A Guidance for Building Dark Energy Models

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COMHEP 2020

UdeSantiago de Chile

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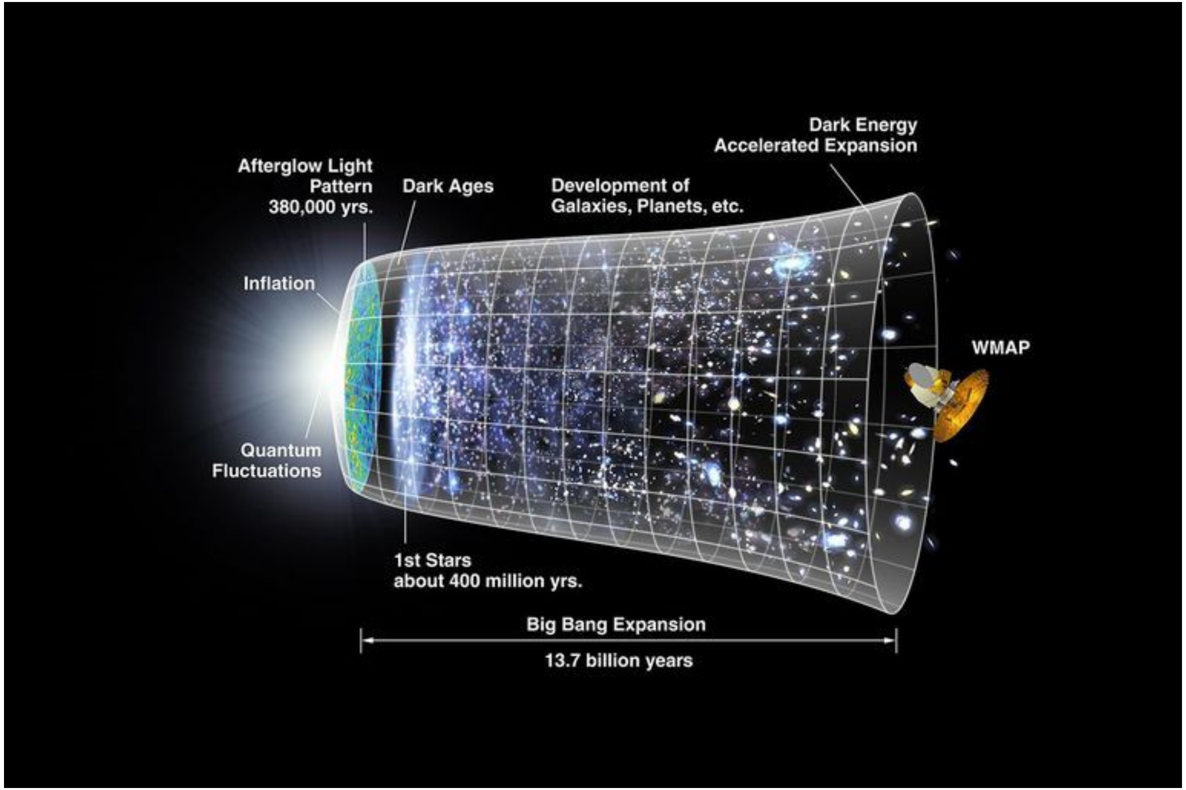
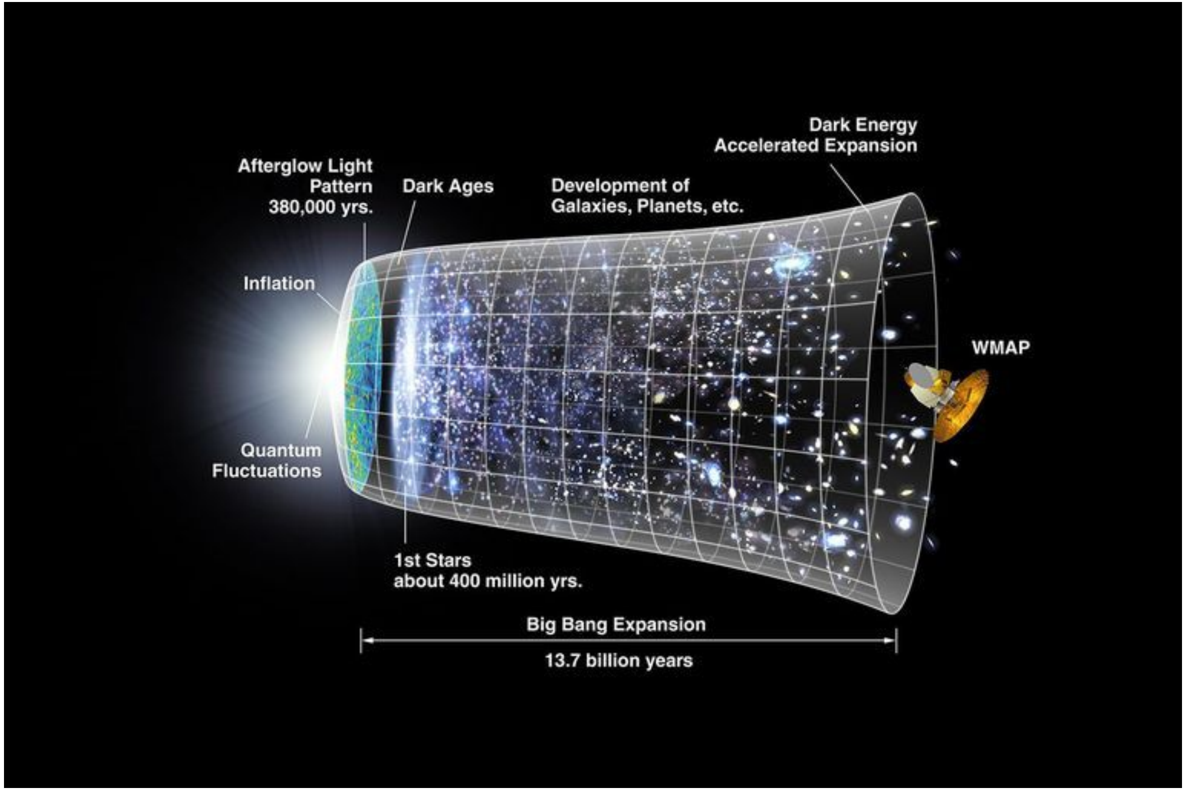
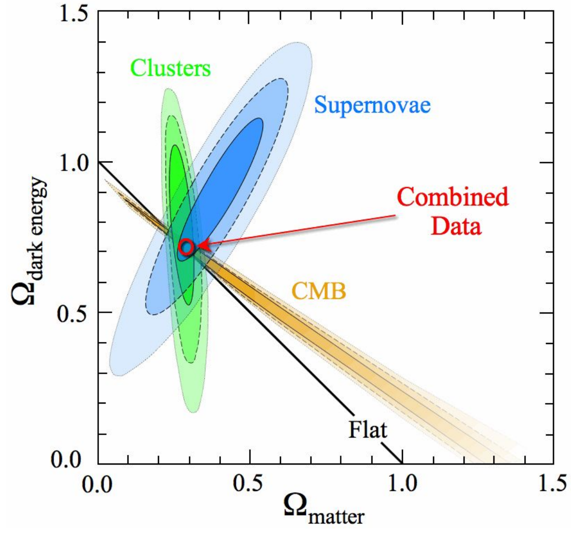

Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years





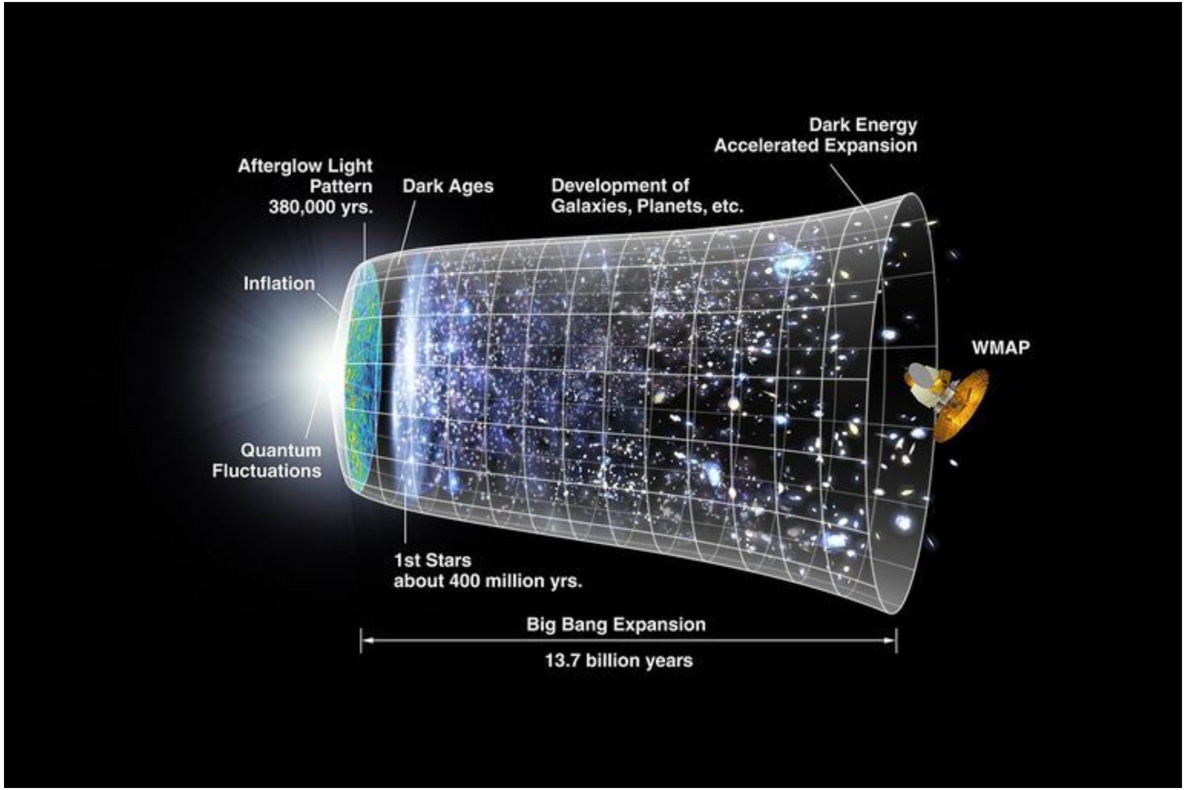
ΛCDM Cosmological model:

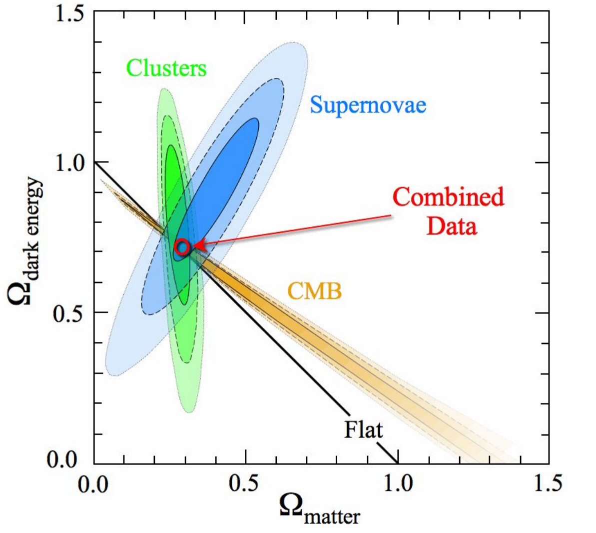
the existence and structure of the cosmic microwave background

the large-scale structure in the distribution of galaxies

the abundances of hydrogen (including deuterium), helium, and lithium

the accelerating expansion of the universe from distant galaxies and supernovae





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the accelerating expansion of the universe from distant galaxies and supernovae

The model assumes general relativity as the correct theory of gravity on cosmological scales!

Problems in the **ΛCDM Cosmological model:**

-The cosmological constant

-The coincidence problem (why not?)

-The Hubble tension

-The growth rate tension between CMB and shear measurements

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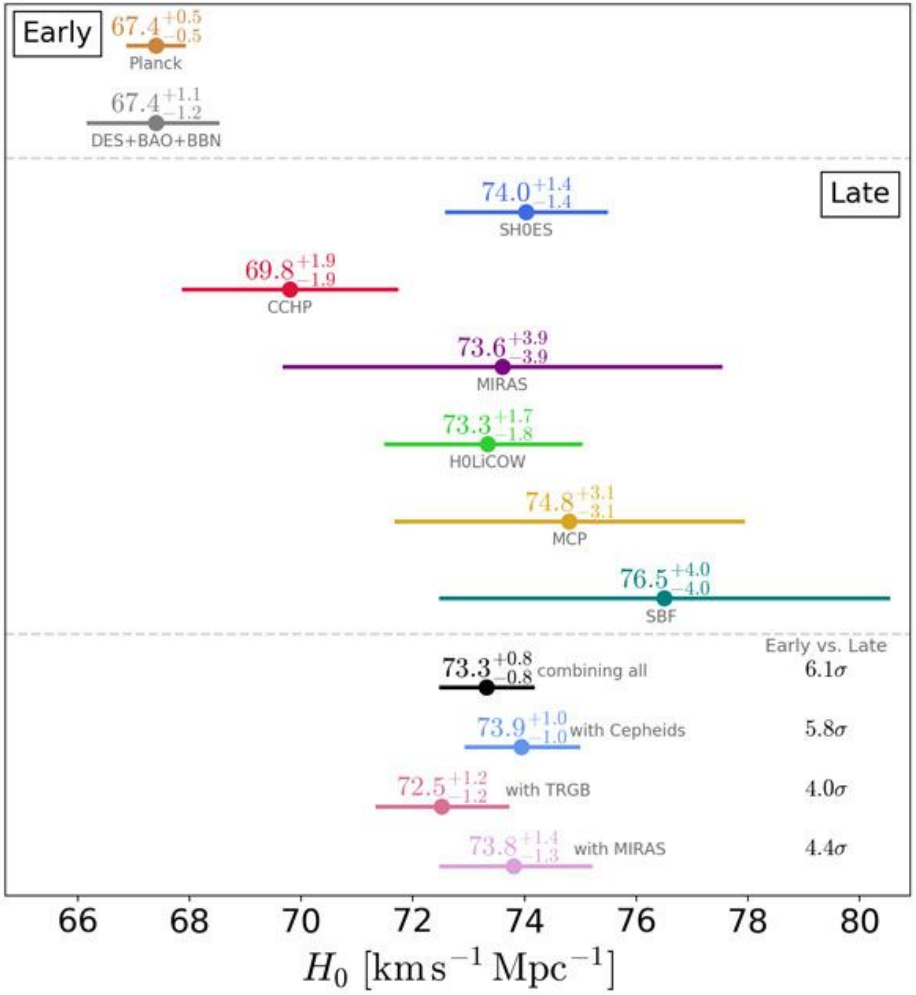

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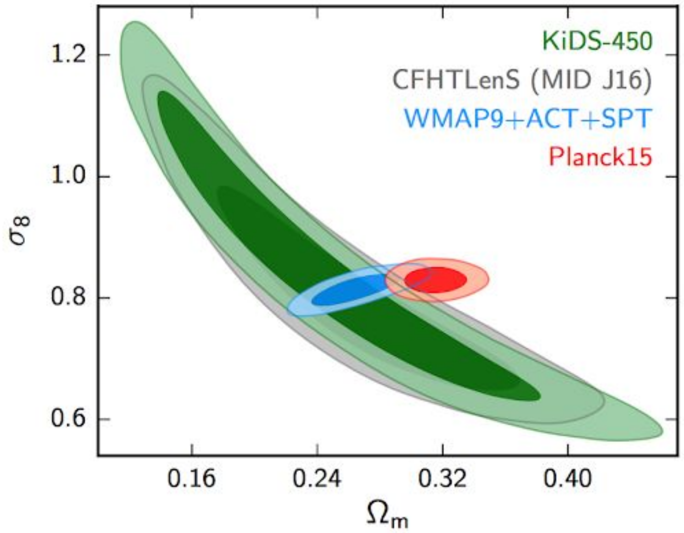
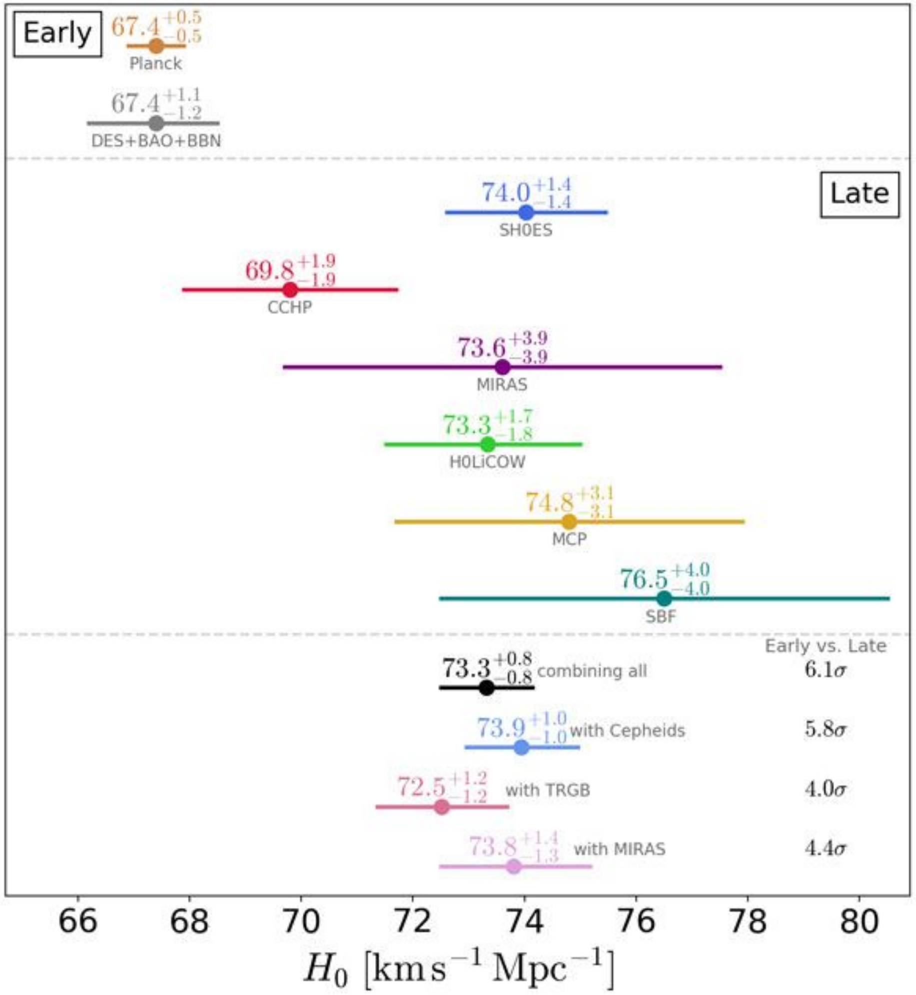
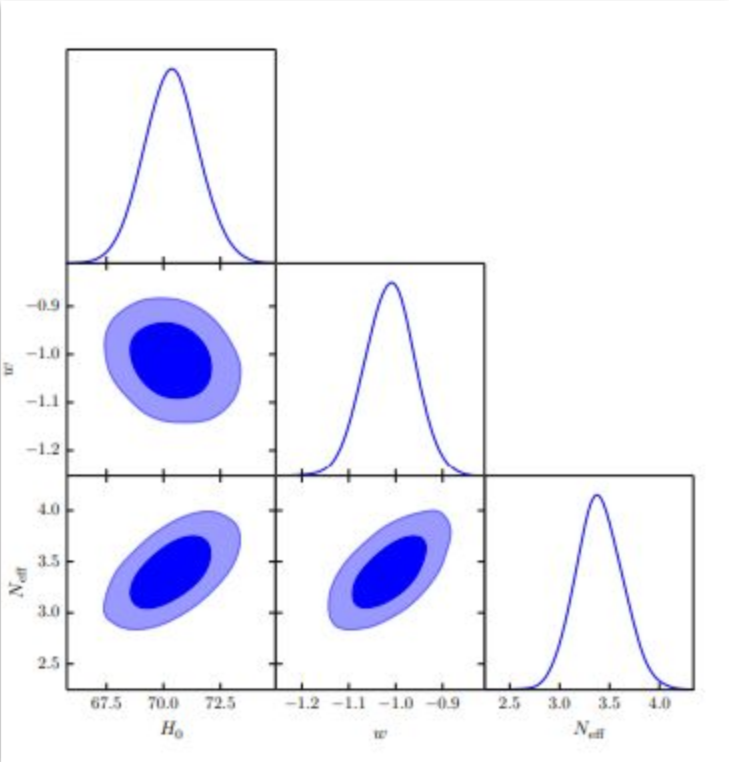
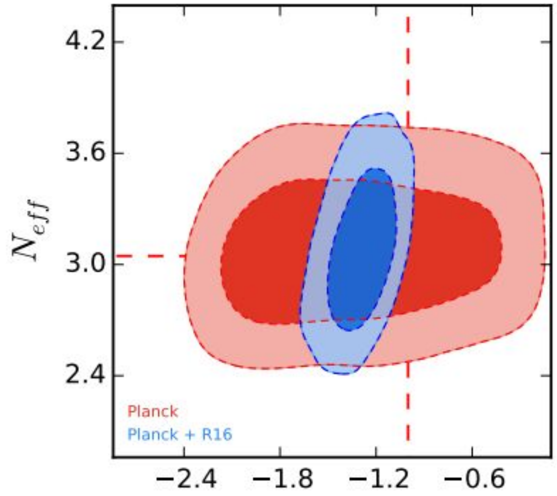




Reconciling Planck with the local value of H_0 in extended parameter space

Eleonora Di Valentino,[1] Alessandro Melchiorri,[2] and Joseph Silk[1,3,4,5]
[1] *Institut d'Astrophysique de Paris (UMR7095: CNRS & UPMC-Sorbonne Universities), F-75014, Paris, France*
[2] *Physics Department and INFN, Università di Roma "La Sapienza", P.le Aldo Moro 2, 00185, Rome, Italy*
[3] *AIM-Paris-Saclay, CEA/DSM/IRFU, CNRS, Univ. Paris VII, F-91191 Gif-sur-Yvette, France*
[4] *Department of Physics and Astronomy, The Johns Hopkins University Homewood Campus, Baltimore, MD 21218, US*
[5] *BIPAC, Department of Physics, University of Oxford, Keble Road, Oxford OX1 3RH, UK*





New physics in light of the H_0 tension: an alternative view

Sunny Vagnozzi[1,*]
[1] *Kavli Institute for Cosmology (KICC) and Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, United Kingdom*
(Dated: June 29, 2020)

$$(w_{de} < -1)$$

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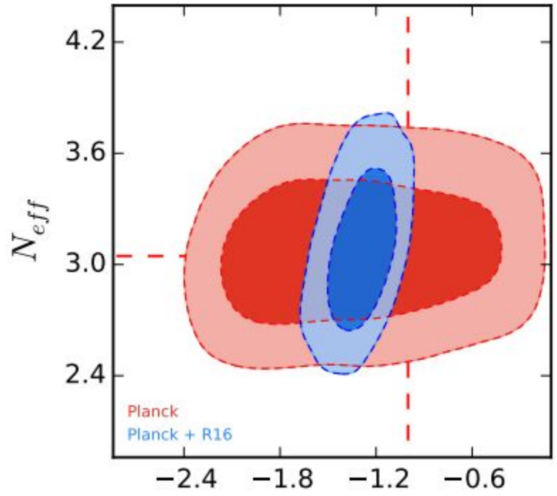

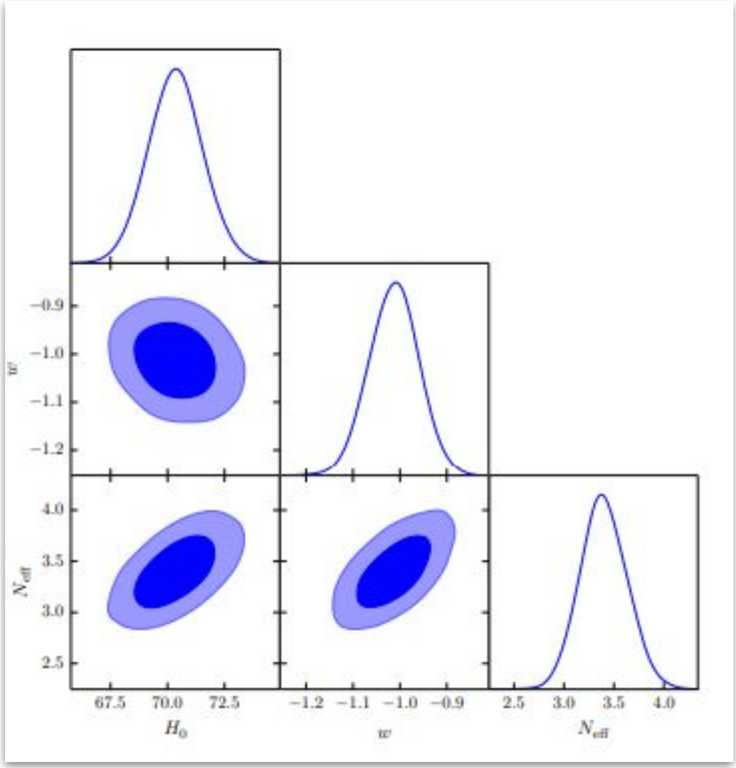

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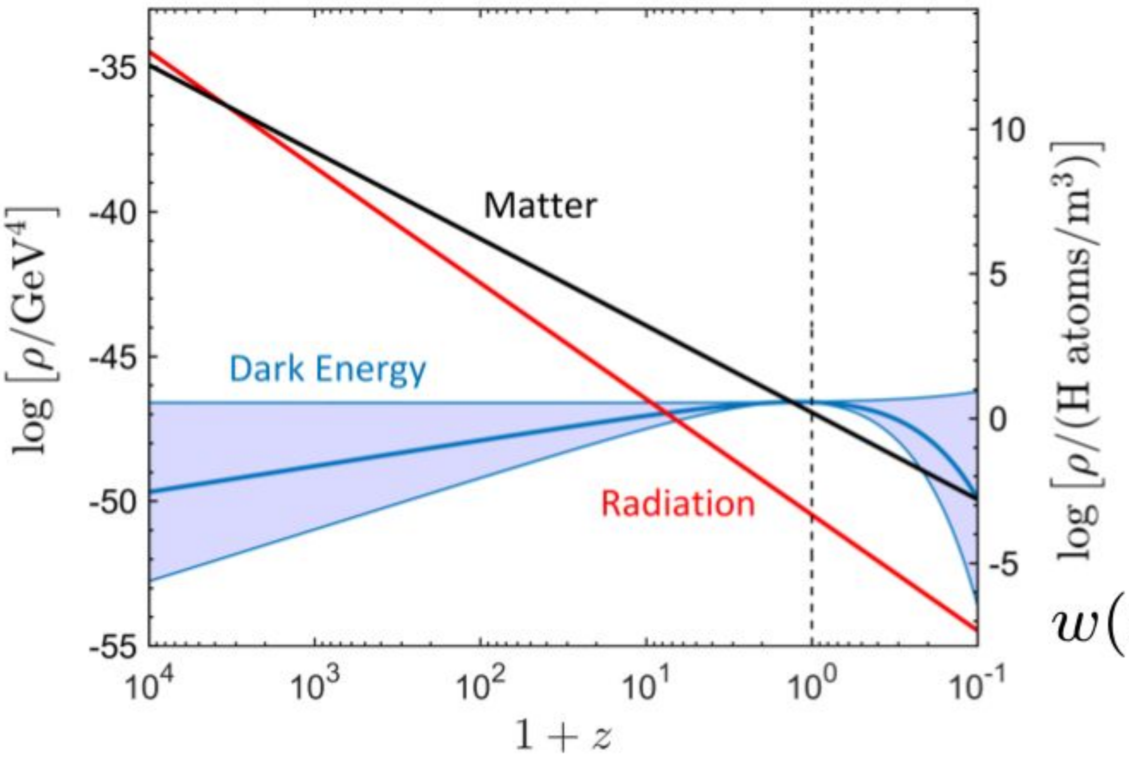
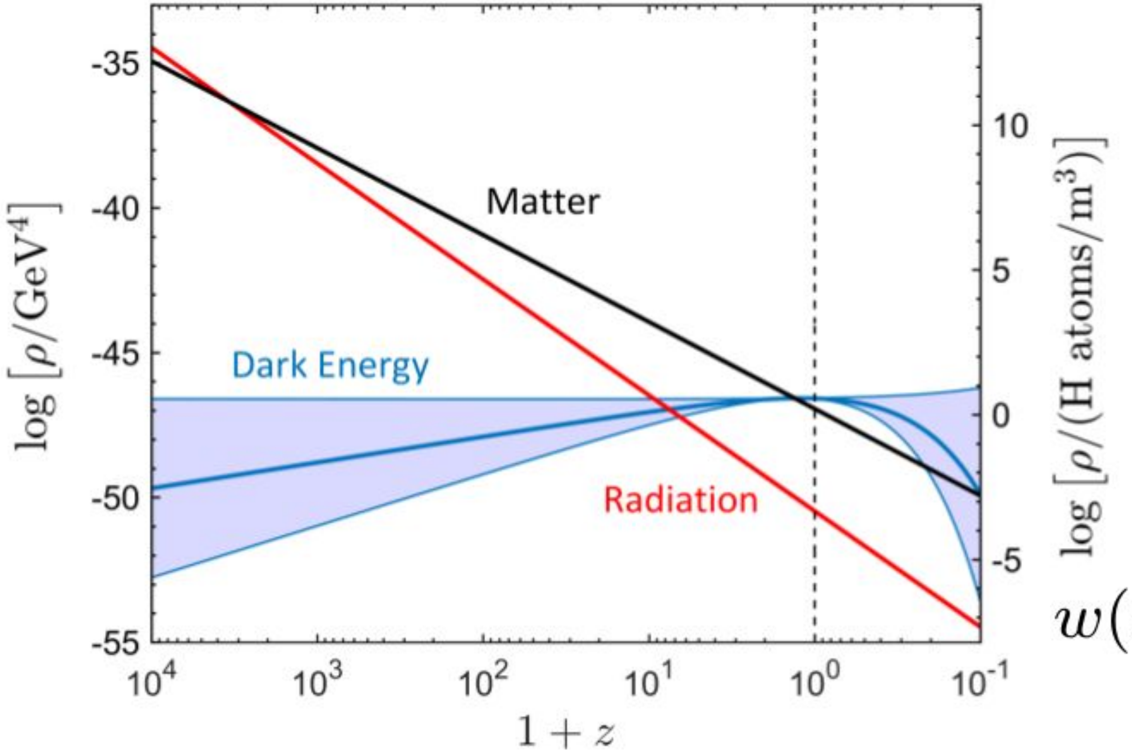
Interacting dark energy, massive neutrinos
Dark Matter-neutrino interactions, etc...



Prog. Phys. 81 (2018) 0169011

$$w(z) = w_0 + w_a\, z/(1+z).$$



Prog. Phys. 81 (2018) 0169011

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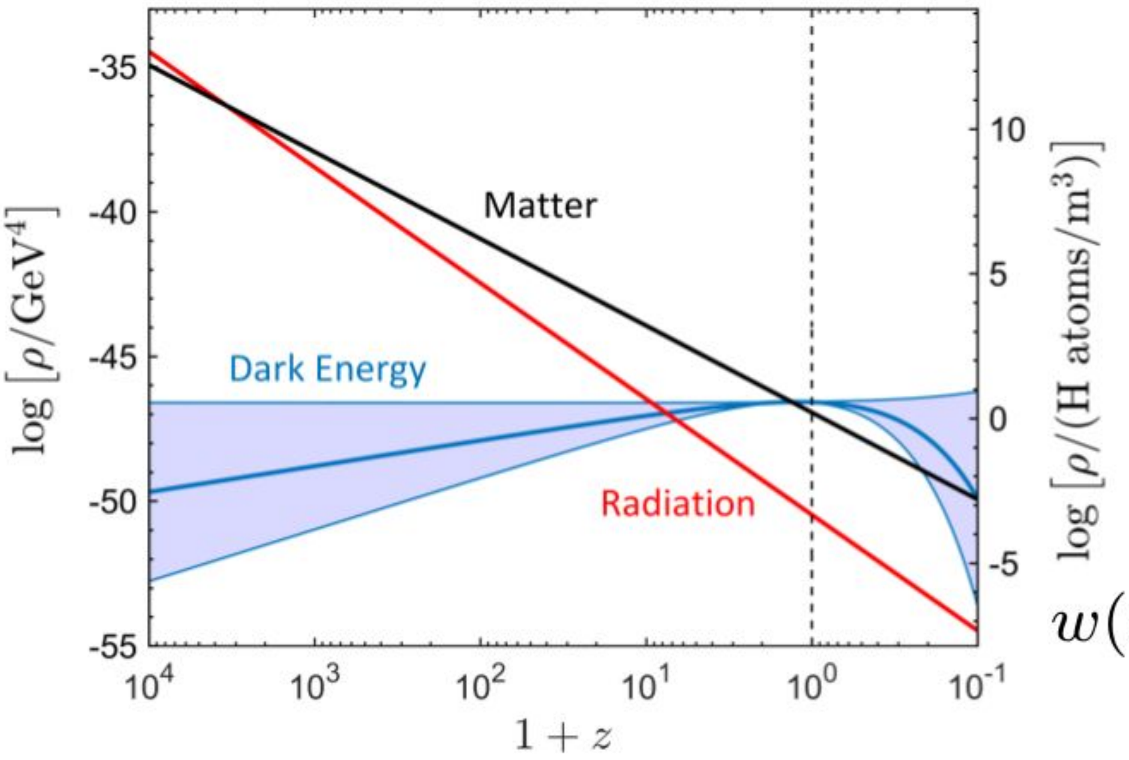
...but what is the physical mechanism behind the accelerating universe?



Prog. Phys. 81 (2018) 0169011

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$$w < -1/3\,(1+\rho_m/\rho_{\rm de});$$

The Einstein Tensor and Its Generalizations

Journal of Mathematical Physics **12**, 498 (1971); https://doi.org/10.1063/1.1665613

David Lovelock

The Four-Dimensionality of Space and the Einstein Tensor

Journal of Mathematical Physics **13**, 874 (1972); https://doi.org/10.1063/1.1666069

David Lovelock

LOVELOCK'S THEOREM: if we try to create any gravitational theory in a 4D Riemannian space from an action principle involving the metric tensor and its derivatives only, then the only field equations that are second order (or less) are Einstein's equations and/or a cosmological constant.

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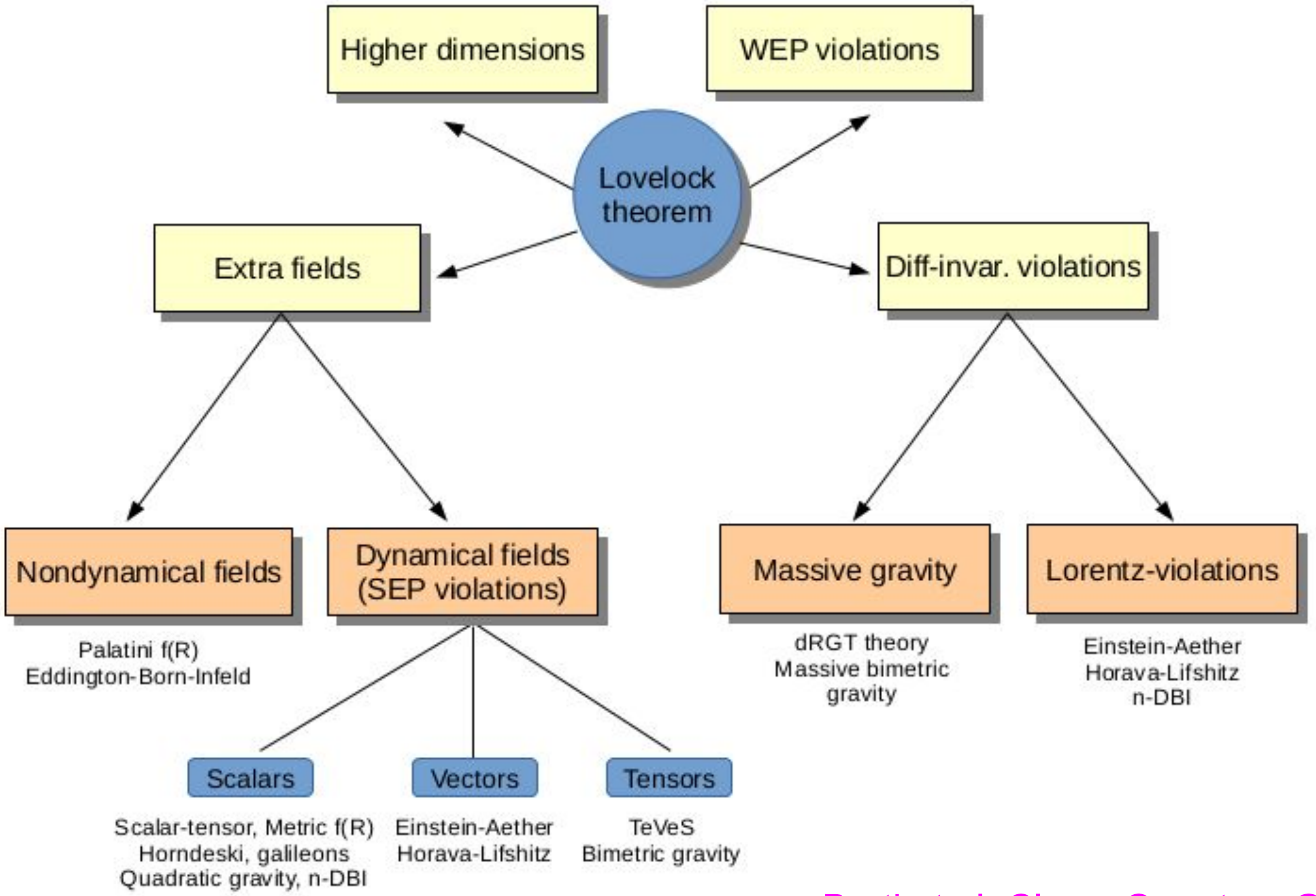
The main energy conditions in general relativity for the energy-momentum tensor are expressed as

Null energy condition $\Leftrightarrow \rho - p \geq 0$

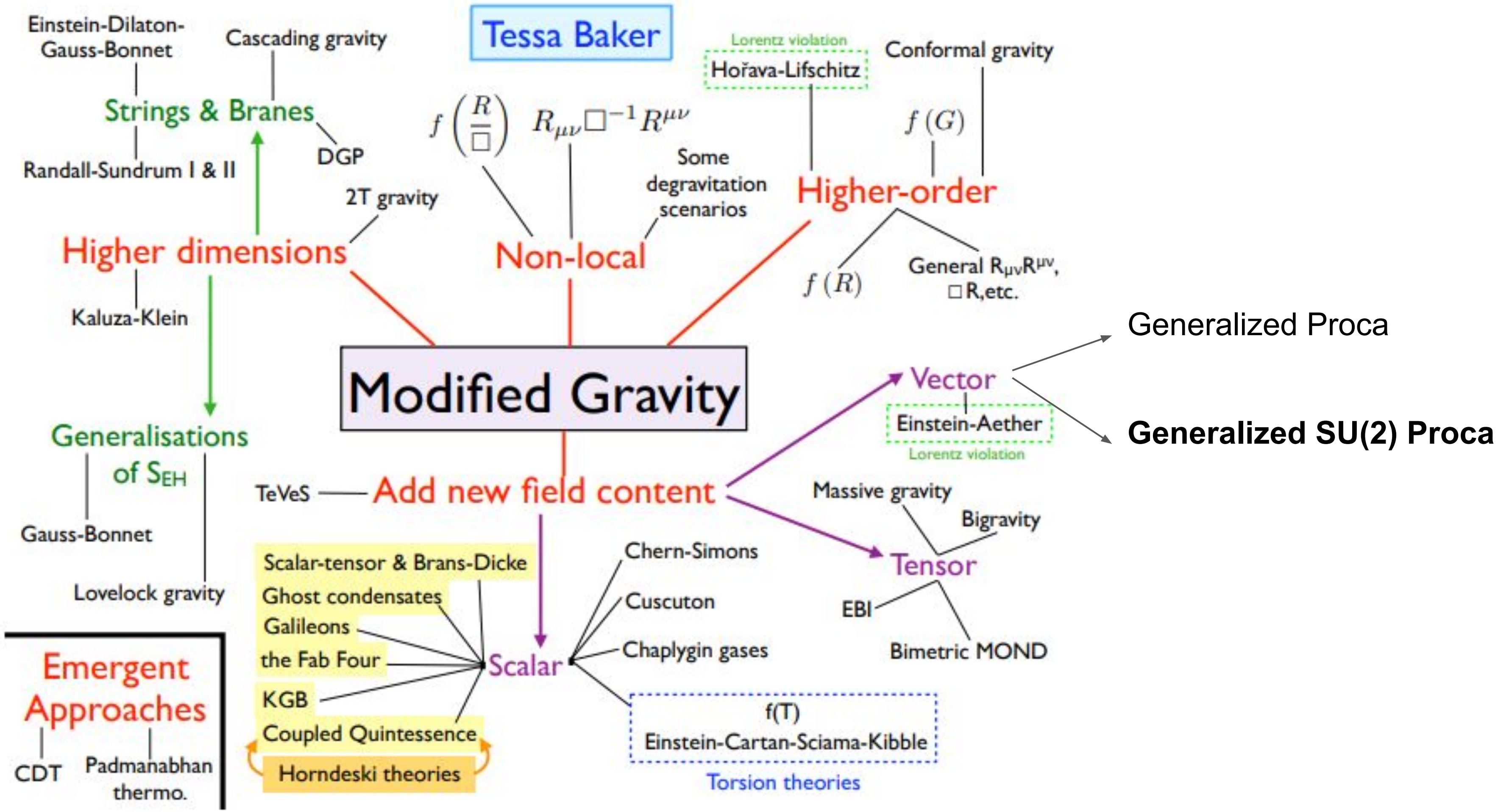
Weak energy condition $\Leftrightarrow \rho \geq 0$

Dominant energy condition $\Leftrightarrow \rho + p \geq 0$

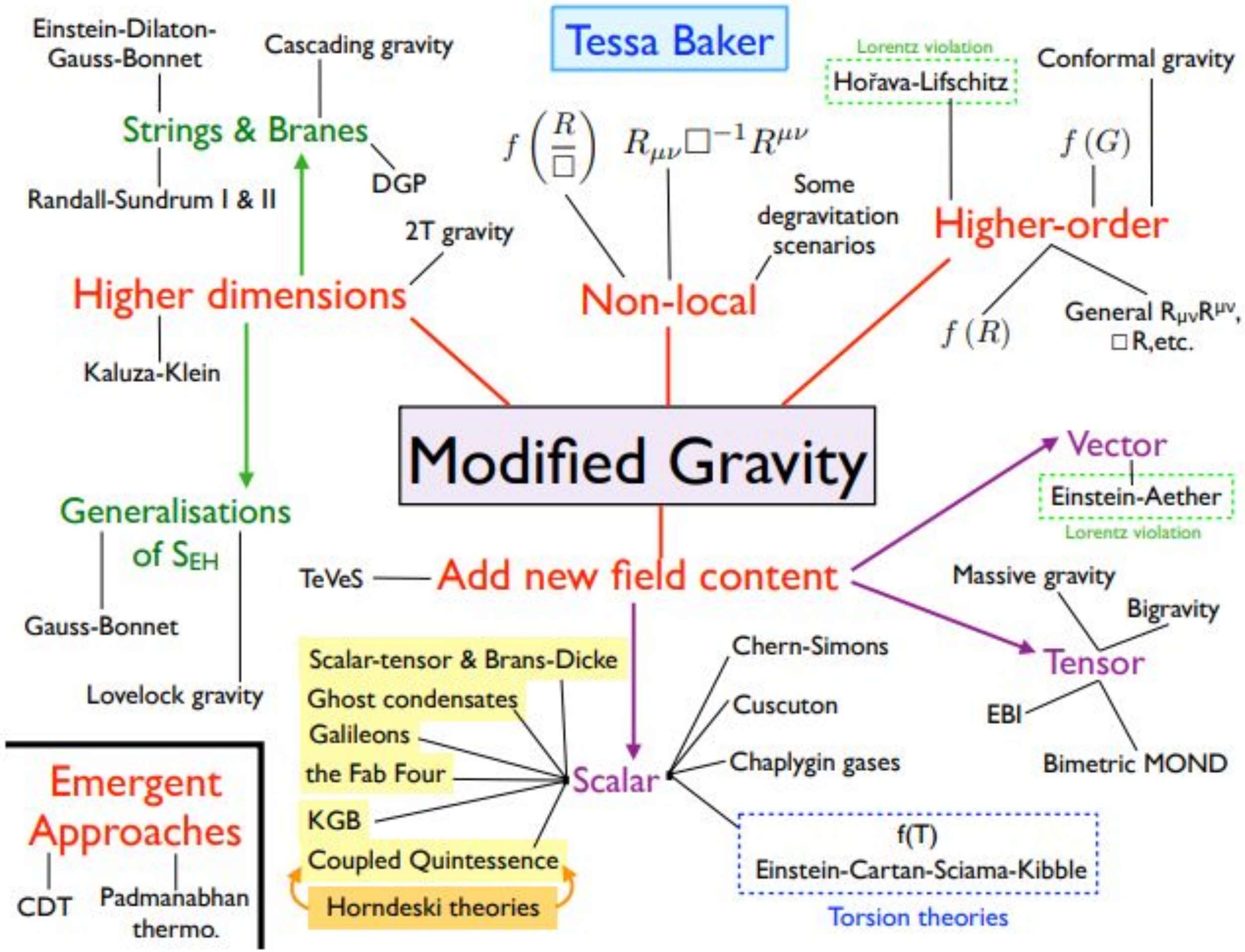
Strong energy condition $\Leftrightarrow \rho + 3p \geq 0$



Berti et al. Class. Quantum Grav. 32, 243001 (2015)



Tessa Baker
Einstein-Dilaton-Gauss-Bonnet
Cascading gravity
Strings & Branes
DGP
Randall-Sundrum I & II
2T gravity
Higher dimensions
Kaluza-Klein
Generalisations of S_{EH}
Gauss-Bonnet
Lovelock gravity
Emergent Approaches
CDT
Padmanabhan thermo.
$f\left(\frac{R}{\Box}\right)$
$R_{\mu\nu}\Box^{-1}R^{\mu\nu}$
Some degravitation scenarios
Non-local
Lorentz violation
Hořava-Lifschitz
Conformal gravity
$f(G)$
Higher-order
$f(R)$
General $R_{\mu\nu}R^{\mu\nu}$, $\Box R$, etc.
Modified Gravity
Vector
Einstein-Aether
Lorentz violation
Generalized Proca
Generalized SU(2) Proca
TeVeS
Add new field content
Massive gravity
Bigravity
Tensor
EBI
Bimetric MOND
Scalar-tensor & Brans-Dicke
Ghost condensates
Galileons
the Fab Four
KGB
Coupled Quintessence
Horndeski theories
Scalar
Chern-Simons
Cuscuton
Chaplygin gases
f(T)
Einstein-Cartan-Sciama-Kibble
Torsion theories

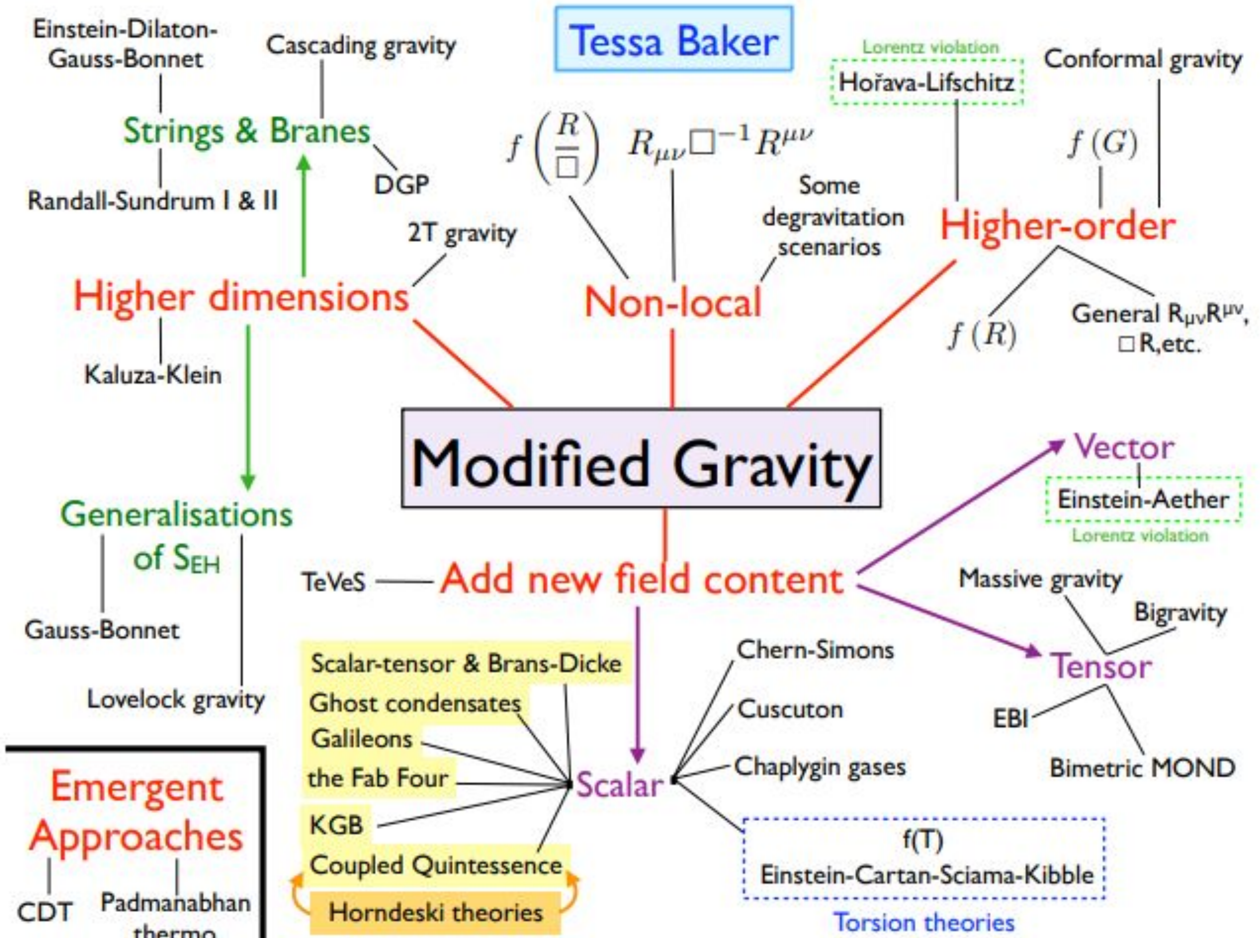


Quintessence:

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R+\mathcal{L}_\phi\right]+S_M\,,$$

$$\left|\frac{M_{\rm pl}^2 V_{,\phi\phi}}{V}\right|\lesssim 1\,,\quad m_\phi^2\equiv V_{,\phi\phi},$$

$$|m_\phi|\lesssim H_0\approx 10^{-33}\ \mathrm{eV}\,,$$



Phys. Rev. D 102, 104045 (2020)

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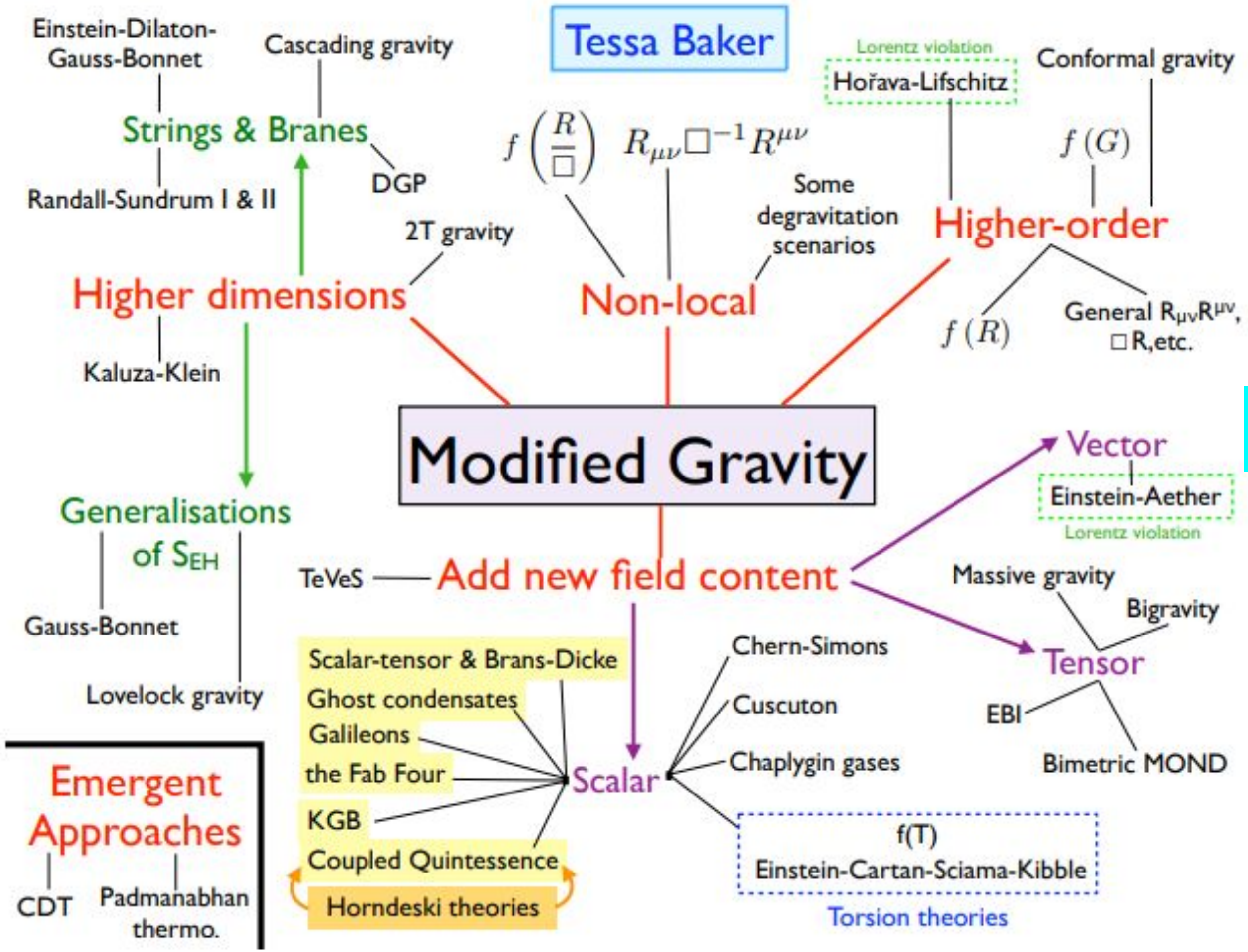
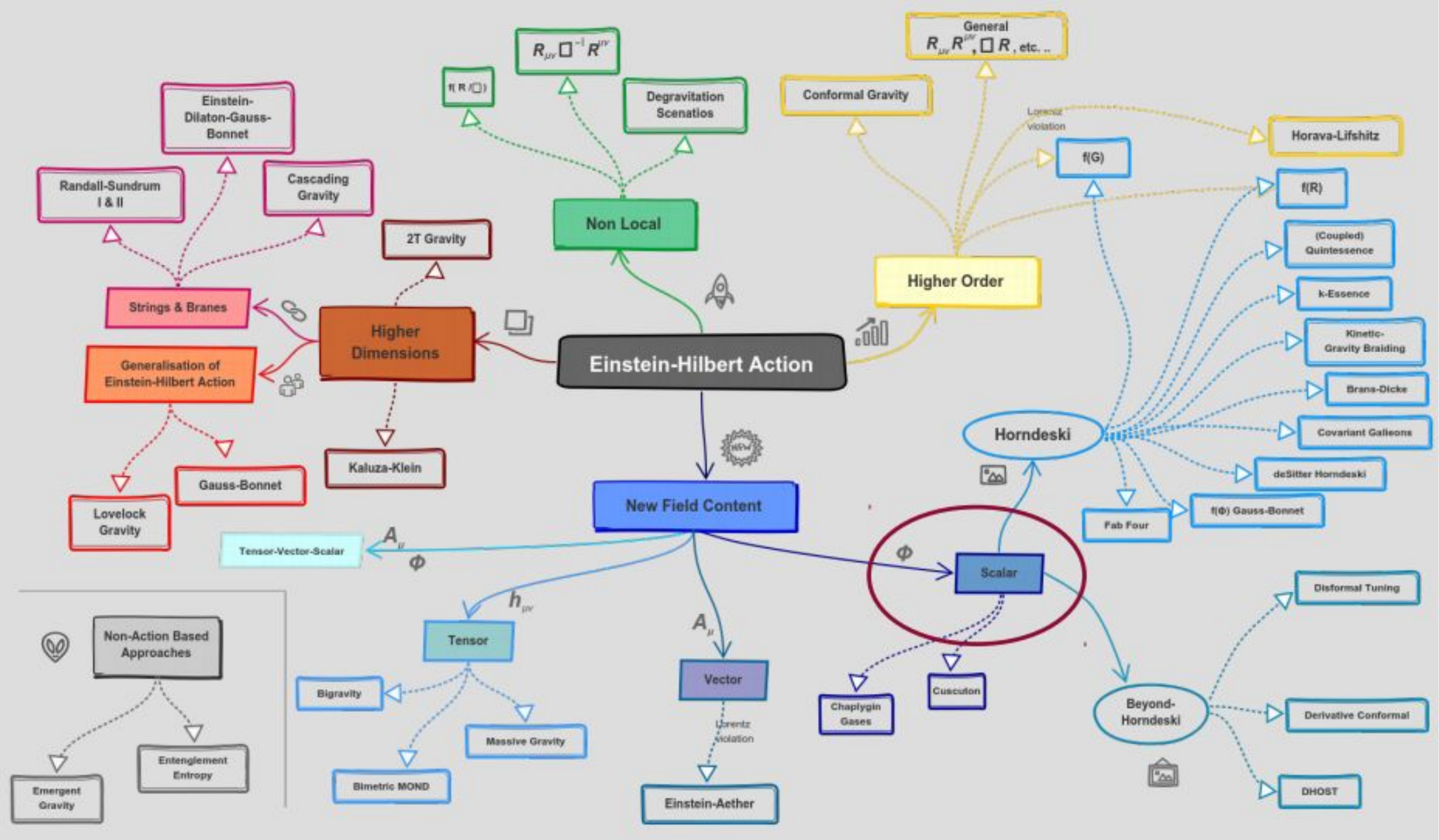
$$|m_\phi| \lesssim H_0 \approx 10^{-33}\ \mathrm{eV} ,$$

the mass is unstable against radiative corrections!



Self-accelerating solutions


Einstein-Hilbert Action
Non Local
f(R /□)
$R_{\mu\nu} \Box^{-1} R^{\mu\nu}$
Degravitation Scenatios
Higher Order
Conformal Gravity
General $R_{\mu\nu} R^{\mu\nu}, \Box R$, etc. ..
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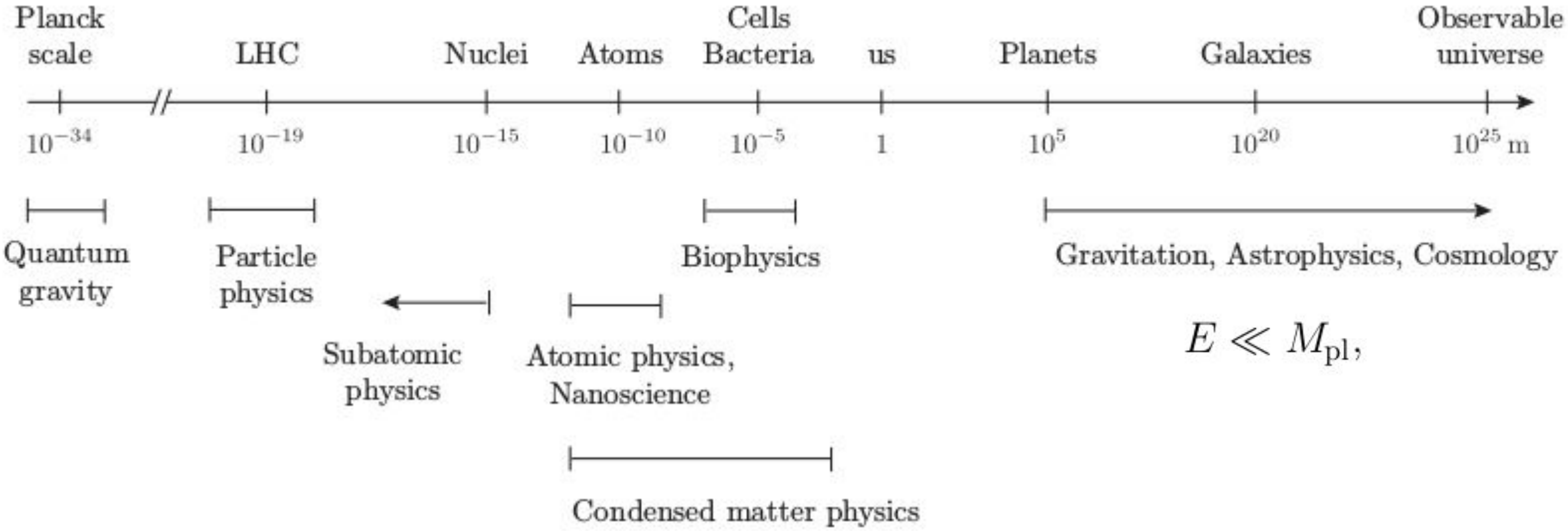
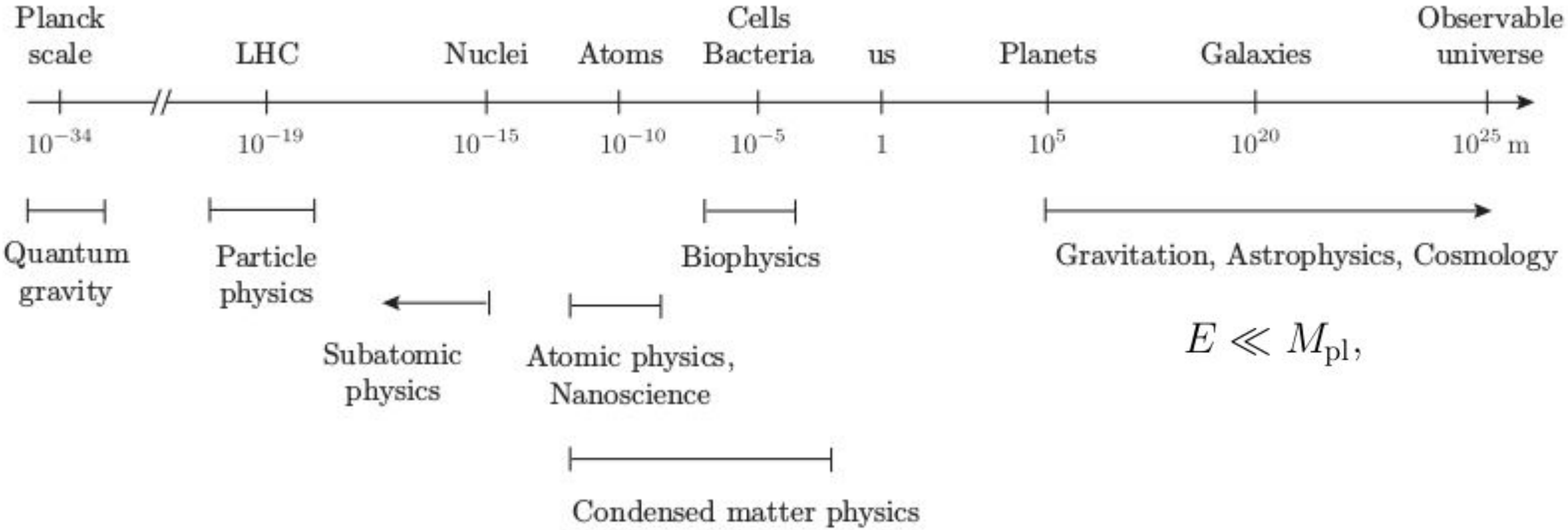
Cascading Gravity
Generalisation of Einstein-Hilbert Action
Lovelock Gravity
Gauss-Bonnet
New Field Content
Tensor-Vector-Scalar
A_μ
Φ
$h_{\mu\nu}$
Tensor
Bigravity
Massive Gravity
Bimetric MOND
A_μ
Vector
Lorentz violation
Einstein-Aether
Φ
Scalar
Chaplygin Gases
Cuscuton
Horndeski
(Coupled) Quintessence
k-Essence
Kinetic-Gravity Braiding
Brans-Dicke
Covariant Galileons
deSitter Horndeski
f(Φ) Gauss-Bonnet
Fab Four
Beyond-Horndeski
Disformal Tuning
Derivative Conformal
DHOST
Non-Action Based Approaches
Emergent Gravity
Entenglement Entropy

Theory	Field content	Strong EP	Massless graviton	Lorentz symmetry	Linear $T_{\mu\nu}$	Weak EP	Well-posed?	Weak-field constraints
Extra scalar field								
Scalar-tensor	S	✗	✓	✓	✓	✓	✓ [34]	[35–37]
Multiscalar	S	✗	✓	✓	✓	✓	✓ [38]	[39]
Metric $f(R)$	S	✗	✓	✓	✓	✓	✓ [40,41]	[42]
Quadratic gravity								
Gauss-Bonnet	S	✗	✓	✓	✓	✓	✓?	[43]
Chern-Simons	P	✗	✓	✓	✓	✓	✗✓? [44]	[45]
Generic	S/P	✗	✓	✓	✓	✓	?	
Horndeski	S	✗	✓	✓	✓	✓	✓?	
Lorentz-violating								
Æ-gravity	SV	✗	✓	✗	✓	✓	✓?	[46–49]
Khronometric/ Hořava-Lifshitz	S	✗	✓	✗	✓	✓	✓?	[48–51]
n-DBI	S	✗	✓	✗	✓	✓	?	none ([52])
Massive gravity								
dRGT/Bimetric	SVT	✗	✗	✓	✓	✓	?	[17]
Galileon	S	✗	✓	✓	✓	✓	✓?	[17,53]
Nondynamical fields								
Palatini $f(R)$	–	✓	✓	✓	✗	✓	✓	none
Eddington-Born-Infeld	–	✓	✓	✓	✗	✓	?	none
Others, not covered here								
TeVeS	SVT	✗	✓	✓	✓	✓	?	[37]
$f(R)\mathcal{L}_m$	?	✗	✓	✓	✓	✗	?	
$f(T)$	?	✗	✓	✗	✓	✓	?	[54]

Berti et al. Class. Quantum Grav. 32, 243001 (2015)


Planck scale
LHC
Nuclei
Atoms
Cells Bacteria
us
Planets
Galaxies
Observable universe
10^{-34}
10^{-19}
10^{-15}
10^{-10}
10^{-5}
1
10^{5}
10^{20}
10^{25} m
Quantum gravity
Particle physics
Biophysics
Gravitation, Astrophysics, Cosmology
Subatomic physics
Atomic physics, Nanoscience
Condensed matter physics

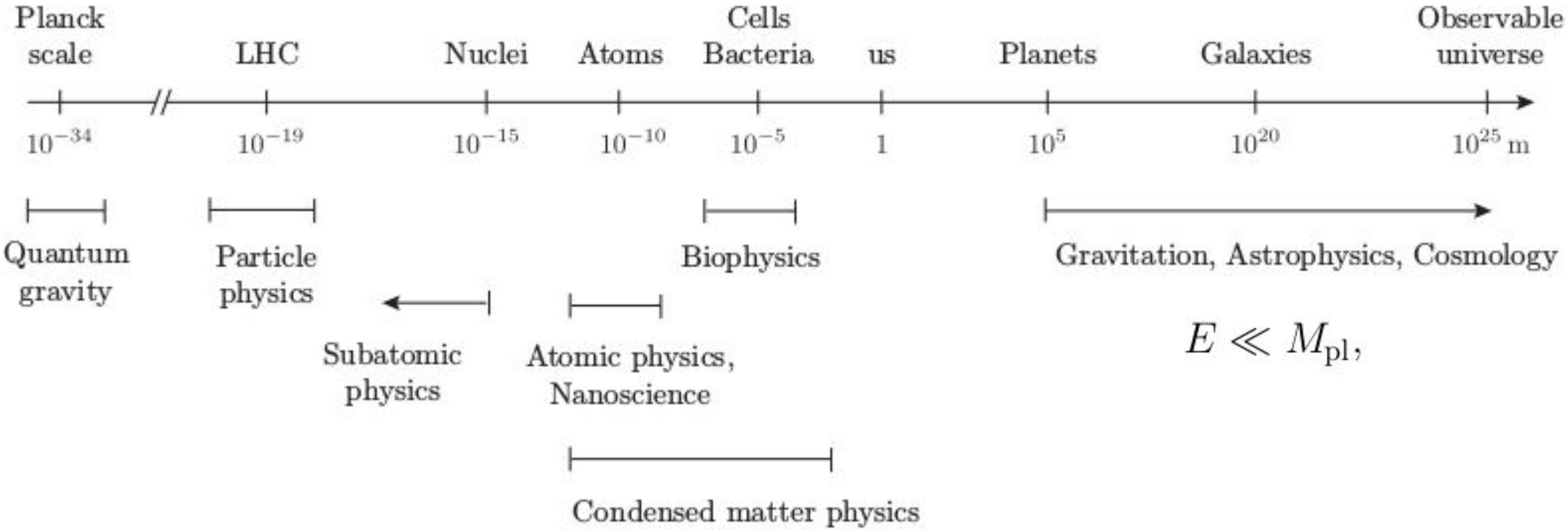
$$E \ll M_{\rm pl},$$



$$E \ll M_{\rm pl},$$

$$\mathcal{L}_{\rm eff}[g_{\mu\nu}] = \sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + c_1 R^2 + c_2 R_{\mu\nu}R^{\mu\nu} + \frac{1}{\Lambda^2}\left(d_1 R^3 + \cdots\right) + \cdots\right],$$

where $\Lambda \lesssim M_{\rm pl}$.



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where $\Lambda \lesssim M_{\rm pl}$.

We can parameterize our ignorance as an EFT

HORNDESKI'S THEORY:

$$S[g_{\mu\nu}, \phi] = \int d^4x \sqrt{-g} \left[\sum_{i=2}^{5} \mathcal{L}_i + \mathcal{L}_m \right] ,$$

$$\mathcal{L}_2 = G_2(\phi, X) , \qquad \mathcal{L}_3 = G_3(\phi, X)\Box\phi ,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4,X}(\phi, X) \left[(\Box\phi)^2 - \phi_{;\mu\nu}\phi^{;\mu\nu} \right]$$

$$\mathcal{L}_5 = G_5 G_{\mu\nu}\phi^{;\mu\nu} - \frac{1}{6} G_{5,X}(\phi, X) \Big[(\Box\phi)^3 - 3\phi_{;\mu\nu}\phi^{;\mu\nu}\Box\phi + 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\alpha}\phi_{;\alpha}{}^{\mu} \Big]$$

Self-accelerating Galileon models!

Horndeski, Int. J. Theor. Phys. 10, 363 (1974).
Kobayashi et al. Prog. Theor. Phys. 126, 511 (2011)
Deffayet et al. Phys. Rev. D 80, 064015 (2009).

Theoretical considerations are a very powerful tool in testing new models:

the study of classical and quantum fluctuations
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-Do the quantum fluctuations become strongly coupled at some unacceptably low energy scale?

Theoretical considerations are a very powerful tool in testing new models:

the study of classical and quantum fluctuations about classical solutions.

-Do the classical fluctuations propagate super-luminally?

LAPLACIAN INSTABILITIES negative squared propagation speed for high enough frequencies

-Can we excite a ghost?

GHOST: describe physical excitations with a wrong sign in the kinetic energy. ghost will generate instabilities if it couples to other, more conventional, fields since its energy is unbounded from below.

-Do the quantum fluctuations become strongly coupled at some unacceptably low energy scale?

STRONG COUPLING:

-quantum fluctuations on a classical solution becomes strongly coupled at an unacceptably low scale

-Classical solution itself is meaningless at distances below $1/\Lambda$ (loss of predictivity).

building up a ghost-free theory demands a positive-definite kinetic matrix.

building up a Laplacian-free theory demands a positive propagation speed

Generalized SU(2) Proca theory

Erwan Allys, Patrick Peter, and Yeinzon Rodríguez
Phys. Rev. D **94**, 084041 – Published 26 October 2016

$$\mathcal{L}_4^1 = \frac{1}{4}(A_b \cdot A^b)[S_\mu^{\mu a} S_{\nu a}^\nu - S_\nu^{\mu a} S_{\mu a}^\nu + A_a \cdot A^a R]$$
$$+ \frac{1}{2}(A_a \cdot A_b)[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + 2A^a \cdot A^b R],$$
$$\mathcal{L}_4^2 = \frac{1}{4}(A_b \cdot A_b)[S_\mu^{\mu a} S_\nu^{\nu b} - S_\nu^{\mu a} S_\mu^{\nu b} + A^a \cdot A^b R]$$
$$+ \frac{1}{2}(A^{\mu a} A^{\nu b})[S_{\mu a}^\rho S_{\nu\rho b} - S_{\nu a}^\rho S_{\mu\rho b} - A_a^\rho A_b^\sigma R_{\mu\nu\rho\sigma}$$
$$- (\nabla^\rho A_{\mu a})(\nabla_\rho A_{\nu b}) + (\nabla^\rho A_{\nu a})(\nabla_\rho A_{\mu b})],$$
$$\mathcal{L}_4^3 = \tilde{G}_{\mu\sigma}^b A_\alpha^\mu A_{\nu b} S^{\nu\sigma a}, \;\; \mathcal{L}_4^{\text{curv}} = L_{\mu\nu\rho\sigma} A^{\mu a} A^{\nu b} A_a^\rho A_b^\sigma$$

Generalized SU(2) Proca theory

Erwan Allys, Patrick Peter, and Yeinzon Rodríguez
Phys. Rev. D **94**, 084041 – Published 26 October 2016

$$
\begin{aligned}
\mathcal{L}_4^1 =& \frac{1}{4}(A_b \cdot A^b)[S^{\mu a}_{\mu} S^{\nu}_{\nu a} - S^{\mu a}_{\nu} S^{\nu}_{\mu a} + A_a \cdot A^a R] \\
&+ \frac{1}{2}(A_a \cdot A_b)[S^{\mu a}_{\mu} S^{\nu b}_{\nu} - S^{\mu a}_{\nu} S^{\nu b}_{\mu} + 2A^a \cdot A^b R], \\
\mathcal{L}_4^2 =& \frac{1}{4}(A_b \cdot A_b)[S^{\mu a}_{\mu} S^{\nu b}_{\nu} - S^{\mu a}_{\nu} S^{\nu b}_{\mu} + A^a \cdot A^b R] \\
&+ \frac{1}{2}(A^{\mu a} A^{\nu b})[S^{\rho}_{\mu a} S_{\nu \rho b} - S^{\rho}_{\nu a} S_{\mu \rho b} - A^{\rho}_a A^{\sigma}_b R_{\mu\nu\rho\sigma} \\
&- (\nabla^{\rho} A_{\mu a})(\nabla_{\rho} A_{\nu b}) + (\nabla^{\rho} A_{\nu a})(\nabla_{\rho} A_{\mu b})], \\
\mathcal{L}_4^3 =& \tilde{G}^{b}_{\mu\sigma} A^{\mu}_{\alpha} A_{\nu b} S^{\nu \sigma a}, \;\; \mathcal{L}_4^{\mathrm{curv}} = L_{\mu\nu\rho\sigma} A^{\mu a} A^{\nu b} A^{\rho}_a A^{\sigma}_b
\end{aligned}
$$

Generalized SU(2) Proca theory reconstructed and beyond

Phys. Rev. D

Alexander Gallego Cadavid, Yeinzon Rodríguez, and L. Gabriel Gómez

Accepted 9 October 2020

THE NEXT TALK

Stability conditions in the generalized $SU(2)$ Proca theory

L. Gabriel Gómez and Yeinzon Rodríguez
Phys. Rev. D **100**, 084048 – Published 21 October 2019

$$\mathcal{S} = \int d^4x \sqrt{-g}(\mathcal{L}_{\text{E-H}} + \mathcal{L}_{\text{YM}} + \alpha_1 \mathcal{L}_4^1 + \alpha_{\text{Curv}} \mathcal{L}_4^{\text{Curv}}),$$

Stability conditions in the generalized $SU(2)$ Proca theory

L. Gabriel Gómez and Yeinzon Rodríguez
Phys. Rev. D **100**, 084048 – Published 21 October 2019

$$\mathcal{S} = \int d^4x \sqrt{-g}(\mathcal{L}_{\text{E-H}} + \mathcal{L}_{\text{YM}} + \alpha_1 \mathcal{L}_4^1 + \alpha_{\text{Curv}} \mathcal{L}_4^{\text{Curv}}),$$

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

$$A_\mu^a \equiv a(t)\phi(t)\delta_\mu^a,$$

Second order action:

$$S^s = \frac{1}{(2\pi)^3}\int d^3k\,dt\,a^3\left(\dot{\vec{\chi}}^t\mathbf{A}\dot{\vec{\chi}} - k^2\vec{\chi}^t\mathbf{G}\vec{\chi} - \dot{\vec{\chi}}^t\mathbf{B}\vec{\chi} - \vec{\chi}^t\mathbf{M}\vec{\chi}\right),$$

$$S^T = \frac{1}{(2\pi)^3}\int d^3k\,dt\,a^3\,\frac{M^2(t)}{8}\left[(\dot{h}^T_{ij})^2 - c_t(t)^2\frac{k^2}{a^2}(h^T_{ij})^2\right],$$

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tensor perturbations:
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Coupled oscillator:

$$\partial^i h_{ij} = h^i_i = 0 \text{ and } \delta^i_a\partial_i t^a_j = \delta^i_a t^a_i = 0.$$

$$\delta g_{11} = -\,\delta g_{22} = a^2h_+,\ \delta g_{12} = a^2h_\times,$$

$$\delta A^1_\mu = a(0, t_+, t_\times, 0, 0),\ \delta A^2_\mu = a(0, t_\times, -t_+, 0, 0).$$

Ghostfree conditions :

$$S_K^2 = \int d^3x dt a^3 \dot{\vec{x}}^T K \dot{\vec{x}},$$

$$K_{11} = K_{13} = \frac{1}{4} + \left(\frac{61\alpha + 19\kappa}{8} - 2\theta \right) \phi^4$$

$$K_{22} = K_{44} = 1 + (-5\alpha + \kappa + 2\lambda)\phi^2$$

$$K_{12} = K_{21} = \frac{1}{2}(10\alpha - -3\kappa + 8\theta - 2\lambda)\phi^3,$$

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Gradient-instability free conditions:

$$S_L^2 = \int d^3x dt(-a\partial\vec{x}^T L \partial\vec{x}).$$

$$L_{11} = L_{13} = \frac{1}{4} + \left(\frac{81\alpha - \kappa}{8}\right)\phi^4$$

$$L_{22} = L_{44} = 1 + (-5\alpha + \kappa + 2\lambda)\phi^2$$

$$L_{12} = L_{21} = \frac{(10\alpha + \kappa - 4\lambda)}{2}\phi^3,$$

$$\det(c_s^2 K - L) = 0.$$

Gradient-instability free conditions:

$$
\begin{aligned}
c_{\mathrm{T}_\pm}^2 = &\{\alpha_1\phi^2[-10 + (-71 + 8d)\phi^2 - 5\alpha_1(31 + 24d)\phi^4] \\
&\pm 2[1 + [\alpha_1^2\phi^6(1 + 5\alpha_1\phi^2)[64d^2 + (5 + 4d)^2\phi^2 \\
&+ \alpha_1(125 + 8d(-75 + 254d))\phi^4]]^{1/2}]\}/ \\
&\{-2 + \alpha_1\phi^2[-10 + (-61 + 16d)\phi^2 \\
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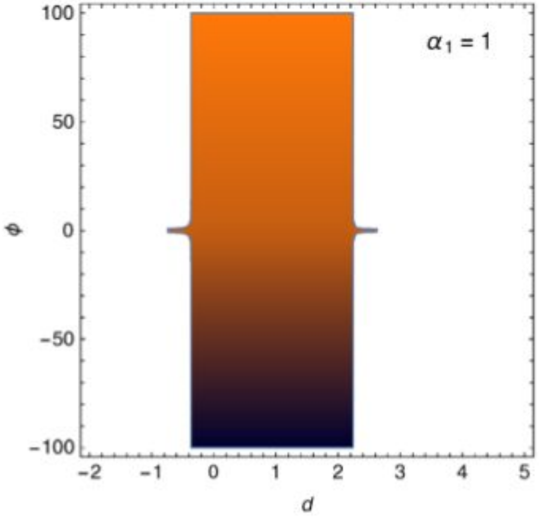

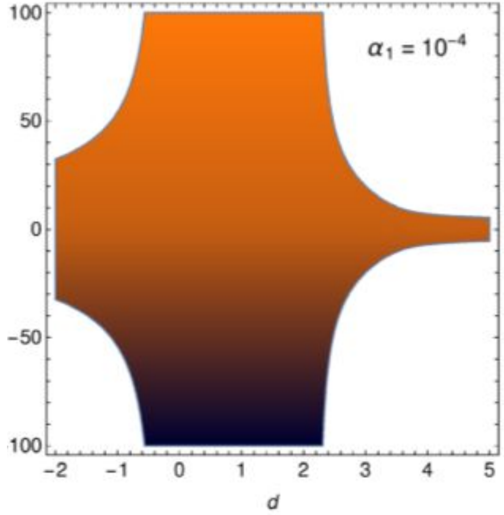

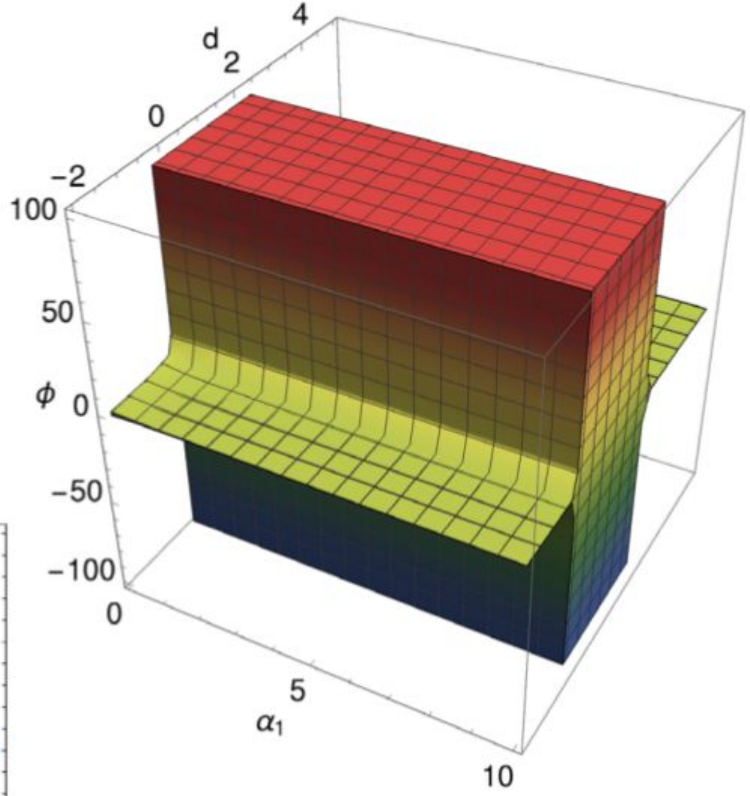

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the safest way to deal with a ghost is to dismiss those unphysical solutions of a theory upon which the ghost can fluctuate

Probing Dark Energy with GWs:

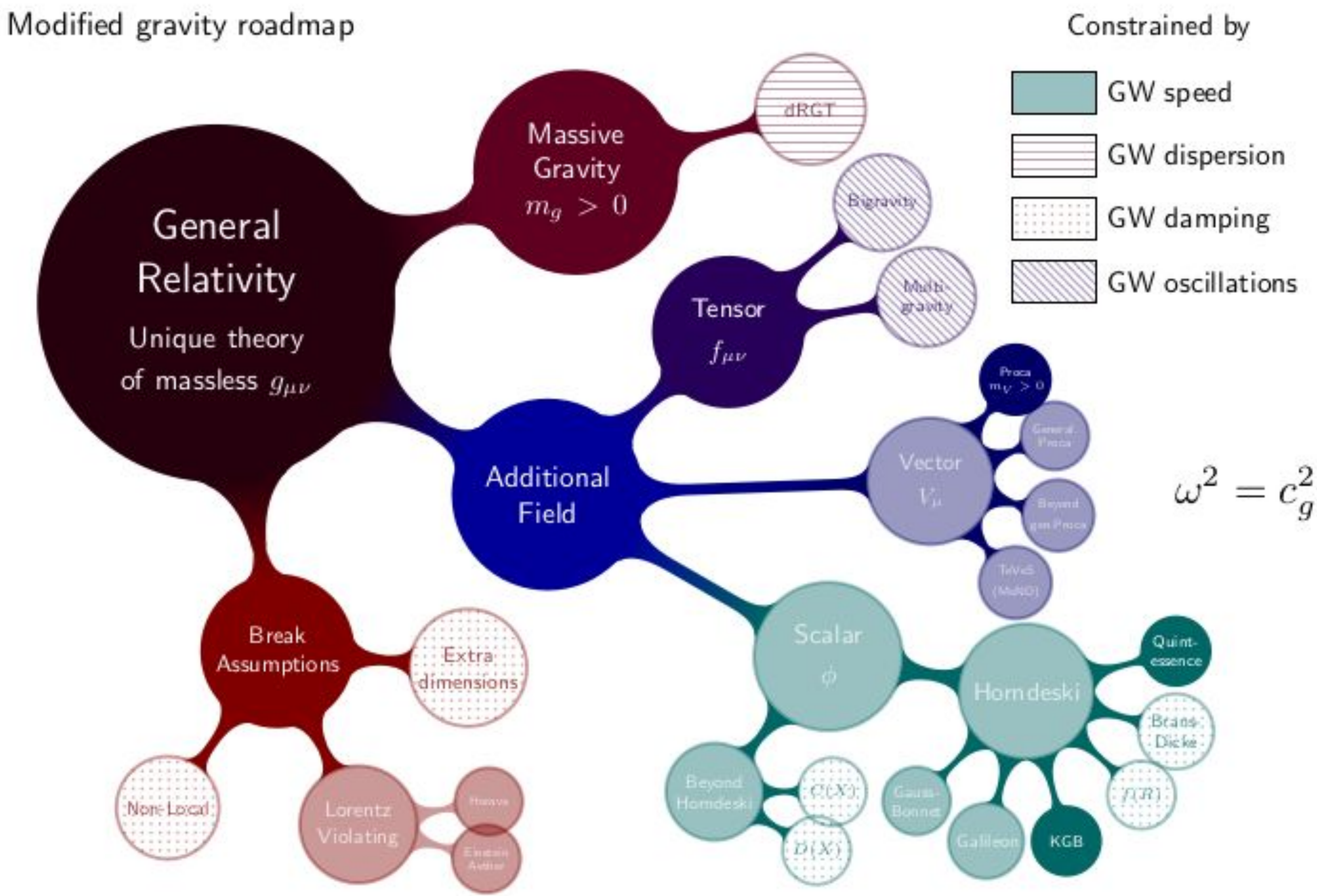
$$\ddot{h}_{ij} + (3 + \alpha_M) H \dot{h}_{ij} + (1 + \alpha_T) k^2 h_{ij} = 0, \quad c_g^2 = 1 + \alpha_T$$

$$\alpha_M \equiv d\log(M_*^2)/d\log(a)$$

$$-3 \cdot 10^{-15} \leq c_g/c - 1 \leq 7 \cdot 10^{-16}.$$

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity	quartic/quintic Galileons [13, 14]
	quintessence/k-essence [46]	Fab Four [15]
	Brans-Dicke/$f(R)$ [47, 48]	de Sitter Horndeski [49]
	Kinetic Gravity Braiding [50]	$G_{\mu\nu}\phi^\mu\phi^\nu$ [51], $f(\phi)\cdot$Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17]	quartic/quintic GLPV [18]
	Disformal Tuning (21)	quadratic DHOST [20] with $A_1 \neq 0$
	quadratic DHOST with $A_1 = 0$	cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

Ezquiaga et al. Phys. Rev. Lett. 119, 251304 (2017)



$$\omega^2 = c_g^2 k^2 + m_g^2 + \sum_{n=3} \mathbb{A}_n k^n ,$$

Ezquiaga et al. Front. Astron. Space Sci. 5:44 (2018)

within the set of theories passing present tests, what interesting phenomenology is still possible?

$$c_g(z = 0) = c.$$

$$\omega^2(k) = c_g^2 k^2 \left(1 + \sum_n c_n \left(\frac{aH}{k}\right)^n\right) .$$

This result leaves us with two ways to construct gravity theories with GWs moving at the speed of light: 1) start with a luminal theory and apply a conformal transformation, D = 0, or 2) compensate the anomalous speed with a disformal factor.

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$$\tilde{g}_{\mu\nu} = \Omega^2(\phi, X) g_{\mu\nu} + \mathcal{D}(\phi, X)\phi_{,\mu}\phi_{,\nu} ,$$

changes the GW-cone

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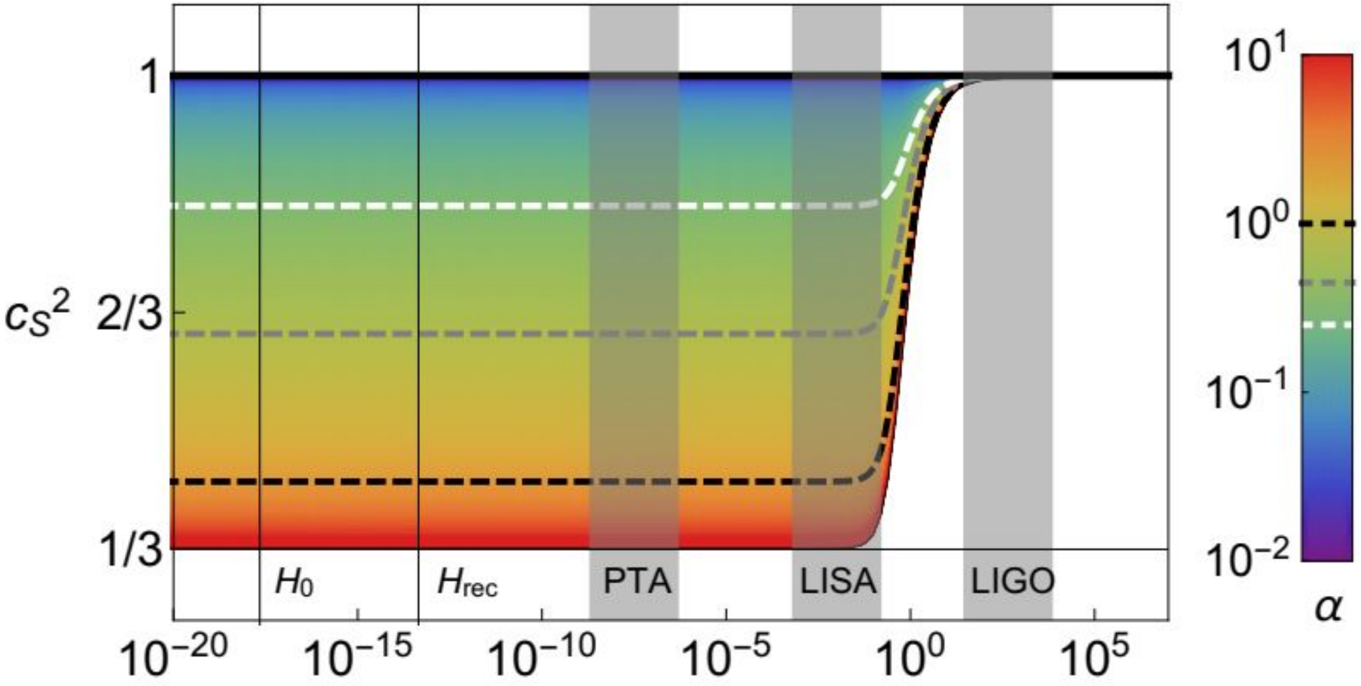
This particular cancellation holds over general background!

Ezquiaga et al. Phys. Rev. Lett. 119, 251304 (2017)

Gravitational Rainbows: $c_T = c_T(k)$.



The EFT can safely describe cosmology from today H0 to before recombination Hrec, but may receive order one corrections in the LIGO band.

$$M \lesssim \Lambda_{\rm Horndeski} \sim (M_{\rm Pl} H_0^2)^{1/3} \sim 260{\rm Hz}$$

De Rham et al. Phys. Rev. Lett. 121, 221101 (2018)

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Take care about all possible instabilities at classical and at quantum level:

avoid: ghost instabilities, Laplacian instabilities, radiative instabilities, strong decay of gravitational waves into dark energy fluctuations and other fundamental issues.

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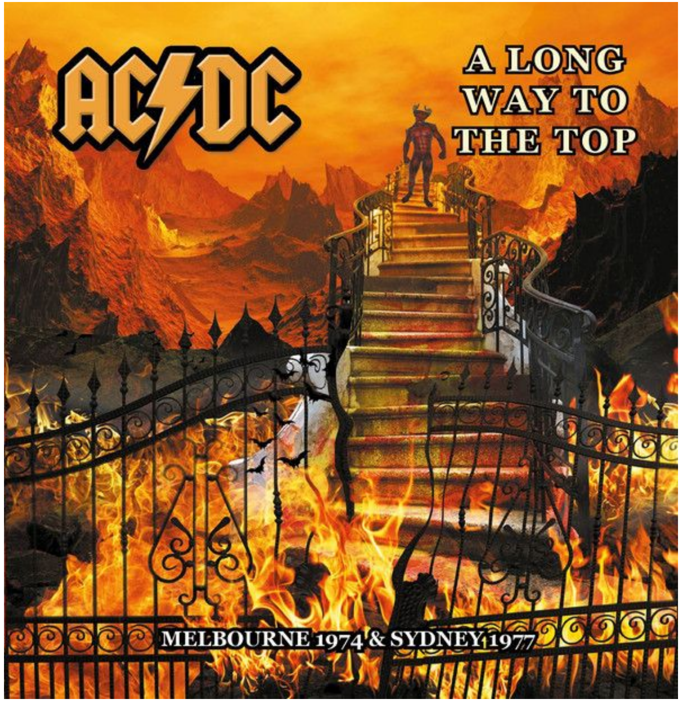

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