Theory of Fast Flavor Conversion of Supernova neutrinos

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CONTENT OF THE TALK ?

- **Equations of motion and a bit of background in the field**

- **Complicacy of the system and our motivation**

- **Our Numerical approach**

- **Theory governing fast conversion :**

  1) Multipole diffusion
  2) Transverse relaxation

- **Main results**

- **Conclusion and its astrophysical importance**

BACKGROUND

- Neglecting momentum changing collisions, EOM in 2 flavor framework for neutrinos of momentum $\vec{p} \sim (\omega, \vec{v})$ in a realistic SN scenario :

$$\left(\partial_t + \vec{v}.\vec{\nabla}\right) \mathbf{S}_{\omega,\vec{v}} = \left(\mathbf{H}^{\mathrm{vac}}_{\omega} + \mathbf{H}^{\mathrm{mat}} + \mathbf{H}^{\mathrm{self}}_{\vec{v}}\right) \times \mathbf{S}_{\omega,\vec{v}}$$

- $\mathsf{S}_{\omega,\vec{v}}[\vec{r}, t]$, with $|\mathsf{S}_{\omega,\vec{v}}| \equiv S_{\omega,\vec{v}} = 1$ is the normalized Bloch vector in the flavor basis :$\{\hat{e}_1, \hat{e}_2, \hat{e}_3\}$ and corresponds to the density matrix : $|\nu_{\omega,\vec{v}}\rangle\langle\nu_{\omega,\vec{v}}|$ varying in space-time coordinates, $(\vec{r}, t)$

- $\hat{e}_3$ component , $\mathsf{S}^{||}_{\omega,\vec{v}}$: $|\langle\nu_e|\nu\rangle|^2 - |\langle\nu_\mu|\nu\rangle|^2$ & $\hat{e}_1 - \hat{e}_2$ components, $\mathsf{S}^{\perp}_{\omega,\vec{v}}$: Amount of flavor conversion

- **Vaccum and Matter terms :**

$$\mathsf{H}^{\mathrm{vac}}_{\omega} = \omega\big(\sin 2\vartheta, 0, \cos 2\vartheta\big)$$

$$\mathsf{H}^{\mathrm{mat}} = \sqrt{2} G_F (n_{e^-} - n_{e^+})\big(0, 0, 1\big)$$

Self-interaction term :

$$\mathsf{H}^{\mathrm{self}}_{\vec{v}} = \int d^3\vec{p}\,'_{\omega',\vec{v}'}/(2\pi)^3\, g_{\omega',\vec{v}'}\big(1 - \vec{v}\cdot\vec{v}'\big)\mathsf{S}_{\omega',\vec{v}'}$$

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- $\mathrm{H}_{\omega}^{\mathrm{vac}} = \omega($…

$$\ldots(n_{e^-} - n_{e^+})(0,0,1) \ll \mathrm{H}_{\vec{v}}^{\mathrm{self}} = \int d^3\vec{p}'_{\omega',\vec{v}'}/(2\pi)^3\, g_{\omega',\vec{v}'} \left(1 - \vec{v}\cdot\vec{v}'\right) \mathrm{S}_{\omega',\vec{v}'}$$

Fast Flavor Conversion !!

BACKGROUND

CHARACTERISTICS :

- **Why fast ?** Flavor conversions can occur with a rate much faster ($\sim \mu$) compared to neutrino flavor conversions in ordinary matter or vacuum.

- **How fast ?** Fast flavor conversion : $\mu \sim \sqrt{2} G_F n_\nu \sim 10\, cm^{-1}$ $\quad [n_\nu \sim (10^{35} - 10^{30}\,\mathrm{cm}^{-3})]$

  Ordinary flavor conversion : $\omega = |\Delta m^2|/(2E) \sim km^{-1}$ $\quad \boldsymbol{\mu \sim 10^5 \omega}$

- It can occur at a distance of about $r \sim (10 - 100)\,\mathrm{km}$ from the supernova core.

- It is quite insensitive to the size or sign of ω and can be triggered even in absence of ω

- **Nature of the initial growth rate ?** $\mathbf{S^\perp \sim e^{Im\,\Omega\, t}}$

  Linear Stability Analysis $\quad |\mathrm{S}^\perp| \ll \mathrm{S}^\parallel \approx 1$

$$(\partial_t + v_x \partial_x)\, \mathrm{S}^\perp_{v_x} = i \int_{-1}^{+1} dv_x{}' \,(1 - v_x v_x{}')\, G_{v_x{}'}\, (\mathrm{S}^\perp_{v_x{}'} - \mathrm{S}^\perp_{v_x})$$

F.T →

$$\det\left(\Pi^{\mu\nu}[K_x, K_y, \Omega]\right) = 0$$

BACKGROUND

- **Why Important ?** Fast conversions can have a dramatic effect on various supernova astrophysics, e.g- the nucleosynthesis of elements, explosion mechanism e.t.c and also on the final neutrino signal reaching on earth.

- **What are the difficulties ?**
  a) Huge phase space dimensionality :

  Momentum ($\vec{p}$) + Space ($\vec{r}$) + Time (t) = 7 dim problem
  (3) (3) (1)

  b) Nonlinear and coupled system of P.D.E's

  c) Linear stability captures the initial linear growth of fast oscillations but fails to compute the final outcome

  d) Stellar evolution and neutrino signal is sensitive to the final fluxes and a required theory prediction is lacking

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What did we do?

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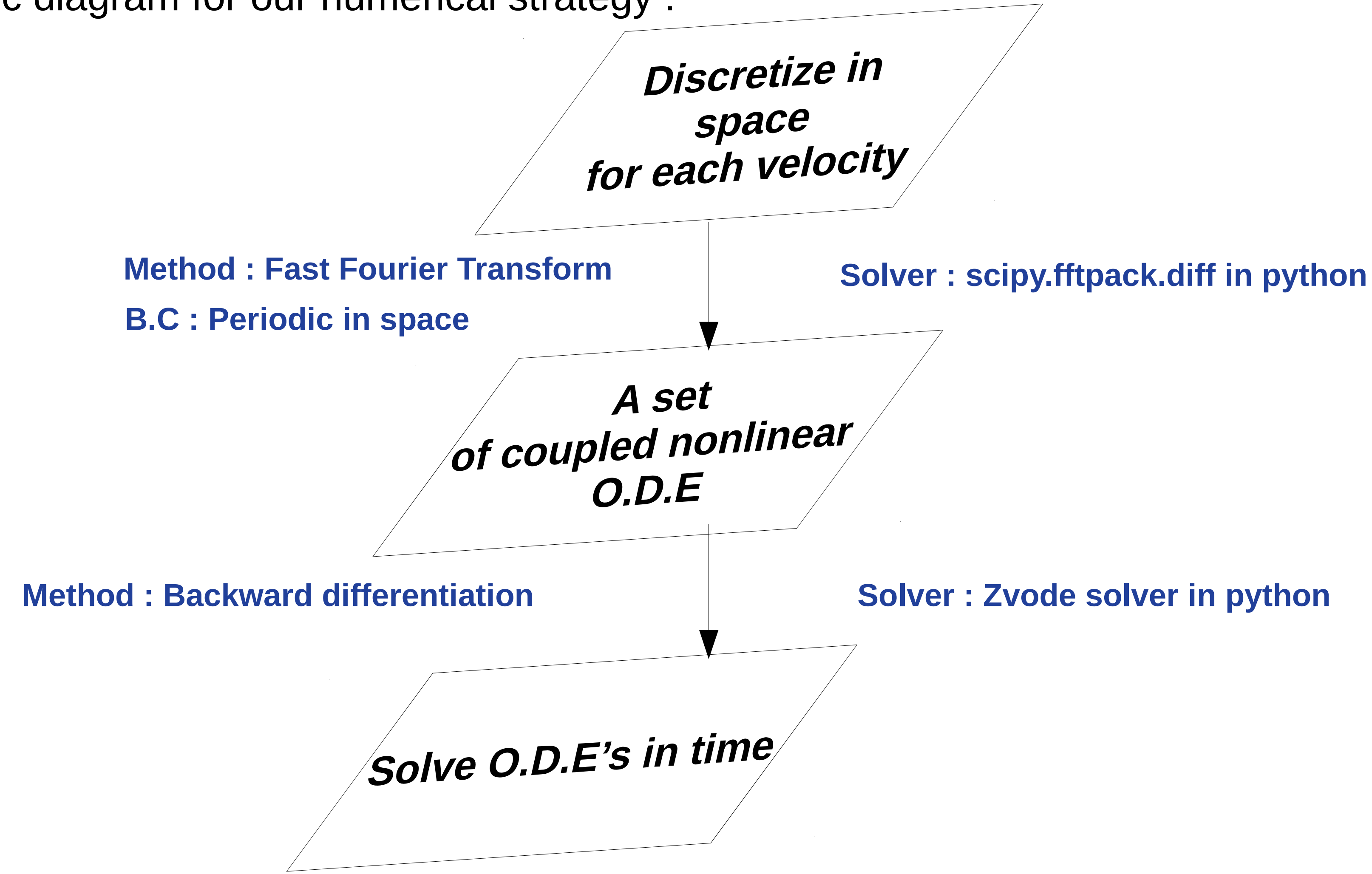
1) Developed a Numerical Code

2) Developed a Theory of FFC in the nonlinear regime

3)Predicted how, when and to what extent flavor conversion can happen

OUR NUMERICAL RECIPE

Schematic diagram for our numerical strategy :



OUR TOY MODEL

- In our model we choose, $\mathsf{H}^{\rm vac}_{\omega}, \mathsf{H}^{\rm mat} \ll \mathsf{H}^{\rm self}_{\vec{v}}$ for fast oscillations to occur.

- In this limit $g_{\omega,\vec{v}}$ becomes ω independent and $\mathsf{S}_{\omega,\vec{v}}$ enters in $\mathsf{H}^{\rm self}_{\vec{v}}$ through :

$$G_{\vec{v}} = \int d\omega g_{\omega,\vec{v}}$$

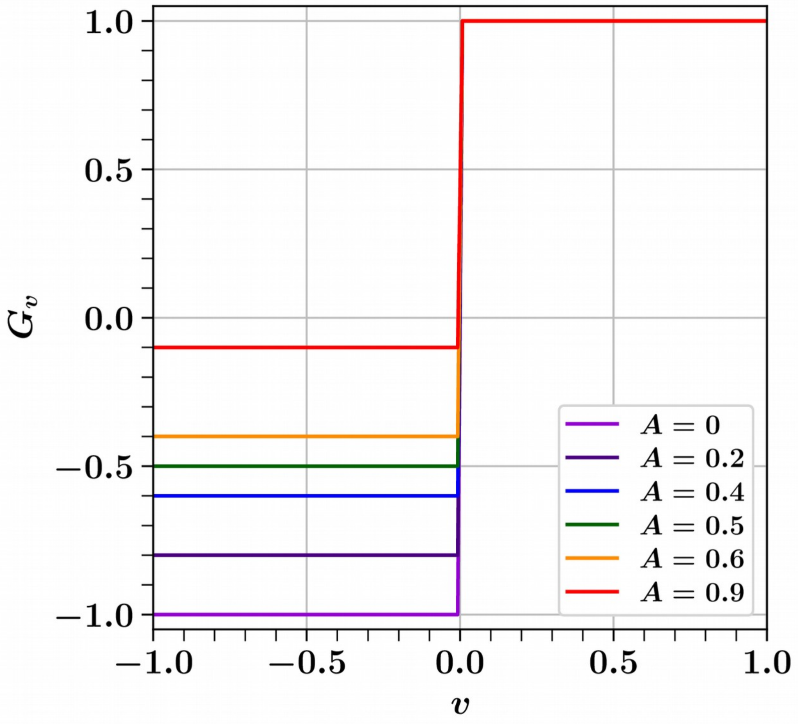
- Reduced phase space : 1+1+1D

$$(\partial_t + v\partial_z)\mathsf{S}_v = \mu_0 \int_{-1}^{+1} dv' G_{v'} (1 - vv')\, \mathsf{S}_{v'} \times \mathsf{S}_v$$

- Initial conditions :

$$\left|\mathsf{S}_v^{\perp}\right|^{\rm ini} = 10^{-6}\delta(z) \qquad \mathsf{S}_v^{\parallel}|^{\rm ini} = +1$$

$$\mu_0 = 33\,{\rm cm}^{-1}$$



OUR ANALYTICAL APPROACH

- In terms of multipole moments, $\mathsf{M}_n = \int_{-1}^{+1} dv\, G_v L_n \mathsf{S}_v$ and considering n as continuum we get :

$$\partial_t \mathsf{M}_n - \mathsf{M}_0 \times \mathsf{M}_n = \partial_z \left(\mathsf{M}_n + \partial_n \mathsf{M}_n/(2n+1) + \partial_n^2 \mathsf{M}_n/2\right) - \mathsf{M}_1 \times \left(\mathsf{M}_n + \partial_n \mathsf{M}_n/(2n+1) + \partial_n^2 \mathsf{M}_n/2\right)$$

- Further approximating $|\mathsf{M}_n \times \mathsf{M}_1| \gg \mathsf{M}_n \cdot \mathsf{M}_1$, averaging over z , and using $2n+1 \approx 2n$

$$\partial_t \langle M_n \rangle = \frac{\langle M_1 \rangle}{2} \left(\partial_n^2 \langle M_n \rangle + \frac{1}{n} \partial_n \langle M_n \rangle \right)$$

Diffusion in multipole space

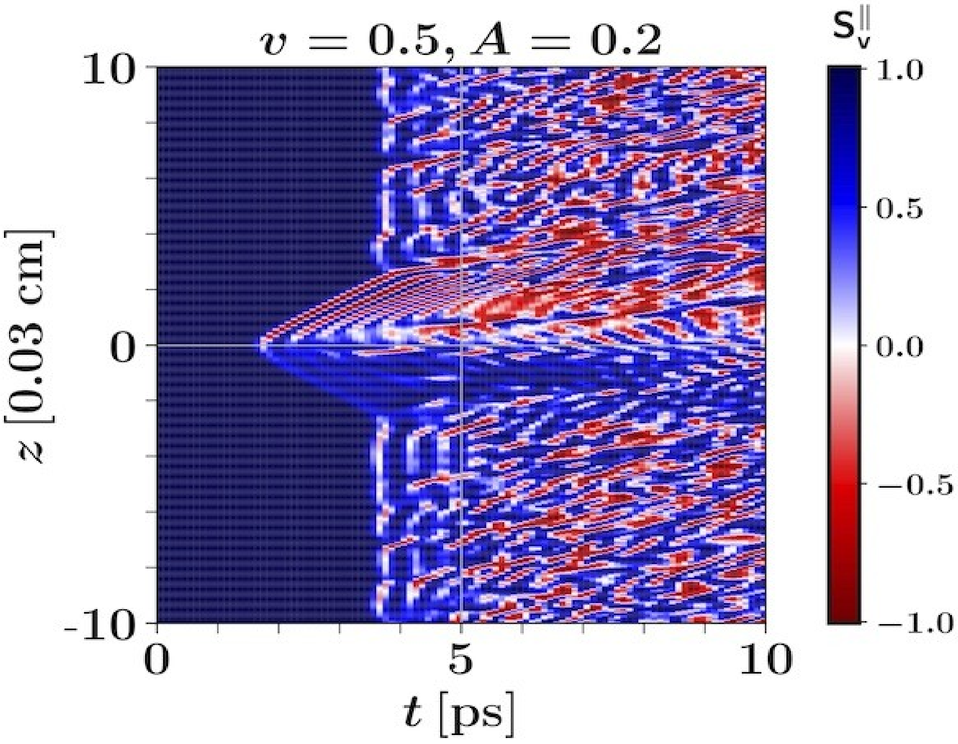
- The above equation remains same under $n \to an\,,\ t \to a^2 t$ ⟶ $\langle M_n(t) \rangle = f\left(\frac{n^2}{t}\right) = f(\xi)$

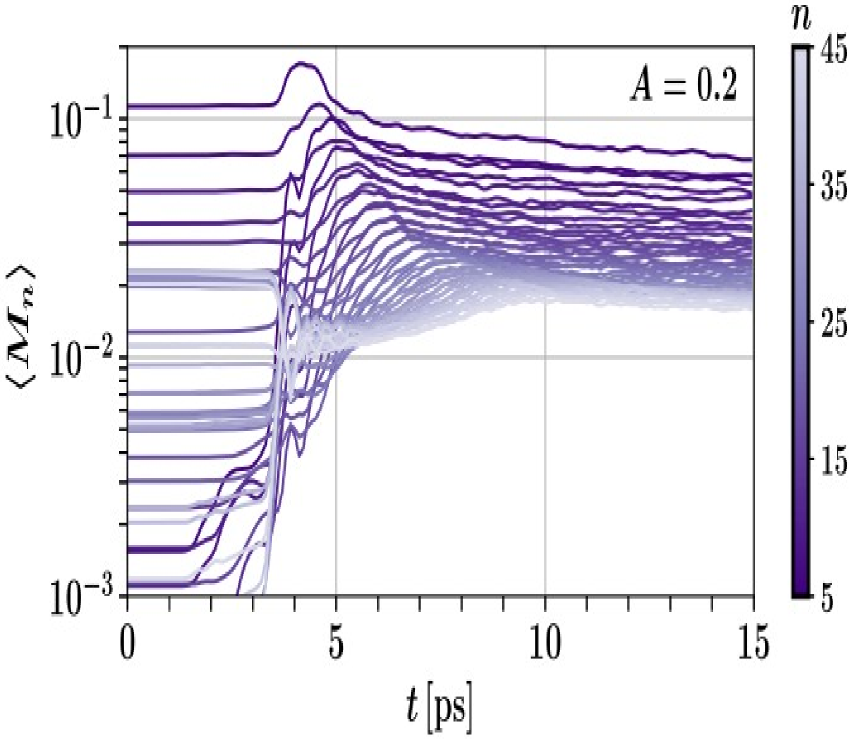
$$2\frac{d^2}{d\xi^2} f(\xi) + (1/\langle M_1 \rangle + 2/\xi) \frac{d}{d\xi} f(\xi) = 0$$

$$\langle M_n(t) \rangle = c_1\, \mathrm{Ei}\left[-n^2/\left(2\langle M_1 \rangle t\right)\right] + c_2$$

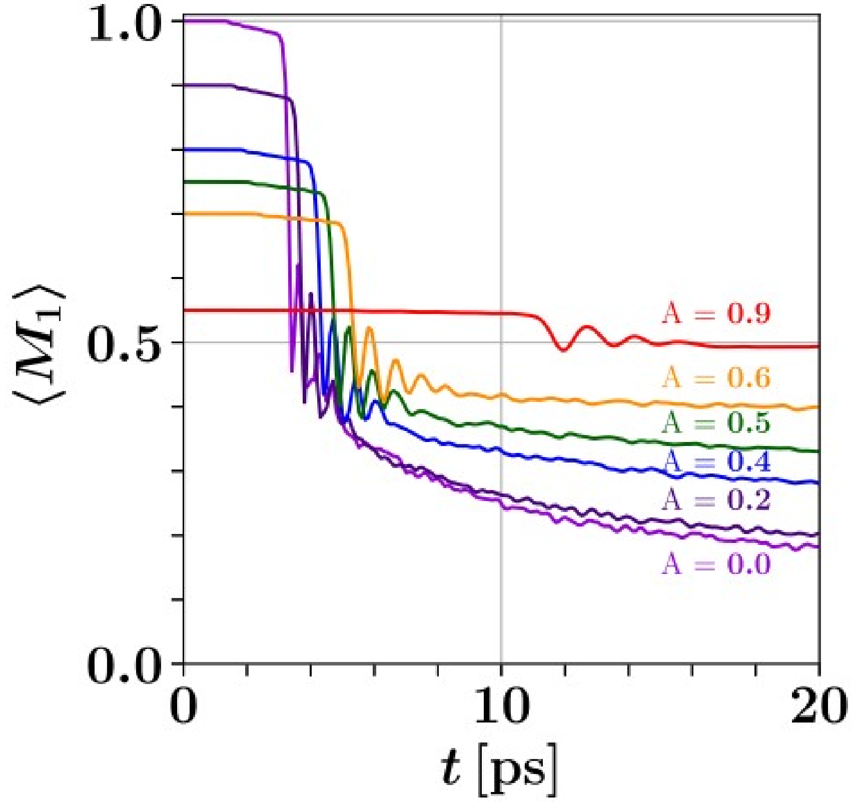
- Power flow in multipole space from lower to higher n values resulting in a time irreversible system and the power for every multipole moment or $|\mathsf{M}_n|^2$ is not conserved.

OUR ANALYTICAL APPROACH





OUR ANALYTICAL APPROACH



- Multipole equation for n = 0 :

$$\partial_t \langle M_0 \rangle = 0 \longrightarrow \langle M_0 \rangle = A$$

- A captures the amount of lepton asymmetry in the system.

- The larger the value of A, the power loss is lesser and vice-versa.

OUR ANALYTICAL APPROACH

- The behaviour of lower order multipoles for e.g : n = 1 is governed through **T2 relaxation**

- The EOM in a spatially averaged sense can be visualized as a spin moving in an effective hamiltonian, $\mathsf{H}_v \approx -\left(\frac{1}{3}\mathsf{M}_0 + v\mathsf{M}_1\right)$

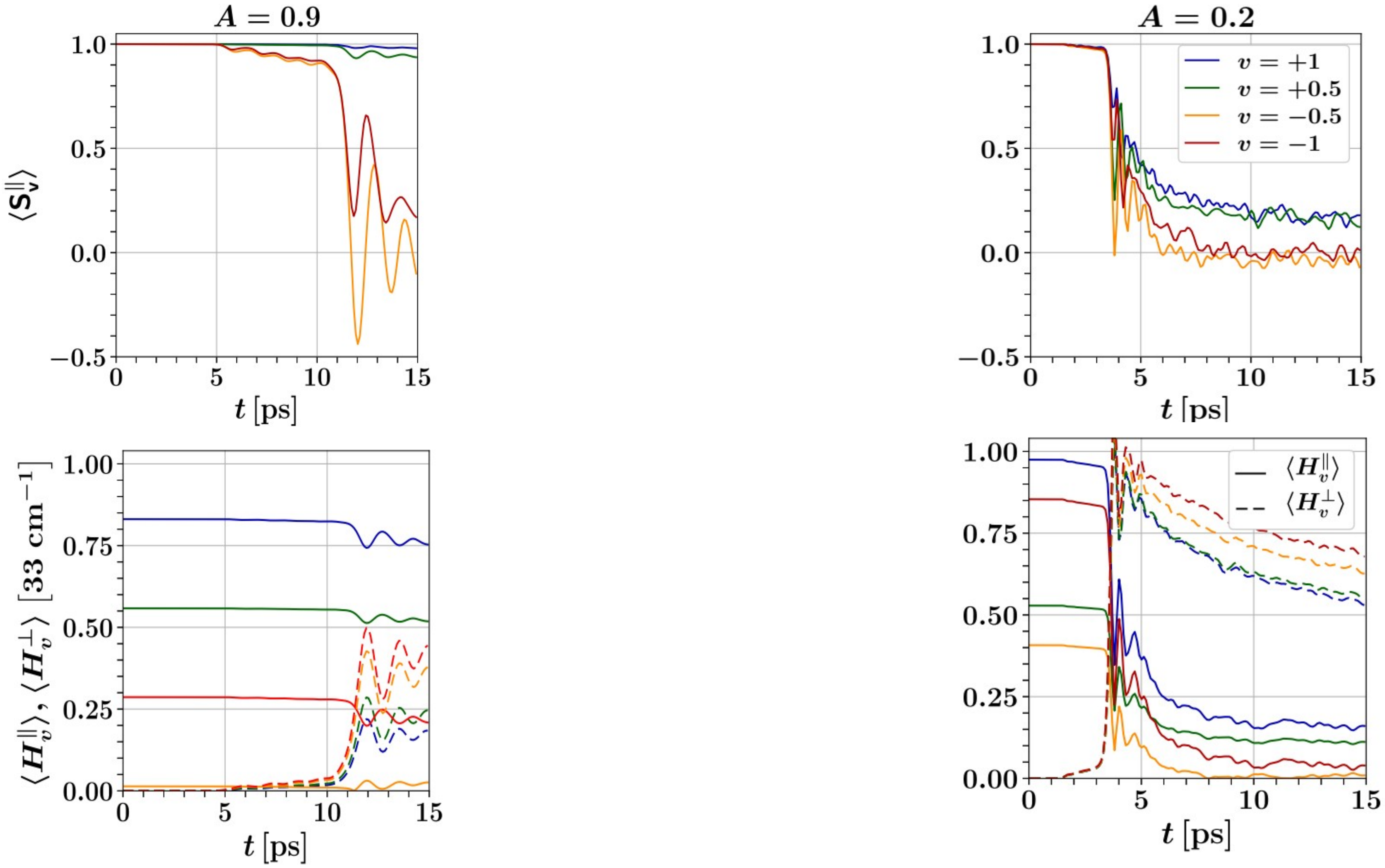
- The epoch of **T2 relaxation** is determined through the condition,

$$\boxed{\left|\mathbf{H}_v^{\perp}\right| \approx \left|\mathbf{H}_v^{\parallel}\right|}$$

- Initially $\langle \mathsf{S}_v \rangle$ is along $\hat{\mathsf{e}}_3$ but it starts tilting away due to $\mathsf{H}_\omega^{\text{vac}}$. So, $\langle H_v^{\perp} \rangle$ starts growing as well. If $\left|\mathsf{H}_v^{\perp}\right| \approx \left|\mathsf{H}_v^{\parallel}\right|$ is satisfied $\langle \mathsf{S}_v \rangle$ makes a large precession angle and reaches the transverse plane.
- Then S_v at different spatial locations relatively dephase and their average $\langle S_v^{\perp} \rangle$ shrinks. As $\langle \mathsf{S}_v \rangle$ swings past the plane, $\langle S_v \rangle$, and thus , $\langle M_1 \rangle$ has shrunk leading to flavor depolarization.

- The amount of flavor depolarization has a dependence on both the velocity mode and the lepton asymmetry A.

OUR ANALYTICAL APPROACH



OUR ANALYTICAL APPROACH

HOW TO CALCULATE THE EXTENT OF FLAVOR DEPOLARIZATION ?

Depolarization factor :

$$f_v^{\mathrm{D}} = \frac{1}{2}\left(1 - \langle S_v \rangle^{\mathrm{fin}} / \langle S_v \rangle^{\mathrm{ini}}\right)$$

$f_v^{\mathrm{D}} = 0.5$ → Complete depolarization

$f_v^{\mathrm{D}} = 0$ → No depolarization

We analytically compute f_v^{D} upto linear order using

a) Lepton number conservation (A)

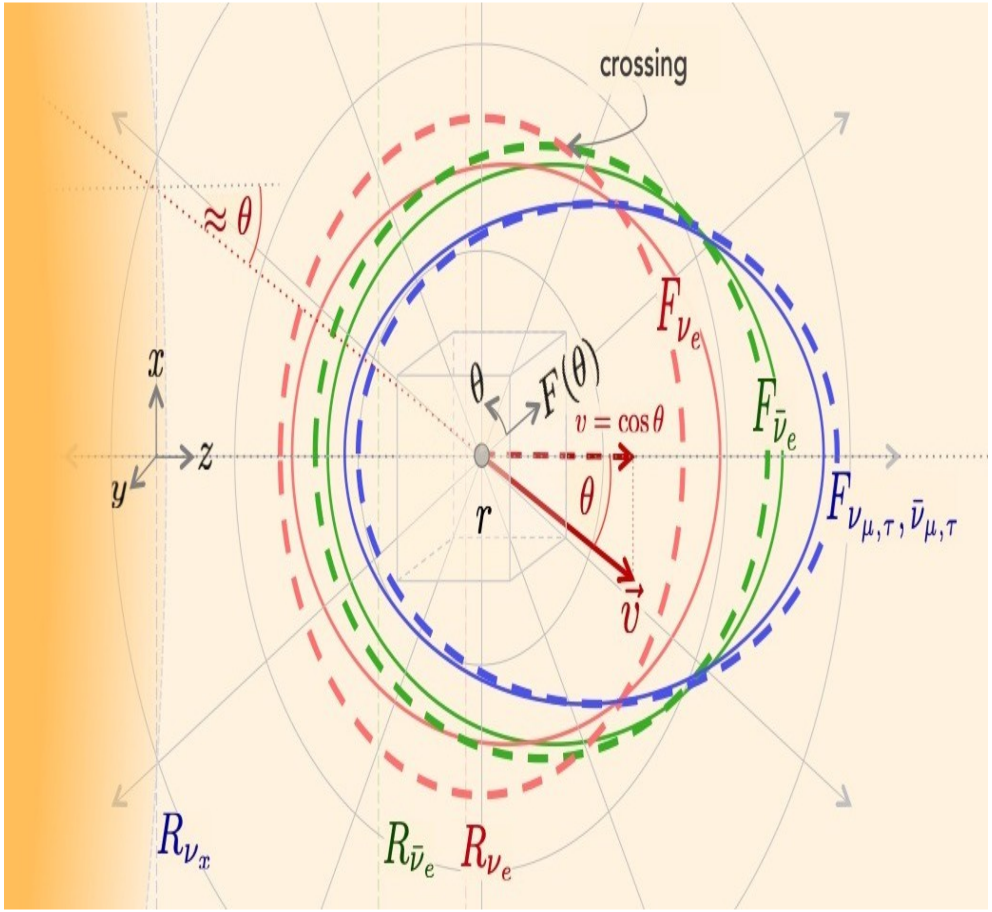
b) Neglecting the higher multipoles as they already diffuse away

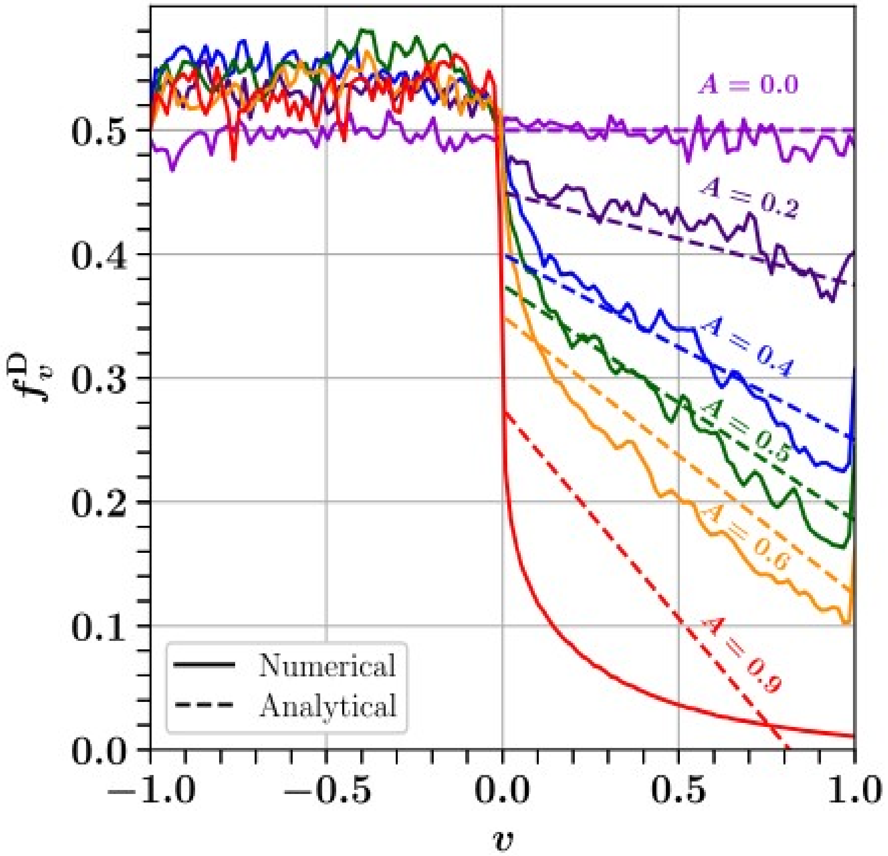
c) Considering v < 0 modes to be completely depolarized.

$$f_v^{\mathrm{D}} \approx \frac{1}{2} - \frac{A}{4} - \frac{3A}{8} v, \text{if } v > 0$$

$$f_v^{\mathrm{D}} \approx 0.5, \text{if } v < 0$$

RESULTS


crossing
$\approx \theta$
F_{ν_e}
$F_{\bar{\nu}_e}$
$F_{\nu_{\mu,\tau},\bar{\nu}_{\mu,\tau}}$
θ
$F(\theta)$
$v = \cos\theta$
x
y
z
r
θ
$\vec{v}$
R_{ν_x}
$R_{\bar{\nu}_e}$
R_{ν_e}


$A = 0.0$
$A = 0.2$
$A = 0.4$
$A = 0.5$
$A = 0.6$
$A = 0.9$
Numerical
Analytical
f_v^{D}
v
0.0
0.1
0.2
0.3
0.4
0.5
−1.0
−0.5
0.0
0.5
1.0

CONCLUSION

- We have presented an ana-lytical theory of fast neutrino flavor conversions in the nonlinear regime.

- We showed ***how***, as time passes, flavor differences over large ranges of velocity diffuse into varia-tions over smaller velocity ranges, or equivalent ranges of emission angles, causing depolarization.

- This introduces loss of information that leads to an apparent arrow of time out of the time-reversible EOM.

- We then showed that the epoch of T2 relaxation determines ***when*** depolarization occurs, and the initial lepton asymmetry A determines the rate of flavor depolarization.

- Finally, we gave a strategy and a formula for computing the ***extent*** of flavor depolarization
- This formula can be directly used in supernova simulations and for computing neutrino fluxes at Earth can have important implications for supernova astrophysics.