EFTs: Bounding the systematic uncertainties of the Inverse Amplitude Method

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In collaboration with A. Salas-Bernárdez, J. Escudero & J.A. Oller
and long term, A. Dobado and R. L. Delgado

Presented at the Vth COMHEP, Dec.1st 2020

And the Higgs was found...

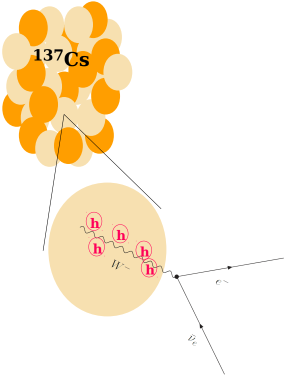
- Explained the size of the atom and of all beautiful things

$$\frac{1}{\lambda_e v_{\rm Higgs}} \propto \frac{1}{m_e} \propto a_{\rm Bohr}$$

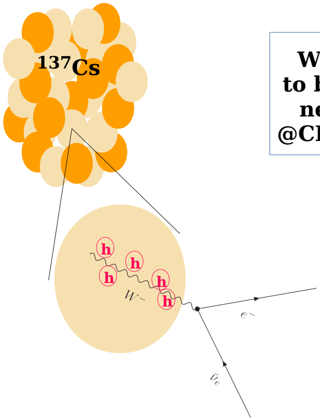
Bohr radius
gives us scale

And the Higgs was found...

- Also, drag on the W explains slowness of β radioactive decay



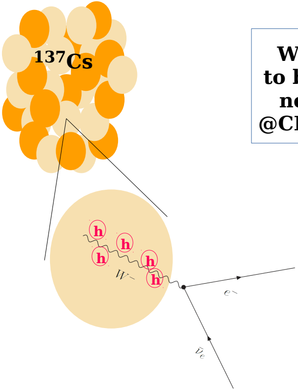
So what now?



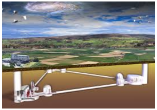
What to build next @CERN?

What to do with the LHC upgrades?

~~FCC~~ look up! High Energy physics w Gravitational Waves



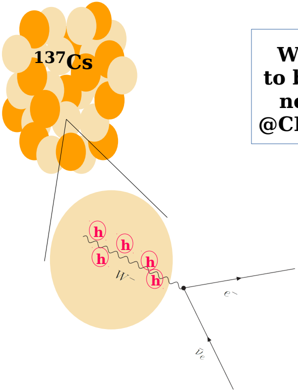
What to build next @CERN?



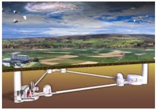
Einstein telescope

What to do with the LHC upgrades?

Meanwhile, exploit the LHC



What to build next @CERN? →



Einstein telescope

What to do with the LHC upgrades? → **This talk**

Too many terms = too much noise

Maybe enough scientists with enough coefficients will find separation from SM...

http://notapipe.biz/quality-quantity-and-infinite-monkeys/

LHC is unique for Electroweak Symmetry Breaking Sector

Cut to the chase: Vector Boson Scattering

- $\vec{W}_L \simeq \vec{\omega}$ Goldstone bosons of symmetry breaking
- h additional scalar particle distinguished by symmetry breaking
- Relevant processes:
  $W_L W_L \to W_L W_L$, $W_L W_L \to (h)h$, $hh \to hh$, $W_L h \to W_L h$

(Under hypothesis of $SU(2)$ isospin custodial symmetry)

R.L. Delgado, A. Dobado, FJLE and others, series of Complutense papers

HEFT Lagrangian for electroweak symmetry breaking

Compact, TeV-scale seven-number version

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}\left(1+2a\frac{h}{v}+b\left(\frac{h}{v}\right)^2\right)\partial_\mu\omega^a\partial^\mu\omega^b\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2}\right)\\ &+ \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b+\frac{4a_5}{v^4}\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b\\ &+ \frac{1}{2}\partial_\mu h\partial^\mu h+\frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2\\ &+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a+\frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^a\partial_\nu\omega^a .\end{aligned}$$

R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015

Derivative expansion ($\simeq$ ChPT)

$$\mathcal{L} = \frac{1}{2}\left(1+2a\frac{h}{v}+b\left(\frac{h}{v}\right)^2\right)\boxed{\partial_\mu\omega^a\partial^\mu\omega^b}\left(\delta_{ab}+\frac{\omega^a\omega^b}{v^2}\right)$$
$$+ \frac{4a_4}{v^4}\partial_\mu\omega^a\partial_\nu\omega^a\partial^\mu\omega^b\partial^\nu\omega^b + \frac{4a_5}{v^4}\boxed{\partial_\mu\omega^a\partial^\mu\omega^a\partial_\nu\omega^b\partial^\nu\omega^b}$$
$$+ \frac{1}{2}\partial_\mu h\partial^\mu h + \frac{g}{v^4}(\partial_\mu h\partial^\mu h)^2$$
$$+ \frac{2d}{v^4}\partial_\mu h\partial^\mu h\partial_\nu\omega^a\partial^\nu\omega^a + \frac{2e}{v^4}\partial_\mu h\partial^\nu h\partial^\mu\omega^a\partial_\nu\omega^a.$$

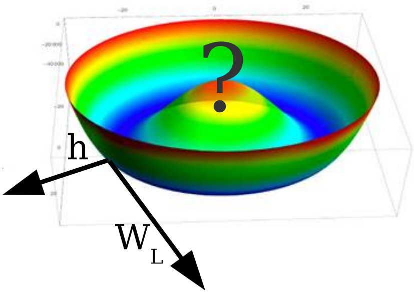
R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015

Note the different counting

- In SMEFT you would think $\partial_\mu \omega^a \partial^\mu \omega^a \partial_\nu \omega^b \partial^\nu \omega^b$ is of order 8
- In HEFT ω does not cost you a power, so this is NLO
- Needed for renormalization of loops of LO terms

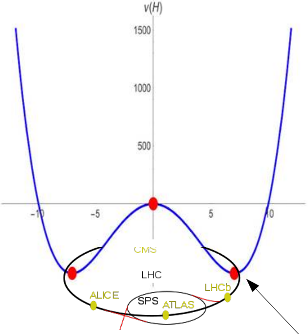
R. L. Delgado, A. Dobado and FJLE, JHEP 02 (2014) 121;
Also G. Buchalla et al. Phys.Lett.B 731 (2014) 80-86, etc.

Note the "SM Higgs potential" might be a red herring



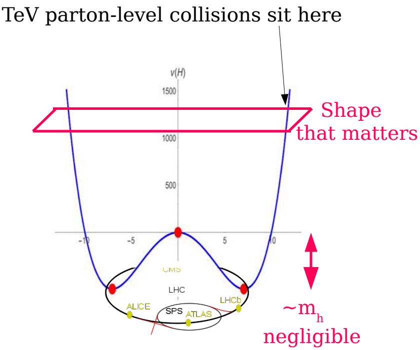
Picture from J. Lorenzo Díaz Cruz, Rev. mex. fis. **65** 2020

Note the "SM Higgs potential" might be a red herring



The LHC sits here

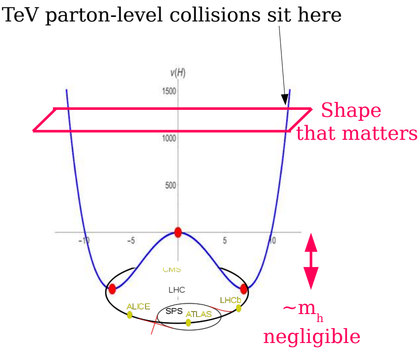
If there is new physics



$$m_h \sim m_W \sim m_Z \ll \sqrt{s}$$

$$\Longrightarrow \quad V(h)_{\rm SM-like} \ll \sqrt{s}$$

If there is new physics



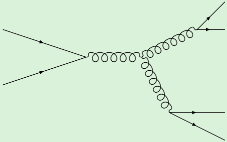
$$m_h \sim m_W \sim m_Z \ll \sqrt{s}$$

$$\Longrightarrow \quad V(h)_{\rm SM-like} \ll \sqrt{s}$$

Seven-parameter EFT description
of what's important @LHC with new EWSBS physics

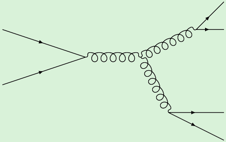
Is the LHC a high- or a low- energy machine?

- High energy scattering: $V << T$, Feynman diagrams, Madgraph, etc.

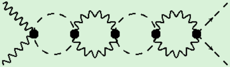


Is the LHC a high- or a low- energy machine?

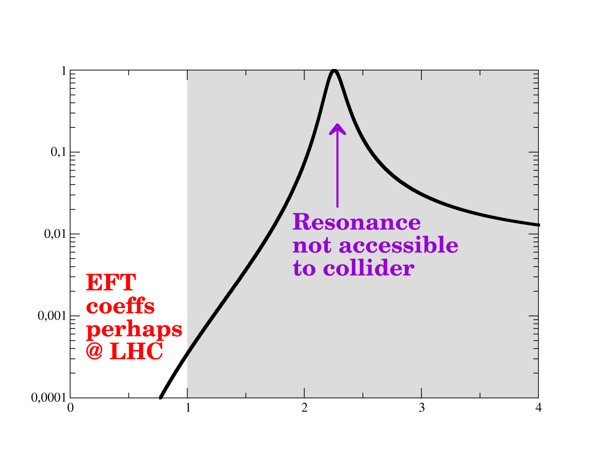
- High energy scattering: $V << T$, Feynman diagrams, Madgraph, etc.



- Low energy respect to new physics (strongly interacting? $V \sim T$ requires resummation)



Is the LHC a high- or a low- energy machine?



Expand partial wave amplitudes

$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty} (2J+1) t_{IJ}(s) P_J(\cos\theta_s)$$

Expand partial wave amplitudes

$$T_I(s,t,u) = 16\eta\pi \sum_{J=0}^{\infty} (2J+1) t_{IJ}(s) P_J(\cos\theta_s)$$

$$t_{IJ}(s) \simeq \underbrace{t_0}_{O(s)} + \underbrace{t_1}_{O(s^2)} + \dots$$

(typical HEFT expansion)

Inverse Amplitude Method

$$\frac{1}{t} \simeq \frac{1}{t_0 + t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \Longrightarrow \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0 - t_1}}$$

Inverse Amplitude Method

$$\frac{1}{t} \simeq \frac{1}{t_0 + t_1} \simeq \frac{1}{t_0} - \frac{t_1}{t_0^2} \implies \boxed{t^{IAM} \simeq \frac{t_0^2}{t_0 - t_1}}$$

Advantage: for $s > s_{\rm th}$,

$$\mathrm{Im}\frac{1}{t_{IJ}(s)} = -\sigma(s) \simeq -1$$

Perturbative vs exact (elastic) unitarity

$$\text{Im}\, t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2$$

Perturbative vs exact (elastic) unitarity

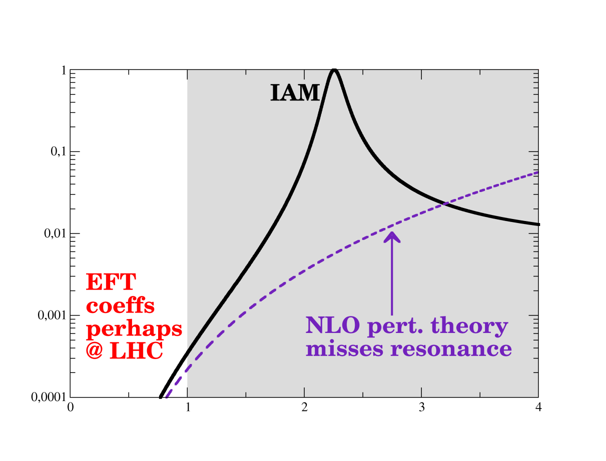
$$\mathrm{Im}\, t_{IJ}(s) = \sigma(s)|t_{IJ}(s)|^2$$

- Exact in IAM
- Only order by order in EFT

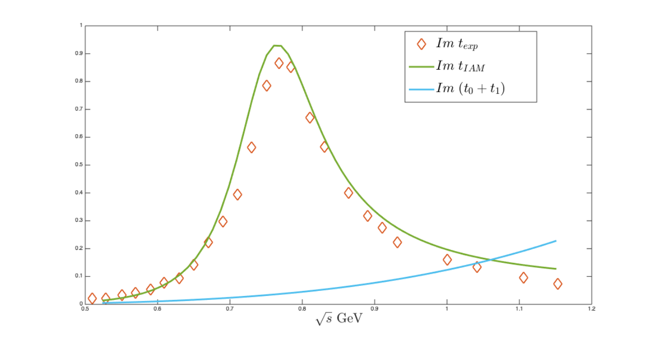
$$\mathrm{Im}\, t_1(s) = \sigma(s)|t_0(s)|^2$$

Why would anyone care?

- EFT reliable only near threshold

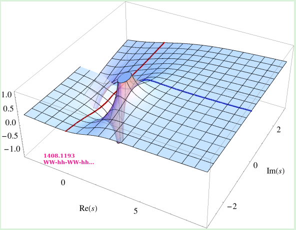


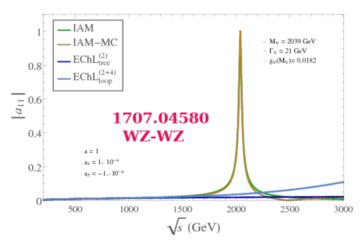
Much used in hadron physics to obtain resonances

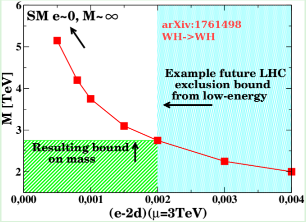


(This is an IAM prediction from threshold data, not a fit)

Prediction of resonances from HEFT

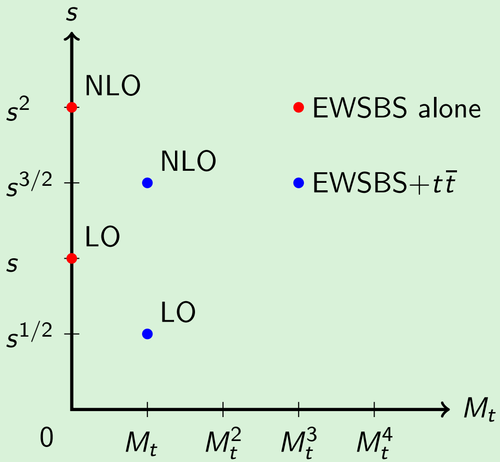






LHC bounds on HEFT coeffs $\Longrightarrow$ bounds on new physics scale

Coupling to top-antitop sector

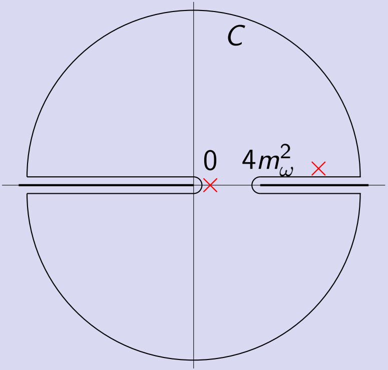


$$\begin{aligned}\mathcal{L}_t = \quad & -M_t\left(1 + c_1\frac{h}{v} + c_2\frac{h^2}{v^2}\right)\\ & \times\left\{\sqrt{1-\frac{\omega^2}{v^2}}\ t\bar{t}\right.\\ & +\frac{i\omega^0}{v}\bar{t}\gamma^5 t\\ & +i\sqrt{2}\frac{\omega^+}{v}\left(-\bar{t}_R b_L\right)\\ & \left.+i\sqrt{2}\frac{\omega^-}{v}\left(\bar{b}_L t_R\right)\right\}\\ & +g_t\frac{M_t}{v^4}(\partial_\mu\omega^i\partial^\mu\omega^j)t\bar{t}\\ & +g_t'\frac{M_t}{v^4}(\partial_\mu h\partial^\mu h)t\bar{t}.\end{aligned}$$

Castillo *et al.* EPJC **77** 436 (2017)

Use its dispersive derivation: 2010.13709

- Causality $\implies$ analiticity
- Large circumference convergence $G \propto e^{-s}$ (1912.08747)
- Can apply Cauchy's theorem



to the function $t_0^2(s')/t(s')(s-s')s'^3$.

Master formula is a dispersion relation for $G(s) \equiv \frac{t_0^2(s)}{t(s)}$

$$G(s) = G(0) + G'(0)s + \frac{1}{2}G''(0)s^2 + PC(G) + \\ + \frac{s^3}{\pi}\int_{RC} ds' \frac{\text{Im } G(s')}{s'^3(s'-s)} + \\ + \frac{s^3}{\pi}\int_{LC} ds' \frac{\text{Im } G(s')}{s'^3(s'-s)}$$

Dispersion relation: approximations

NLO subtraction constants — Neglected

$$G(s) = \underbrace{G(0) + G'(0)s + \frac{1}{2}G''(0)s^2}_{\text{NLO subtraction constants}} + \underbrace{PC(G)}_{\text{Neglected}} +$$

$$+ \frac{s^3}{\pi}\int_{RC} ds' \frac{\text{Im}\, G(s')}{s'^3(s'-s)} +$$

$$+ \frac{s^3}{\pi}\int_{LC} ds' \frac{\text{Im}\, G(s')}{s'^3(s'-s)}$$

NLO imaginary part Im G ⟶ -Im t1

Gives $\boxed{t \simeq t_0^2/(t_0 - t_1) = t_{IAM}}$.

Sources of uncertainty

- Neglected pole contributions of t^{-1}:
  subthreshold Adler zeroes and CDD zeroes of t.

- Inelasticities due to KK (hh in HEFT), 4ω, etc.

- $\mathcal{O}(p^4)$ truncation of subtraction constants.

- Left cut approximation $Im\ G \simeq -Im\ t_1$.

The SM is a very specific theory

$$\mathcal{L} = \frac{1}{2}(v+h)^2(\partial_\mu \vec{\omega})^2 + \frac{1}{2}(\partial_\mu h)^2 + \dots$$

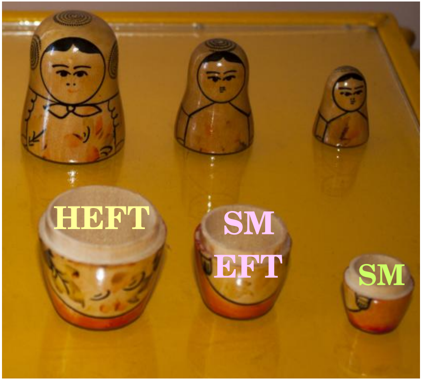
In terms of three Goldstone bosons $\vec{\omega}$ and Higgs h fields

SMEFT extension

$$\mathcal{L} = \frac{1}{2}(v+h)^2 A\left(\frac{(v+h)^2}{\Lambda^2}\right)(\partial_\mu \vec{\omega})^2 + \frac{1}{2}\left(1 + C\left(\frac{(v+h)^2}{\Lambda^2}\right)\right)(\partial_\mu h)^2 + \dots$$

$$A(0) = 1, \; C(0) = 0$$

HEFT extension

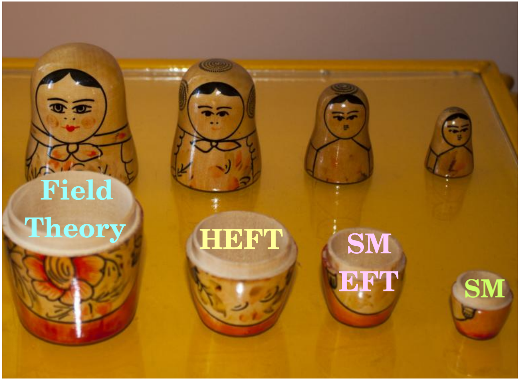


$$\mathcal{L} = \frac{1}{2} v^2 F\left(\frac{h}{v}\right)^2 (\partial_\mu \vec{\omega})^2 + \frac{1}{2}(\partial_\mu h)^2 + \dots$$

$$F(0) = 1$$

(n.b. field redefinition shows *SMEFT* $\subset$ *HEFT*)

If different symmetry breaking pattern... still field theory

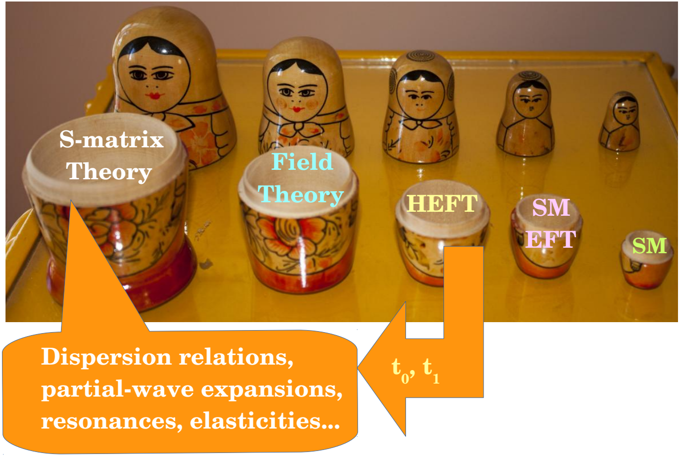


But S-matrix theory more general

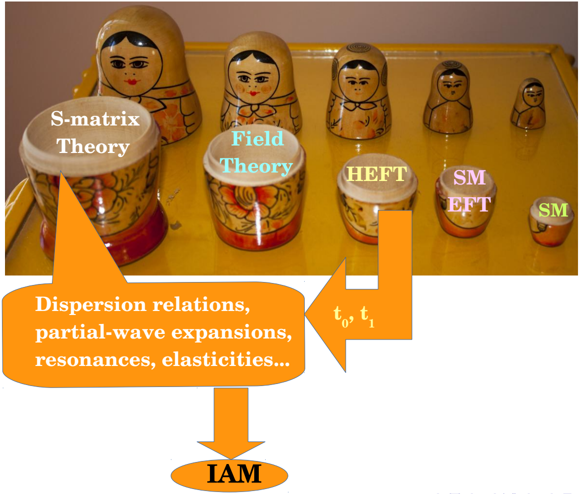
Basic particle concepts part of S-matrix theory

Dispersion relations, partial-wave expansions, resonances, elasticities...

S-matrix too general: ambiguous $\Longrightarrow$ HEFT input



More predictive Inverse Amplitude Method

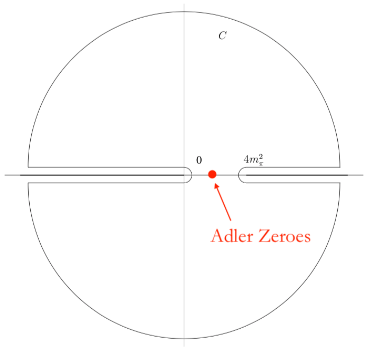


Other unitarization methods

- K-matrix (no good analyticity properties)
- N/D method (integral equations, not algebraic)
- Large N method (only approximate for $O(4) \to O(3)$)
- Inverse Amplitude Method $\Longrightarrow$
  control theory uncertainties (this work)

We have provided improved/simplified versions of all methods
R. L. Delgado, A. Dobado and FJLE, PRD **91**, 075017 2015; J.Phys.G 41 (2014) 025002.

Adler zeroes of t near threshold



EFT feature

$$t_0 + t_1 = a + bs + cs^2$$

vanishes near $s = -a/b$

Adler zeroes of t near threshold

Tiny uncertainty in resonance region because at/below threshold

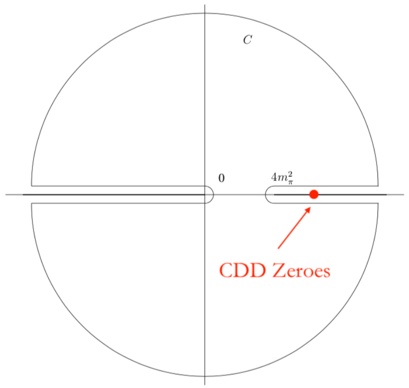
Adler zeroes of t near threshold

Tiny uncertainty in resonance region because at/below threshold

Uncertainty	Behavior	Displacement $\sqrt{s} = m_\rho$	improvable?
Adler zeroes of t	$(m_\omega/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM

0712.2763

CDD poles ($t = 0$ in resonance region: new physics)



("New" physics)

CDD poles ($t = 0$ in resonance region: new physics)

Can affect a resonance calculation dramatically

Need to

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Can affect a resonance calculation dramatically

Need to

1. Check for CDD pole appearance: $t_0(s_C) + \mathrm{Re}\, t_1(s_C) = 0$

CDD poles ($t = 0$ in resonance region: new physics)

Can affect a resonance calculation dramatically

Need to

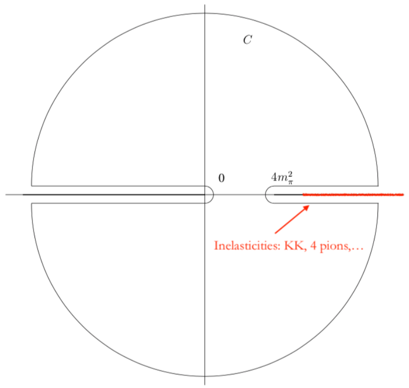
1. Check for CDD pole appearance: $t_0(s_C) + \mathrm{Re}\, t_1(s_C) = 0$
2. If present, modify

$$t_{\mathrm{IAM}} = \frac{t_0^2}{t_0 - t_1} \to \frac{t_0^2}{t_0 - t_1 + \frac{s}{s - s_c}\mathrm{Re}(t_1)} \, .$$

CDD poles ($t = 0$ in resonance region: new physics)

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes

Inelastic 2-body channels



Inelastic 2-body channels

- Hadrons: $\pi\pi \to \pi\pi$ couples to KK

$$\mathrm{Im}\,\frac{1}{t_{\pi\pi}} \to -\sigma_{\pi\pi}\left(1 + \frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}\frac{|t_{\pi\pi\to K\bar{K}}|^2}{|t_{\pi\pi\to\pi\pi}|^2}\right)$$

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suppressed by phase-space $\frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}$ and low inelasticity in $t_{\pi\pi\to K\bar{K}}$

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- In HEFT only inelasticity in $\omega\omega - hh$ (actually zero in SM)

Inelastic 2-body channels

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$$\text{Im}\,\frac{1}{t_{\pi\pi}} \to -\sigma_{\pi\pi}\left(1 + \frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}\frac{|t_{\pi\pi\to K\bar{K}}|^2}{|t_{\pi\pi\to\pi\pi}|^2}\right)$$

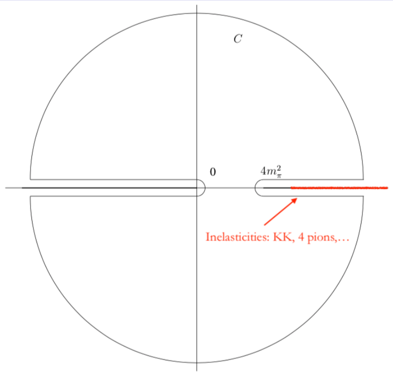
suppressed by phase-space $\frac{\sigma_{K\bar{K}}}{\sigma_{\pi\pi}}$ and low inelasticity in $t_{\pi\pi\to K\bar{K}}$

- In HEFT only inelasticity in $\omega\omega - hh$ (actually zero in SM)
- We can use the coupled channel IAM directly or to estimate uncertainty in elastic IAM

Inelastic 2-body channels

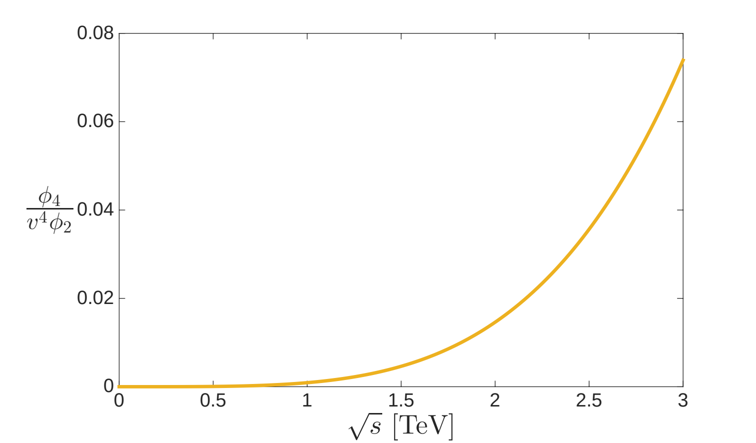
Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes

Inelastic 4-body channels



- Difference with SMEFT: here, in ChPT or HEFT, additional particles *not* suppressed by the chiral counting.
  But phase space helps.

Inelastic 4-body channels

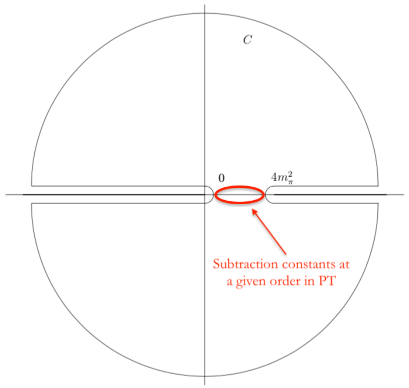


Inelastic 4-body channels

In hadron physics,
(with elastic and 4-π inelastic amplitudes taken as similar)

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes
Inelastic 4-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-4}	Partly

$O(p^4)$ truncation



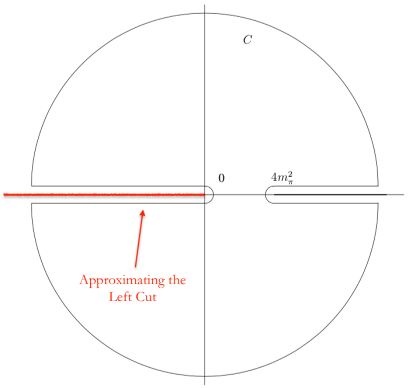
Estimate based on size of NNLO counterterms ($\Longrightarrow$ subtraction constants) from Resonance Effective Field Theory

$O(p^4)$ truncation

Uncertainty	Behavior	Displacement m_ρ	improvable?
Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes: mIAM
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
Inelastic 2-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-3}	Yes
Inelastic 4-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-4}	Partly
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes

$$G(s) = \frac{t_0^2}{t} \simeq t_0 - t_1 - t_2 + \frac{t_1^2}{t_0}$$

Approximate left cut



Need to check

$$\int_{LC} ds' \frac{\text{Im}\, G + \text{Im}\, t_1}{s'^3(s'-s)} \; .$$

i.e., failure of IAM's

$$Im\ G = -Im\ t_1$$

over the left cut

Approximate left cut

Split interval in 3:

- Low-$|s|$ (ChPT/HEFT ✓) $|s|^{\frac{1}{2}} < 470\mathrm{MeV}$.
- Intermediate-$|s|$: Match to ChPT + natural-size counterterm. Currently studying LC parameterizations from GKPY eqns.
- High -$|s|$: Sugawara-Kanazawa relates it to right cut: Regge asymptotics there. Far from RC anyway.

Approximate left cut

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Adler zeroes of t	$(m_\pi/m_\rho)^4$	$10^{-3} - 10^{-4}$	Yes
CDD poles at M_0	M_R^2/M_0^2	0–$\mathcal{O}(1)$	Yes
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Inelastic 4...-body	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-4}	Partly
$O(p^4)$ truncation	$(\sqrt{s}/(4\pi f_\pi))^4$	10^{-2}	Yes
Left Cut	$(\sqrt{s}/(4\pi f_\pi))^4$	0.17	Partly

Conclusion: if you know your EFT...

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- We have laid out (2010.13709 [hep-ph]; VBSCAN-PUB-10-20) its systematic theory uncertainties
- To make it more useful for BSM searches:
  shown evaluation with ρ-hadron,
  reassessable for $(\vec{\omega}, h)$ if/when HEFT coefficients measured.

Funding acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 824093; grants MINECO:FPA2016-75654-C2-1-P, FPA2016-77313-P MICINN: PID2019-108655GB-I00, PID2019-106080GB-C21, PID2019-106080GB-C22 (Spain); UCM research group 910309 and the IPARCOS institute; and the VBSCAN COST action CA16108.

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