Fourier Analysis on two Black Holes Collision -Ligo Hanford

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LIGO

Outline

Learn about:

- Fourier Analysis
- LIGO and Black Holes

Learn the Know-How:

- Data Treatment and Analysis

Conclusions

Fourier Transform

A Fourier Series is a mathematical technique discovered by Jean Baptiste Joseph Fourier (1768-1830) that consists on expressing a function $f(t)$ as a sum of frequency modes or generally a function $f(q)$ as a sum of q's reciprocal modes. The Fourier Transform is the generalization of the Fourier Series when the period of the function goes to infinity and the difference between different frequencies $\Delta\omega \rightarrow 0$.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}[a_n \cos(nt) + b_n \sin(nt)]$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\ \cos(nt)\, dt$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\ \sin(nt)\, dt$$

$$\hat{\mathrm{f}}(\omega) \equiv \mathrm{F}(\omega) \equiv \mathcal{F}\{\mathrm{f(t)}\} \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} \mathrm{f(t)}\ \mathrm{e}^{-\mathrm{i}\omega t}\ \mathrm{dt}$$

Fourier Transform in Time Domain

$$\mathrm{f(t)} \equiv \mathcal{F}^{-1}\{\mathrm{F}(\omega)\} \stackrel{\mathrm{def}}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{F}(\omega)\ \mathrm{e}^{\mathrm{i}\omega t}\ \mathrm{d}\omega$$

W=2π*f (Angular Frequency)

$$\hat{\mathrm{f}}(\kappa) \equiv \mathrm{F}(\kappa) \equiv \mathcal{F}\{\mathrm{f(x)}\} \stackrel{\mathrm{def}}{=} \int_{-\infty}^{\infty} \mathrm{f(x)}\ \mathrm{e}^{-\mathrm{i}\kappa x}\ \mathrm{dx}$$

Fourier Transform in Space Domain

$$\mathrm{f(x)} \equiv \mathcal{F}^{-1}\{\mathrm{F}(\kappa)\} \stackrel{\mathrm{def}}{=} \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{F}(\kappa)\ \mathrm{e}^{\mathrm{i}\kappa x}\ \mathrm{d}\kappa$$

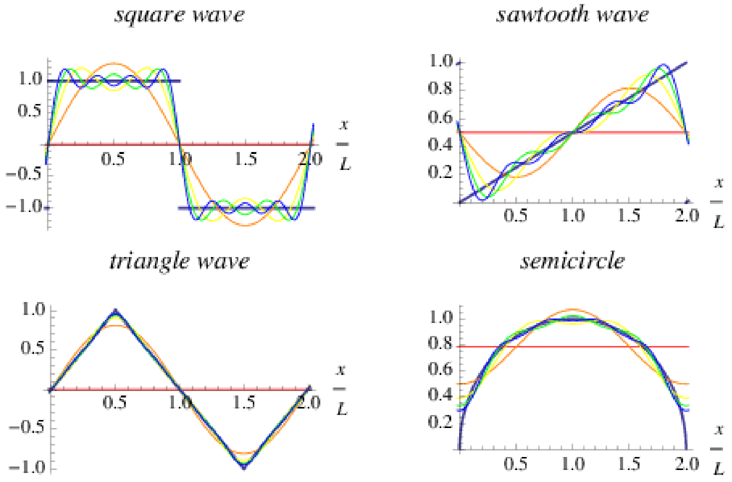
$K = \frac{2\pi}{\lambda}$ (wave vector)

What is Fourier Analysis ?

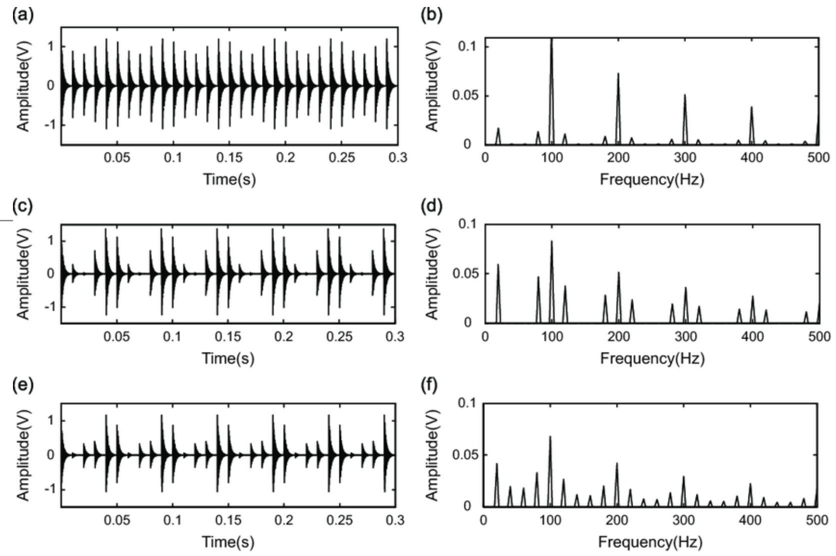
With the Fourier transforms, one can do the so-called Fourier Analysis which is very important on data and signal analysis and is useful for:

1. Eliminating undesirable frequencies from a signal (i.e. white noise);
2. Imaging and sound filtering;
3. Simplifying physical problems that are easy to understand in reciprocal space;
4. Discover more about the hidden properties of signals.

EXEMPLES OF FOURIER ANALYSIS



Signal Processing

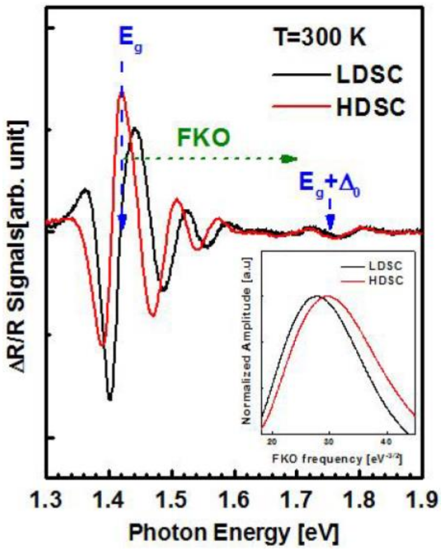


Audio analysis and processing

$$\Delta\chi\Delta\rho \geq \frac{\hbar}{2}$$

Quantum Physics

New Physics !?



LIGO and Black Holes

Laser Interferometer Gravitational Wave Observatory, better known as LIGO, was founded in 1992 by Kip Thorne, Ronald Drever (Caltech) and Rainer Weiss (MIT).

As the name indicates, the LIGO´s main goal is detecting gravitational waves with the help of an interferometer.

For better data acquisition performance LIGO has two interferometers in USA (Hanford Site, Livingston) and in Europe in partnership with VIRGO Europe (Cascina, Italy);

In our work we analyze Hanford-LIGO data.

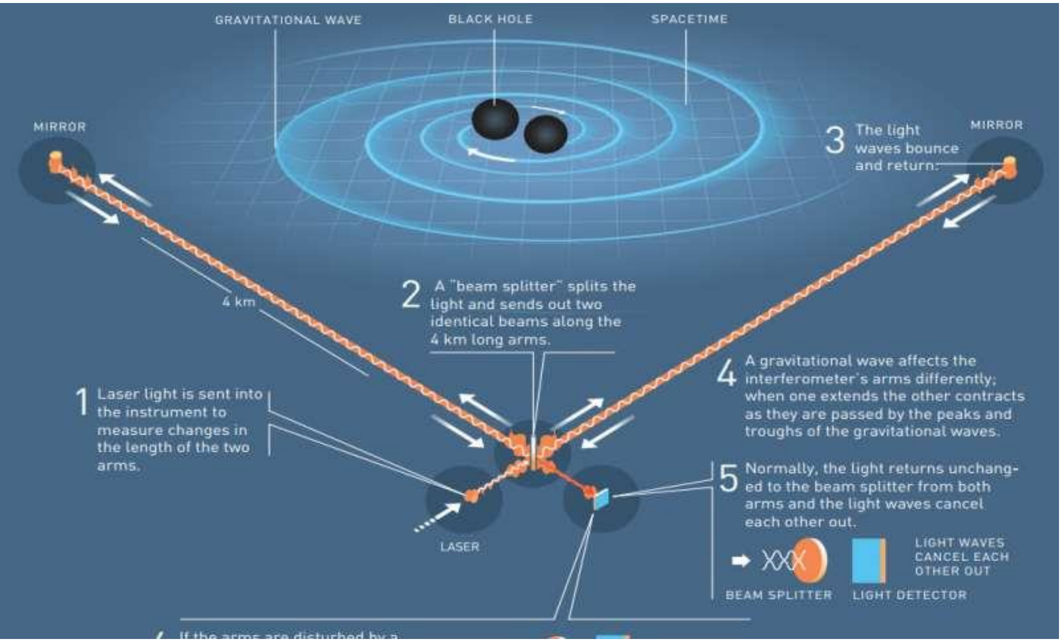
Facts about LIGO

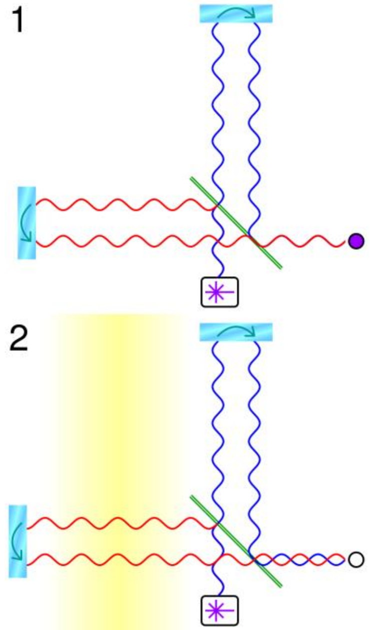
1. Sensitivity: LIGO has the potential to detect changes between mirrors with the size of 1/10000 of the proton length ;
2. Vacuum:
   - The 3rd biggest vacuum chamber in the world with capacity to inflate 1.8 million of football balls;
   - The minimum pressure it can reach is 10^{-9} Torr (1.3×10^{-7}Pa);
   - The time to take all the $10000\mathrm{m}^3$ of air and other residual gases is 40 days;
3. Curvature of the Earth: The 4km arms can be affected by the earth curvature. For this reason, the constructors put a precision concrete to put all arms at the same level;

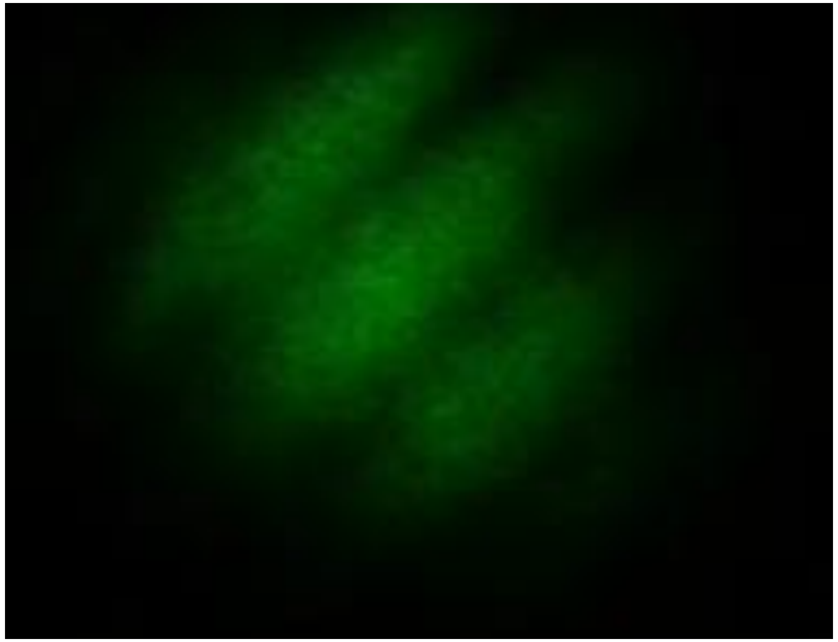
How LIGO works ?

LIGO is a very big Michelson Interferometer with 4Km arm length .

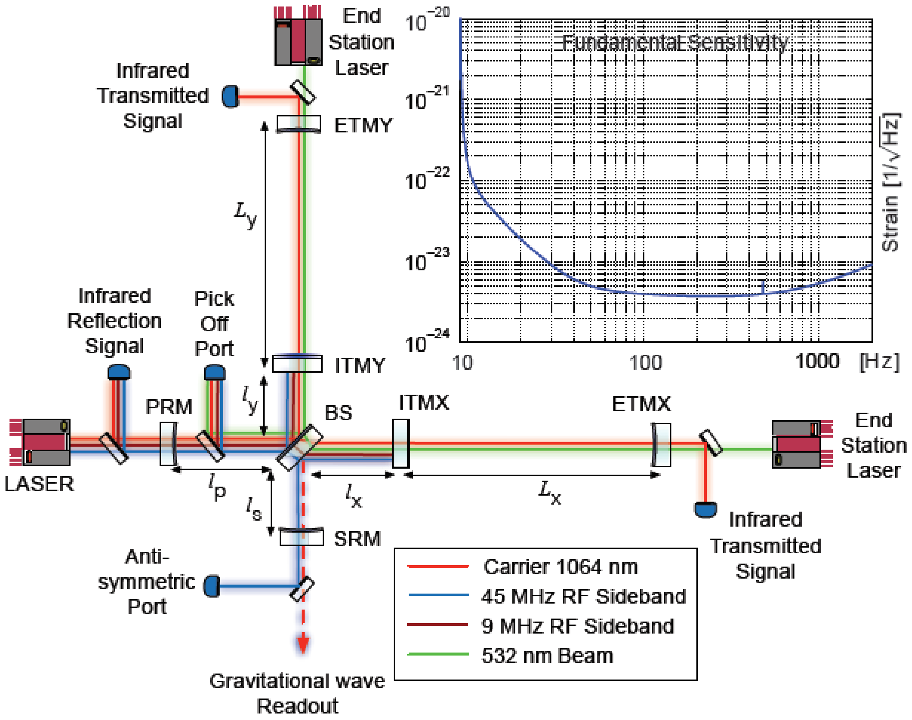
The moment when black holes collide, we have a constructive or a destructive interference that allow us to collect/study data.

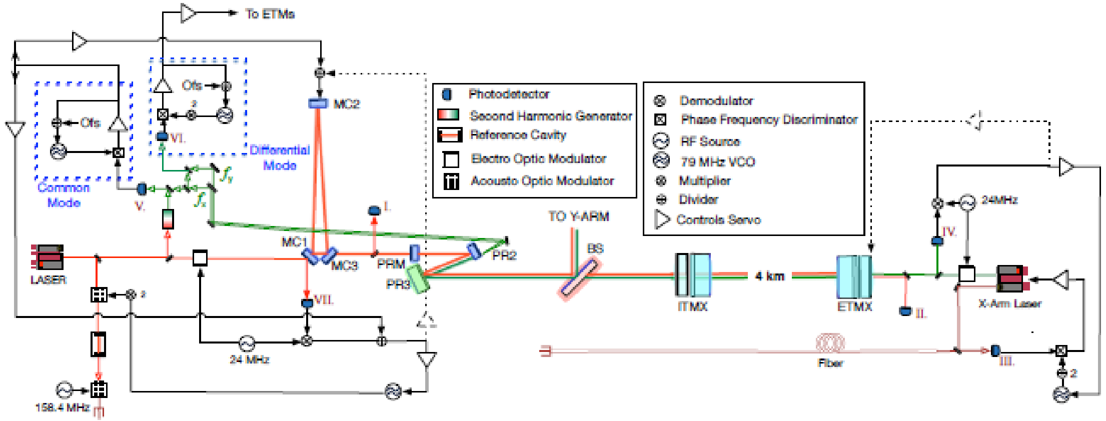






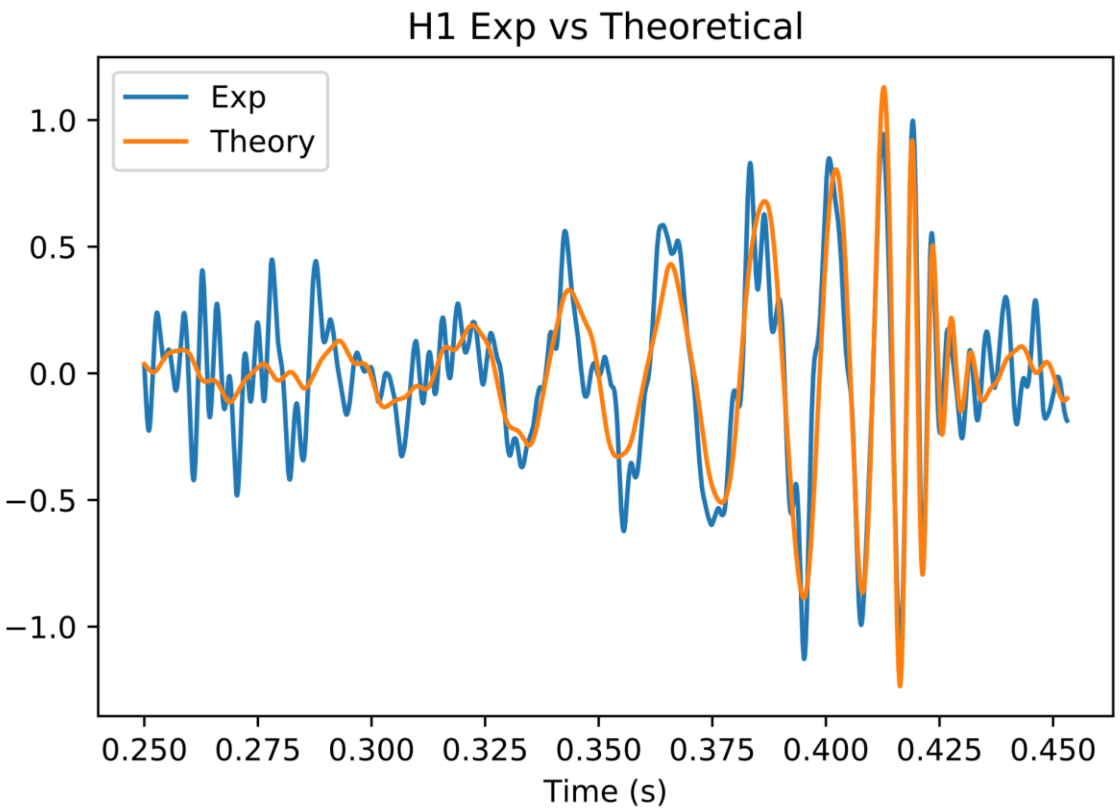
Instrumentation Map

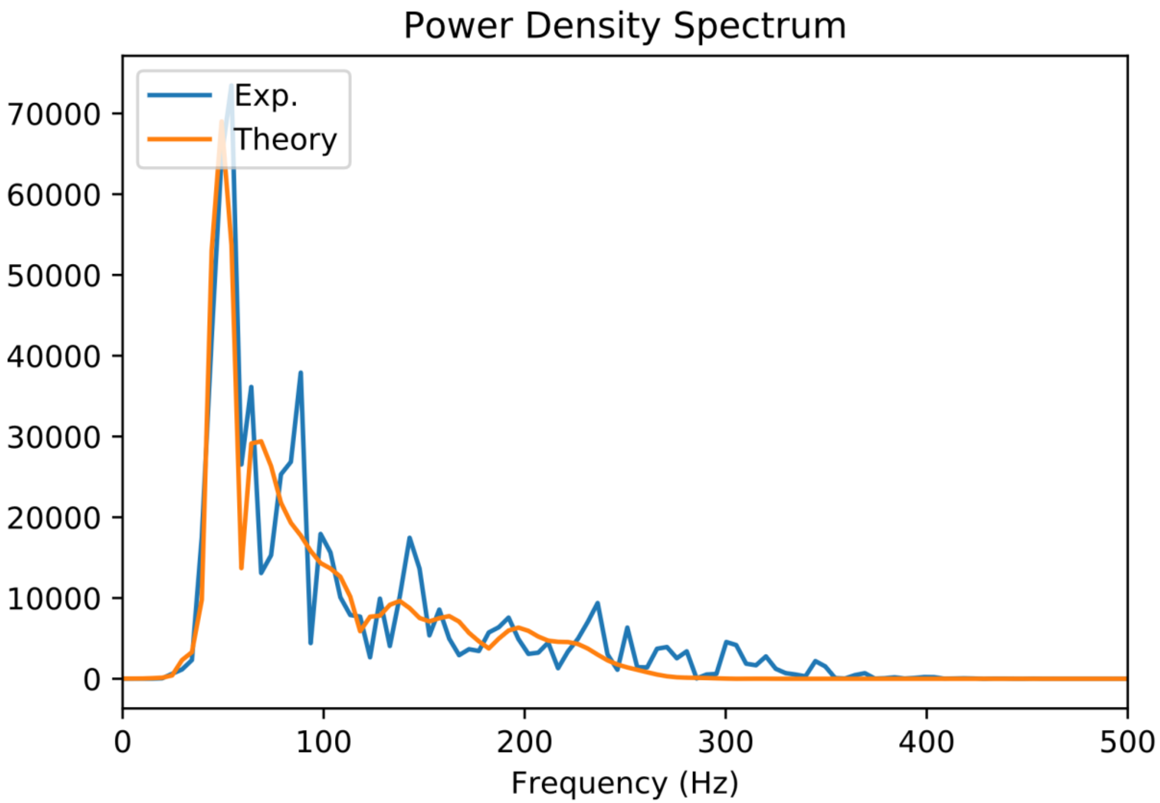




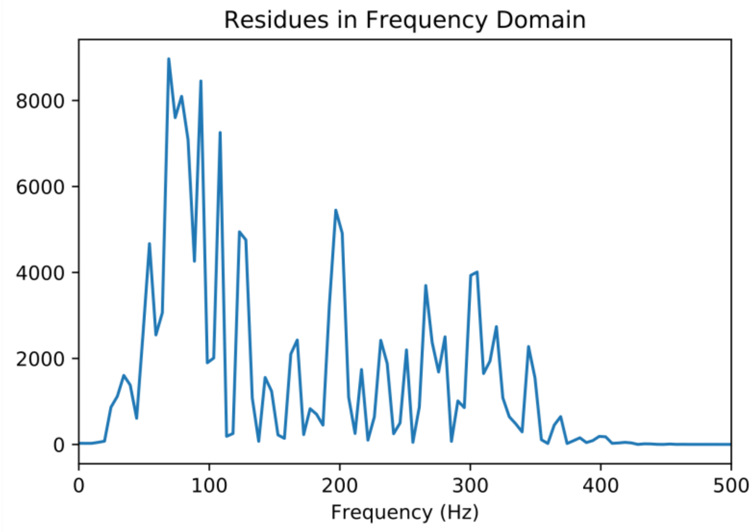
Data treatment and analysis

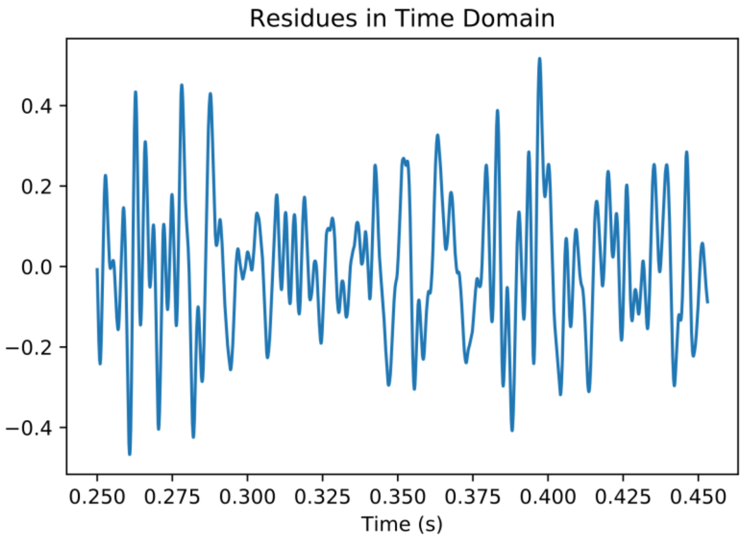
Non-Filtered Data

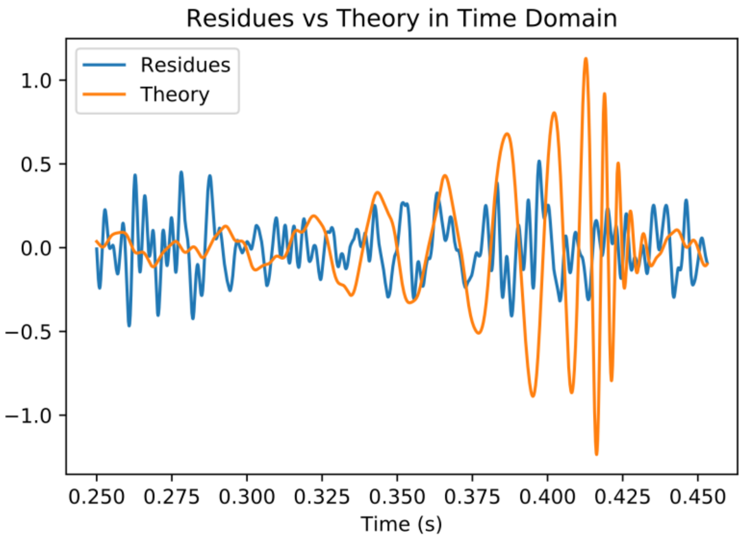




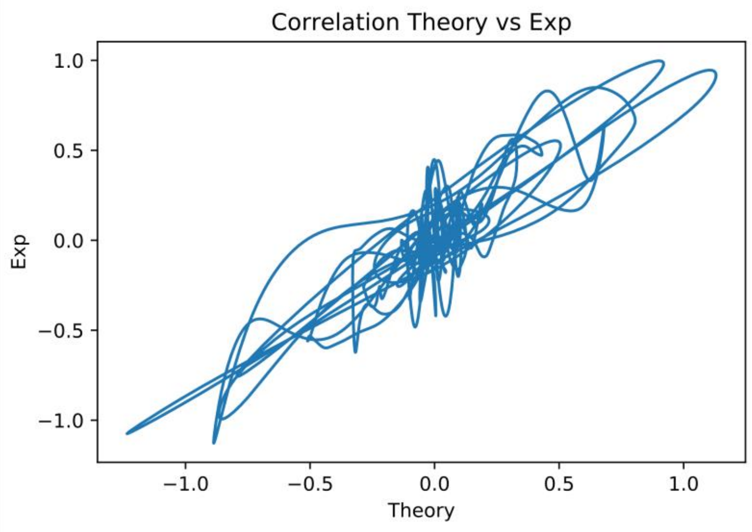
Residues

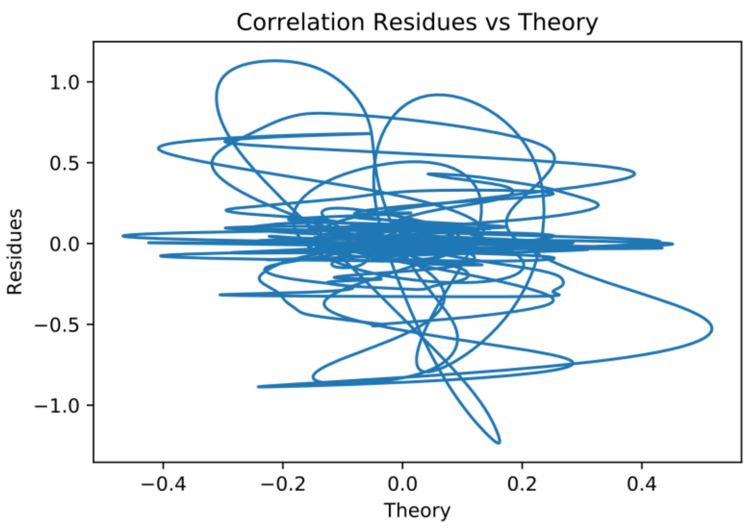

Residues in Frequency Domain
8000
6000
4000
2000
0
0
100
200
300
400
500
Frequency (Hz)

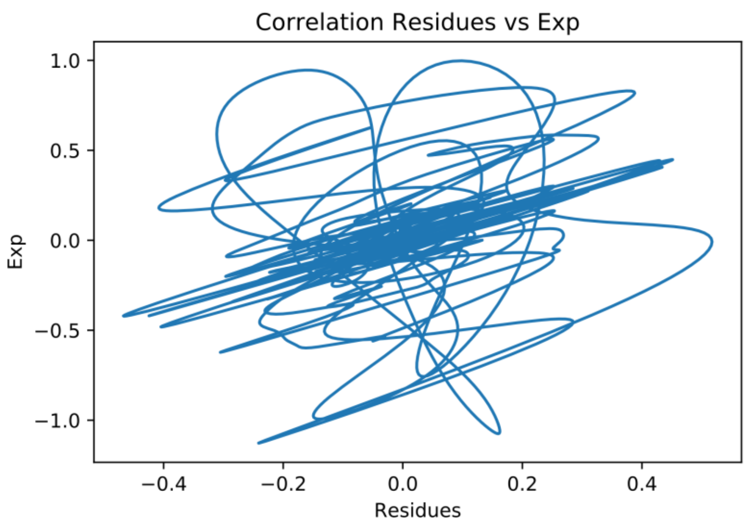

Residues in Time Domain
0.4
0.2
0.0
−0.2
−0.4
0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425 0.450
Time (s)


Residues vs Theory in Time Domain
Residues
Theory
1.0
0.5
0.0
−0.5
−1.0
0.250 0.275 0.300 0.325 0.350 0.375 0.400 0.425 0.450
Time (s)

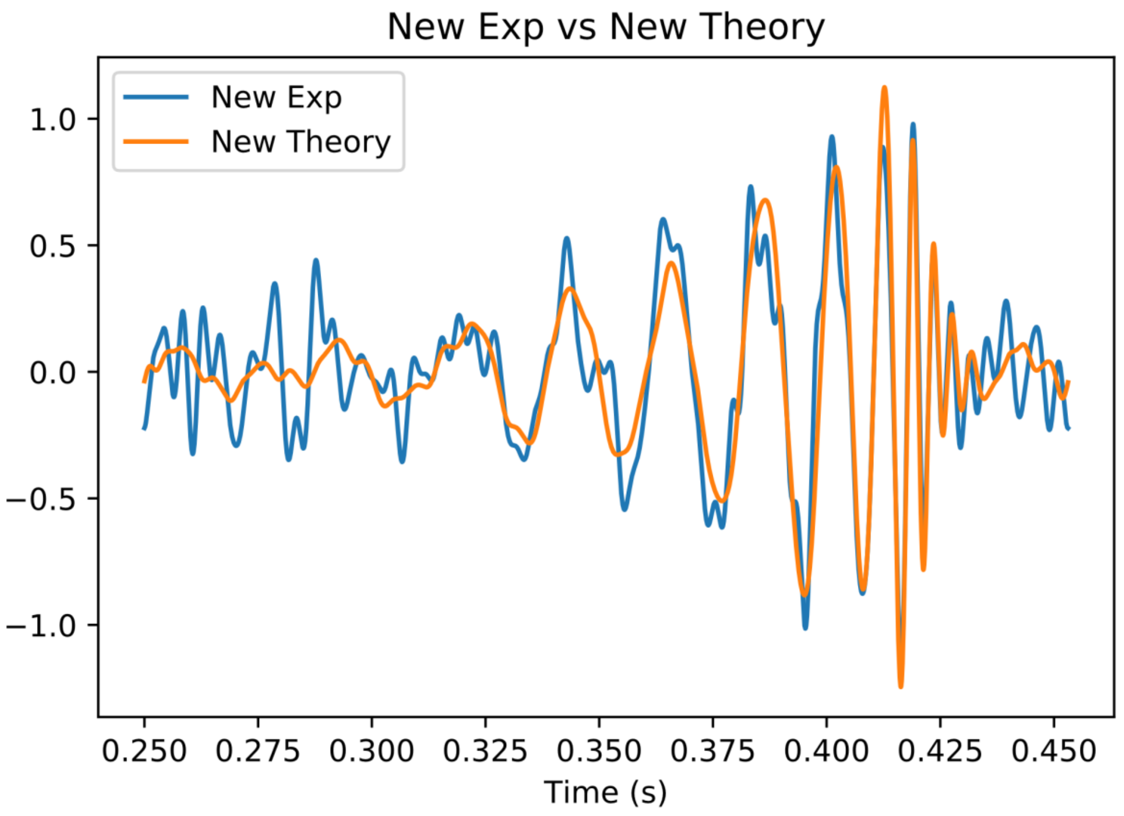
Residues Correlations

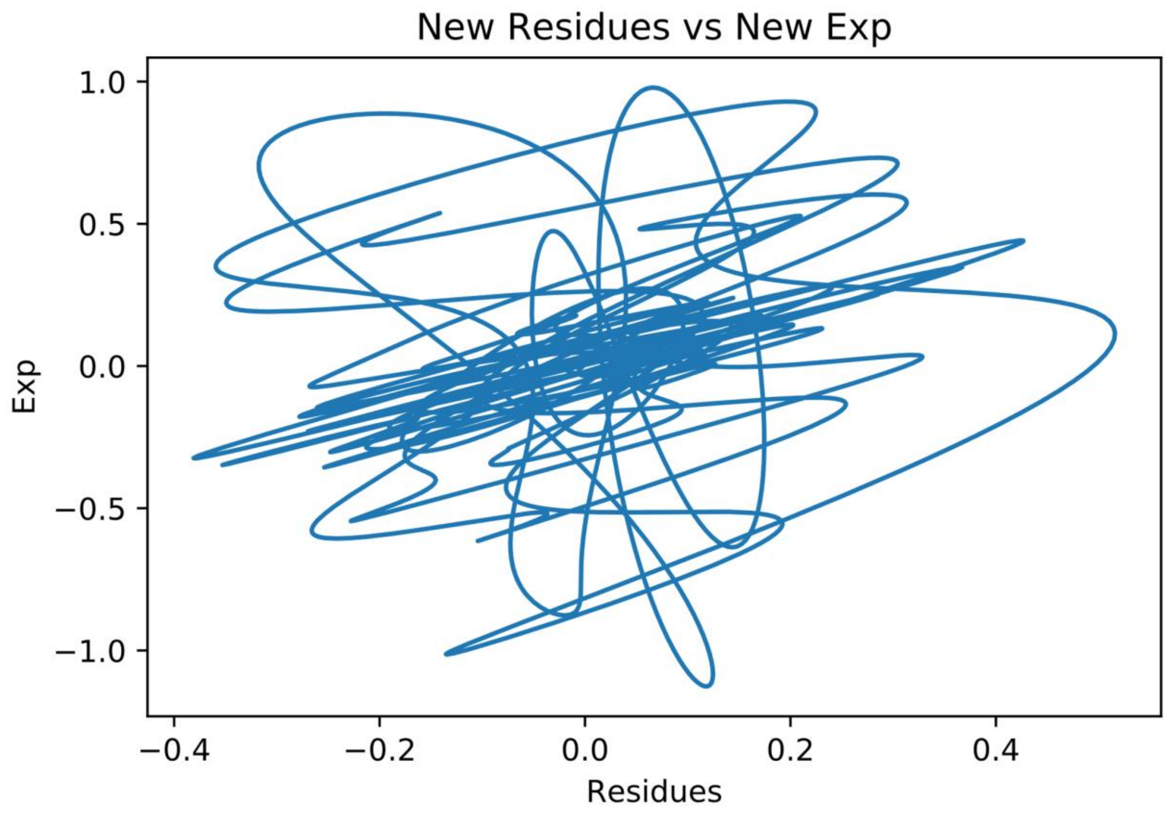




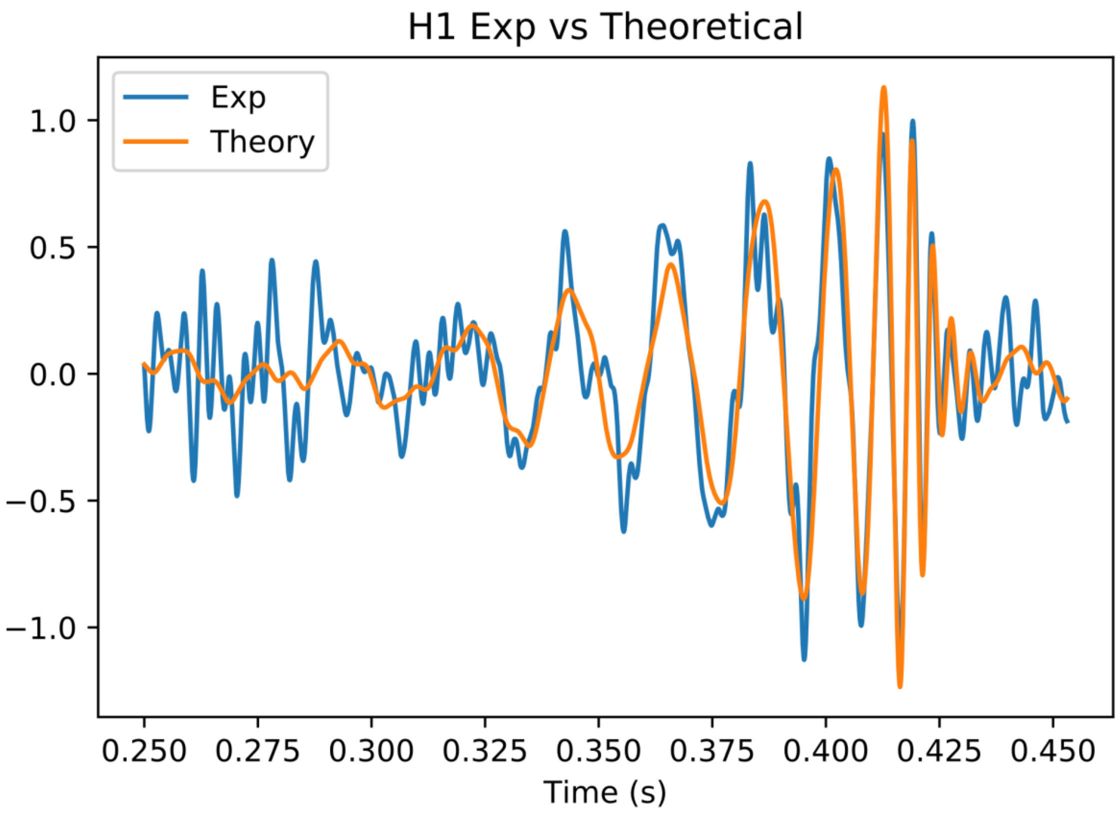


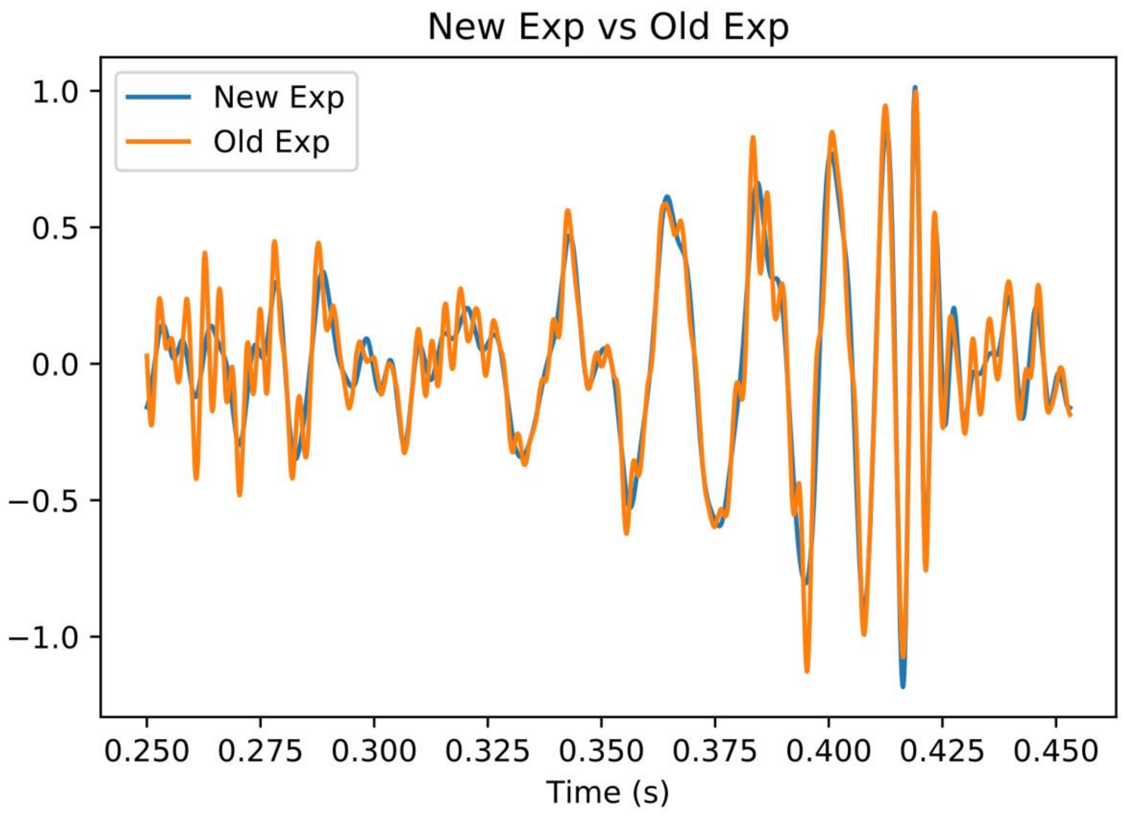
Filtered data – first try



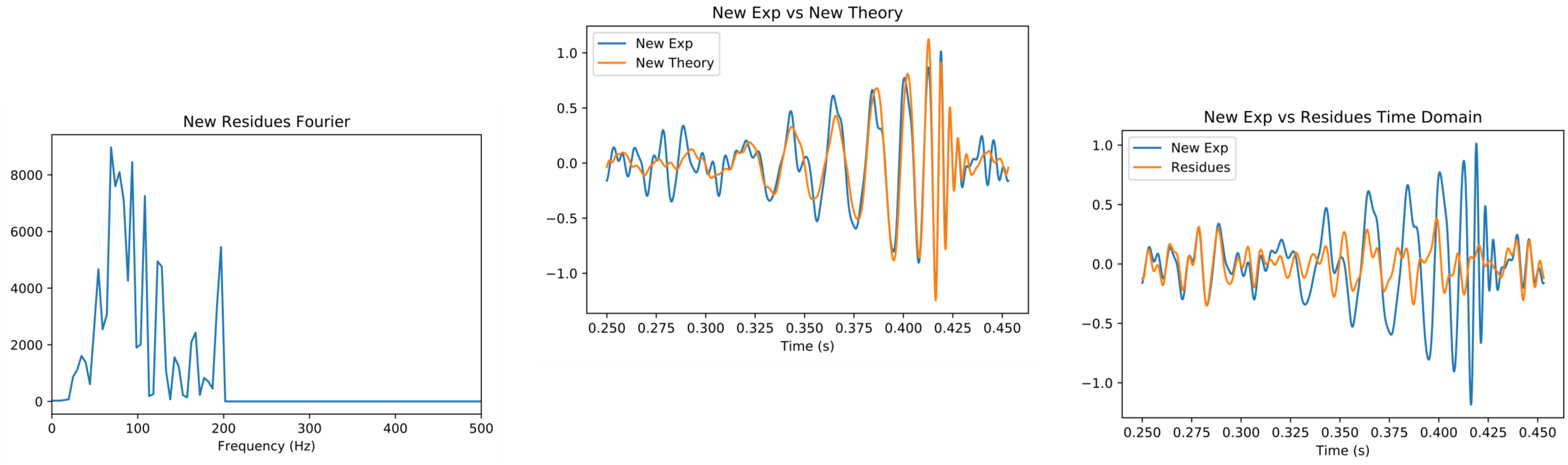


Filtered Data

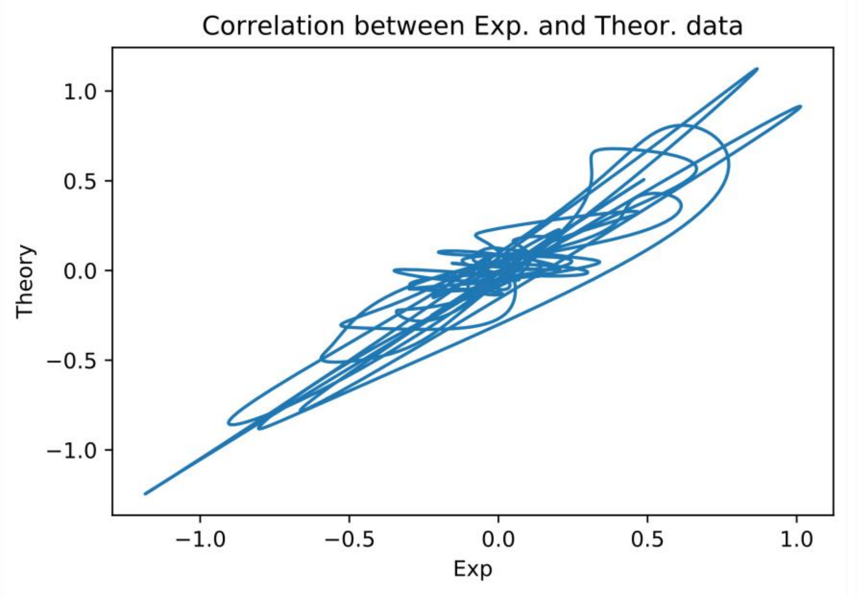


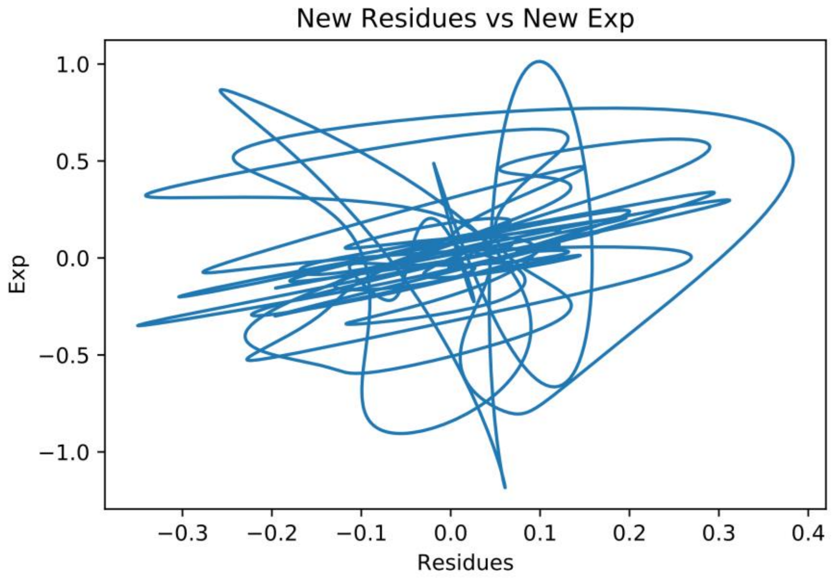


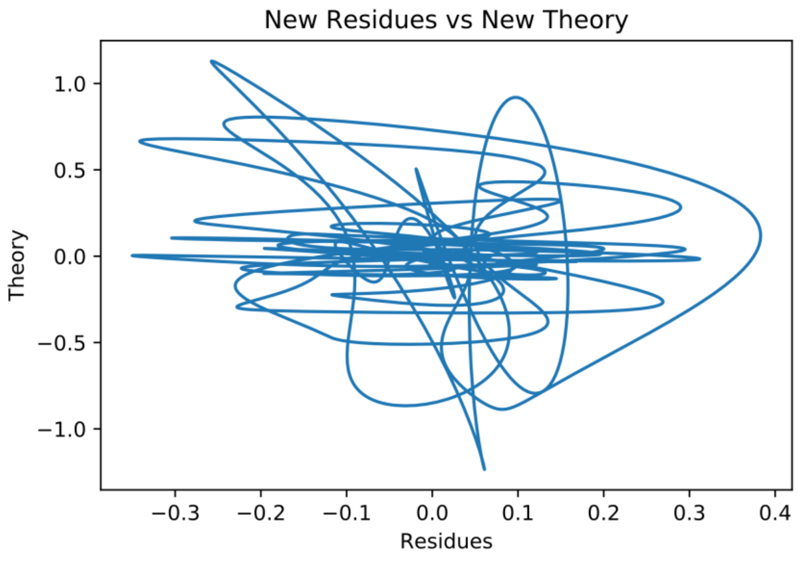
Filtered Data



Residues


Correlation between Exp. and Theor. data
Theory
Exp
1.0
0.5
0.0
−0.5
−1.0
1.0
0.5
0.0
−0.5
−1.0

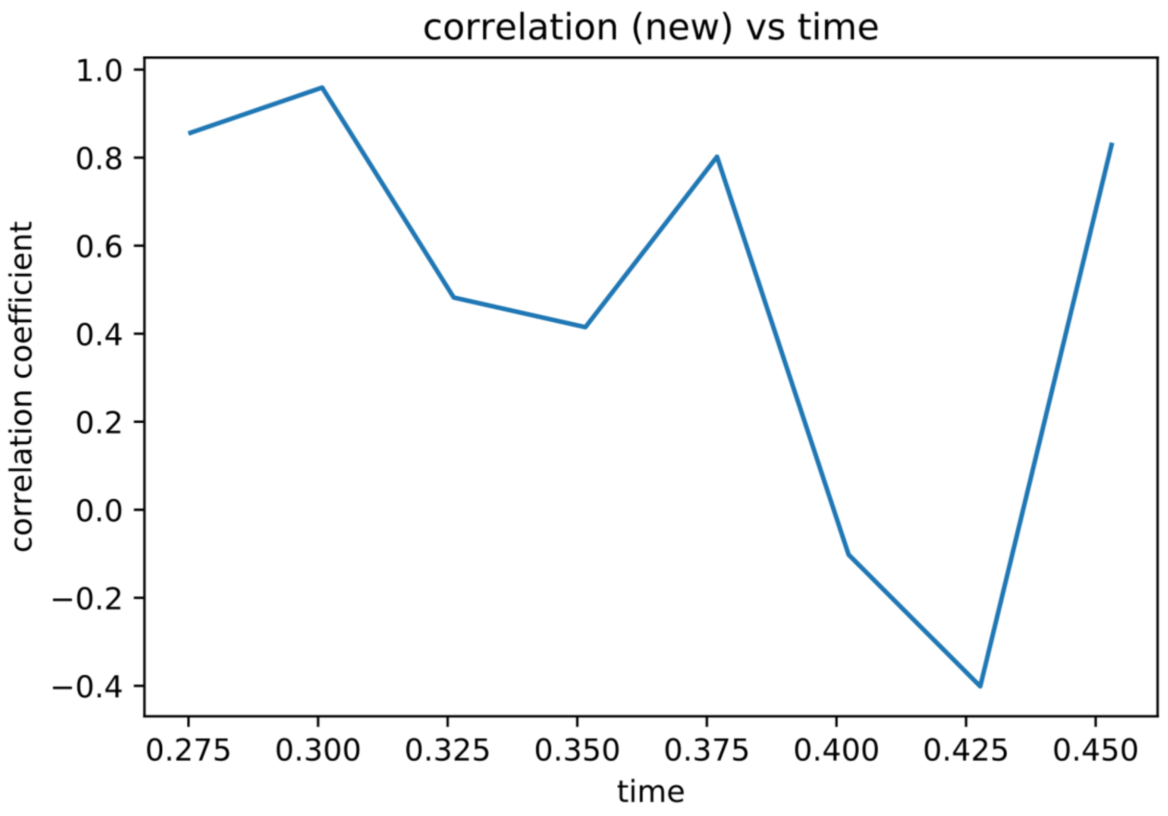

New Residues vs New Exp
Exp
Residues
1.0
0.5
0.0
−0.5
−1.0
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4

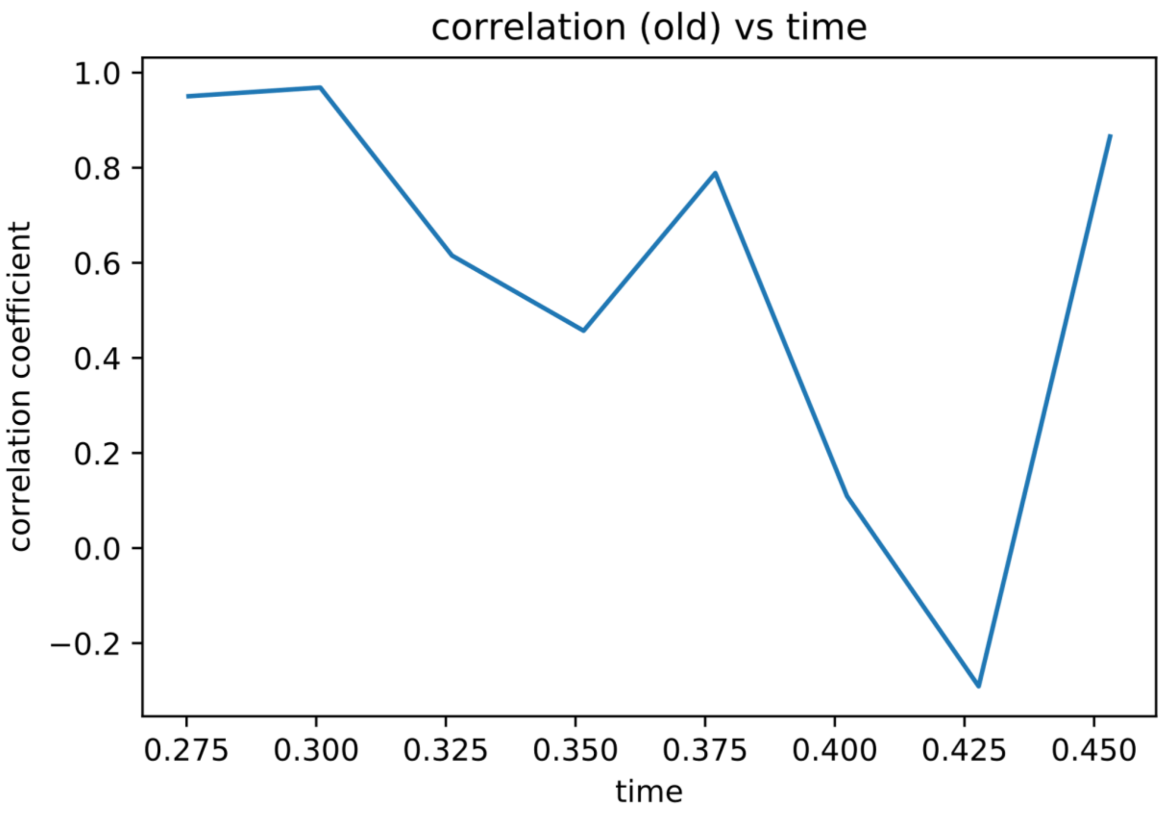

New Residues vs New Theory
Theory
Residues
1.0
0.5
0.0
−0.5
−1.0
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4

Correlations in Time Domain

New Correlations	Time Windows (s)	Old Correlations	Time Windows (s)
0.8565	0.250-0.275	0.9504	0.250-0.275
0.9592	0.275-0.300	0.9687	0.275-0.300
0.4822	0.300-0.326	0.6154	0.300-0.326
0.4148	0.326-0.351	0.4572	0.326-0.351
0.8021	0.351-0.376	0.7888	0.351-0.376
-0.1021	0.376-0.402	0.1094	0.376-0.402
-0.4011	0.402-0.427	-0.2907	0.402-0.427
0.8289	0.427-0.453	0.8657	0.427-0.453
0.2492	Total Correlation	0.3545	Total Correlation

Correlations in Time Domain





Conclusions

1. Before filtering, the giant spikes of theoretical and experimental data already fit together very well (high signal part)
2. Deviation of theoretical and experimental data in the beginning and end is probably noise (more observable in low signal parts)
3. Residues are as expected not correlated with the theory (not predictable)
4. The filtered experimental data fits the theoretical data better (correlation between exp. vs theory is before 0.874; after 0.912)

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Conclusions

5. The residue that stayed behind fluctuates randomly
6. Slight correlation between the residue and experimental signal
7. The correlation between residue and experimenal data is still very high in the beginning and end.

→indication of unremoved noise

8. In the time window between, the residue and experimental data is nearly uncorrelated

→ theoretical model describes our data very well

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