UNIVERSIDADE DO MINHO

CF-UM-UP

LIP

Physics of Nuclei and Particles Exercises

Physics Course, 2º Semester

1. The electromagnetic field can be described, quite conveniently, by a scalar potential (f) and a vector potential (**A**). For simplicity, consider only the scalar potential. Bearing in mind that Maxwell's equations can be described by the equation,

$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho(\mathbf{r}, t)}{\varepsilon_0}$$

   a) show that the equation, at points in space away from the region of charge concentration, has solutions in the form of waves that propagate in space with energy E and linear moment P;

   b) show that these solutions correspond to solutions associated with zero mass particles.

2. The Electromagnetic and Weak interactions were combined in the Electroweak Theory, proposed by Weinberg and Salam. This theory predicts the existence of 2 charged gauge bosons (W + and W-) and a neutral Z boson with a mass of about 91 GeV/c^2 (in addition to the photon that has zero mass). How could you generalize Maxwell's equations (discussed in problem 1) in order to accommodate the Z boson, assuming that their interactions could be described by equations like the one in Problem 1?
   Note: consider only the scalar potential for simplicity.

3. Study the solution of Maxwell's equations, in the static case, in the presence of charge distributions and show the solution can be expressed by,

$$\phi(\mathbf{r}, t) \approx \frac{1}{4\pi\varepsilon_0}\int \frac{\rho_1(\mathbf{r}', t)}{|\mathbf{r} - \mathbf{r}'|}\mathrm{d}^3\mathbf{r}'$$

4. Consider the solutions of the relativistic wave equation for a free particle of mass *m*, of the type

$$\psi(\mathbf{r}, t) = (\text{constant})\mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)} \quad \text{where} \quad \omega^2 = c^2k^2 + (m^2c^4/\hbar^2)$$

   Show that the group velocity of a wave packet that represents a particle of energy is equal to the velocity of a classical relativistic particle with the same energy.

5. A system consisting of an electron-positron pair connected through Coulomb interaction is called a positronium. Determine the minimum number of photons that the system can decay to, at rest, and calculate the energy of those photons.

6. Use energy-momentum conservation to demonstrate that the annihilation of an electron-positron pair with the emission of only one photon is impossible in free space.

7. Show that a muon, in free space, with a kinetic energy of 1MeV, can travel a distance of about 90m before decaying.

8. Consider that the mass of the electron, resulting from the decay of a muon, is negligible when compared to the mass of the muon itself. Show that, if the muon decays at rest and if the energy is distributed equally among all its decay products, the angle between any two of these products is equal to 120°.

9. Identify, in the following processes, possible interactions responsible for each one of the decays and the involved conservation principles.

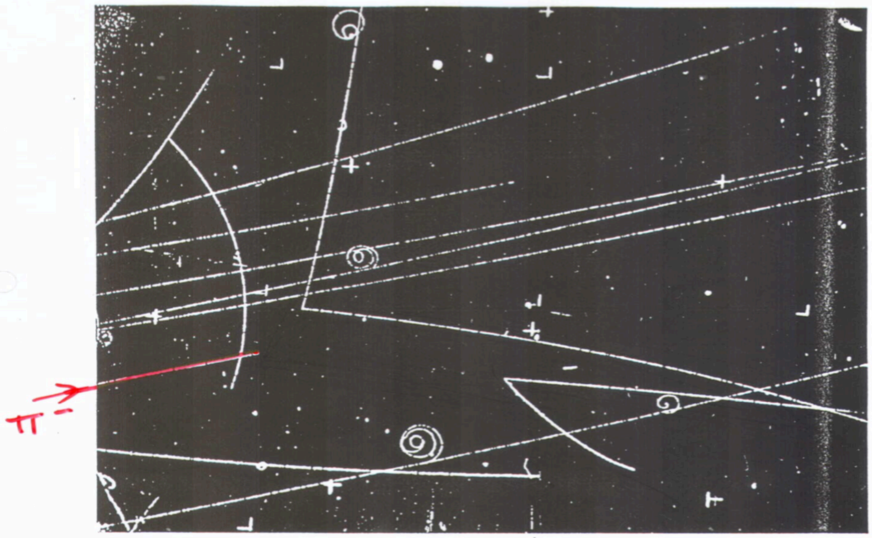


Fig. 10.8 A photograph of an interaction of a 1 GeV/c π^- ...on with a proton in a liquid hydrogen bubble chamber. The reaction is

$$\pi^- + p \rightarrow K^0 + \Lambda,$$ Strong production

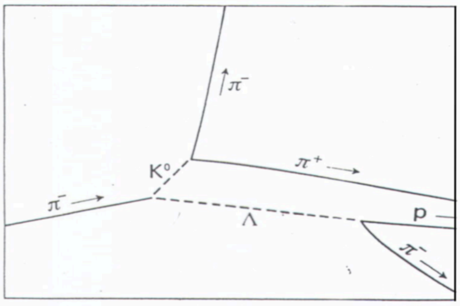
followed by

$$K^0 \rightarrow \pi^+ + \pi^-,$$

and

$$\Lambda \rightarrow \pi^- + p.$$ weak decays

This is an example of associated production of two strange particles.



10. Classify the involved conservation principles and interactions in the decays:

$$\Xi^{*-} \rightarrow K^- + \Sigma^0$$
$$\Sigma^0 \rightarrow \Lambda^0 + \gamma, \quad \Lambda^0 \rightarrow p + e^- + \bar{\nu}_e$$
$$K^- \rightarrow \pi^- + \pi^0, \quad \pi^0 \rightarrow \gamma + \gamma$$
$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\Xi^{*-}(ssd), \quad \Sigma^0(sud), \quad \Lambda^0(sud), \quad p(uud),$$
$$K^-(s,\bar{u}), \quad \pi^-(d\bar{u}), \quad \pi^0(u\bar{u} - d\bar{d}).$$

11. Omega decay was first observed in bubble chambers and was easily identified by its characteristic cascade decay chain. Check, using conservation principles, that the decay chain shown below could match the decay. Represent examples of Feynman diagrams that can give rise to the observed decays, if possible, while also identifying the responsible interaction.

$$\Omega^- \longrightarrow \Xi^0 + \pi^-$$
$$\hookrightarrow \Lambda^0 + \pi^0$$
$$\hookrightarrow p + \pi^- .$$

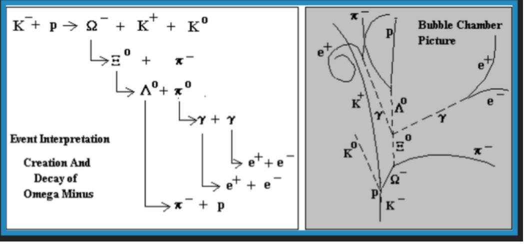
baryon	mass	lifetime	dominant decays	quark content
$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	1232 MeV	6×10^{-24} s	$p\pi$, $n\pi$	uuu, uud, udd, ddd
$\Sigma^{*+}, \Sigma^{*0}, \Sigma^{*-}$	1385 MeV	2×10^{-23} s	$\Lambda\pi$	uus, uds, dds
Ξ^{*0}, Ξ^{*-}	1530 MeV	7×10^{-23} s	$\Xi\pi$	uss, dss
Ω^-	1672 MeV	0.8×10^{-10} s	ΛK^-, $\Xi\pi$	sss

12. Indicate, justifying, whether the following reactions occur through weak, electromagnetic or strong interactions, or if they do not occur at all.

(i) $\pi^- \to \mu^- + \overline{\nu_\mu}$
(ii) $\tau^- \to \mu^- + \nu_\tau$
(iii) $\Sigma^0 \to \Lambda + \gamma$
(iv) $p \to n + e^+ + \nu_e$
(v) $\pi^- + p \to \pi^0 + \Sigma^0$
(vi) $\pi^- + p \to K^0 + \Sigma^0$
(vii) $e^+ + e^- \to \mu^+ + \mu^-$

n(udd) 940, p(uud) 938
Σ^- (dds) 1197, Σ^0(uds) 1192, Σ^+(uus) 1189
K^0($d\bar{s}$) 498, K^+($u\bar{s}$) 494.

Ξ^{*-}(ssd), Σ^0(sud), Λ^0(sud), p(uud),
K^-(s, $\bar{u}$), π^-($d\bar{u}$), π^0($u\bar{u} - d\bar{d}$).



13. Determine which of the following decays are possible, according to the conservation laws you know:

(a) $n \to p + \gamma$,
(b) $p \to e^+ + \gamma$,
(c) $p \to \pi^+ + \gamma$,
(d) $\bar{p} + n \to \pi^- + \pi^0$.

14. The Coulomb self-energy of a hadron with charge $+e$ or $-e$ is about 1 MeV. The quark content and rest energies (in MeV) of some hadrons are:

n(udd) 940, p(uud) 938
Σ^- (dds) 1197, Σ^0(uds) 1192, Σ^+(uus) 1189
K^0($d\bar{s}$) 498, K^+($u\bar{s}$) 494.

The u and d quarks make different contributions to the rest energy. Estimate this difference.

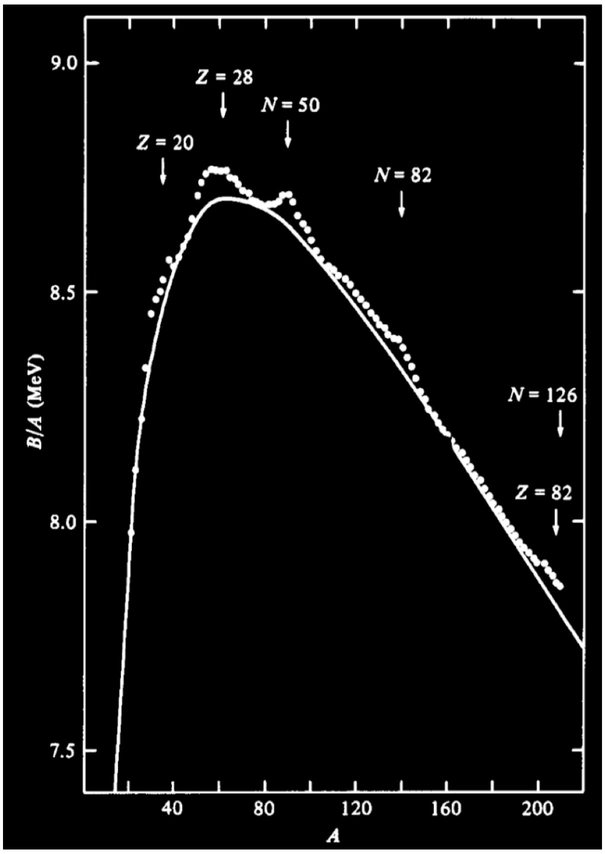
15. The carbon isotope $^{16}_{4}C$ is produced in nuclear reactions of cosmic rays in the atmosphere. It is β-unstable,

$$^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + \mathrm{e}^{-} + \bar{\nu}_{\mathrm{e}} + 0.156\ \mathrm{MeV},$$

with a mean life of 8270 years. It is found that a gram of carbon, newly extracted from the atmosphere, gives on average 15.3 such radioactive decays per minute. What is the proportion of ^{14}C isotope in the carbon? What count rate would you expect from one gram of carbon extracted from the remains of a wooden hut thought to be 4000 years old?

16. Establish a condition so that alpha decay (emission of a helium nucleus) from a nucleus of mass number *A* and atomic number *Z*, is possible.

17. Observe the graph represented in the Figure below, which represents the binding energy per nucleon, for nuclei belonging to the valley of stability, and which are stable from the point of view of beta decay. What conclusions can you draw from the graph? (the solid line represents the estimate obtained based on the liquid drop model).



18. Consider a relativistic electron (whose rest mass can be neglected) that has an energy E, and suffers an elastic scattering caused by the interaction with a particle of mass M, which is at rest. If after the interaction, the electron has an energy E' and a scattering angle $\boldsymbol{\theta}$, determine:

a) the energy of the M particle, after spreading,
b) its momentum,
c) the fraction of energy lost by the electron in the interaction.

19. Using the information in the Table below, show that $^{8}_{4}Be$ can decay in two alpha particles, with an energy release equal to 0.1MeV, but that $^{12}_{6}C$ cannot decay in 3 alpha particles. Calculate the energy released in $^{2}_{1}H + ^{4}_{2}He \rightarrow ^{6}_{3}Li$

Nucleus	Binding energy (MeV)	Binding energy of last nucleon (MeV)	Binding energy per nucleon (MeV)	Spin and parity
$^{2}_{1}H$	2.22	2.2	1.1	1^+
$^{3}_{2}H$	8.48	6.3	2.8	$\frac{1}{2}^+$
$^{4}_{2}He$	28.30	19.8	7.1	0^+
$^{5}_{2}He$	27.34	−1.0	5.5	$\frac{3}{2}^-$
$^{6}_{3}Li$	31.99	4.7	5.3	1^+
$^{7}_{3}Li$	39.25	7.3	5.6	$\frac{3}{2}^-$
$^{8}_{4}Be$	56.50	17.3	7.1	0^+
$^{9}_{4}Be$	58.16	1.7	6.5	$\frac{3}{2}^-$
$^{10}_{5}B$	64.75	6.6	6.5	3^+
$^{11}_{5}B$	76.21	11.5	6.9	$\frac{3}{2}^-$
$^{12}_{6}C$	92.16	16.0	7.7	0^+
$^{13}_{6}C$	97.11	5.0	7.5	$\frac{1}{2}^-$
$^{14}_{7}N$	104.66	7.6	7.5	1^+
$^{15}_{7}N$	115.49	10.8	7.7	$\frac{1}{2}^-$
$^{16}_{8}O$	127.62	12.1	8.0	0^+
$^{17}_{8}O$	131.76	4.1	7.8	$\frac{5}{2}^+$

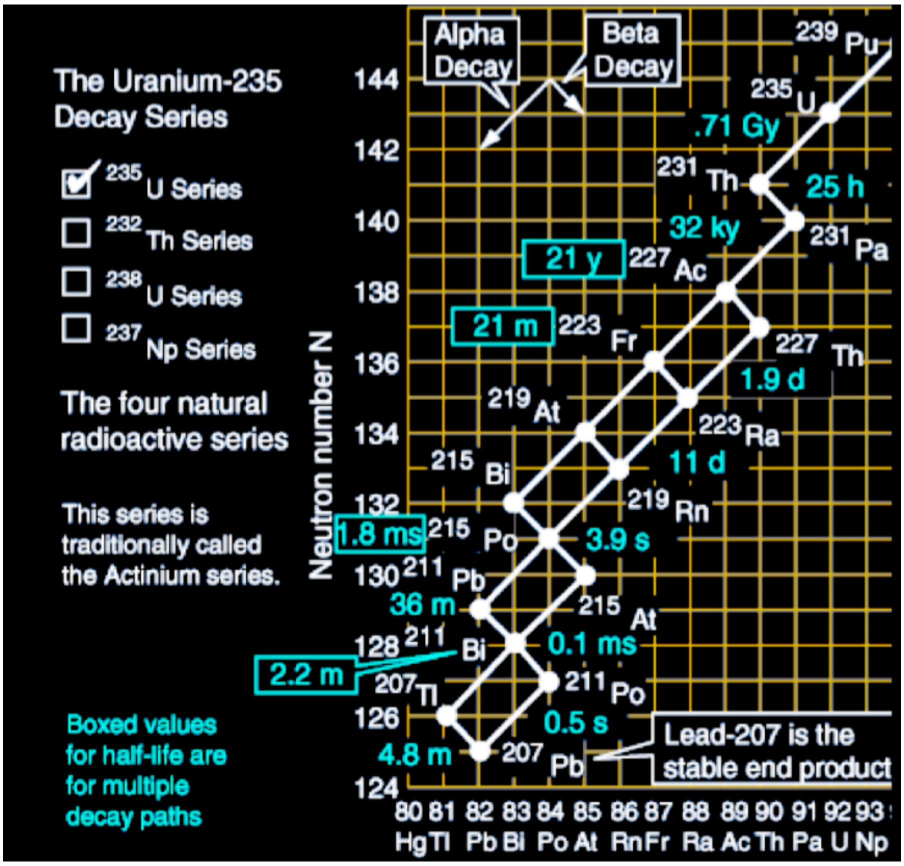
20. Determine the average energy required to remove a nucleon from the following nuclei: $^{40}_{20}Ca, ^{27}_{13}Al$ and $^{27}_{14}Si$. Justify, using the liquid drop model.

21. $^{235}_{92}U$ (m_U=235.94 a.m.u.) decays to lighters nuclei according to the radioactive decay chain represented in the Figure below (1 a.m.u.=931.49 MeV/c^2, m_p=938.27 MeV/c^2, m_n=939.57 MeV/c^2)

a) define binding energy of an atomic nucleus and explain the terms contributing to it;

b) calculate the $^{235}_{92}U$ binding energy;

c) If, in a rock sample, we find a ratio 1:7 between the abundances of $^{235}_{92}U$ and $^{207}_{82}Pb$, would it be possible to calculate the age of the sample? If so, calculate its age.



22. Consider the following decay process, $^{235}_{92}U \rightarrow ^{231}_{90}Th + ^{4}_{2}He$. Check if the decay is possible by establishing the necessary condition for the decay to happen.

23. Determine the kinetic energy an alpha particle must have to be absorbed by a $^{238}_{92}U$, in a collision experiment.

24. The spectrometric analysis of the potassium (*K*) and argon (*Ar*) abundances of a sample of rocks from the Moon showed that, the ratio between the number of ^{40}Ar atoms (stable) present in the sample and the number of ^{40}K atoms in the sample (radioactive), was 10.3. Suppose that all the argon atoms were produced by the decay of the potassium and that the half-life for this decay was determined to be 1.25×10^9 years. How old is the rock?

25. Natural uranium, as found on Earth, consists of 2 isotopes in the proportion $^{235}_{92}U$ / $^{238}_{92}U$ = 0.7%. Assuming that these two isotopes were produced in equal amounts when the Earth was formed, estimate the age of the Earth [$\tau(^{238}_{92}U)$ = 6.52Gy; $\tau(^{235}_{92}U)$ = 1.02Gy].

26. The ^{90}Sr decays to ^{90}Y with a half-life of 28 years. In turn, the ^{90}Y decays to ^{90}Zr with a half-life of 64 hours. If you initially have a pure ^{90}Sr sample, what should the sample composition be after (a) 1 h (b) 10 years?

27. A sealed box with an alloy was found that has two radioactive elements, A and B, which are known to have existed in equal parts (by weight) when the alloy was formed. Knowing that elements A and B, have a half-life of 12 and 18 years, respectively, and that the box, when opened, contained 0.53 kg of A and 2.20 kg of B, determine the age of the alloy.

28. Consider that for nuclei with a mass number of A≲40, the number of protons and neutrons is approximately equal. Knowing that the nuclear density is equal to 0.17 fm^{-3}, determine the Fermi energies for the neutron and proton population.

29. If the separation energy of the last neutron (S_n) is given by

$$S_n(N, Z) = B(N, Z) - B(N-1, Z),$$

will it be possible to assess the total height of the neutron potential well?

30. Calculate the contribution of the Coulomb term to the potential well, of a nucleus with Z protons.

31. If the separation energy of the last proton (S_p) is given by

$$S_p(N, Z) = B(N, Z) - B(N, Z-1)$$

will it be possible to assess the total height of the proton potential well?