Five Texture Zeros for Dirac Neutrino Mass Matrices.

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Outline

Introduction

- Standard Model + 3 right handed neutrinos (SMRHN). Assuming Dirac masses for the neutrinos.
- Texture zeros.
- V_{PMNS} and CP-violating phase, the neutrino mass squared differences $\delta m^2_{21}, \delta m^2_{31}$, and the three charged lepton masses.
- Rotation Matrix as function of the mass values of the particles.
- Ramond, Robert and Ross are the type of textures that we are analysing. [Nucl. Phys. B, 19:406. 1993.]

Counting parameters

Yukawa Lagrangian for lepton sector

$$-\mathcal{L}_D = \bar{\nu}_L M_n \nu_R + \bar{\ell}_L M_\ell \ell_R + \text{h.c}, \tag{1}$$

where $\nu_{L,R} = (\nu_e, \nu_\mu, \nu_\tau)^T_{L,R}$ and $\ell_{L,R} = (e, \mu, \tau)^T_{L,R}$.

The matrices M_n and M_l are in general 3×3 complex matrices.

In the most general case, they contain 36 free parameters. In the context of the SMRHN, such a large number of parameters can be drastically cut by making use of the polar theorem of matrix algebra, by which, one can always decompose a general complex matrix as the product of a Hermitian and a unitary matrix.

Parameters

$$A = SU$$

S is a hermitian and U is an unitary matrix.

$$36 \xrightarrow{polar \quad theorem} 18.$$

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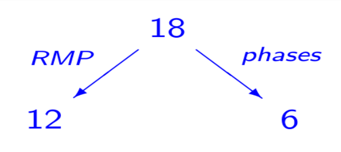

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$$\begin{tikzcd}
 & 18 \arrow[dl, "RMP"'] \arrow[dr, "phases"] & \\
12 & & 6
\end{tikzcd}$$

$$6 \xrightarrow{5 \quad are \quad absorbed} 1 \text{ phase.}$$

Weak Basis Transformation

In the context of the SMRHN, it is always possible to implement the so-called weak basis transformation (WBT).

$$M_n \to M_n^R = U M_n U^\dagger, \qquad M_\ell \to M_\ell^R = U M_\ell U^\dagger, \tag{2}$$

where U is an arbitrary unitary matrix. We say then that the two representations (M_n, M_ℓ) and (M_n^R, M_ℓ^R) are equivalent in the sense that they are related to the same Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix).

$$V_{PMNS}^R = U_l^R U_\nu^{R\dagger} = U_l U^\dagger U U_\nu^\dagger = U_l U_\nu^\dagger = V_{PMNS},$$

Always it is possible to find three non-physical texture zeros, that means, the number of free mathematical parameters in M_n and M_ℓ reduces from twelve to nine real parameters and one phase.

On the other hand, we have physical values for the six Dirac lepton masses, the three mixing angles, and the CP violation phase.

However, for the case of neutrinos, their masses are not known, and only the square mass differences are experimentally available, i.e., $\delta m_{21}^2 = m_2^2 - m_1^2$ and $\delta m_{31}^2 = m_3^2 - m_1^2$. Then have 9 free parameters and 8 experimental restrictions.

Five texture zeros: first case

Lagrangian

$$-\mathcal{L}_D = \bar{\nu}'_L M'_n \nu'_R + \bar{\nu}'_R M'^{\dagger}_n \nu'_L + \bar{\ell}'_L M'_\ell \ell'_R + \bar{\ell}'_R M'^{\dagger}_\ell \ell'_L, \tag{3}$$

where M'_n and M'_ℓ are the neutrino and charged lepton mass matrices respectively.

$$M'_n = \begin{pmatrix} c_n & a_n & 0 \\ a_n^* & 0 & b_n \\ 0 & b_n^* & 0 \end{pmatrix}, \qquad M'_\ell = \begin{pmatrix} 0 & a_\ell & 0 \\ a_\ell^* & d_\ell & b_\ell \\ 0 & b_\ell^* & c_\ell \end{pmatrix}. \tag{4}$$

Lagrangian

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The first step is to remove the phases; this can be done by the following unitary transformation:

$$M'_{n,\ell} = \lambda^{\dagger}_{n,\ell} M_{n,\ell} \lambda_{n,\ell}, \tag{5}$$

which is achieved by using the diagonal matrices
$\lambda_n = (1, e^{i\alpha_{n_1}}, e^{i\alpha_{n_1}+i\alpha_{n_2}})$ and $\lambda_\ell = (1, e^{i\alpha_{\ell_1}}, e^{i\alpha_{\ell_1}+i\alpha_{\ell_2}})$,

If we rotate these matrices by using the orthogonal transformation $R_{n,\ell}$ ($R^T_{n,\ell}\, R_{n,\ell} = 1$)

$$M'_{n,\ell} = \lambda^\dagger_{n,\ell} R^T_{n,\ell} \begin{pmatrix} m_{1,e} & 0 & 0 \\ 0 & -m_{2,\mu} & 0 \\ 0 & 0 & m_{3,\tau} \end{pmatrix} R_{n,\ell} \lambda_{n,\ell} \equiv U_{n,\ell} M^{\mathrm{diag}}_{n,\ell} U^\dagger_{n,\ell}, \quad (6)$$

If we rotate these matrices by using the orthogonal transformation $R_{n,\ell}$ ($R_{n,\ell}^T R_{n,\ell} = 1$)

$$M'_{n,\ell} = \lambda_{n,\ell}^\dagger R_{n,\ell}^T \begin{pmatrix} m_{1,e} & 0 & 0 \\ 0 & -m_{2,\mu} & 0 \\ 0 & 0 & m_{3,\tau} \end{pmatrix} R_{n,\ell}\lambda_{n,\ell} \equiv U_{n,\ell} M_{n,\ell}^{\text{diag}} U_{n,\ell}^\dagger, \quad (6)$$

We use $U_n \equiv (R_n \lambda_n)^\dagger$ and $U_\ell \equiv (R_\ell \lambda_\ell)^\dagger$, that are two unitary matrices used to rotate from the weak basis to the physical basis.

$$\nu'_{L,R} = U_n \nu_{L,R}, \qquad \ell'_{L,R} = U_\ell \ell_{L,R}. \quad (7)$$

Replacing these expressions in the lepton sector of the weak current, we obtain

$$\mathcal{L}_{W^-} = -\frac{g}{\sqrt{2}} W^- \bar{\ell}'_L \gamma^\mu \nu'_L + \text{h.c} = -\frac{g}{\sqrt{2}} W^- \bar{\ell}_L \gamma^\mu U_\ell^\dagger U_n \nu_L + \text{h.c}, \qquad (8)$$

in such a way that the PMNS matrix is given by

$$V_{\text{PMNS}} = U_\ell^\dagger U_n = R_\ell \, \Phi \, R_n^T \,, \qquad (9)$$

normal ordering is assumed, i.e.,: $m_3 > m_2 > m_1$, where:
$m_2^2 = m_1^2 + \delta m_{21}^2$, and $m_3^2 = m_1^2 + \delta m_{31}^2$, with $\delta m_{21}^2, \delta m_{31}^2 > 0$

By imposing the invariance of the trace and the determinant of the mass matrices ($\mathrm{tr}[M'_{n,\ell}] = \mathrm{tr}[M^{\mathrm{diag}}_{n,\ell}]$, $\mathrm{tr}\left[\left(M'_{n,\ell}\right)^2\right] = \mathrm{tr}\left[\left(M^{\mathrm{diag}}_{n,\ell}\right)^2\right]$, and $\mathrm{Det}[M'_{n,\ell}] = \mathrm{Det}[M^{\mathrm{diag}}_{n,\ell}]$), the following relations are obtained for this particular texture:

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$$
\begin{aligned}
c_n &= m_1 - m_2 + m_3, & d_\ell &= m_e - m_\mu + m_\tau - c_\ell, \\
|a_n| &= \sqrt{\frac{(m_1 - m_2)(m_1 + m_3)(m_2 - m_3)}{m_1 - m_2 + m_3}}, & |b_\ell| &= \sqrt{\frac{(c_\ell - m_e)(c_\ell + m_\mu)(m_\tau - c_\ell)}{c_\ell}}, \\
|b_n| &= \sqrt{\frac{m_1\, m_2\, m_3}{m_1 - m_2 + m_3}}, & |a_\ell| &= \sqrt{\frac{m_e\, m_\mu\, m_\tau}{c_\ell}}.
\end{aligned}
$$

From the previous identifications, it is possible to obtain an explicit form for the mass matrices of leptons that allows us to obtain, through diagonalization of M_n and M_ℓ, the orthogonal matrices in Eq. (9),

$$R_n = \begin{pmatrix} -\sqrt{\frac{m_1(m_2-m_1)(m_1+m_3)}{(m_1+m_2)(m_3-m_1)(m_1-m_2+m_3)}} & \sqrt{\frac{m_1(m_3-m_2)}{(m_1+m_2)(m_3-m_1)}} & \sqrt{\frac{m_2 m_3(m_3-m_2)}{(m_1+m_2)(m_3-m_1)(m_1-m_2+m_3)}} \\ \sqrt{\frac{m_2(m_1-m_2)(m_2-m_3)}{(m_1+m_2)(m_2+m_3)(m_1-m_2+m_3)}} & -\sqrt{\frac{m_2(m_1+m_3)}{(m_1+m_2)(m_2+m_3)}} & \sqrt{\frac{m_1 m_3(m_1+m_3)}{(m_1+m_2)(m_2+m_3)(m_1-m_2+m_3)}} \\ \sqrt{\frac{m_3(m_1+m_3)(m_3-m_2)}{(m_3-m_1)(m_2+m_3)(m_1-m_2+m_3)}} & \sqrt{\frac{m_3(m_2-m_1)}{(m_2+m_3)(m_3-m_1)}} & \sqrt{\frac{m_1 m_2(m_2-m_1)}{(m_3-m_1)(m_2+m_3)(m_1-m_2+m_3)}} \end{pmatrix},$$

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$$R_\ell = \begin{pmatrix} -\sqrt{\frac{m_\mu m_\tau(c_\ell-m_e)}{c_\ell(me+m_\mu)(m_\tau-m_e)}} & -\sqrt{\frac{m_e(c_\ell-m_e)}{(m_e+m_\mu)(m_\tau-m_e)}} & \sqrt{\frac{m_e(c_\ell+m_\mu)(c_\ell-m_\tau)}{c_\ell(m_e+m_\mu)(m_e-m_\tau)}} \\ \sqrt{\frac{m_e m_\tau(c_\ell+m_\mu)}{c_\ell(m_e+m_\mu)(m_\mu+m_\tau)}} & -\sqrt{\frac{m_\mu(c_\ell+m_\mu)}{(m_e+m_\mu)(m_\mu+m_\tau)}} & \sqrt{\frac{m_\mu(m_e-c_\ell)(c_\ell-m_\tau)}{c_\ell(m_e+m_\mu)(m_\mu+m_\tau)}} \\ \sqrt{\frac{m_e m_\mu(c_\ell-m_\tau)}{c_\ell(m_e-m_\tau)(m_\mu+m_\tau)}} & \sqrt{\frac{m_\tau(m_\tau-c_\ell)}{(m_\tau-m_e)(m_\mu+m_\tau)}} & \sqrt{\frac{m_\tau(c_\ell-m_e)(c_\ell+m_\mu)}{c_\ell(m_\tau-m_e)(m_\mu+m_\tau)}} \end{pmatrix}.$$

From Eq. (9), we know that $V_{\mathrm{PMNS}} = R_\ell \, \Phi \, R_n^T$, with Φ as the following diagonal matrix:

$$\Phi = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{pmatrix} .$$

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We use an χ^2 analysis, using as free parameters $(m_1, c_\ell, \phi_1, \phi_2)$., in this way:

$$\chi^2 = P_J^2 + \sum_{i,j=1,2,3} P_{ij}^2,$$

where the pulls are

$$P_{ij} = \frac{U_{ij} - \bar{U}_{ij}}{\delta U_{ij}}$$

The Jarlskog invariant, is given by

$$\bar{J} = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta = -0.0270054$$

and the corresponding 1σ uncertainty is $\delta J = 0.0106304$

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and the corresponding 1σ uncertainty is $\delta J = 0.0106304$ The theoretical prediction is given by $J = \mathrm{Im}\left(U_{\mu 3}U_{\tau 3}^{*}U_{\mu 2}U_{\tau 2}^{*}\right)$, where in this expression U stands for the PMNS mixing matrix.

Our best fit results are

$m_1\,(\mathrm{eV})$	$c_\ell\,(\mathrm{eV})$	$\phi_1\,(\mathrm{rad})$	$\phi_2\,(\mathrm{rad})$	χ^2_{min}
$0.00395\pm^{0.00062}_{0.00078}$	523176.	0.0190664	1.56122	12.4204

P_{11}	P_{12}	P_{13}	P_{21}	P_{22}	P_{23}	P_{31}	P_{32}	P_{33}	P_J
0.428531	−0.385085	0.0430767	−0.205321	−1.2577	1.91336	0.290472	1.25036	−2.2701	0.0228083

The fit goodness is $\chi^2/\mathrm{d.o.f} = 2.07$

Five texture zeros: second case

Now the textures are

$$M'_n = \begin{pmatrix} 0 & C_n & 0 \\ C_n^* & E_n & B_n \\ 0 & B_n^* & A_n \end{pmatrix}, \quad M'_\ell = \begin{pmatrix} 0 & C_\ell & 0 \\ C_\ell^* & 0 & B_\ell \\ 0 & B_\ell^* & A_\ell \end{pmatrix}. \tag{10}$$

$M_\ell = U_\ell D_\ell U_\ell^\dagger$ $(M_\ell)_{i,j} = |(M'_\ell)_{i,j}|)$, where $D_\ell =$Diag.$(m_e, -m_\mu, m_\tau)$, Now the textures are

$$M_n' = \begin{pmatrix} 0 & C_n & 0 \\ C_n^* & E_n & B_n \\ 0 & B_n^* & A_n \end{pmatrix}, \quad M_\ell' = \begin{pmatrix} 0 & C_\ell & 0 \\ C_\ell^* & 0 & B_\ell \\ 0 & B_\ell^* & A_\ell \end{pmatrix}. \tag{10}$$

$M_\ell = U_\ell D_\ell U_\ell^\dagger \ (M_\ell)_{i,j} = |(M_\ell')_{i,j}|)$, where $D_\ell =$Diag.$(m_e, -m_\mu, m_\tau)$,

$$U_\ell = \begin{pmatrix} e^{i\theta_1}\sqrt{\frac{m_\mu m_\tau (A_\ell - m_e)}{A_\ell (m_\mu + m_e)(m_\tau - m_e)}} & -e^{i\theta_2}\sqrt{\frac{m_e m_\tau (m_\mu + A_l)}{A_\ell (m_\mu + m_e)(m_\tau + m_\mu)}} & \sqrt{\frac{-m_e m_\mu (A_\ell - m_\tau)}{A_\ell (m_\tau - m_e)(m_\tau + m_\mu)}} \\ e^{i\theta_1}\sqrt{\frac{m_e (m_e - A_\ell)}{(-m_\mu - m_e)(m_\tau - m_e)}} & e^{i\theta_2}\sqrt{\frac{m_\mu (A_\ell + m_\mu)}{(m_\mu + m_e)(m_\tau + m_\mu)}} & \sqrt{\frac{m_\tau (m_\tau - A_\ell)}{(m_\tau - m_e)(m_\tau + m_\mu)}} \\ -e^{i\theta_1}\sqrt{\frac{m_e (A_l + m_\mu)(A_\ell - m_\tau)}{A_\ell (-m_\mu - m_e)(m_\tau - m_e)}} & -e^{i\theta_2}\sqrt{\frac{m_\mu (A_\ell - m_e)(m_\tau - A_\ell)}{A_\ell (m_\mu + m_e)(m_\tau + m_\mu)}} & \sqrt{\frac{m_\tau (A_l - m_e)(A_\ell + m_\mu)}{A_\ell (m_\tau - m_e)(m_\tau + m_\mu)}} \end{pmatrix}, \tag{11}$$

where θ_1 and θ_2 are arbitrary phases and $A_\ell = m_e - m_\mu + m_\tau$.

To obtain the three texture zeros in the lepton mass matrix, the following relations are also necessary:

$$|B_\ell| = \sqrt{\frac{(A_\ell - m_e)(A_\ell + m_\mu)(m_\tau - A_\ell)}{A_\ell}} \qquad \text{and} \qquad |C_\ell| = \sqrt{\frac{m_e\, m_\mu\, m_\tau}{A_\ell}}.$$

For the neutrino sector, we are subject to the condition $U_\ell^\dagger U_n = V_{\text{PMNS}}$, and necessarily, the diagonalizing matrix must be given by $U_n = U_\ell V_{\text{PMNS}}$,

$$M_n' = \begin{pmatrix} 0 & C_n & 0 \\ C_n^* & E_n & B_n \\ 0 & B_n^* & A_n \end{pmatrix} = U_\ell (V_{\text{PMNS}}) D_n (V_{\text{PMNS}})^\dagger U_\ell^\dagger \equiv U_n D_n U_n^\dagger. \qquad (12)$$

For this case the free parameters are: m_1 from the diagonal matrix D_n=Diag.$(m_1, -m_2, m_3)$, and θ_1 and θ_2 from U_ℓ.

From these expressions, we can obtain useful relations by identifying U_ℓ with the WBT U in (2).

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When solving numerically to obtain the textures for the neutrino mass matrix in the normal hierarchy, we obtain

$$\begin{aligned} m_1 &= (0.00354 \pm 0.00088)\,\mathrm{eV}, \\ m_2 &= (0.00930 \pm 0.00036)\,\mathrm{eV}, \\ m_3 &= (0.05040 \pm 0.00030)\,\mathrm{eV}. \end{aligned} \tag{13}$$

With $A_n = 0.0251821$, $B_n = (-0.0122955 + 0.0244187i)$, $C_n = (0.00427236 + 0.00689527i)$, $E_n = 0.0194623$,

$A_\ell = 1671.71 \times 10^6$, $|B_\ell| = 432.237 \times 10^6$, $|C_\ell| = 7.57544 \times 10^6$, and phases $\theta_1 = 0.154895$ and $\theta_2 = 2.01797$. The phases of B_ℓ and C_ℓ were absorbed in B_n and C_n by means of a redefinition, through a WBT, in a previous step.

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And,

$\theta_{12}(^\circ)$	$\theta_{23}(^\circ)$	$\theta_{13}(^\circ)$	$\delta_{CP}(^\circ)$	δm^2_{21} (eV^2)	δm^2_{31} (eV^2)	m_e(MeV)	m_μ(MeV)	m_τ(MeV)
33.82	48.6	8.60	221	7.39×10^{-5}	2.528×10^{-3}	0.510 999	105.658	1776.86

Conclusions

- Using textures zeros it is possible to determine the neutrino masses.
- An ansatz for the lepton mass matrices emerges from the quark-lepton similarity, allowing us to extend the analysis of the mass matrices from the quark sector to the lepton sector, which is a natural.
- Without losing generality, the mass matrices of the lepton sector can be Hermitian in such a way that it is possible to apply the WBT formalism without any restriction
- In both cases, the mass of the lightest neutrino can be considered as a prediction of the models studied.

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Thank you!