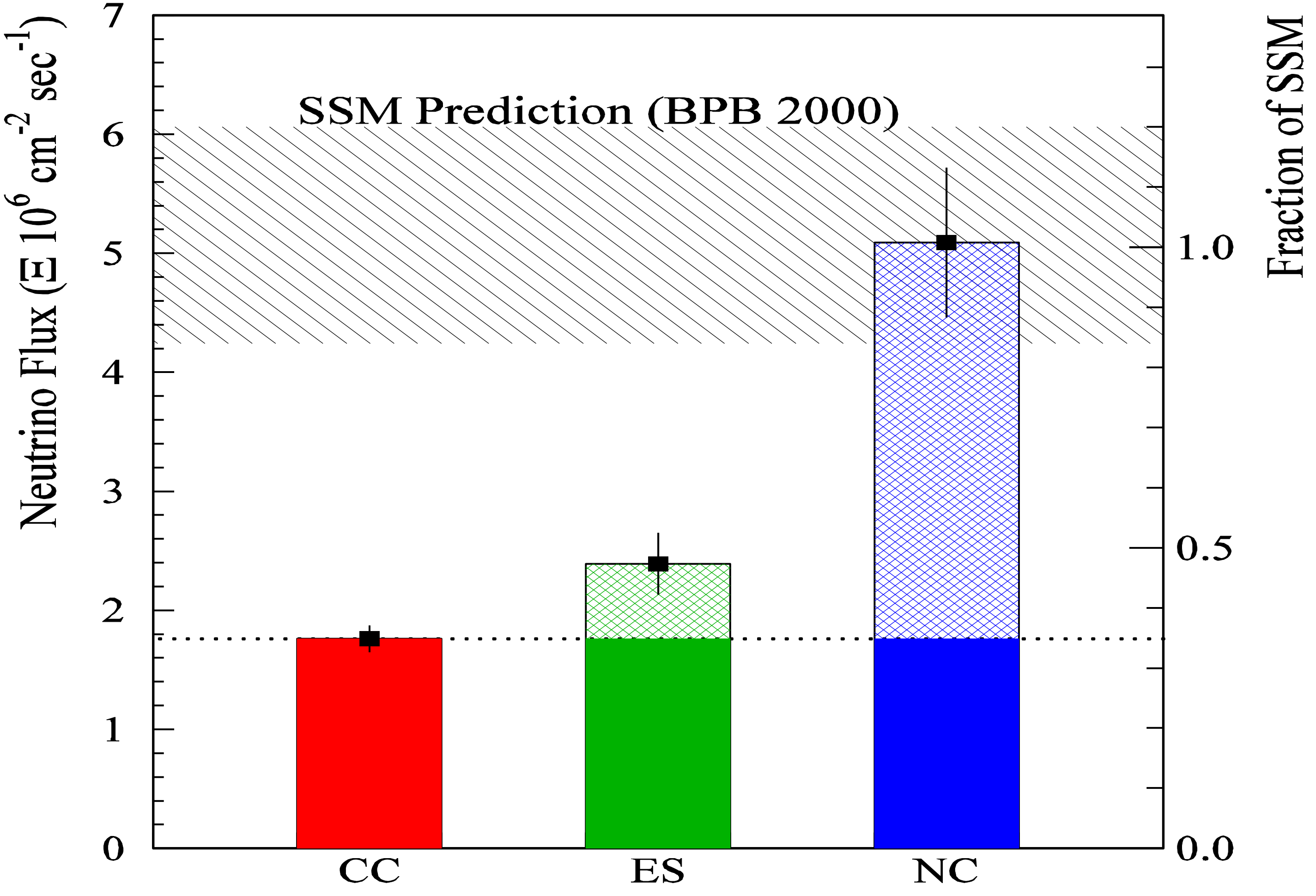
ES $\nu_x + e^- \rightarrow \nu_x + e^-$

Produces Cherenkov Light Cone in $D_2O$

$\nu_e+0.15*(\nu_\mu+\nu_\tau)$

- low statistics
- mainly sensitive to $\nu_e$, some $\nu_\mu$ and $\nu_\tau$
- strong directional sensitivity

# SNO Results




5.3 σ appearance of $v_{\mu,\tau}$ in a $v_e$ beam

Roughly 70% of $n_e$ oscillates away

# Naively...

First instinct is to assume that neutrinos leave the sun as $\nu_e$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8\, km,\, E_\nu < 10\, MeV \rightarrow \Delta m^2 \sim 3 \times 10^{-10}\, eV^2$$

# Naively...

First instinct is to assume that neutrinos leave the sun as $n_e$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8\, km,\, E_\nu < 10\, MeV \rightarrow \Delta m^2 \sim 7 \times 10^{-5}\, eV^2$$

# Naively...

First instinct is to assume that neutrinos leave the sun as $\nu_e$ and oscillate on their way to the earth. Assuming this

$$L \sim 10^8\, km\,,\ E_\nu < 10\ MeV \rightarrow \Delta m^2 \sim \boxed{7 \times 10^{-5}\ eV^2}$$

Oscillations come from phase difference between mass states. In a vacuum the phase diff comes from free particle Hamiltonian. In a material there are interaction potentials as well

$$-i\hbar \frac{\partial \psi}{\partial t} = \boxed{E\psi} = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \rightarrow -i\hbar \frac{\partial \psi}{\partial t} = \boxed{(E+V)}\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$E^2 - p^2 = m_{vac}^2 \rightarrow (E+V)^2 - p^2 = m_{mat}^2 \rightarrow \boxed{m_{mat} \approx \sqrt{m_{vac}^2 + 2EV}}$$

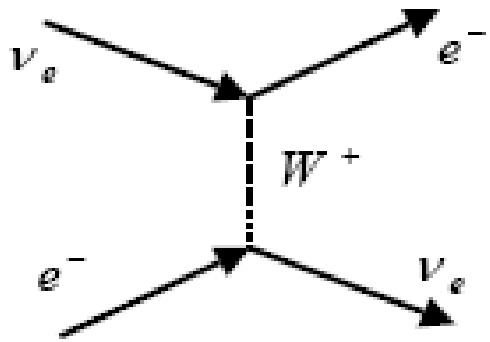
c.f. effective mass of an electron in a semiconductor or light in glass

# Oscillations in Matter

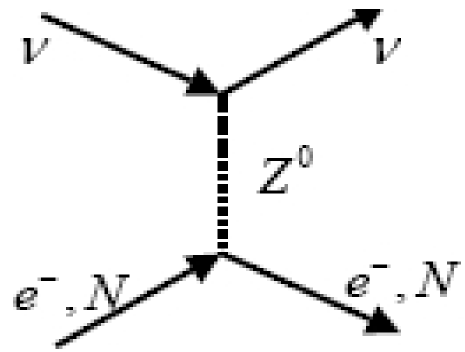
Electrons exist in standard matter – μ/τ do not. Electron neutrinos travelling in matter can experience an extra charged current interaction that other flavours cannot.

$$V_W = \sqrt{2}\, G_F N_e$$

Interaction Potential




$\nu_e$




$\nu_e, \nu_\mu, \nu_\tau$

$$V_Z = -\frac{\sqrt{2}}{2} G_F N_n$$

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta_M)\sin^2\left(\frac{\Delta m_M^2 L}{4E}\right)$$

Oscillation probability modified by matter effects

$$\Delta m_M^2 = \Delta m_V^2 \sqrt{\sin^2(2\theta) + (\cos 2\theta - \zeta)^2}$$

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2}$$

$$\zeta = \frac{2\sqrt{2}\, G_F N_e E}{\Delta m_V^2}$$

# Implications

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta_V}{\sin^2 2\theta_V + (\cos 2\theta_V - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2_{Vac}}$$

- If $\Delta m^2_{Vac} = 0$ or matter is very dense, $\zeta = \infty$ and $\theta_M = 0$
- Similarly, if $\theta_V = 0$, then $\theta_M = 0$
- If there is no matter, then $\zeta = 0$ and we have vacuum mixing
- At a particular electron density, dependent on $\Delta m^2$,

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m^2} = \cos 2\theta_V \;\Rightarrow\; \sin^2 2\theta_M = 1$$

Even if the vacuum mixing angle is tiny, there is a density for which the matter mixing is large

# Mass heirarchy

$$\sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + (\cos 2\theta - \zeta)^2} \qquad \zeta = \frac{2\sqrt{2} G_F N_e E}{\Delta m_V^2}$$

- If mass of $\nu_1$ < mass of $\nu_2$, $\Delta m^2 = m_1^2 - m_2^2 < 0$

$$\zeta = -\frac{2\sqrt{2} G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(cos2\theta + |\zeta|\right)^2}$$

Positive definite – no resonance

- If mass of $\nu_1$ > mass of $\nu_2$, $\Delta m^2 = m_1^2 - m_2^2 > 0$

$$\zeta = \frac{2\sqrt{2} G_F N_e E}{|\Delta m^2|} \rightarrow \sin^2 2\theta_M = \frac{\sin^2 2\theta}{\sin^2 2\theta + \left(cos2\theta - |\zeta|\right)^2}$$

# Mixing matrix

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

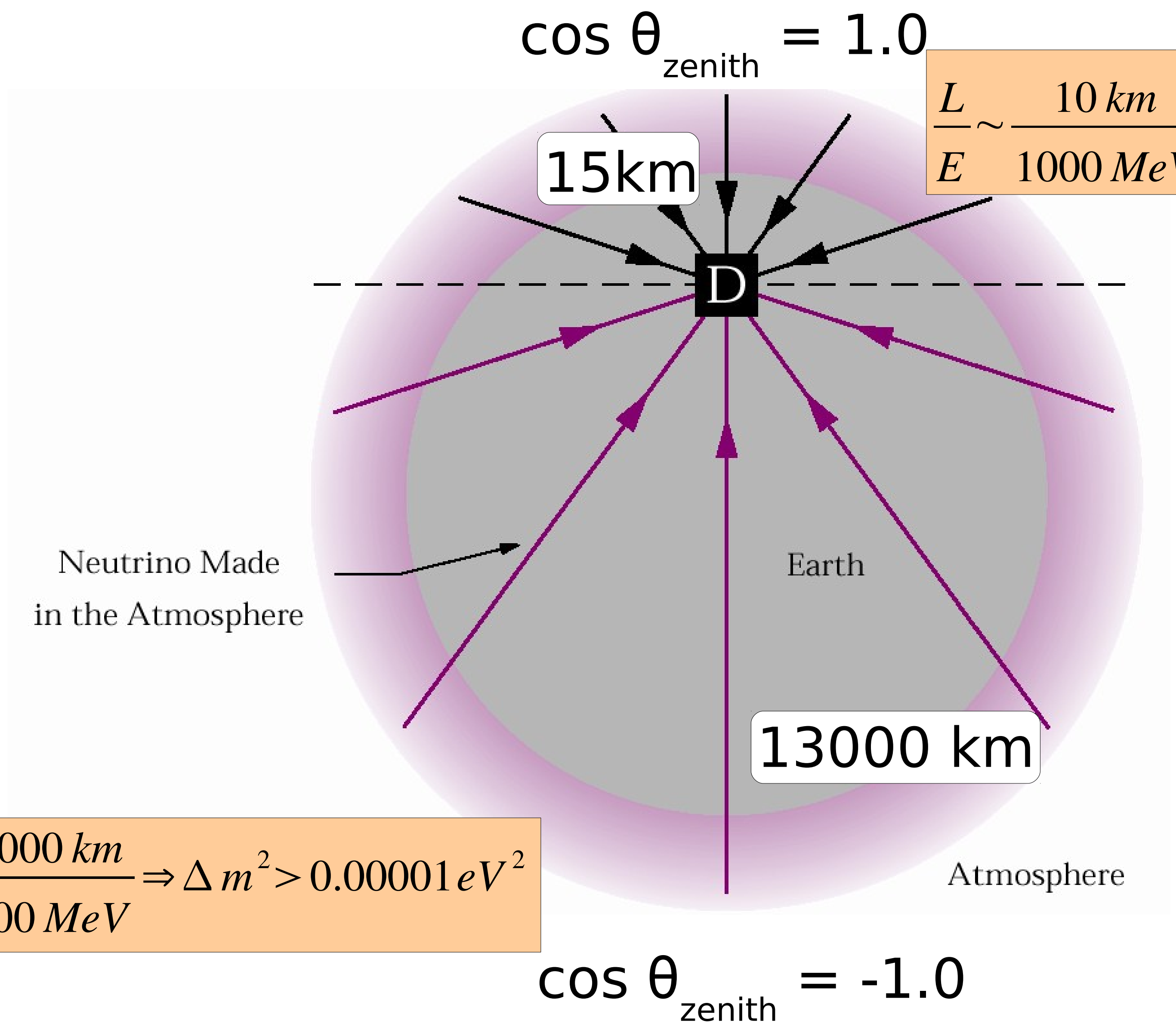
Solar sector

$$\theta_{e\mu}=32.5^{o}\pm 2.4^{o}$$

$$\Delta m_{12}^{2}=+7.9\times 10^{-5}eV^{2}$$

# Explaining the atmospheric data

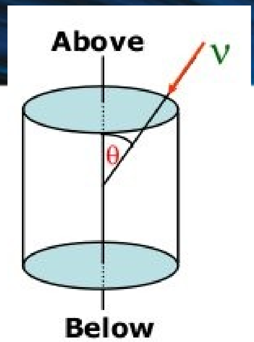
# Cosmic Labs

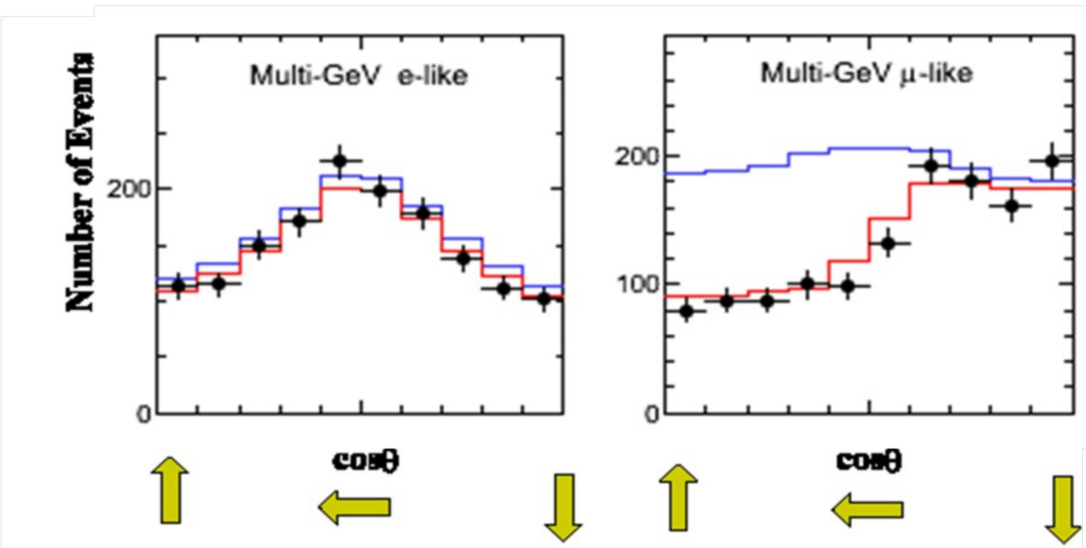



$$\frac{L}{E} \sim \frac{10\,km}{1000\,MeV} \Rightarrow \Delta m^2 > 0.01\,eV^2$$

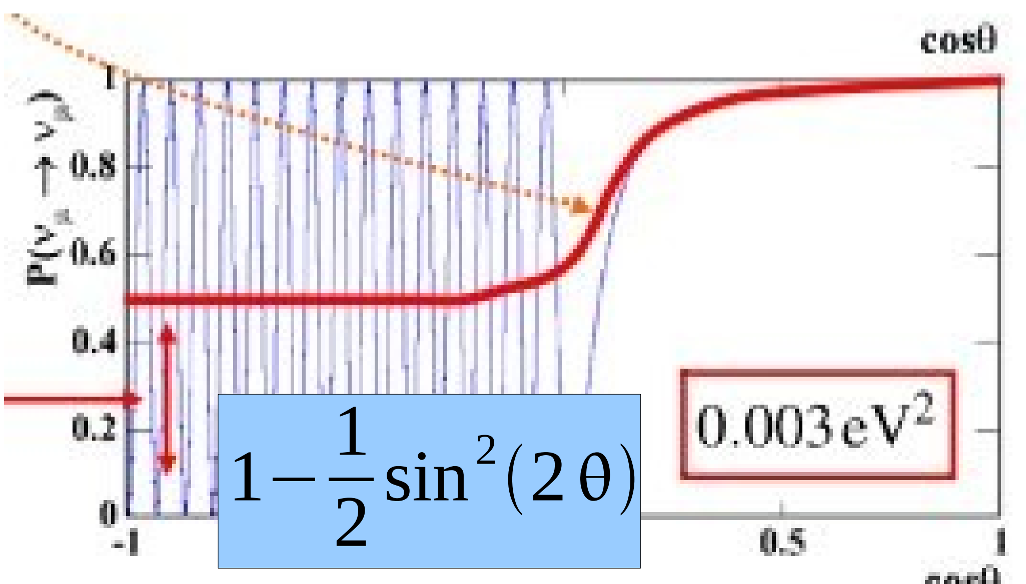
$$\frac{L}{E} \sim \frac{10000\,km}{1000\,MeV} \Rightarrow \Delta m^2 > 0.00001\,eV^2$$

# Atmospheric results




- Prediction for $\nu_e$ rate agrees with data.
- $\nu_\mu$ disappear at large baseline consistent with $\nu_\mu \rightarrow \nu_\tau$
- Don't detect $\nu_\tau$ as
  - -below t mass threshold
  - -SuperK is awful at τ detection




$$1-\frac{1}{2}\sin^2(2\theta)$$

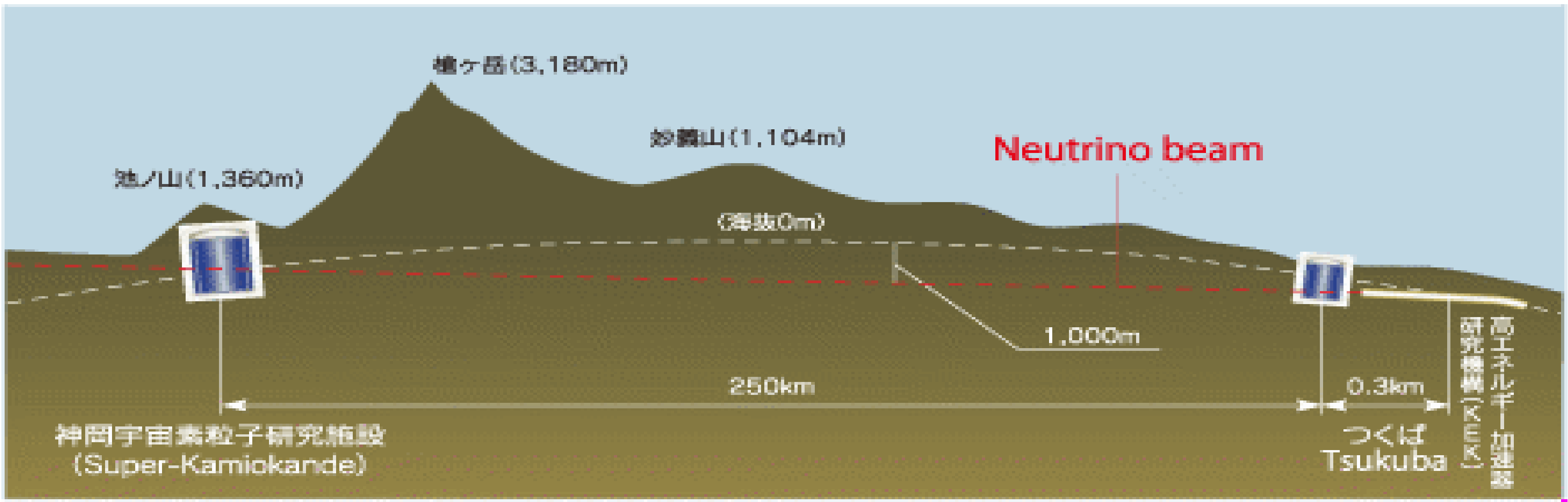
$$|\Delta m^2_{atmos}| \approx 0.0025\, eV^2$$

$$\sin^2(2\theta_{atmos}) \approx 1.0$$

# Accelerator Cross-check

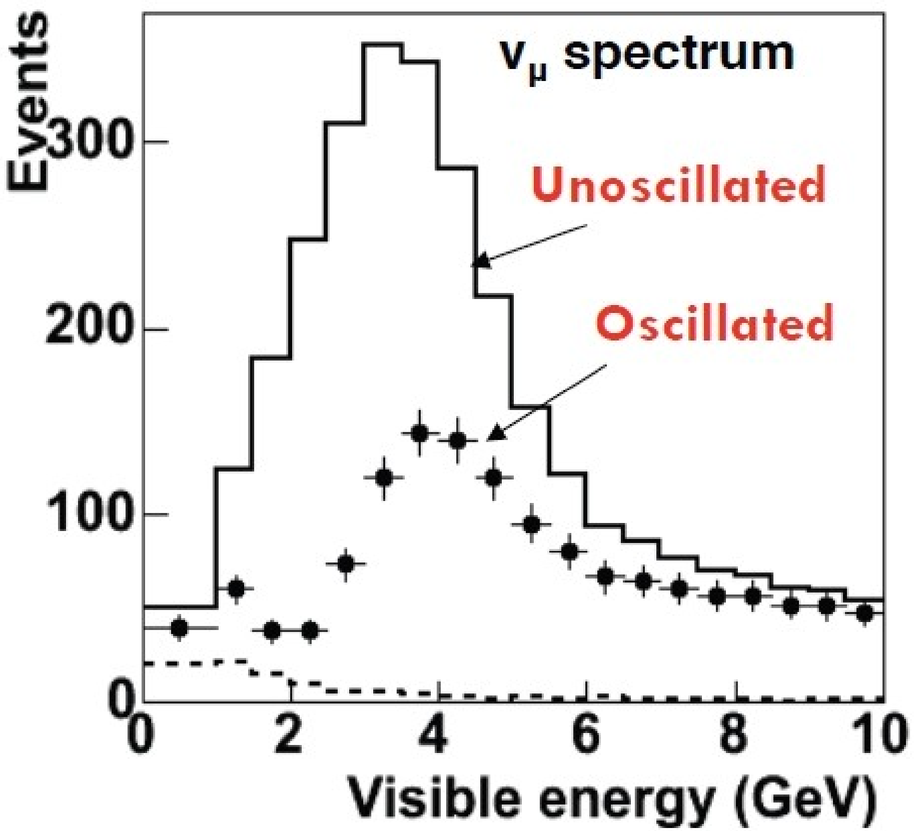
$$\Delta m^2_{atmos} \approx 3 \times 10^{-3} eV^2 \rightarrow L/E \approx 400\, km\, GeV^{-1}$$

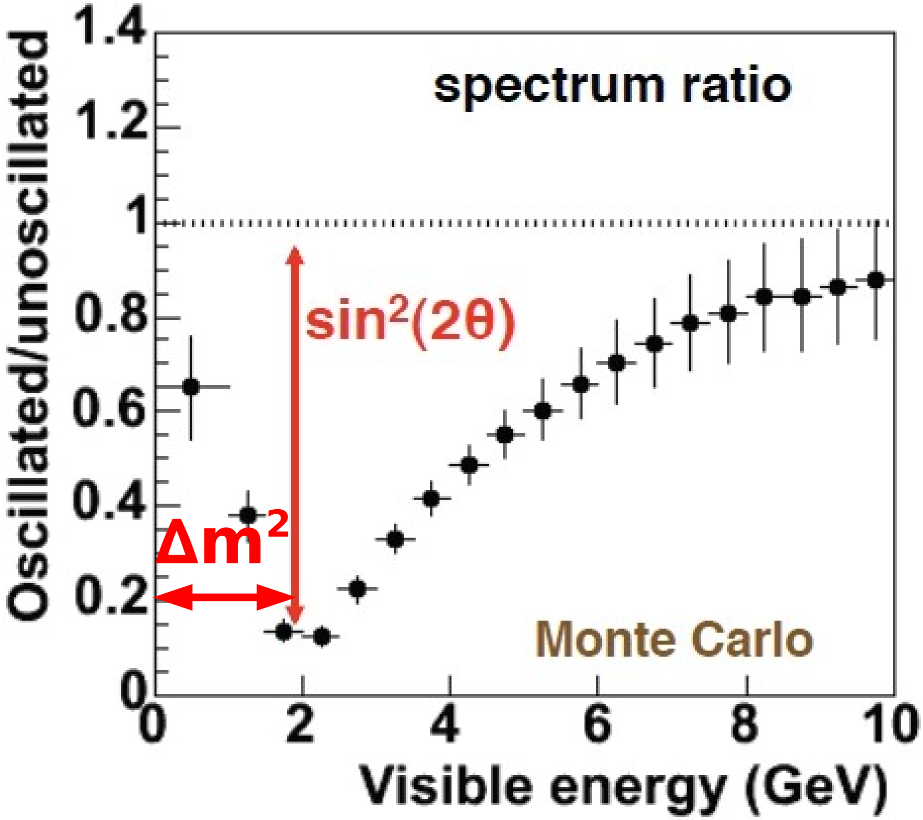
$$L = 250\, km \rightarrow E_\nu \approx 0.6\, GeV$$




Beam events tagged using GPS at both near and far detector sites

# Disappearance Experiments



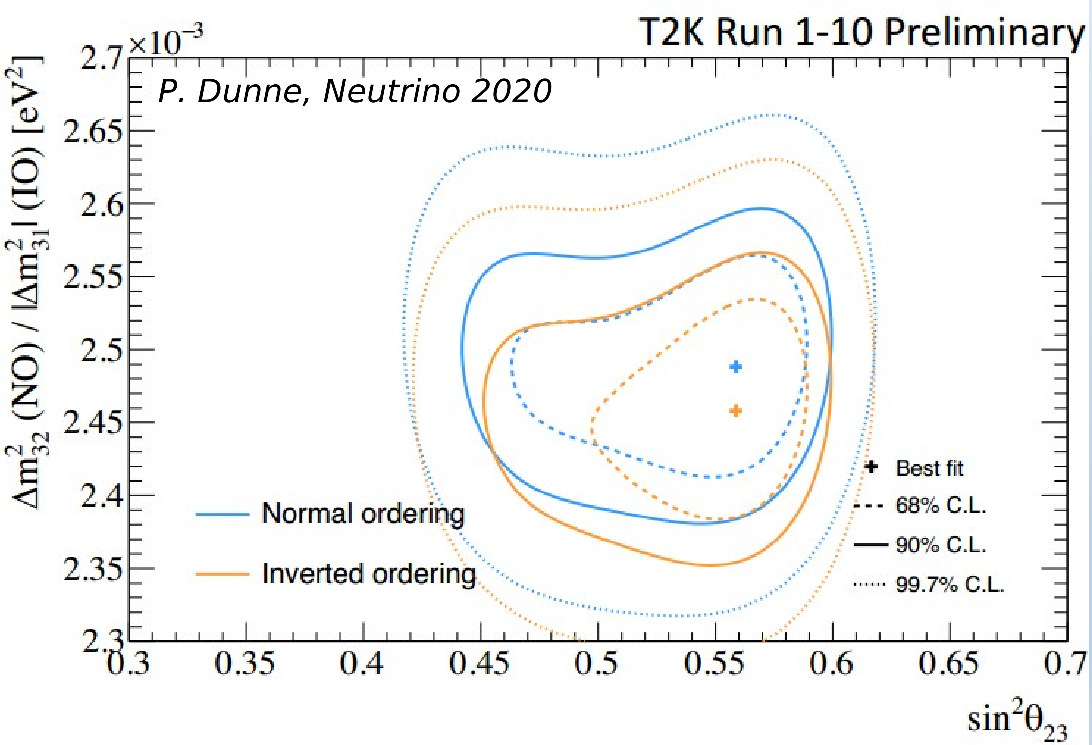

$$P(\nu_\alpha \rightarrow \nu_\alpha) \rightarrow \frac{\Phi_\nu(@FD)}{\Phi_\nu(@ND)}$$

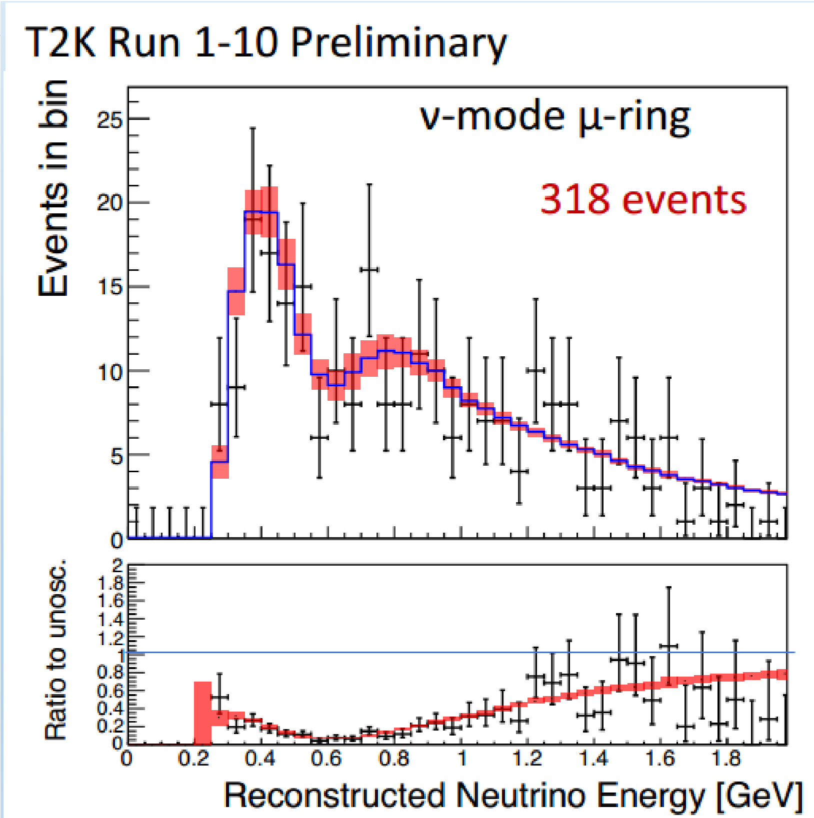
$\Phi_\nu$ : Neutrino Flux

Use Near Detector to measure $\Phi_\nu$(@ND)

# T2K verification

# T2K Disappearance




$$\frac{\#\,events\,observed}{\#\,events\,expected} = P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E}\right)$$

$$|\Delta m^2_{23}| = (2.49 \pm 0.07) \times 10^{-3}\, eV^2$$

(best fit)

$$\sin^2(\theta_{23}) = (0.546 \pm 0.035) \rightarrow \theta_{23} = 47.6 \pm 2.6$$

# Mixing matrix

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

Solar sector : $\nu_\mu \rightarrow \nu_e$

$$\theta_{e\mu} = 34.3^\circ \pm 1.0^\circ$$

$$\Delta m^2_{12} = +(7.50 \pm 0.21) \times 10^{-5}\, eV^2$$

Atmospheric sector

$\nu_\mu \rightarrow \nu_\tau$

$$\theta_{\mu\tau} = 48.7^\circ \pm 1.0^\circ$$

$$\Delta m^2_{23} = \left| (2.56 \pm 0.04) \times 10^{-3} \right| eV^2$$

# How do we measure $\theta_{13}$?

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$\nu_\mu \rightarrow \nu_e$ oscillations with atmospheric L/E

$$P(\nu_\mu \rightarrow \nu_e) = \sin^2 2\theta_{13} \sin^2 \theta_{23} \sin^2 (1.27 \Delta m_{23}^2 \frac{L}{E})$$

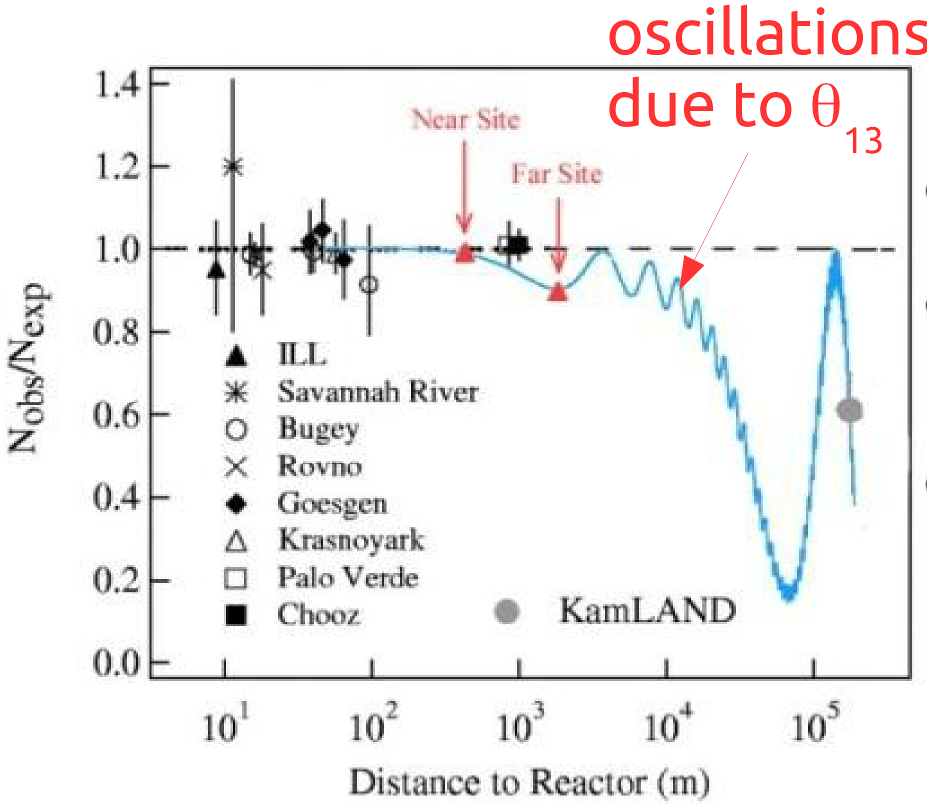
$\nu_e$ appearance in a $\nu_\mu$ beam – ideal for *accelerator experiments*

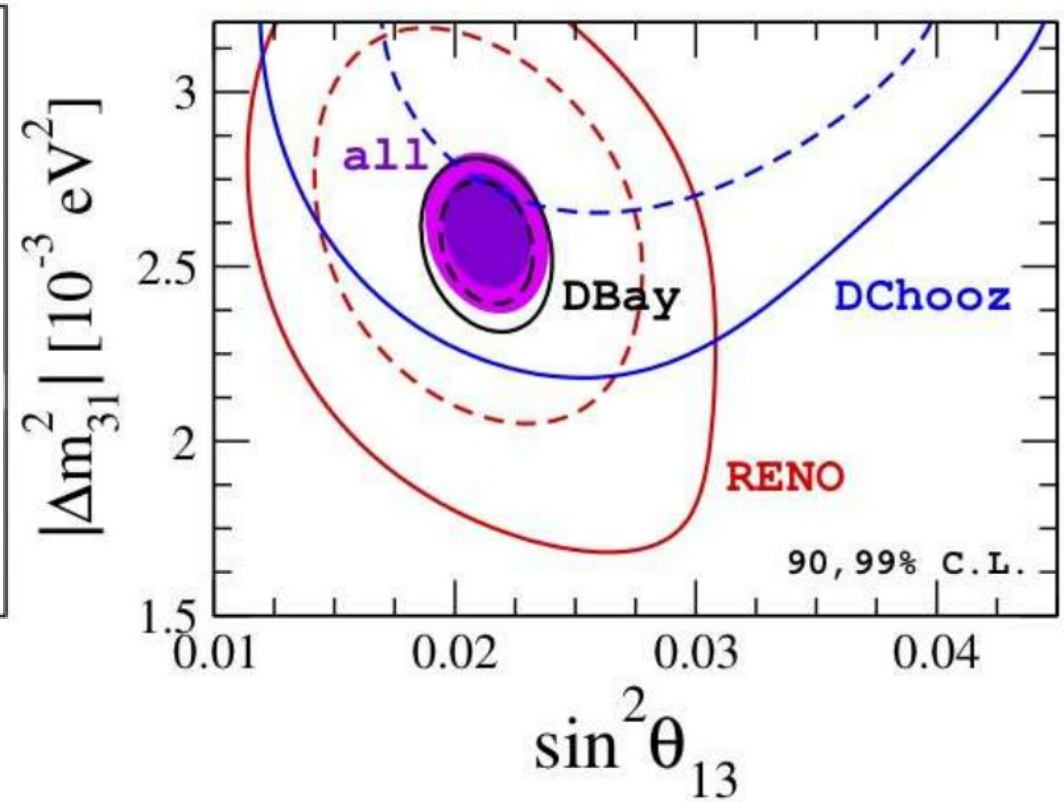
$\nu_e \rightarrow \nu_x$ disappearance oscillations with atmospheric L/E

$$p(\overline{\nu}_e \rightarrow \overline{\nu}_x) \stackrel{\hat{C}\hat{P}}{=} P(\nu_e \rightarrow \nu_x) = 1 - \sin^2(2\theta_{13}) \sin^2(1.27 \Delta m_{23}^2 \frac{L}{E})$$

$\overline{\nu}_e$ disappearance – ideal for *reactor experiments*

# $\theta_{13}$ from reactors




$$\theta_{13} = (8.44(41) \pm 0.16)^\circ \,(\mathrm{NO(IO)})$$

# Global results

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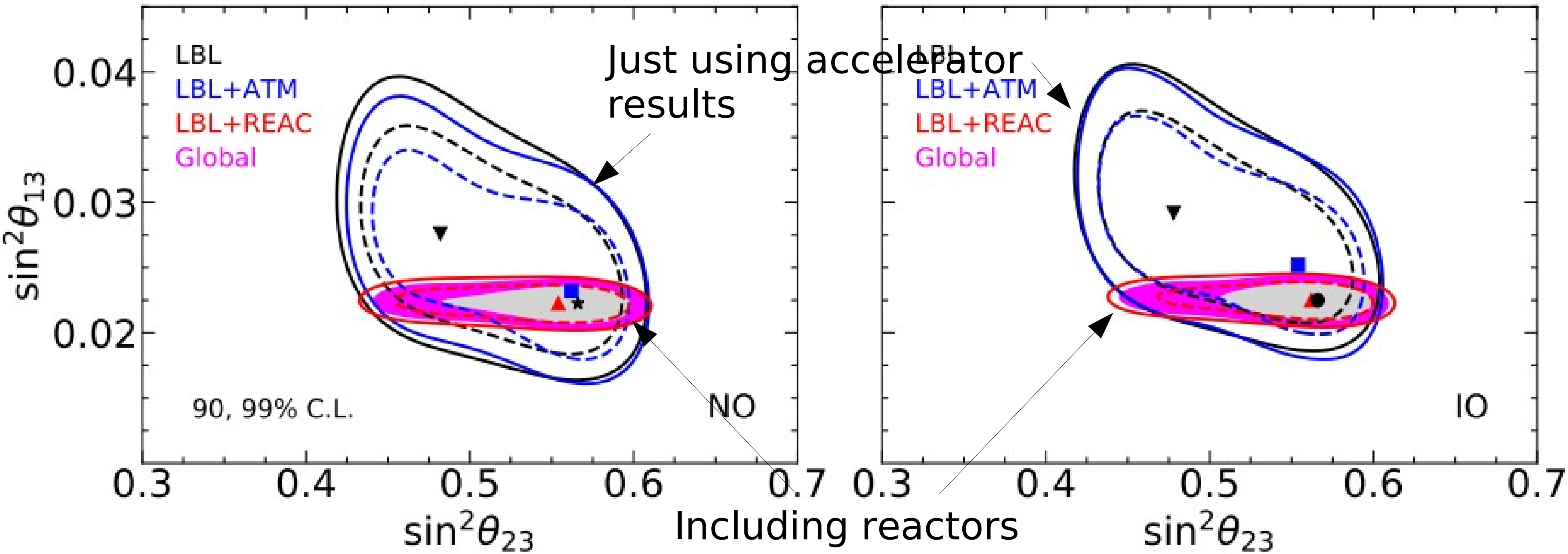
Normal hierarchy Inverted hierarchy




*P. F. de Salis et al*
*https://arxiv.org/abs/2006.11237*

# 3-Neutrino Mixing

$$U=\begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

**Solar sector**

$$\nu_e \rightarrow \nu_\mu$$

$$\theta_{12}=34.3^\circ \pm 1.0^\circ$$

$$\Delta m^2_{12}=+7.50\times 10^{-5}\,eV^2$$

**13 Sector**

$$\nu_\mu \rightarrow \nu_e$$

$$\theta_{13}=8.58^\circ \pm 0.11^\circ$$

$$\Delta m^2_{23}=\left|2.56\times 10^{-3}\right| eV^2$$

**Atmospheric sector**

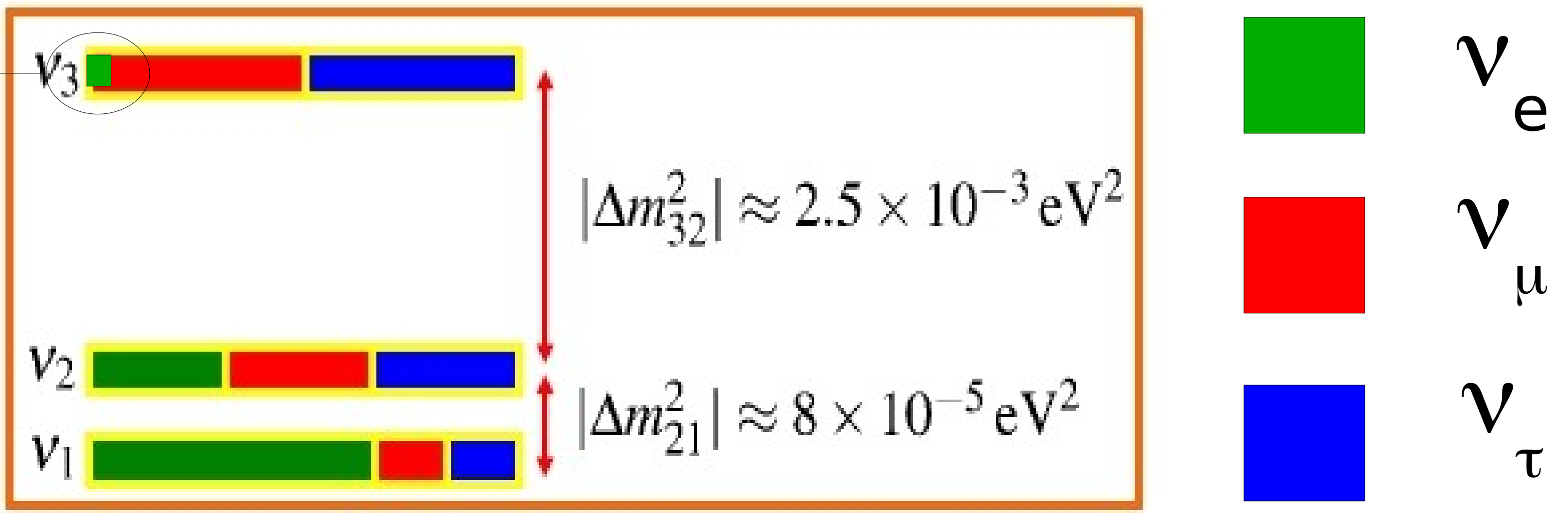
$$\nu_\mu \rightarrow \nu_\tau$$

$$\theta_{23}=48.8^\circ \pm 1.0^\circ$$

$$\Delta m^2_{23}=\left|2.56\times 10^{-3}\right| eV^2$$

# Summary of Current Knowledge

$\theta_{13}$ : how much $\nu_e$ is in $\nu_3$




$$U_{MNSP} = \begin{pmatrix} 0.8 & 0.5 & -0.15 \\ -0.4 & 0.7 & 0.6 \\ 0.4 & -0.5 & 0.7 \end{pmatrix}$$

Some elements only known to 10-30%

Very very different from the quark CKM matrix