Physics Advanced Laboratory

FOURIER ANALYSIS – GRAVITATIONAL WAVES

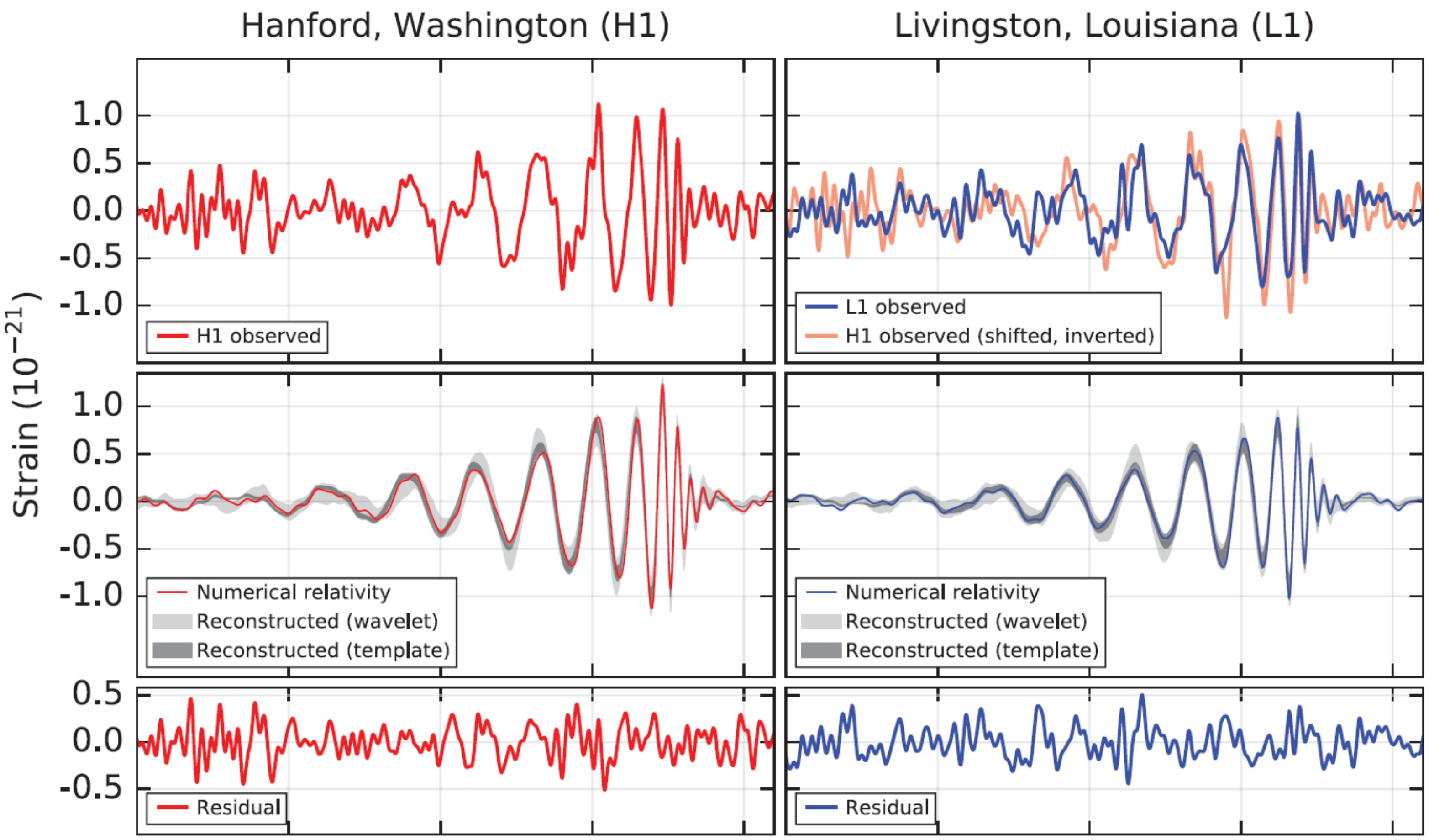
ANTÓNIO OLIVEIRA
SOLANGE NUNES

Outline

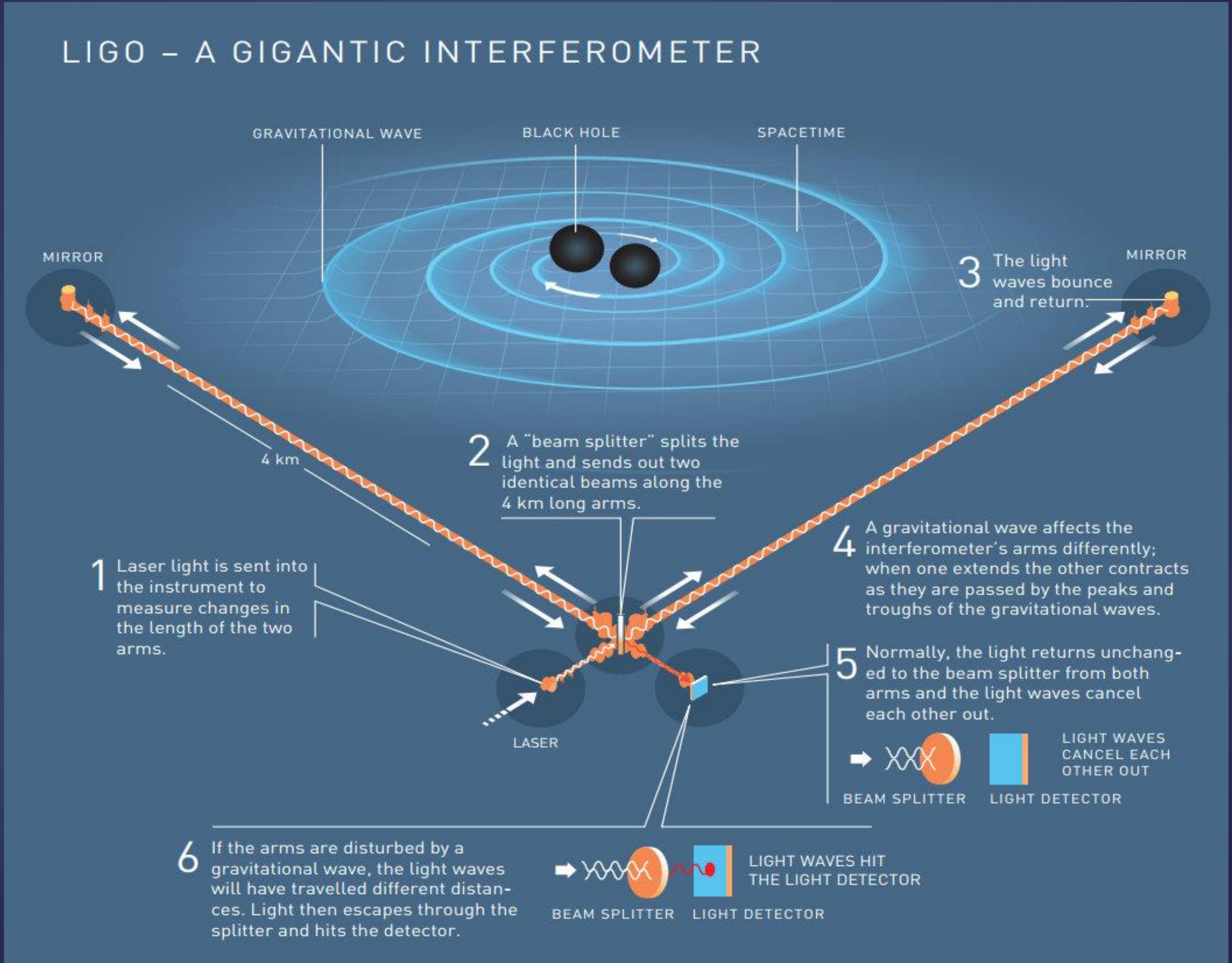
- Description of the experiment;
- Fourier Analysis: main tool to analyze data from the experiment;
- Identifying and removing noise;
- Correlation analysis;
- Analysis in a selected window;
- Conclusions.

Gravitational Waves

- Disturbances in the spacetime curvature;
- Generated by the merger of two black holes;
- Strain: $\sim 10^{-21}$



Gravitational Waves: How can we "see" them?



Fourier Analysis

1. Tool for decomposing a signal into periodic components (sines and cosines) and recover it from those components.
- Allow us to decompose signal in components characterized by a different choice of frequencies, ex. noise in high frequencies.
- Signal x[n] is discrete, given only in N sampling times n∆t , over a period T: need to use Discrete Fourier Transform

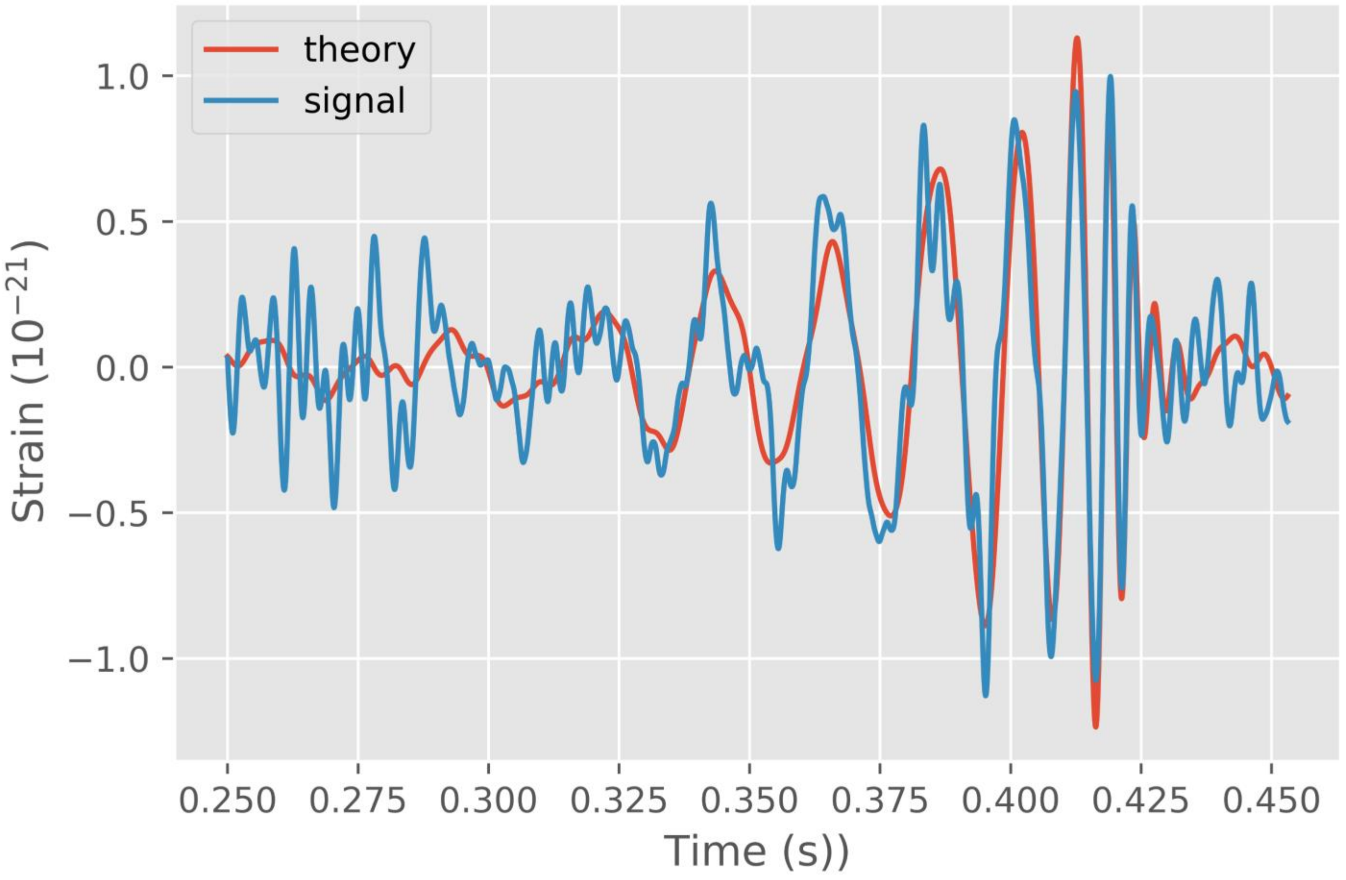
$$x(t) = \sum_{k=-\infty}^{\infty} y_k e^{i\frac{2\pi kt}{T}} \longrightarrow x[n] = \frac{1}{N}\sum_{k=0}^{N-1} e^{2\pi i\frac{kn}{N}} y[k]$$

$$y_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-i\frac{2\pi kt}{T}} x(t)\ dt \longrightarrow y[k] = \sum_{n=0}^{N-1} e^{-2\pi i\frac{kn}{N}} x[n]$$

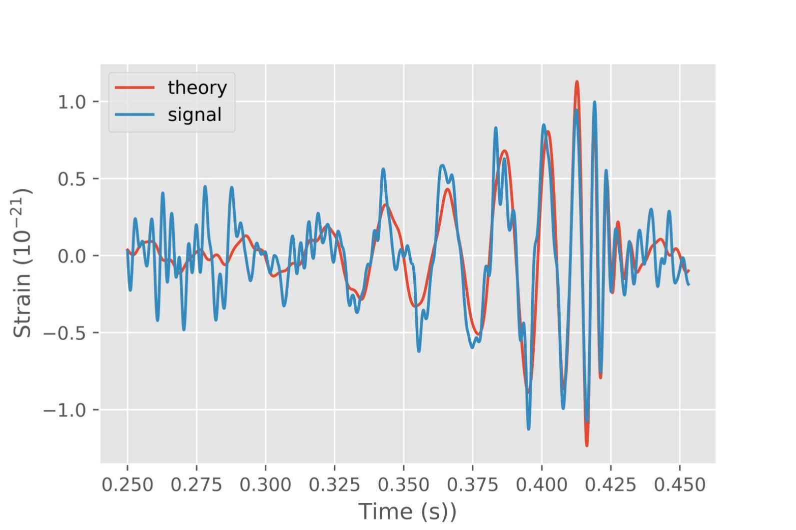
$$i\frac{2\pi}{T}kt \to i\frac{2\pi}{N\Delta t}kn\Delta t \to 2\pi i\frac{kn}{N}$$

Gravitational Waves: Theory vs Experiment

- Analysis of the correlation between theory (expected signal) and observation (actual signal)
- Analysis of the correlation between residue and observation

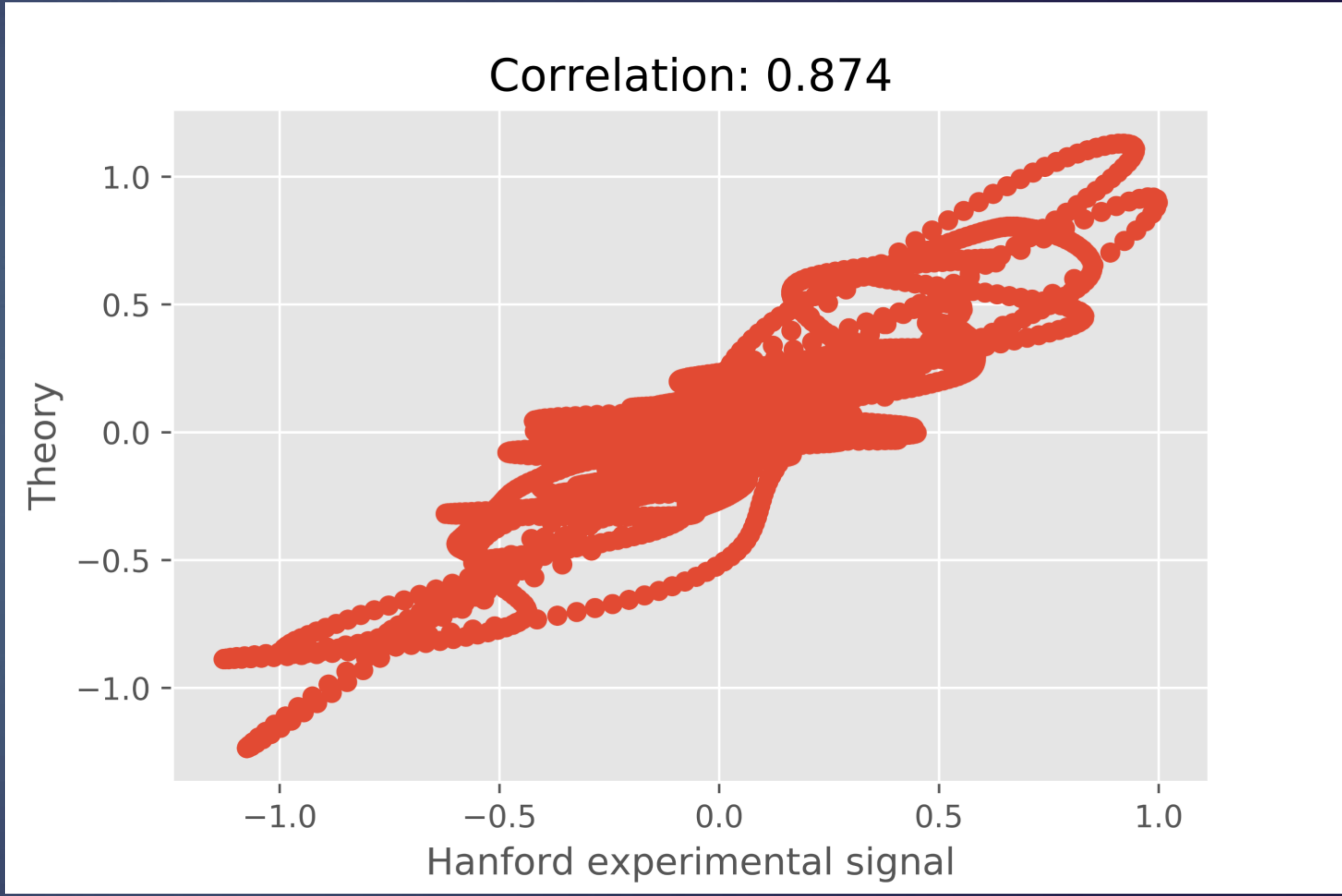


Gravitational Waves theory vs observation

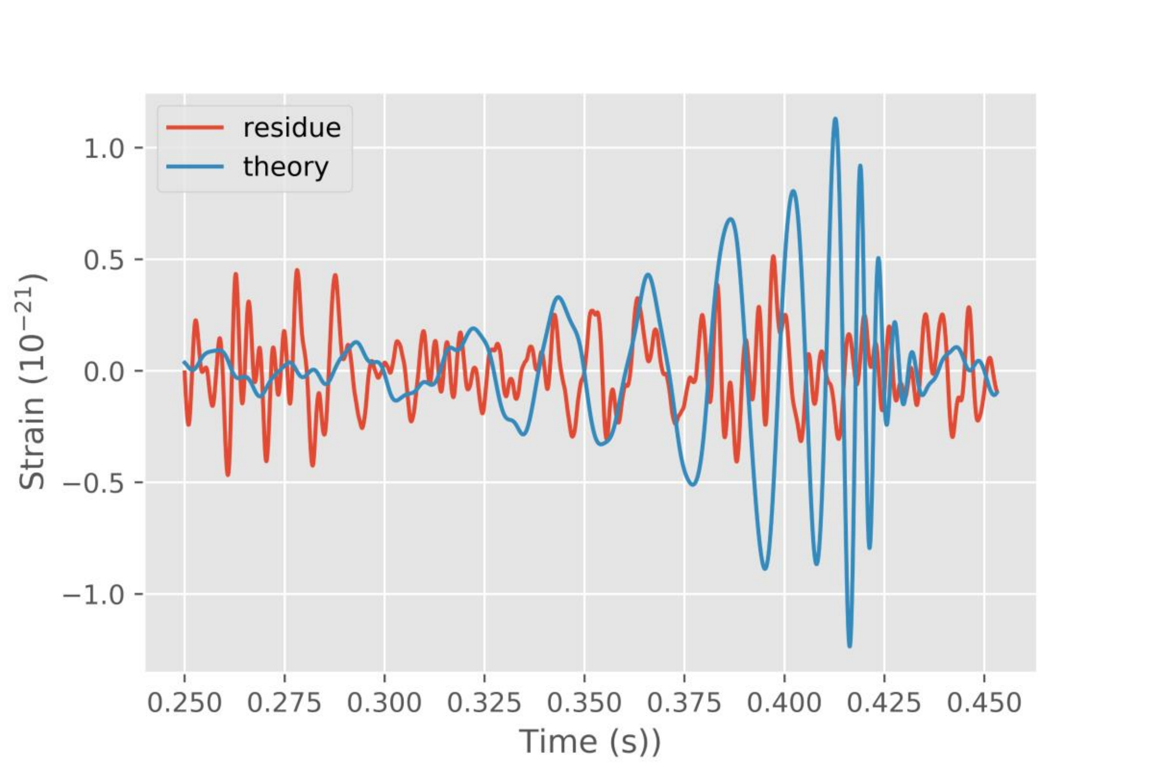


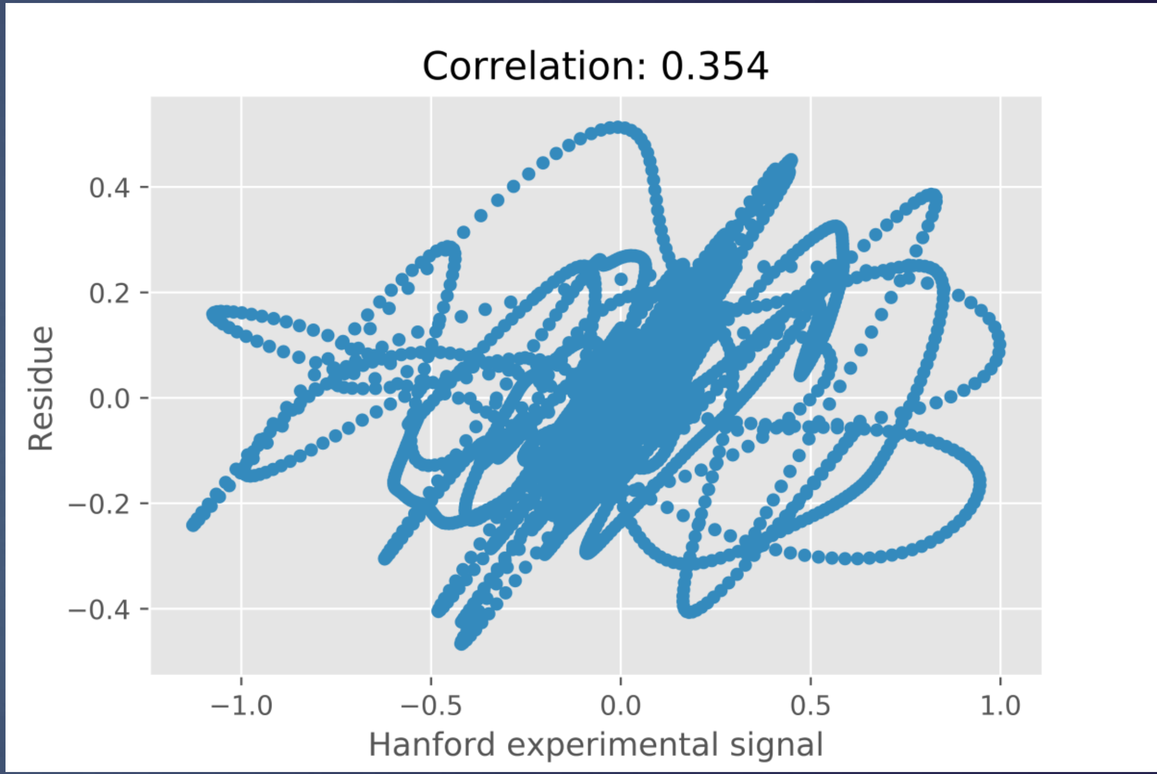
Pearson correlation coefficient

$$r_{xy} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$



Residues: difference between signal and theory

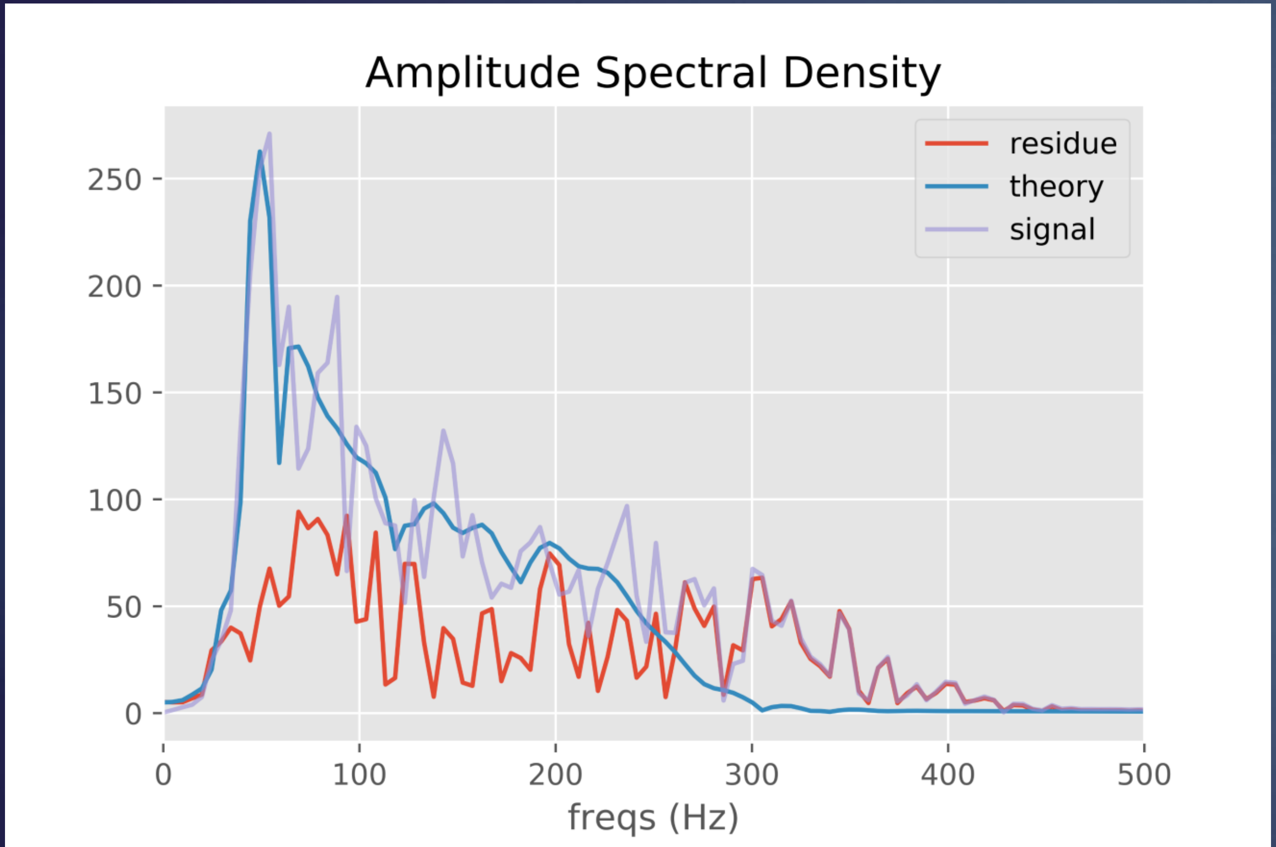


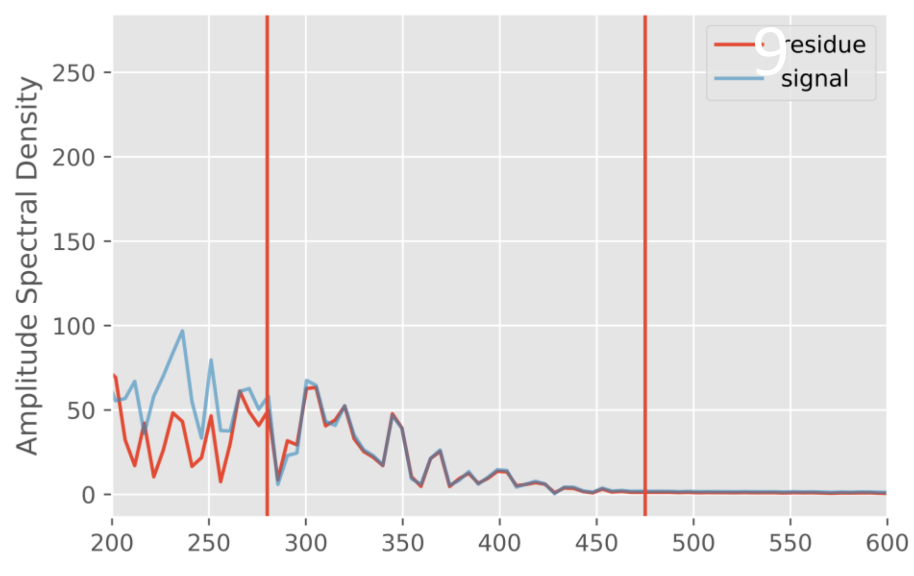


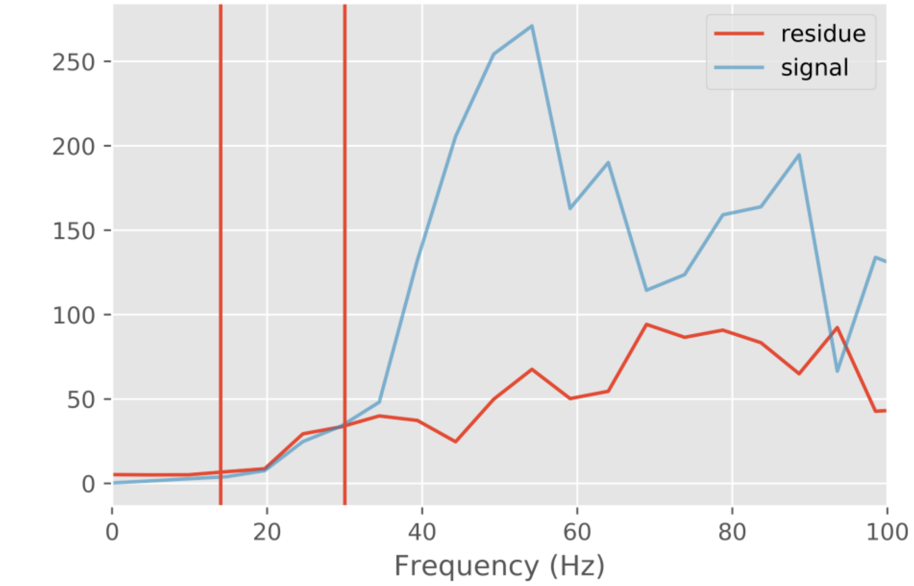
Residue varies too much in amplitude (contains more than noise)

Some correlation between residue and signal: where signal is not expected, actual signal coul[d] pure noise (ex: high frequencies)

Fourier Transforms:
finding noise (raw windows)



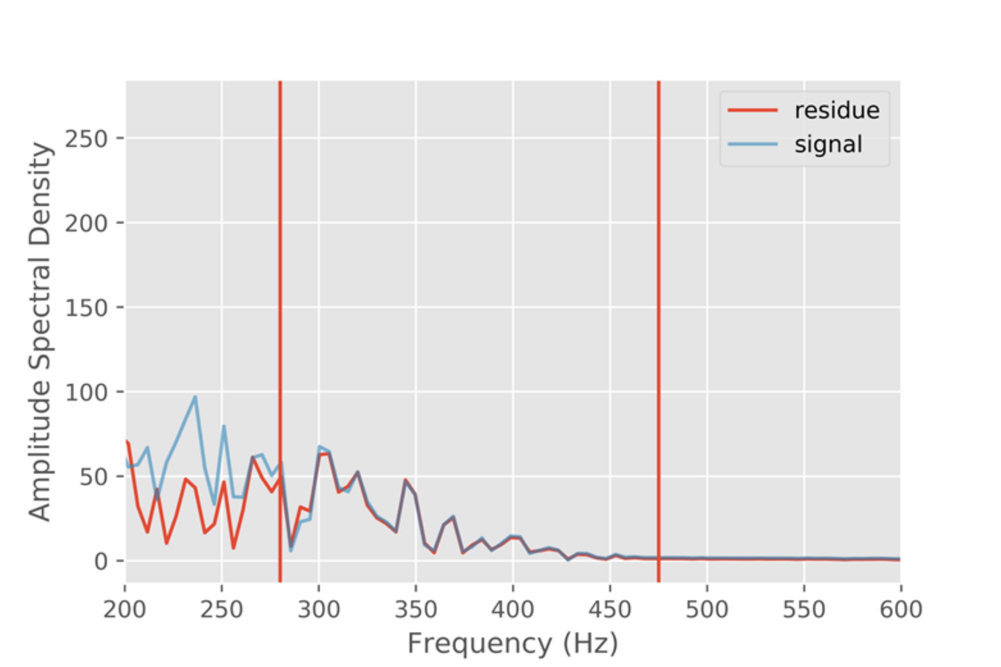




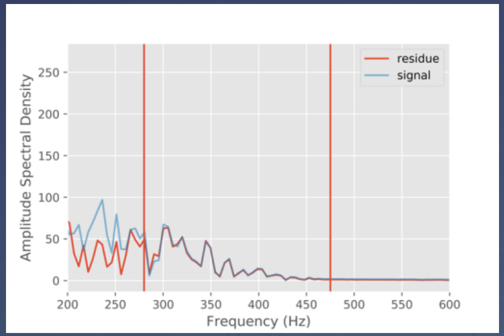
Optimizing the extraction of noise

Procedure

- Departing from crude estimates for the limits of frequency window, search methodically for values that give the best fit for Residue = Signal
- For the cases where the fit is almost perfect (slope > 0.99; intercept ~ 0), find the larger window.



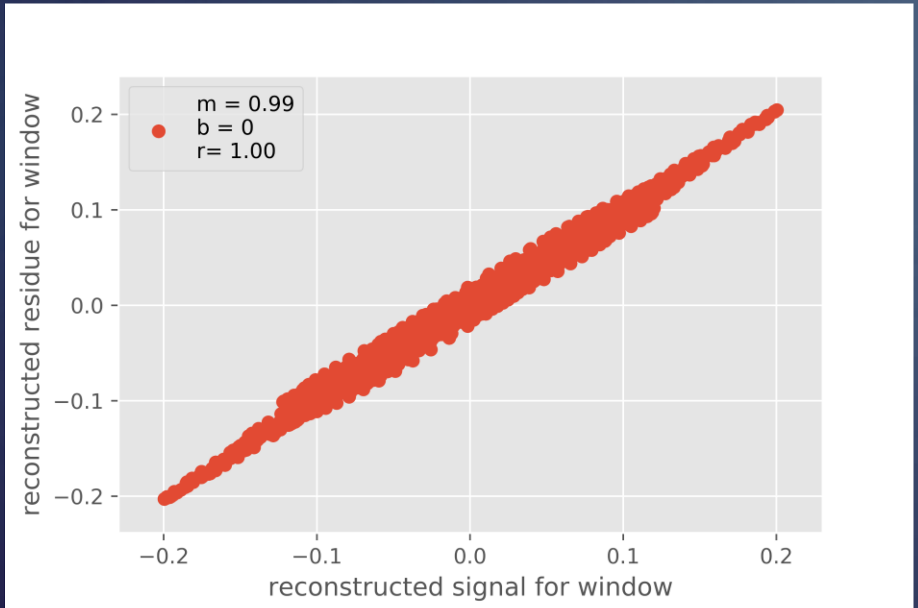
Optimizing the extraction of noise results

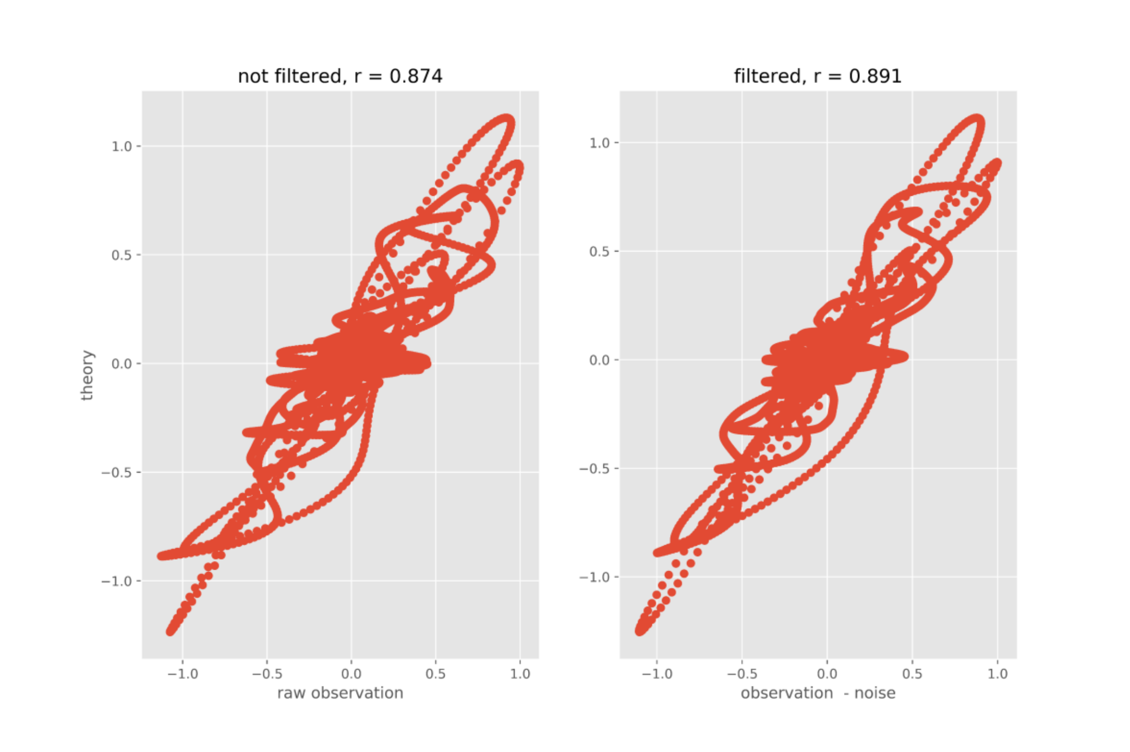


Correlation:

0.874 → 0.891

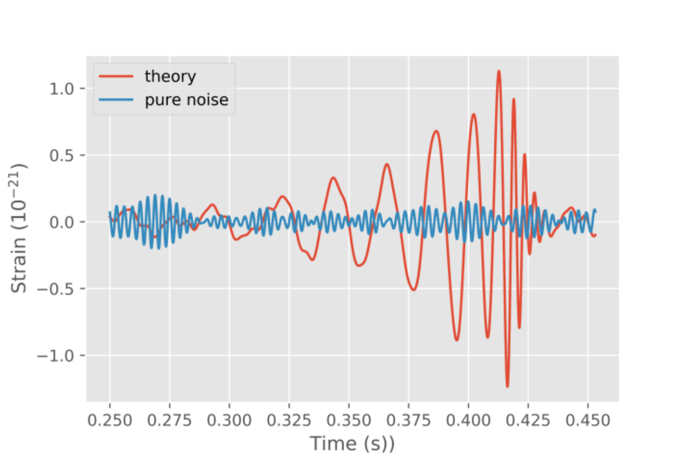
High frequencies window :
286 Hz - 398 Hz

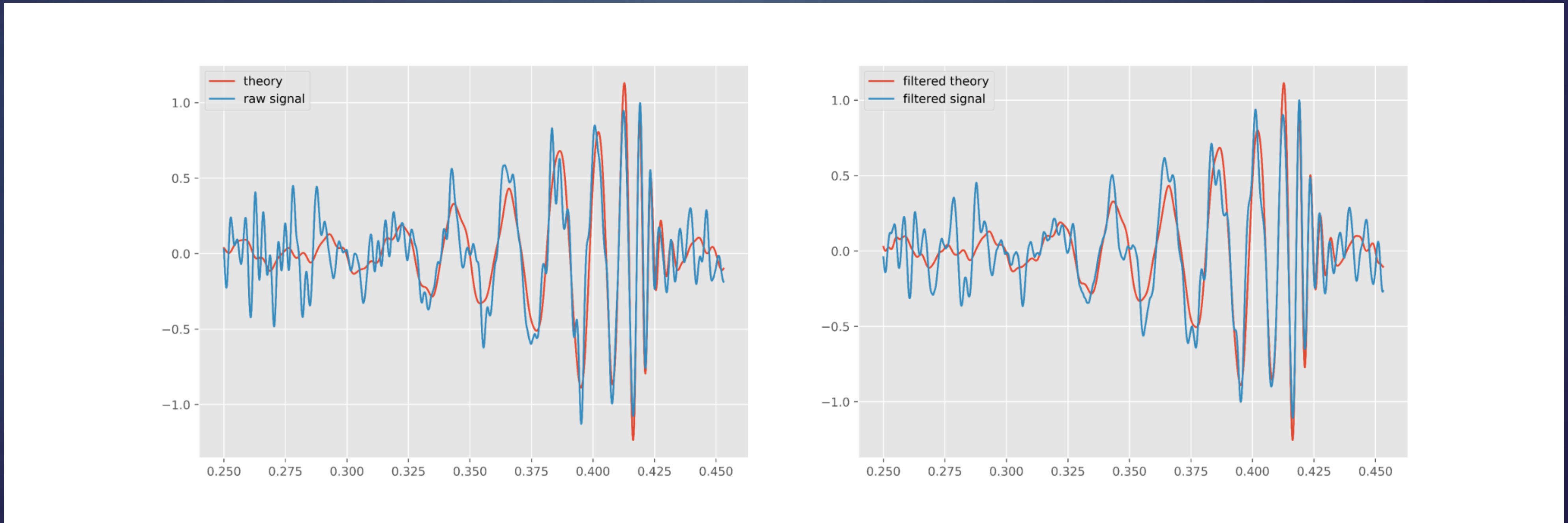




Before and After

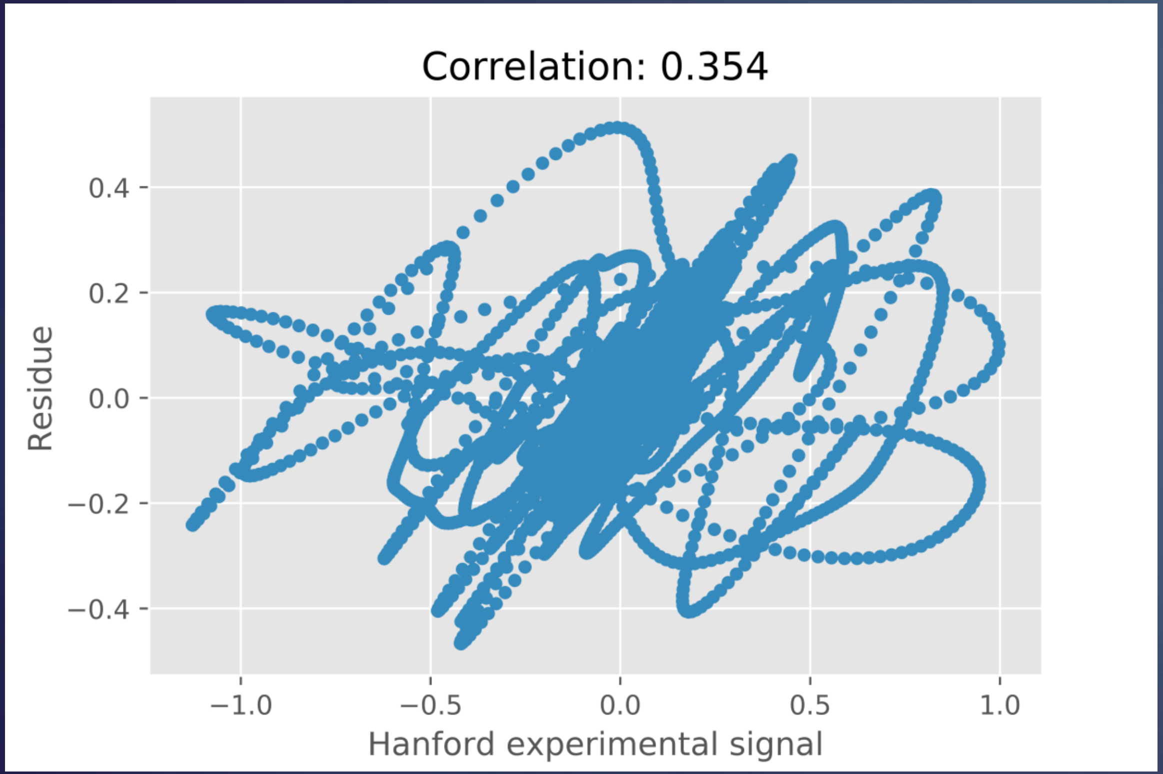
Mostly attenuation of spiky behavior in the signal

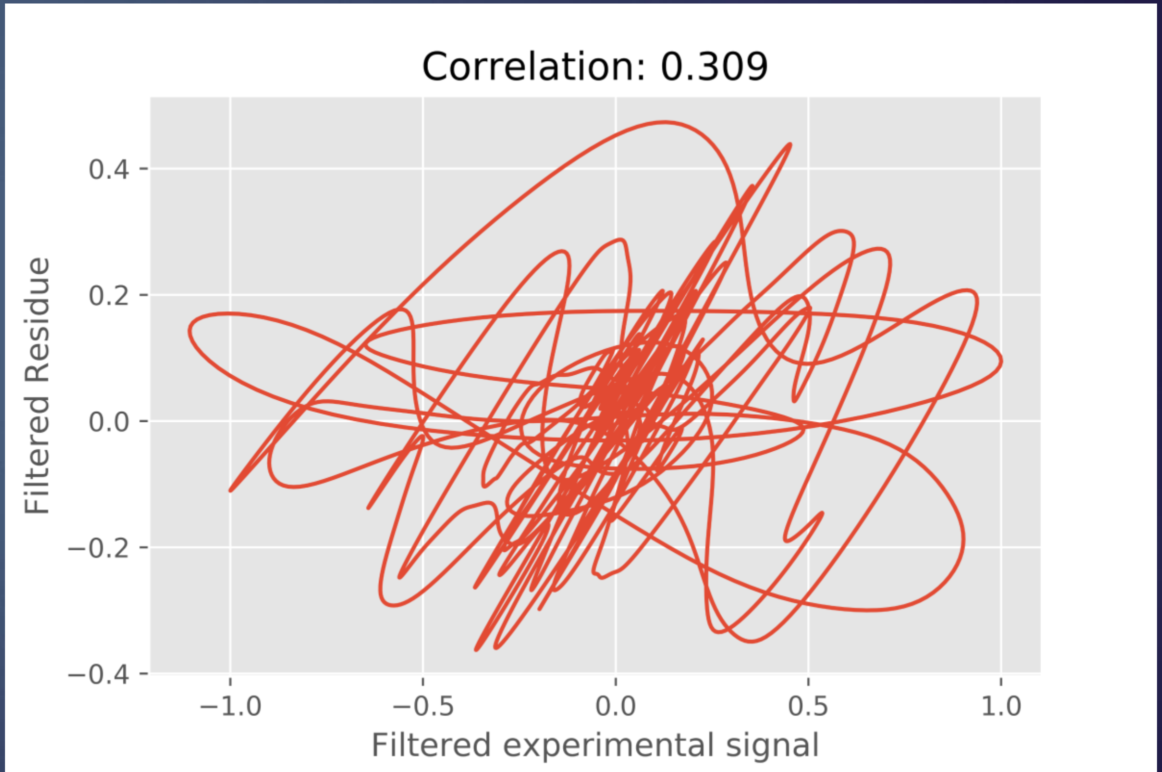




Residue vs Signal

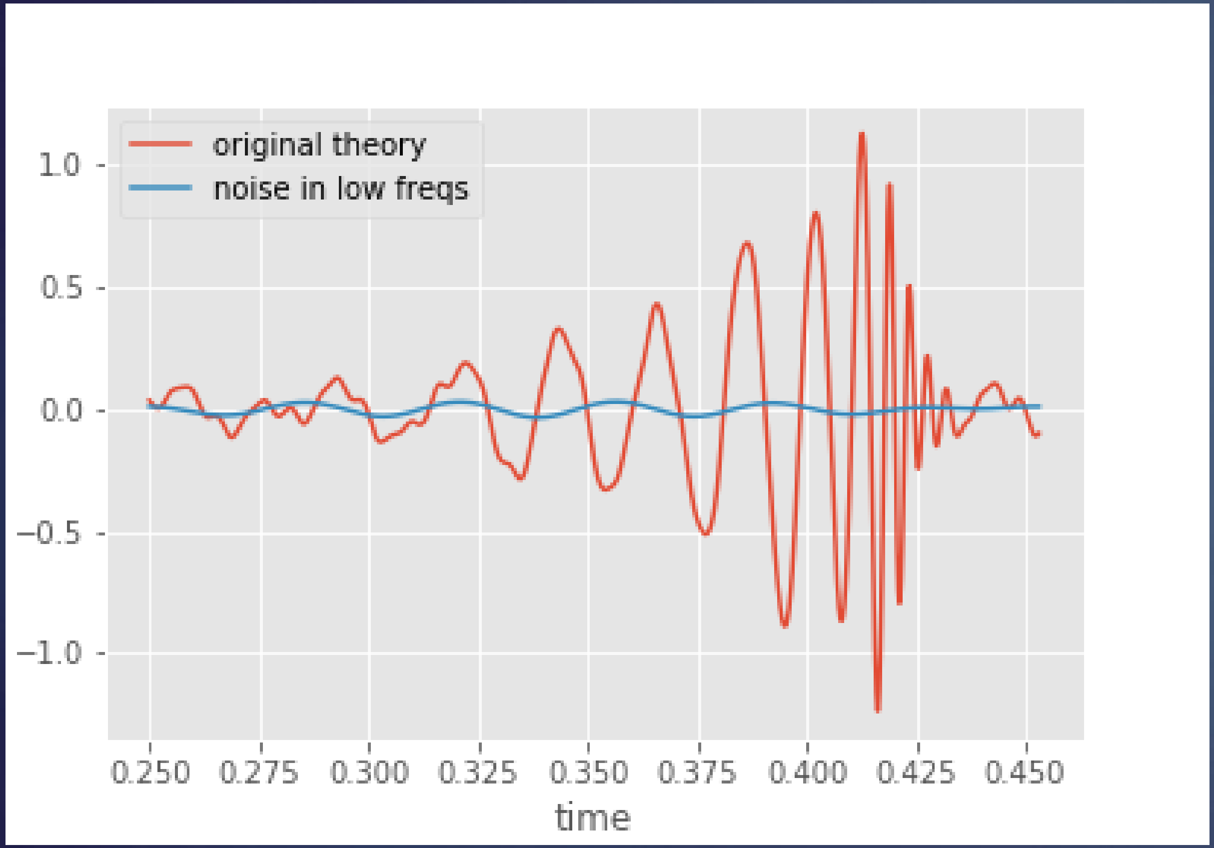
Before and after removing high frequency noise

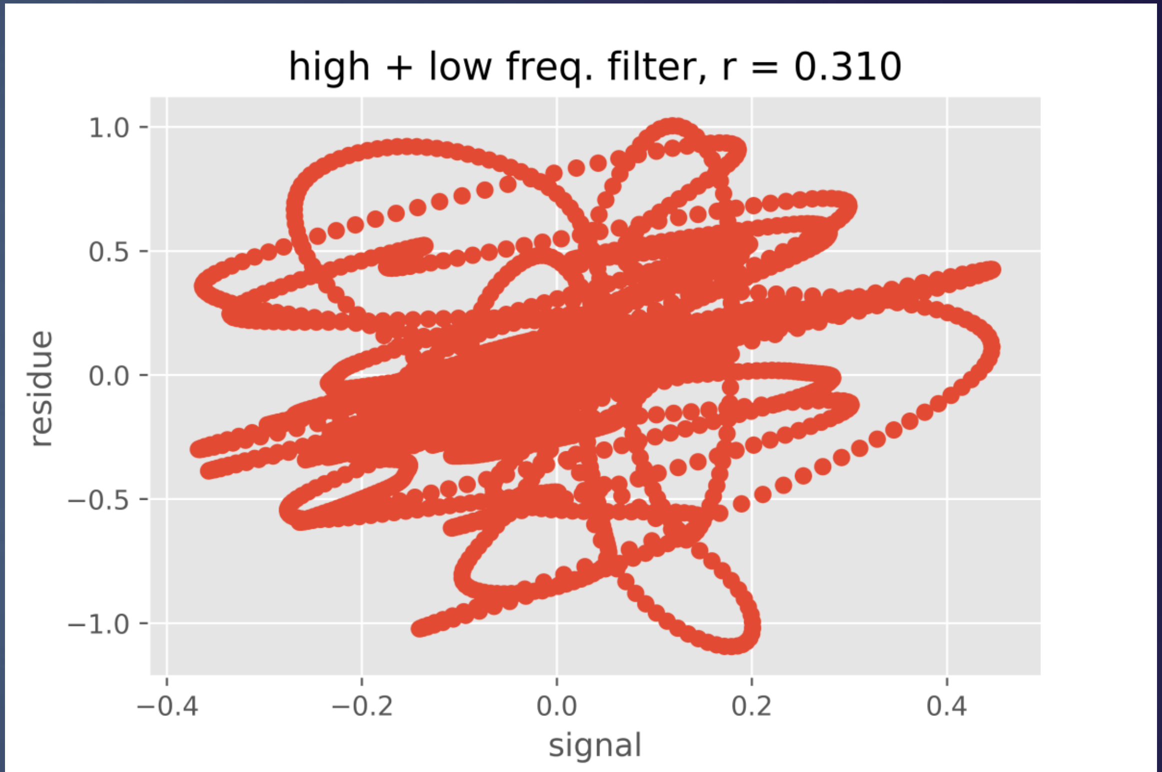




Low frequencies noise

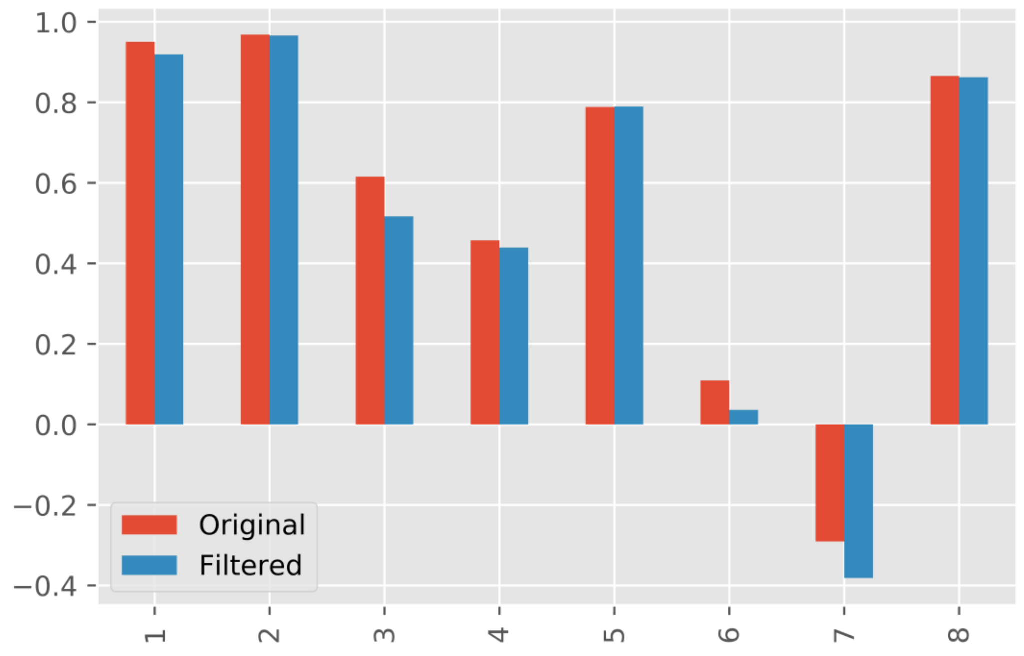
correlation residue/signal actually increases

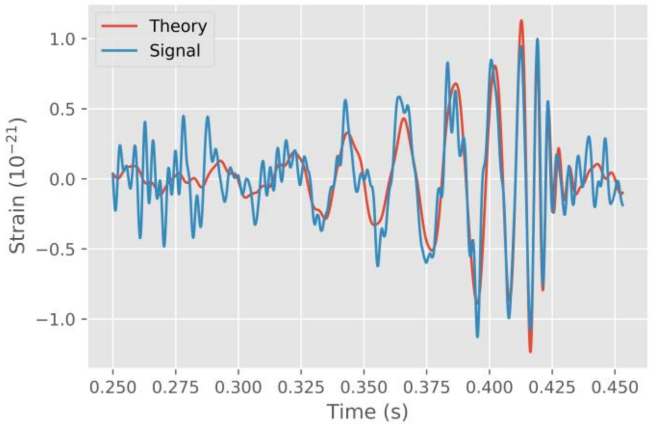


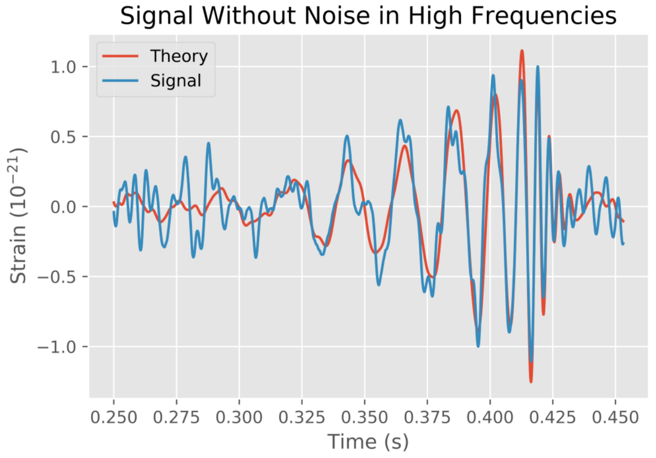


Correlation Analysis

- 1, 2 & 8 – low amplitudes, theory is almost zero
- 3 & 4 – correlation decreases
- 5 – high correlation
- 7 – negative correlation; filtered has bigger correlation

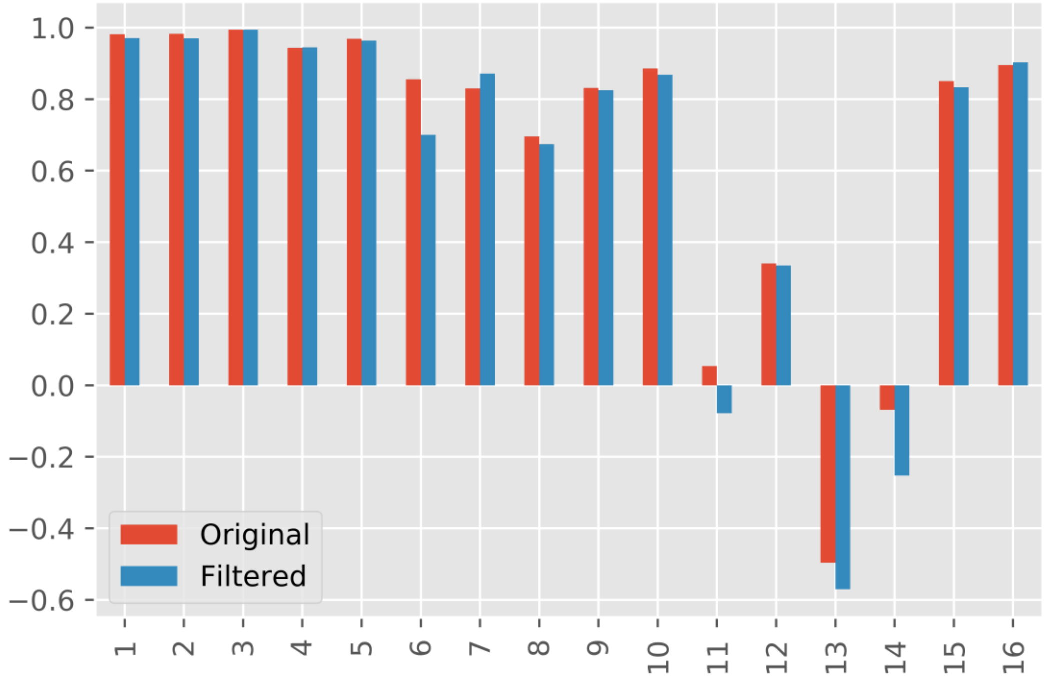


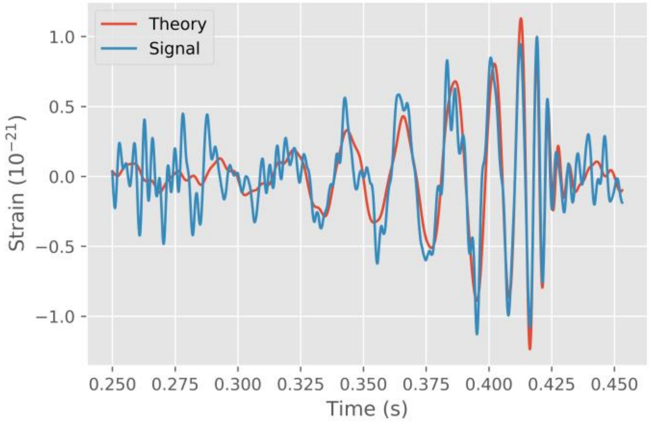


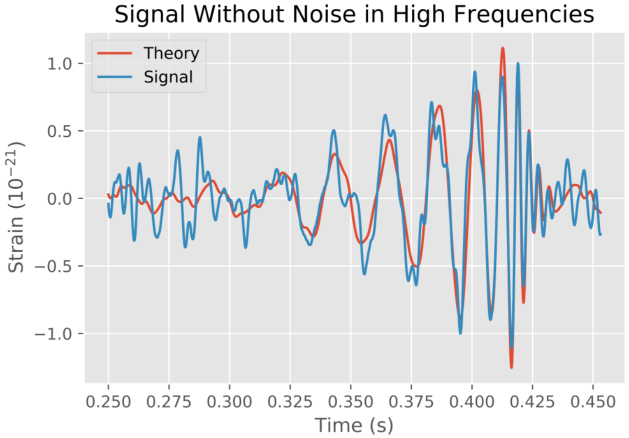


Correlation Analysis

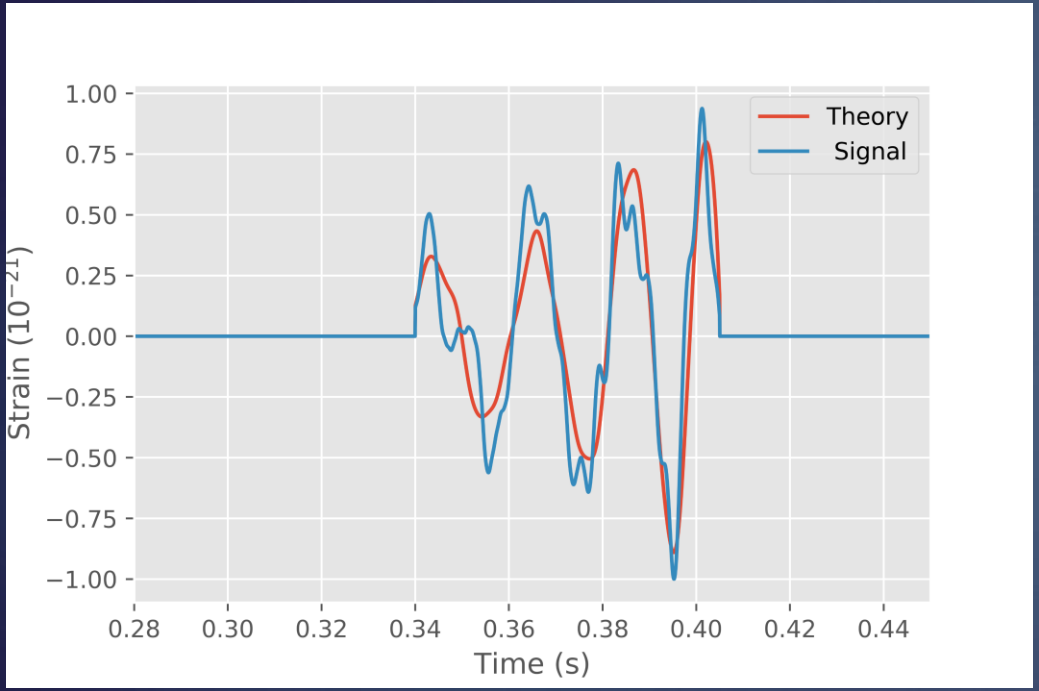
- Sometimes filtered data has bigger correlation than original data;
- Different behaviors for different windows;
- We could analyze the data in different windows…

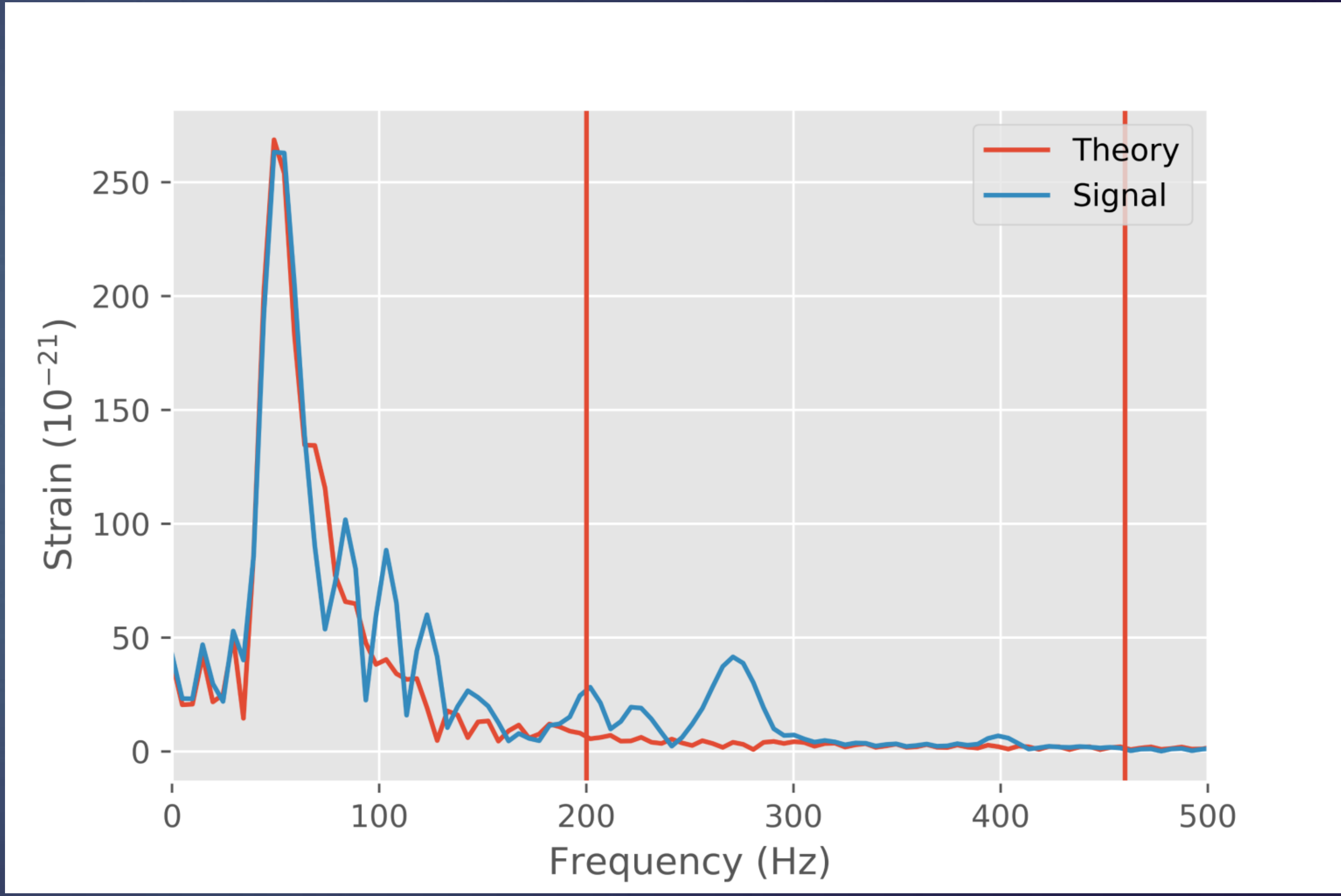




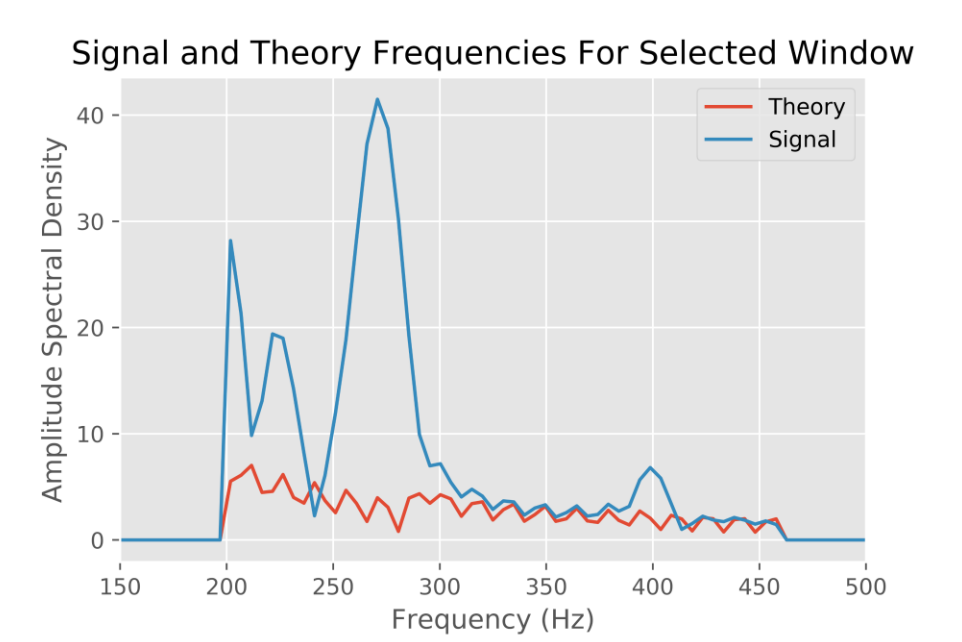


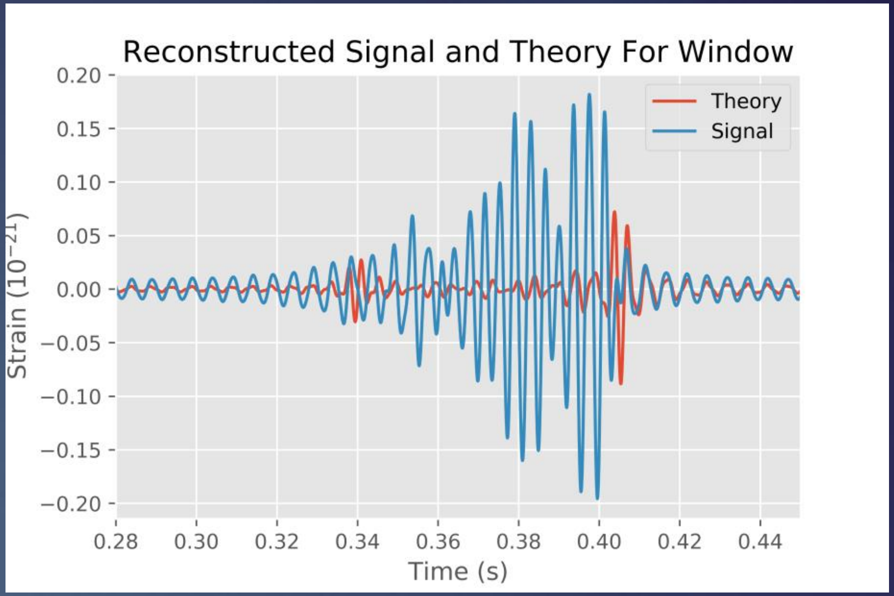
Inspiral Fase Analysis

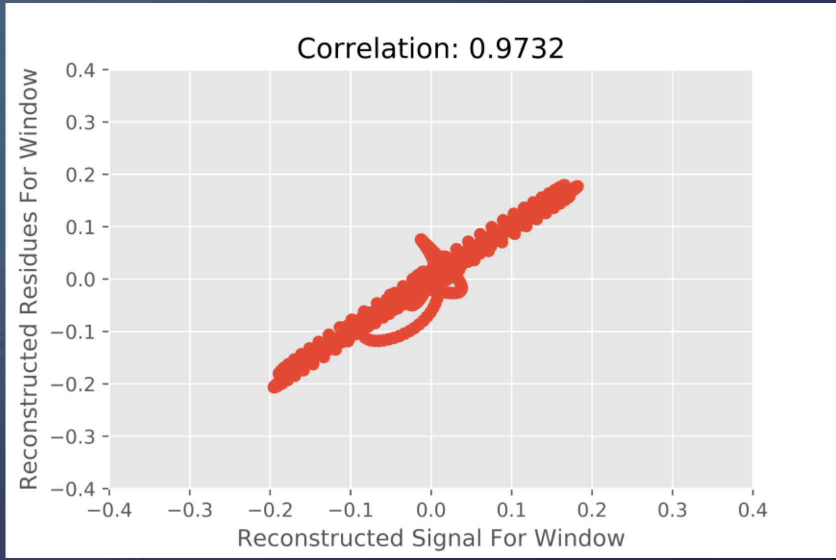




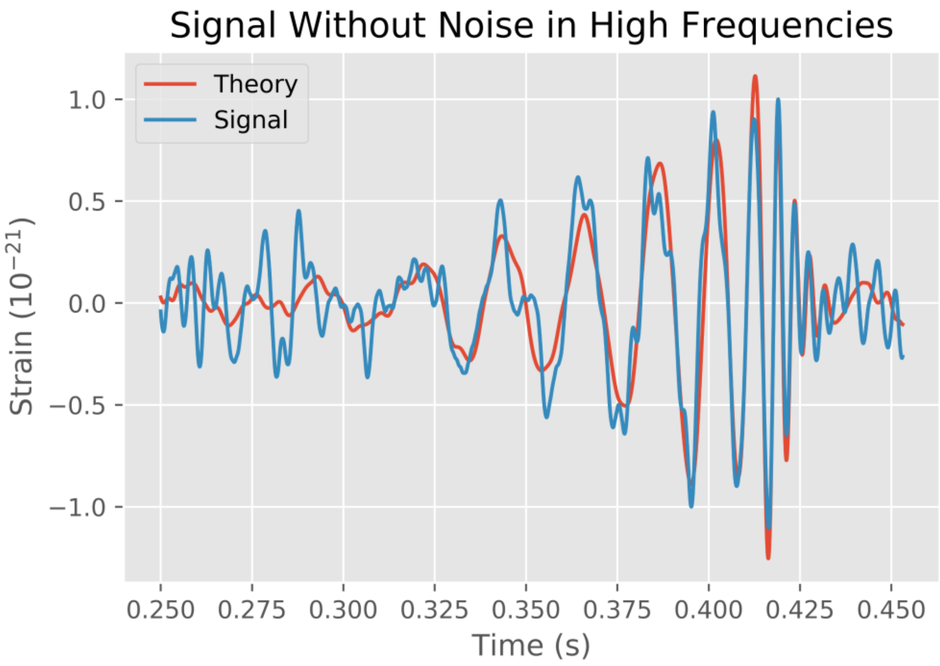
More Noise?

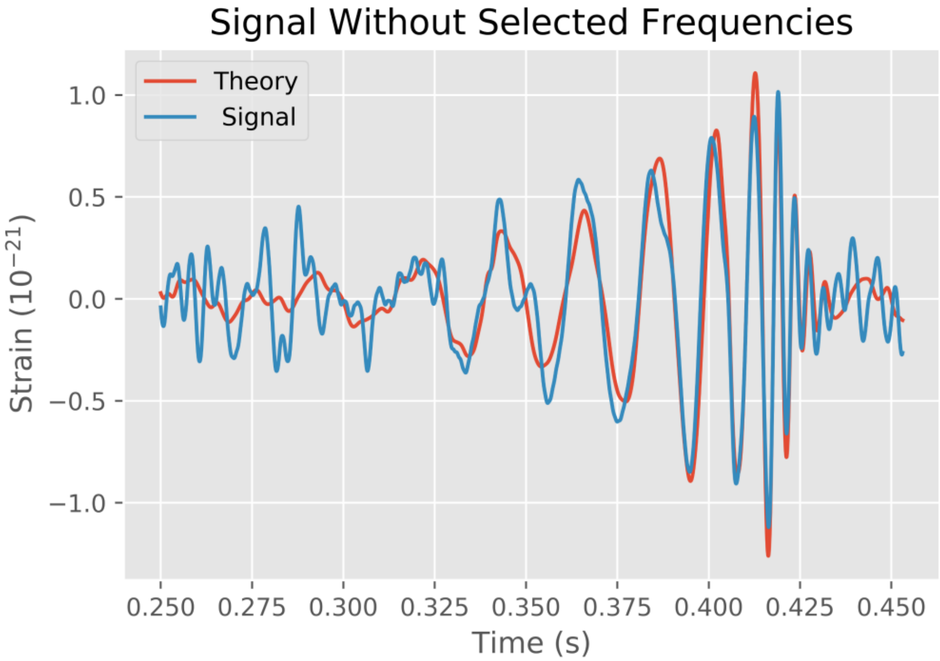

Signal and Theory Frequencies For Selected Window
Theory
Signal
Amplitude Spectral Density
0
10
20
30
40
150
200
250
300
350
400
450
500
Frequency (Hz)


Reconstructed Signal and Theory For Window
Theory
Signal
Strain (10^{-21})
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
−0.20
0.28
0.30
0.32
0.34
0.36
0.38
0.40
0.42
0.44
Time (s)

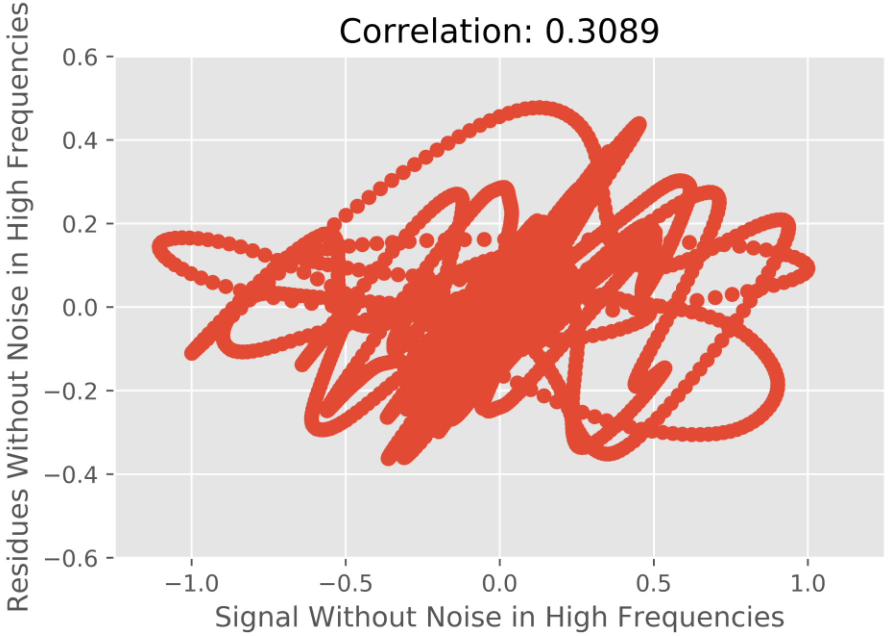

Correlation: 0.9732
Reconstructed Residues For Window
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.4
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
0.4
Reconstructed Signal For Window

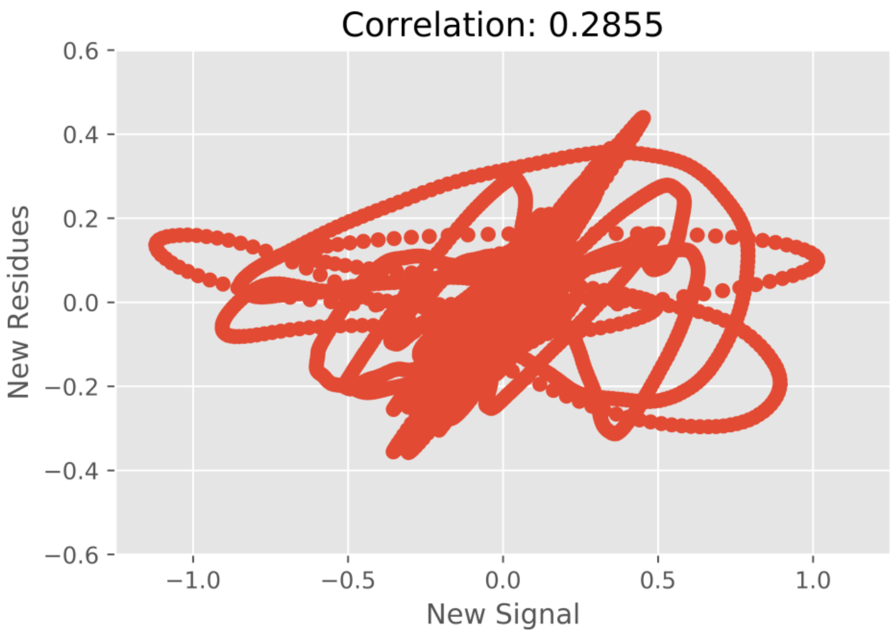
Data Evolution



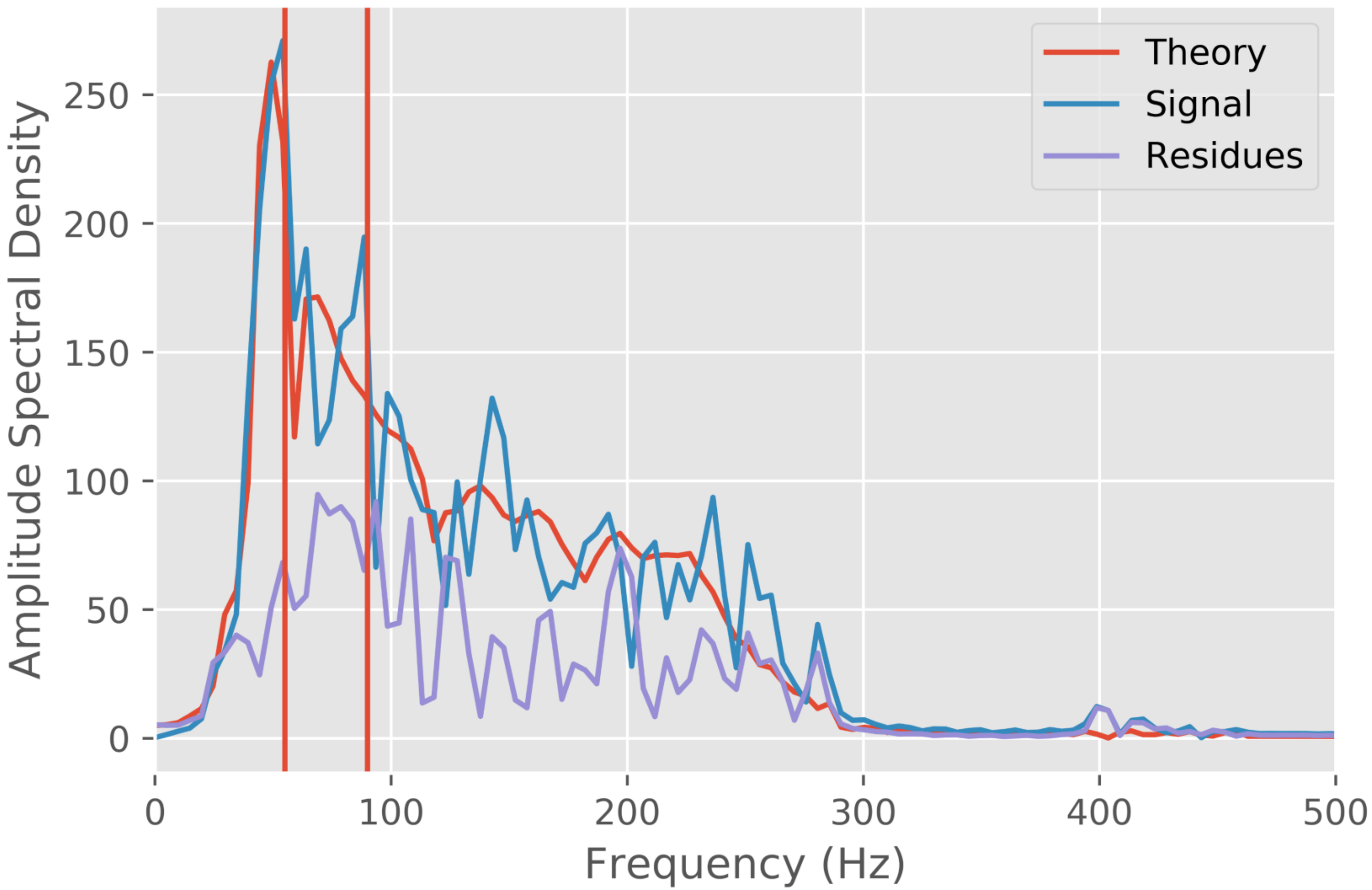


Correlation Evolution

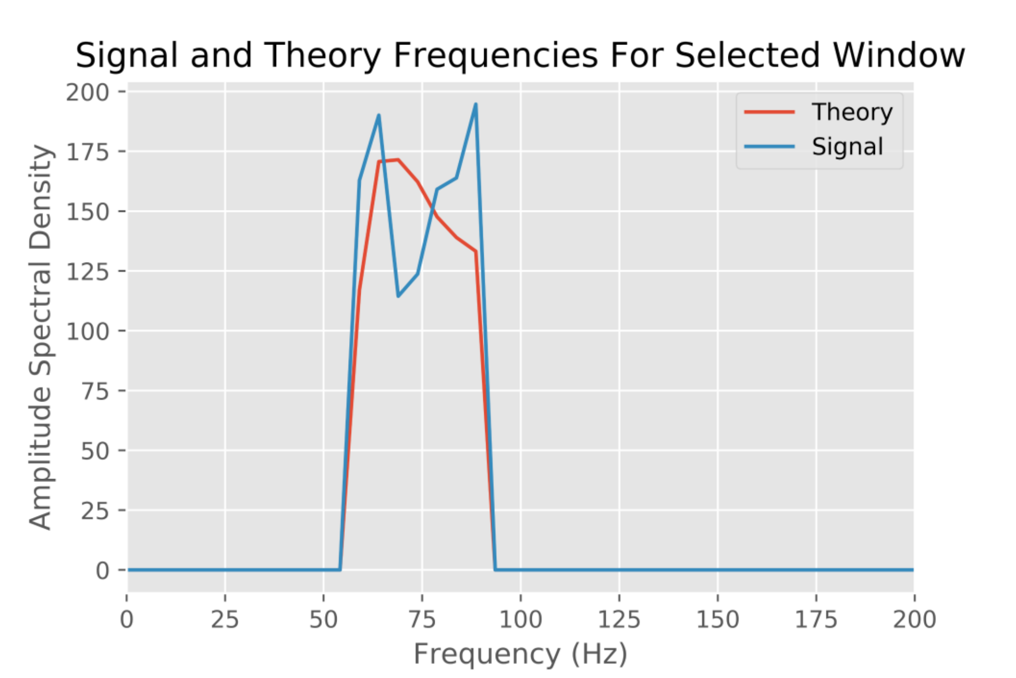


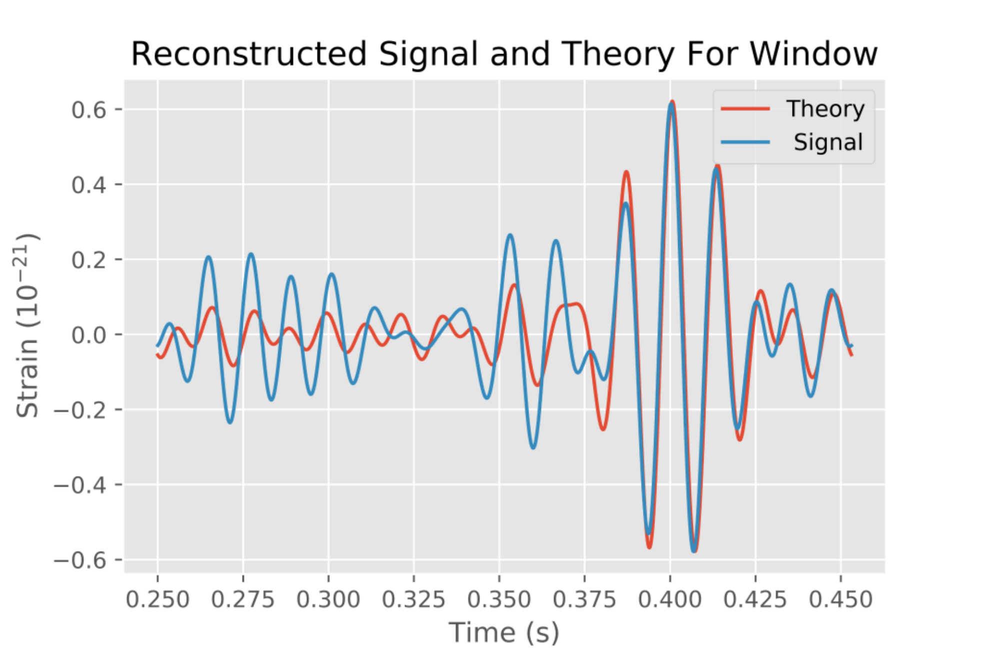


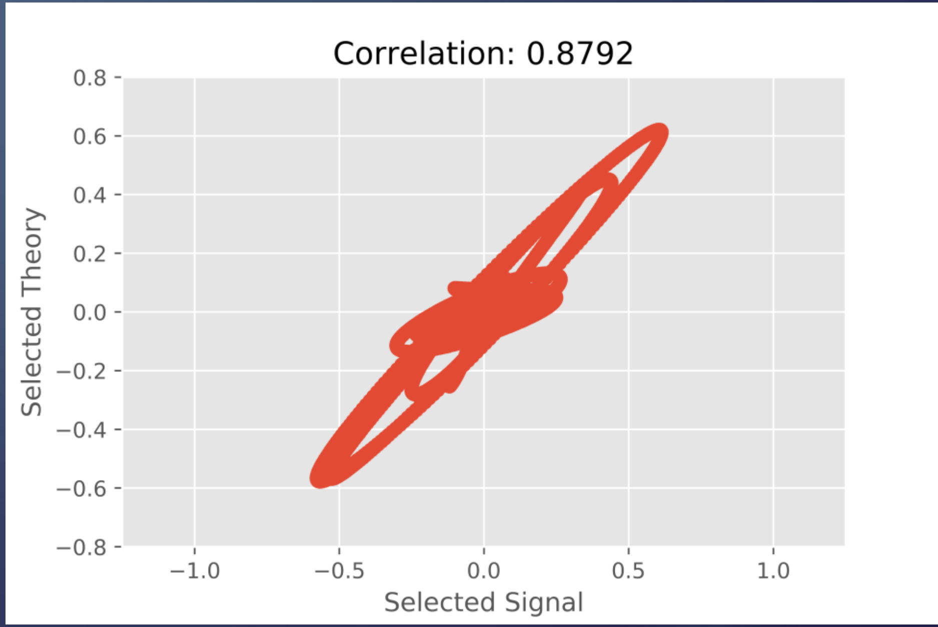
A New look at the Frequency Domain



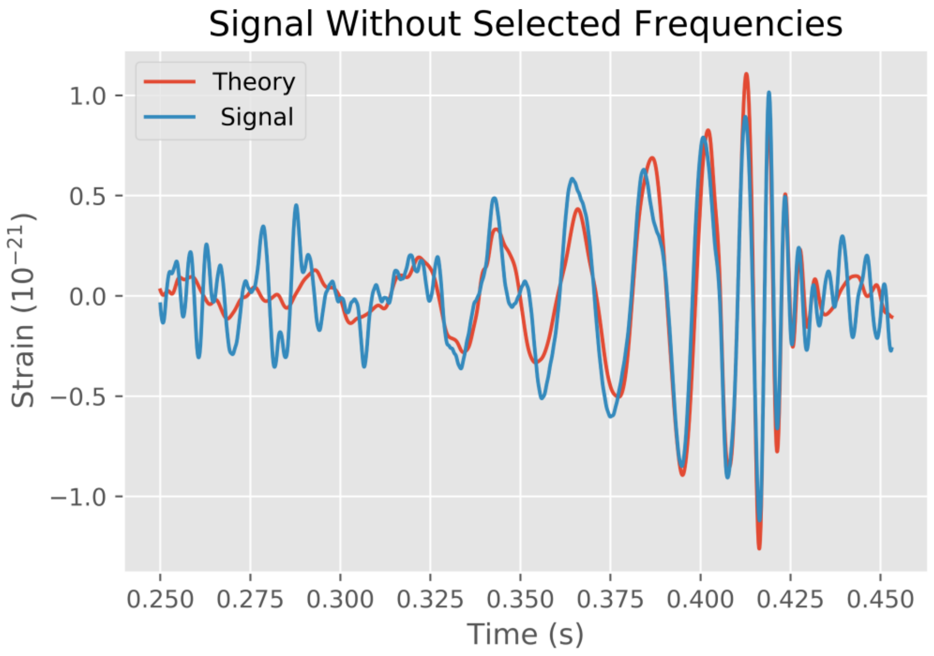
High Residues

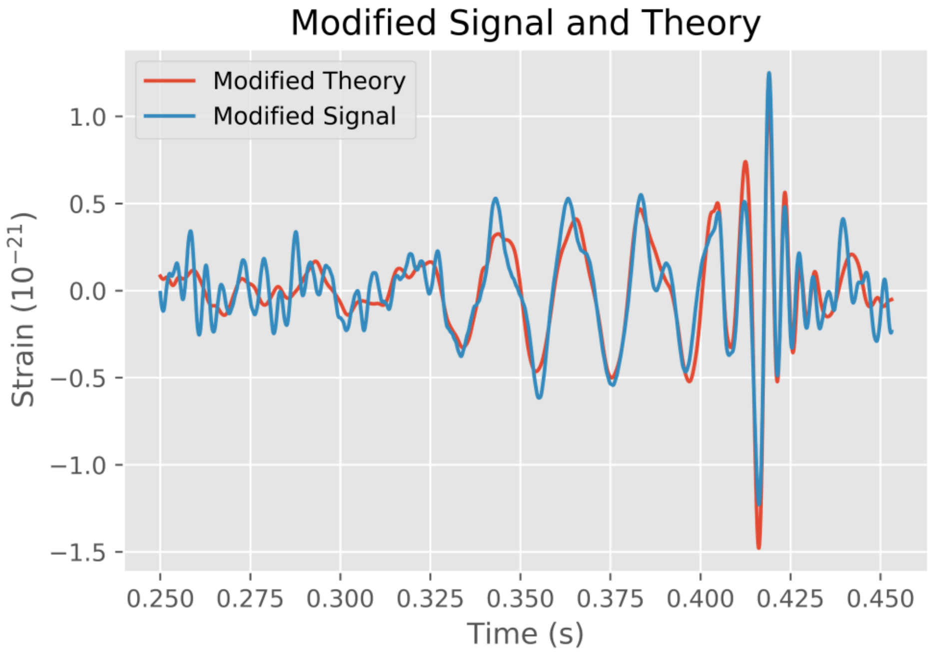




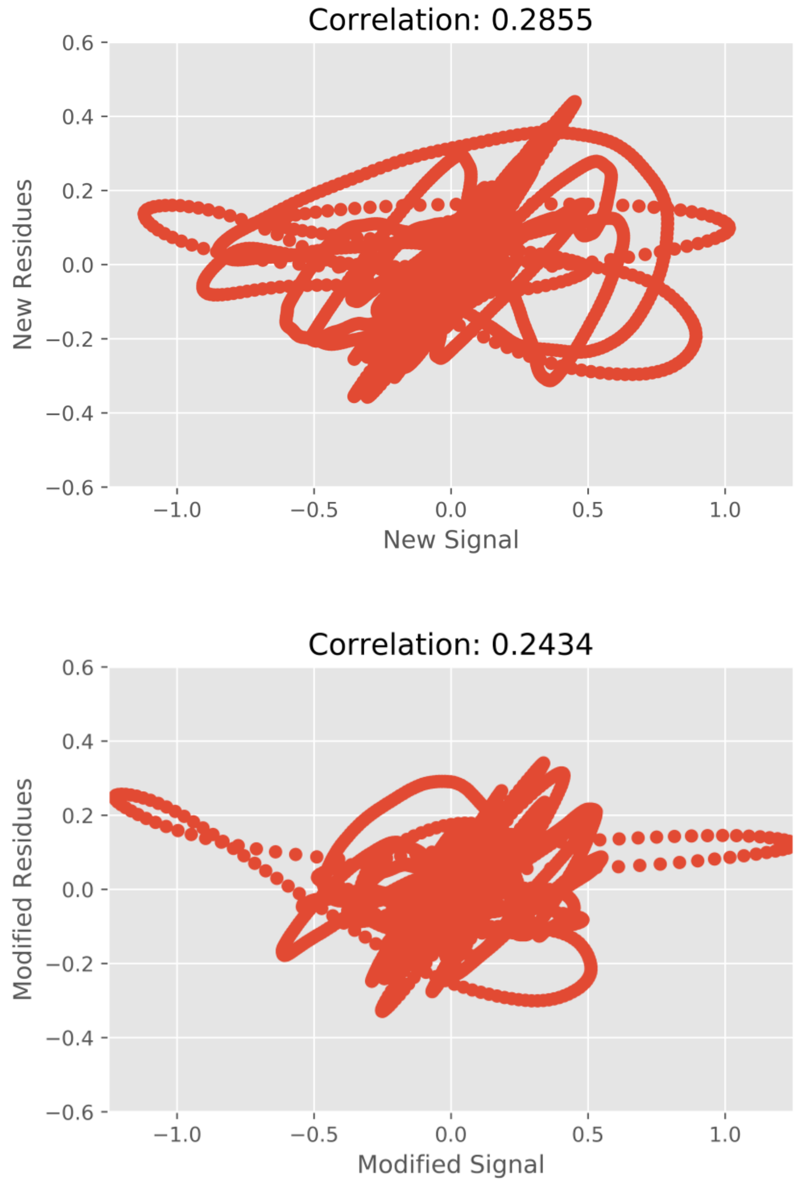


Data Evolution

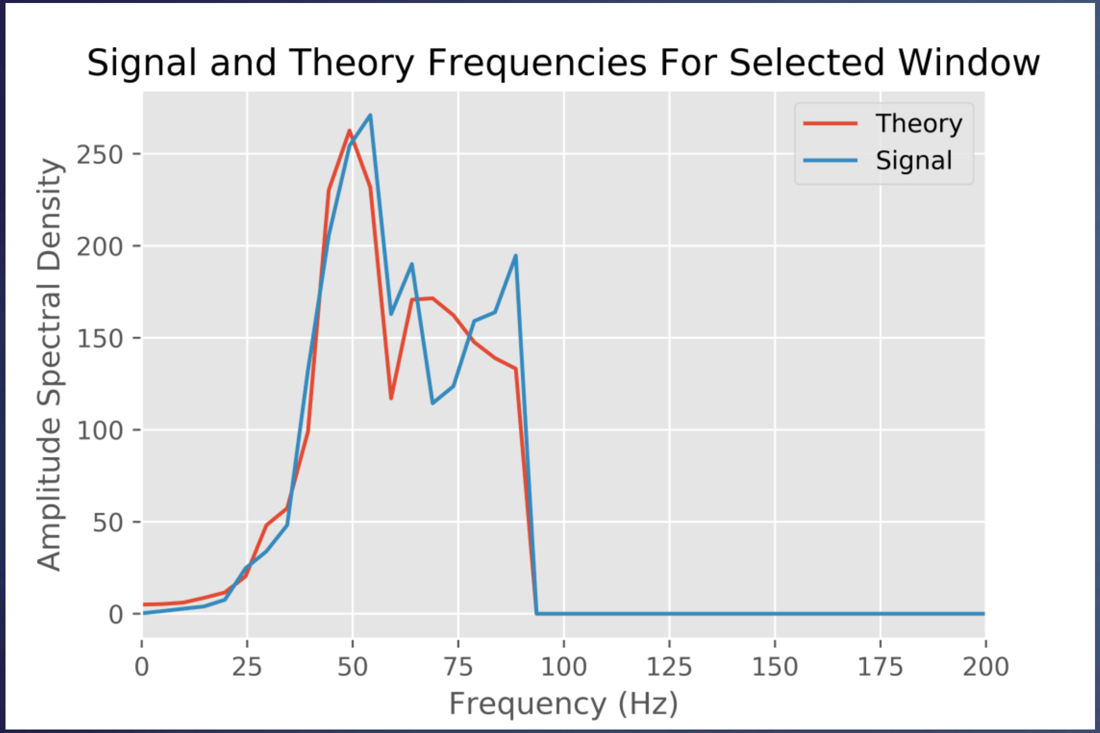


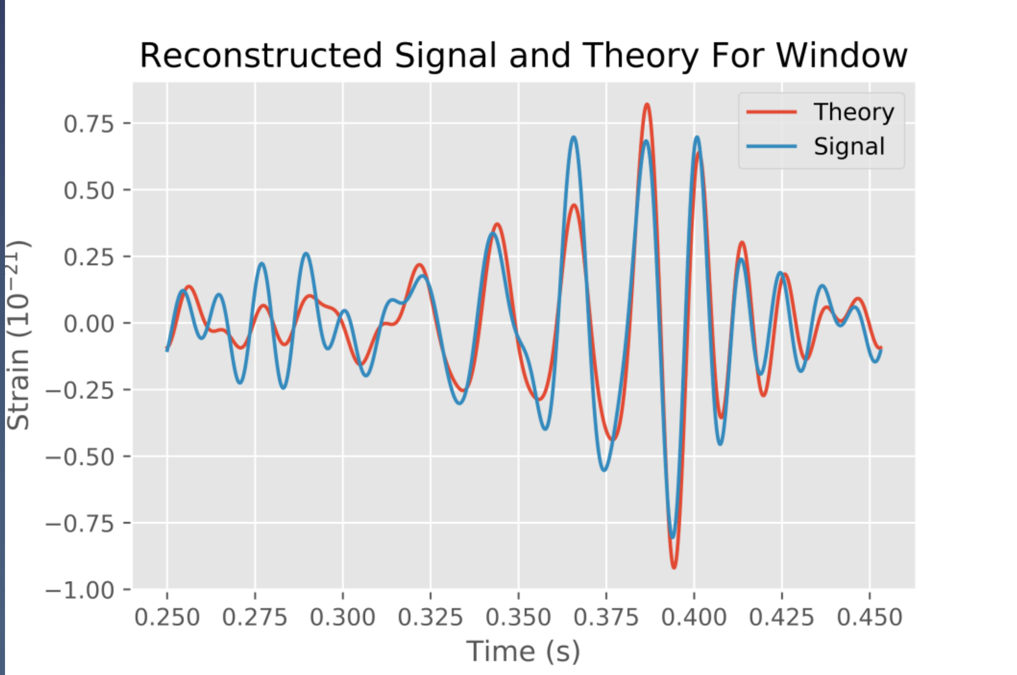


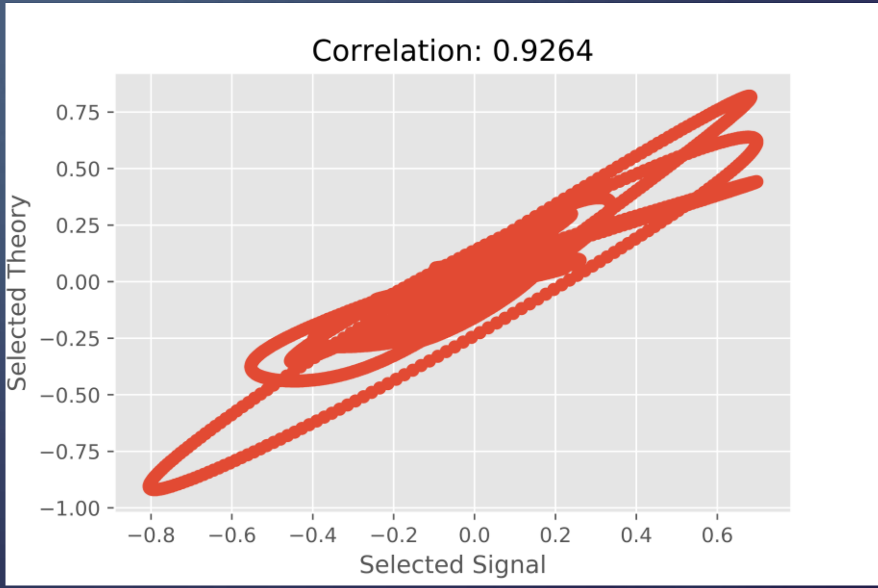
Correlation Evolution



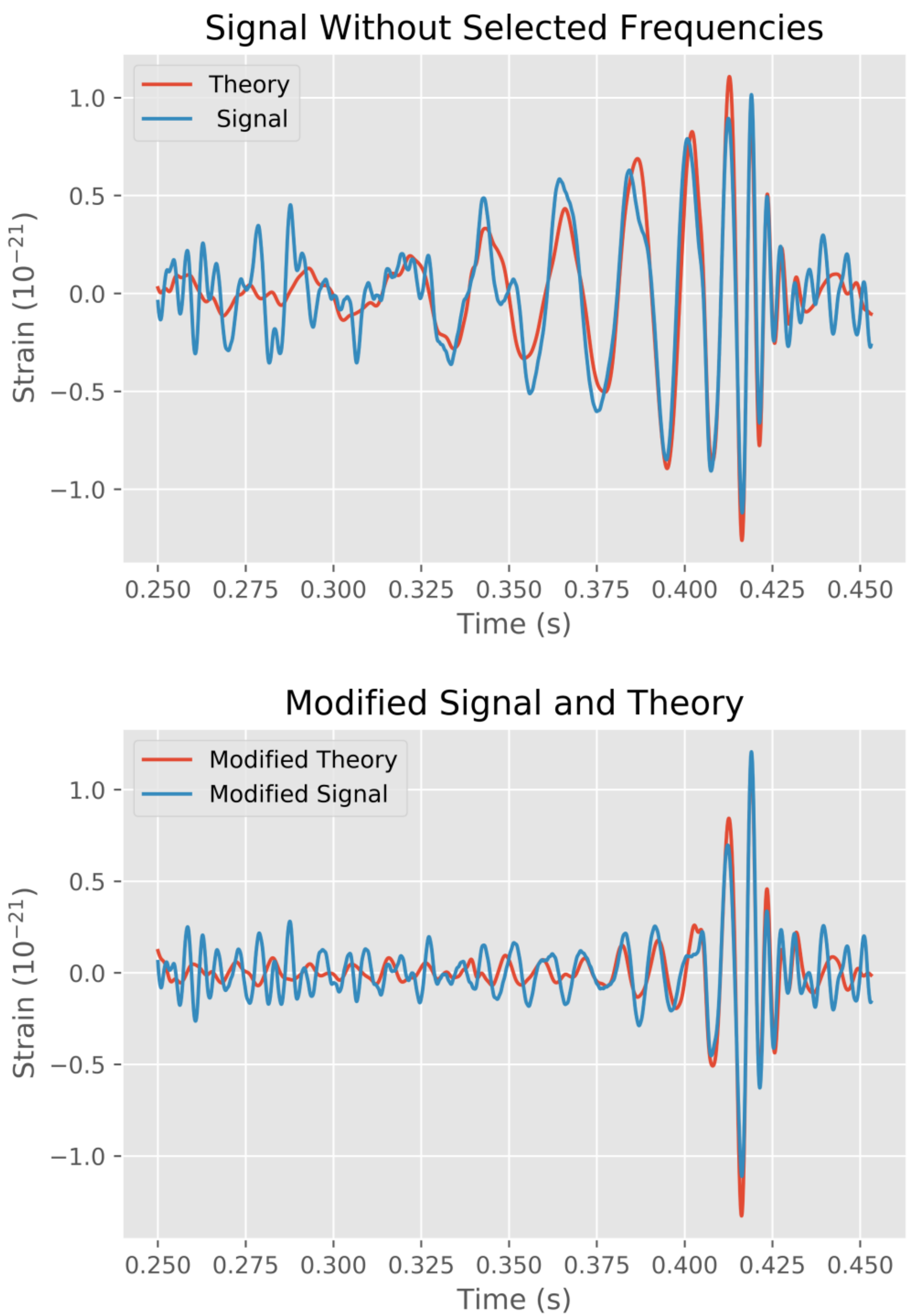
Lower Frequencies



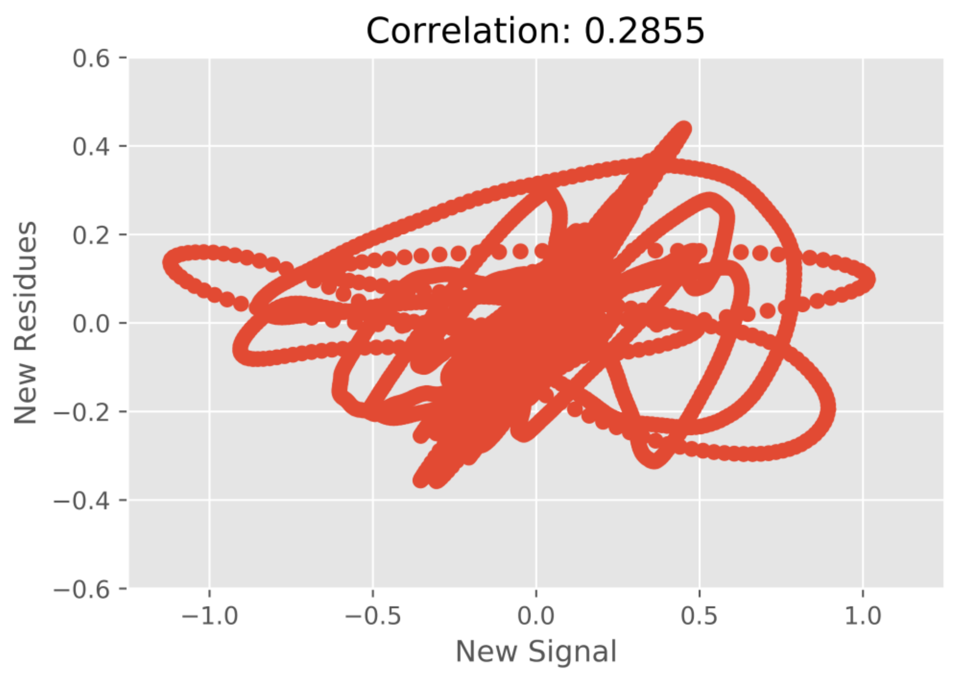




Data Evolution



Correlation Evolution



Conclusions

- We were able to find a window of noise in the high frequency region.
- In low frequencies, we didn't consider any noise because what we isolated was weak and by using that filter the correlation between residue and signal increased.
- Looking at the correlation plots we see that, by clearing the noise, not all zones got better correlations, some even got a little worse.
- This can happen because this noise has some modulation and does not behave the same way along the signal.
- By looking at different regions of the signal, we can find new noise or regions were our model seems to not describe the signal very well.