Non-linear Regge trajectories with AdS/QCD

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   - From holographic grounds
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Why non-linear trajectories?

Regge Trajectories

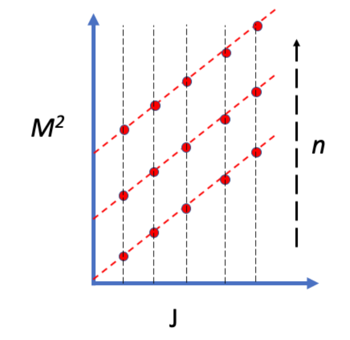

Figure: Regge Trajectory

Regge trajectories are a systematic form to organize hadronic states in terms of their quantum numbers. We will focus on the *radial trajectories*, which can be generically written as

$$M^2 = a(n+b)^\nu.$$

These sort of trajectories come naturally in potential models, Bethe-Salpeter equation analysis and other effective approaches (See Anisovich's nice book!). These objects also provide a tool to test confinemenent in effective models for hadrons.

Why non-linear trajectories

Important Remarks coming from the experimental data:

Light hadron case:

- Linearity ($\nu = 1$) works ($R^2 = 0.999$) for light (unflavored) meson spectra.
- The mass gap between states in the trajectory for light hadrons seems to be constant.

Flavored and heavy Hadrons case:

- Adding *s*-quarks will cause deviations from linearity (Gershtein et al., 2006).
- For heavy quarkonia, linear trajectories do not have the same accuracy, but they are still a good approach.
- The mass gap between the ground state and the first excitation in heavy quarkonia is bigger than the gap between higher excitations.
- Linearity emerges in the high excitation number limit.

Our Hadronic lab

We can observe this phenomenological landscape in the *isovector meson family* containing ρ, ω, ϕ, J/ψ, and Υ mesons.

Some examples: Isovector $I^G J^{PC} = 0^- (1^{--})$

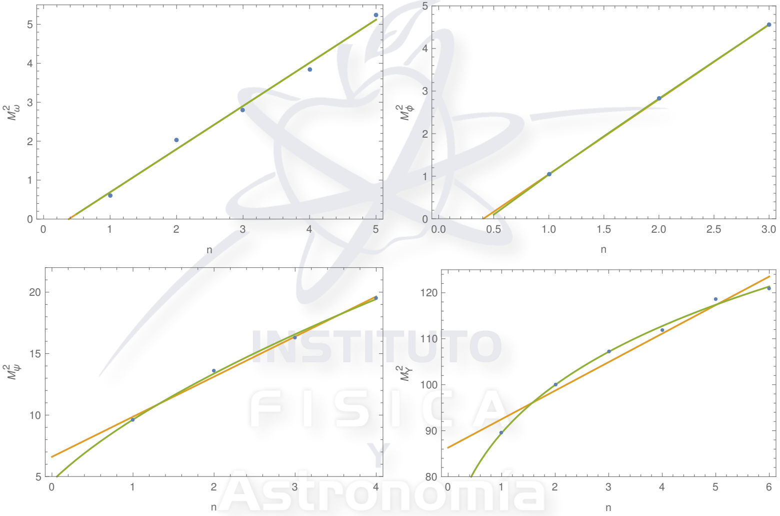

Figure: M^2 vs n for isovector mesons $(\omega, \phi, \psi$ and $\Upsilon)$. Dots represent experimental data (PDG).

Non-linearity Hypothesis

Hypothesis

Linearity is connected with the hadron constituent mass: when constituent mass raises, linearity ceases. The linear case appears when constituent quark masses are supposed to be zero, i.e., they are small enough compared with the meson mass.

From Bethe-Salpeter (Afonin and Pusenkov, 2014; J. K. Chen, 2018) we can write the trajectory as

$$(M_n - m_{q_1} - m_{q_2})^2 = a(n+b).$$

When the limit $m_{q_{1,2}\to\infty}$ (as in heavy quarkonium), the trajectory acquires a generic non-linear form:

$$M_n^2 \propto n^{2/3}.$$

Also in heavy-light systems, non-linearity is expected (J. K. Chen, 2018).

Our goal is how to translate this hypothesis into a holographic bottom-up language.

AdS and Confinement

AdS/CFT correspondence original ideas deal with the duality between a conformal FT and a gravity theory.

Non-conformality is translated into the existence of confinement: for some energies, hadrons are bounded. For others, they break apart.

QCD is not a conformal theory (Oops!).

Therefore, one evidence of the presence of confinement is the Regge Trajectories. These objects *inherit* the stringy behavior from the strong interaction ancient times. This feature is recovered in holographic QCD.

How do we include confinement into AdS/CFT?

Just "break" the conformal invariance by introducing an energy scale. In our particular case, this scale defines the Regge slope.

This is why we study Regge trajectories in AdS/QCD!

Bottom-up and Confinement

How is confinement realized in bottom-up models?

In the particular bottom-up case, confinement is realized via many approaches, roughly summarized as

- explicitly, by introducing a cutoff to the AdS space. This is the hardwall model (Braga and Boshi-Filho, 2005, Polchinski-Strassler 2006).
- softly by introducing a smooth quadratic and static dilaton field (Karch et. al. 2006).
- mixing both approaches: a UV cutoff and a static and quadratic dilaton (Braga, M.A. Martin and Diles, 2014).
- deforming AdS geometry (Forkel, 2006; Capossoli et al., 2020).
- These methods are just the tip of the iceberg! We have other proposals, including dynamical dilatons in fixed AdS backgrounds, interpolating dilatons, dynamical AdS-like backgrounds, *et cetera*.

General Bottom-up Algorithm

AdS-like Background

$$dS^2 = \frac{R^2}{z^2}\, e^{h(z)} \left[dz^2 + \eta_{\mu\nu}\, dx^\mu\, dx^\nu\right]$$

with R defined as the AdS radius and $h(z)$ a geometric deformation.

General Action with minimal coupling

$$I_{\text{SWM}} = \int d^5x\, \sqrt{-g}\, e^{-\Phi(z)}\, \mathcal{L}_{\text{Hadron}},$$

with $\Phi(z)$ defined as a static dilaton field, responsible for inducing the conformal symmetry breaking, i.e, confinement. The lagrangian $\mathcal{L}_{\text{Hadron}}$ has the information about the bulk field dual to hadrons.

General Bottom-up Algorithm

Holographic Potential

The action written above defines a set of equation of motion for the bulk fields that, in general, has a Schrödinge-like form:

$$-\psi''(z) + V(z)\,\psi(z) = M_n^2\,\psi(z),$$

where $V(z)$ is the holographic potential written in terms of the deformation and the dilaton. In the case of $p-$form bulk fields as follows:

$$V(z) = \frac{1}{4}B'(z)^2 - \frac{1}{2}\,B''(z) + \frac{M_5^2\,R^2}{z^2}e^{h(z)}, \tag{1}$$

with $B(z) = \Phi(z) + \beta\,[\log\frac{R}{z} + \frac{1}{2}h(z)]$, M_5 is the bulk mass associated to $\psi(z)$ and $\beta = -(3 - 2\,p)$. Latter we will connect β with the hadronic (integer) spin.

In our particular case, since we want to deal with no geometric deformations, we will fix $h(z) = 0$. This is the case of the so-called softwall-like models (Karch *et al.* 2006).

General Bottom-up Algorithm

Holographic Regge Trajectories

Regge trajectories will emerge as the eigenvalue spectrum associated to the Sturm-Liouville problem defined by $V(z)$:

$$M_n^2 = A\,(n + B)^{\nu},$$

where A is an energy scale defined by the dilaton and/or deformation, B carries information about the hadronic angular momentum, and ν measures linearity. If the deformations and dilatons are quadratic at the high-z limit, the out-coming trajectory will be linear This is supported by WKB analysis.

Hadronic Identity: how we constuct hadrons in bottom-up holographic QCD

Hadrons are characterized by the scaling dimension of the boundary operator $\hat{\mathcal{O}}$ that creates them. This quantity is fixed to match the bulk field conformal dimension, Δ. According to the original AdS/CFT, the bulk mass M_5 carries the information Δ as follows:

- $q\,\bar{q}$ and non-$q\,\bar{q}$states:

$$M_5^2\,R^2 = (\Delta - S)(\Delta + S - 4)$$

- Baryons of spin $1/2$ with $\Delta = 3/2$:

$$m_5 = \Delta + 2.$$

Thus, the bulk mass defines the *hadronic identity* of the state at hand. For example, for mesons:

$$\hat{\mathcal{O}} = \bar{q}(x)\,q(x) \;\to \Delta = 3, \text{ thus: } M_5^2\,R^2 = (3 - S)(S - 1).$$

Hadronic Identity

Scalar hadrons		Vector hadrons	
$(n\text{Q})(m\text{G})$	Δ	$(n\text{Q})(m\text{G})$	Δ
(2Q)	3	(2Q)	3
(2G)	4	(2Q)(1G)	5
(2Q)(1G)	5	(4Q) or (3G)	6
(4Q)	6	(2Q)(2G)	7
(2Q)(2G)	7	(4Q)(1G)	8
(4Q)(1G) or (4G)	8	(6Q) or (2Q)(3G)	9
(6Q) or (2Q)(3G)	9	(5G) or (4Q)(2G)	10

Table: Possible hadronic states composed by n quarks (or antiquarks) and m gluons and their conformal dimensions.

Holographic algorithm: a summary

The road so far:

- Define a geometry background, deformations and dilaton field.
- Define an action for the bulk fields dual to hadronic states.
- Obtain equations of motion.
- Solve the associated Sturm-Liouville problem (Boundary Value Problem).
- Find the mass spectrum as the eigenvalues of the BVP.
- Evaluate the Regge Trajectory.

Example: Holographic potential in SWM for Vector Mesons

In the case of vector mesons, fix $M_5^2 R^2 = 0$, $\beta = -1$ and $\Delta = 3$. Thus:

From the action (Karch et al., 2006):

$$I = -\frac{1}{4 g_5^2} \int d^5x \sqrt{-g}\, e^{-\kappa^2 z^2} F_2^2,$$

with $F_2 = d A_1$, we obtain the following potential and spectrum:

$$V(z) = \frac{3}{4 z^2} + \kappa^4 z^2$$

$$M_n^2 = 4 \kappa^2 (n+1).$$

Notice that the trajectory depends on the the energy scale, and therefore it is flavor-dependent.

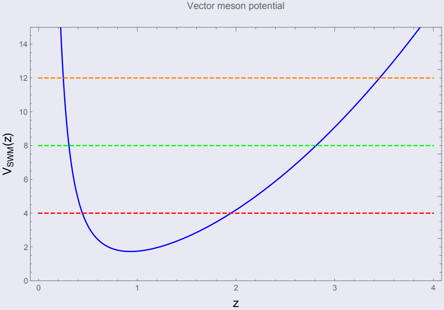

Figure: Holographic potential for vector mesons, along with the ground state and the first two excited states.

Non-quadratic Dilaton

Our proposal

In order to induce non-linear Regge Trajectories we define

$$\Phi(z) = (\kappa\, z)^{2-\alpha}$$

as a deformation of softwall model (quadratic) dilaton field.

Holographic potential for vector hadrons ($\beta = -1$)

This dilaton defines the following potential:

$$V(z,\kappa,\alpha) = \frac{3}{4z^2} - \frac{1}{2}\alpha^2\,\kappa^2\,(\kappa\, z)^{-\alpha} + \frac{1}{4}\alpha^2\,\kappa^2(\kappa\, z)^{2-2\alpha} + \frac{3}{2}\alpha\,\kappa^2\,(\kappa\, z)^{-\alpha} - \kappa^2(\kappa\, z)^{-\alpha} - \alpha\,\kappa^2\,(\kappa\, z)^{2-2\alpha} + \kappa^2\,(\kappa\, z)^{2-2\alpha} + \frac{\kappa}{z}(\kappa\, z)^{1-\alpha} - \frac{\alpha\,\kappa}{2\,z}(\kappa\, z)^{1-\alpha} + \frac{M_5^2(\Delta)\,R^2}{z^2}$$

Non-quadratic dilaton

Initial Parameters

- κ: sets the energy scale for the trajectory. It is flavor dependent.
- α: runs with the quark constituent mass. Heavier hadrons will have bigger deviations from linearity in their trajectories.

Our "playground"

We will prove this approach with the isovector meson family (ω, ϕ, J/ψ, and Υ) labeled as $I^G\left(J^{PC}\right) = 0^-(1^{--})$.
We do not analyze ρ mesons, which is the lightest isovector, since it is described in the usual softwall model (Karch et al., 2006). For us, this case implies $\alpha = 0$.

Light unflavored and flavored Isovector Fitting

ω with $\alpha = 0.04$ and $\kappa = 498$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	782.65 ± 0.12	981.43	25.4
2	$1400 - 1450$	1374	3.6
3	1670 ± 30	1674	0.25
4	1960 ± 25	1967	1.7
5	2290 ± 20	2149	6.2
$M_n^2 = 0.9514(0.012 + n)^{0.9798}$ with $R^2 = 0.999$			
ϕ with $\alpha = 0.07$ and $\kappa = 585$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	1019.461 ± 0.016	1139.43	11.8
2	1698 ± 20	1583	5.8
3	$2135 \pm 8 \pm 9$	1921	10
$M_n^2 = 1.268(0.0244 + n)^{0.9650}$ with $R^2 = 0.999$			

Table: Summary of results for heavy isovector radial mesonic states considered in this work. Experimental results are read from PDG.

Heavy Isovector Fitting

ψ **with** $\alpha = 0.54$ **and** $\kappa = 2150$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	3096.916 ± 0.011	3077.09	0.61
2	3686.109 ± 0.012	3689.62	0.1
3	4039 ± 1	4137.5	2.44
4	4421 ± 4	4499.4	1.77
$M_n^2 = 8.07(0.287 + n)^{0.6315}$ with $R^2 = 0.999$			

Υ **with** $\alpha = 0.863$ **and** $\kappa = 11209$ MeV			
n	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	9460.3 ± 0.26	9438.5	0.23
2	10023.26 ± 0.32	9923.32	0.78
3	10355 ± 0.5	10277.2	0.75
4	10579.4 ± 1.2	10558.6	0.19
5	$10889.9^{+3.2}_{-2.6}$	10793.5	0.88
6	$10992.9^{+10.0}_{-3.1}$	10995.7	0.03
$M_n^2 = 76.511(0.901 + n)^{0.2369}$ with $R^2 = 0.999$			

Table: Summary of results for light isovector radial mesonic states considered in this work. Experimental results are read from PDG.

Non-linear fitting

The non-quadratic approach induces a running of the parameters κ and α in terms of the inner mesonic structure, parametrized by the average constituent mass $\bar{m}$.

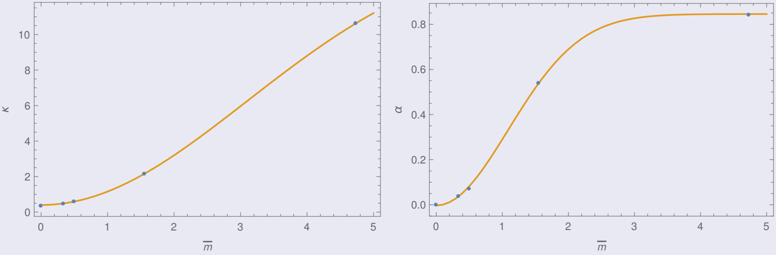

Figure: Running of κ and α in terms of $\bar{m}$.

where for mesons we have

$$\bar{m}(q_1, q_2) = \frac{1}{2}\left(m_{q_1} + m_{q_2}\right).$$

Running of α and κ with $\bar{m}$

Methodology

This specific running allows us to extend this model to other hadronic species by using a particular choice of $\bar{m}$ as the input to read off the associated value of κ and α.

Then, we use these new data set and the scaling dimension Δ of the operator that creates such states to feed up the algorithm in order to get the associated mass spectrum.

Recall that the hadronic identity for vector states is given by:

$$M_5^2 R^2 = (\Delta - 1)(\Delta - 3)$$

Testing our model

We will use this idea to explore the mass spectrum of vector kaons and heavy-light mesons (B and D systems).

Vector Kaons

Vector kaons are mesonic states labeled by $I(J^P) = 1/2(1^-)$, with $S = \pm 1$ and $C = B = 0$ and $\Delta = 3$. Also we define

$$\bar{m}_{K^*} = \frac{m_s + m_d}{2}, \tag{2}$$

with $m_s = 0.486$ and $m_d = m_u = 0.336$ GeV. The numerical results are summarized in the following table

K^* **with** $\bar{m} = 413$ **MeV,** $\alpha = 0.055$**, and** $\kappa = 531.24$ MeV				
n	**State**	M_{Exp} **(MeV)**	M_{Th} **(MeV)**	**R. E. (%)**
1	$K^*(892)$	895.55 ± 0.8	1038.4	16.2
2	$K^*(1410)$	1414 ± 15	1451.0	2.6
3	$K^*(1680)$	1718 ± 18	1754.5	2.1
Experimental Linear R. T.:	$M^2 = 1.075(-0.2157 + n)$ with $R^2 = 0.9992$.			
Experimental Non-Linear R. T.:	$M^2 = 1.157(-0.6102 + n)^{0.718}$ with $R^2 = 1$.			
Theoretical Non-Linear R. T.:	$M^2 = 1.175(-0.0911 + n)^{0.902}$ with $R^2 = 1$.			

Table: Summary of results for the vector kaon K^* radial states. The last column is the relative error per state. Experimental results are read from PDG

Vector Heavy-light mesons

These mesons are labeled as $I(J^P) = 1/2(1^-)$, with $\Delta = 3$. The average constituent mass is

$$\bar{m}_{qQ} = \frac{m_q + m_Q}{2},$$

with $m_u = m_d = 336$ MeV, $m_s = 0.486$ MeV, $m_c = 1550$ MeV, and $m_b = 4730$ MeV. The numerical results are summarized as:

State	$q_1\, q_2$	$\bar{m}$ (MeV)	κ (MeV)	α	M_{Exp} (MeV)	M_{Th} (MeV)	R. E. (%)
$K^*(782)$	$d\,\bar{s}$	413	531.24	0.055	895.55 ± 0.8	1038.4	16.2
$D^{*0}(2007)$	$c\,\bar{u}$	943	1070.8	0.261	2006.85 ± 0.05	1902.5	5.20
$D^{+0}(2010)$	$c\,\bar{d}$	945	1073.6	0.262	2010.26 ± 0.05	1906.4	5.16
D_s^{*+}	$c\,\bar{s}$	1018	1179.1	0.296	2112.2 ± 0.4	2051.7	2.86
B^{*+}	$u\,\bar{b}$	2533	4681.2	0.800	5324.70 ± 0.22	4561.2	14.3
B^{*0}	$d\,\bar{b}$	2535	4687.3	0.801	5324.70 ± 0.22	4564.4	14.27
B_s^{*0}	$s\,\bar{b}$	2608	4901.2	0.809	$5415^{+1.8}_{-1.5}$	4683.0	13.52

Table: Summary of results for vector heavy-light mesonic states contrasting our theoretical results with the available experimental data. The last column is the relative error per state. Experimental results are read from PDG. Although D_s^{*+} has not been fully identified, their decay modes are consistent with $J^P = 1^-$. See PDG.

Bonus: Non-$q\,\bar{q}$ states

Another interesting test

We also can study non-$q\,\bar{q}$ states and test which structure works better from holographic grounds. We will focus on the description of tetraquarks in the context of multiquark and gluonic excitation models. (See Bambrilla et al. nice review, 2014).

Average constituent mass parametrizations

Gluonic excitations and multiquark states can be summarized in a single parametrization in terms of the number of constituents N per state:

$$\bar{m}_{\text{non-}q\bar{q}} = \sum_{i=1}^{N} \left(P_i^{\text{quark}}\,\bar{m}_{q_i} + P_i^{\text{meson}}\,m_{\text{meson}_i} + P_i^{\text{gluon}}\,m_{\text{gluon}_i}\right)$$

$$1 = \sum_{i=1}^{N} \left(P_i^{\text{quark}} + P_i^{\text{meson}} + P_i^{\text{gluon}}\right),$$

To switch between models consider the following:

- Multiquark states: take $P_i^{\text{gluon}} = 0$.
- Gluonic excitations: take $P_i^{\text{gluon}} \neq 0$.

Bonus: Non-$q\bar{q}$ states

Non-$q\bar{q}$ taxonomy

- Multi-quark states:
  - Diquarks.
  - Hadroquarkonium.
  - Hadronic molecule.
- Gluonic excitations:
  - Hybrid vector mesons.

Non-$q\bar{q}$ candidates (PDG)

We will analyze tetraquark candidates split into:

- Multiquark states:
  - Heavy sector: Z_C, Z_B, and ψ states.
- Gluonic excitations:
  - Light sector: π_1 states.
  - Heavy sector: Z_C and Z_B states.

Methodology

We will test each non-$q\bar{q}$ state with our model by using $\bar{m}$ as entry to the κ and α curves, and then computing the corresponding mass spectrum. After that, we will compare with the experimental data.

Bonus: Tetraquarks as Multi-quark states

Holographic spectrum		Non-$q\bar{q}$ states			
$\Delta = 6$ and $\bar{m}_{\text{diquark-antidiquark}}$		Multiquark state			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	4004.8	1	$Z_c(3900)$	3887.2 ± 2.3	3.0
2	4384.9	2	$Z_c(4200)$	4196^{+35}_{-32}	4.5
3	4706.6	3	$Z_c(4430)$	4478^{+15}_{-18}	5.1
$\Delta = 6$ and $\bar{m}_{\text{hadronic molecule}}$		Multiquark state			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	3816.3	1	$Z_c(3900)$	3887.2 ± 2.3	1.82
2	4213.9	2	$Z_c(4200)$	4196^{+35}_{-32}	0.43
3	4551.4	3	$Z_c(4430)$	4478^{+15}_{-18}	1.64
$\Delta = 6$ and $\bar{m}_{\text{Hadrocharmonium}}$		Multiquark state			
$\alpha = 0.604$ and $\kappa = 2523$ MeV		$I^G(J^{CP}) = 0^+(1^{--})$ Y or ψ mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	4228.3	1	$\psi(4260)$	4230 ± 8	0.25
2	4577.3	2	$\psi(4360)$	4368 ± 13	4.8
3	4871.8	3	$\psi(4660)$	4643 ± 9	4.9
$\Delta = 6$ and $\bar{m}_{\text{Hadronic Molecule}}$		Multiquark state			
$\alpha = 0.538$ and $\kappa = 1548.7$ MeV		$I^G(J^{CP}) = 0^+(1^{--})$ Y or ψ mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	40027.8	1	$\psi(4260)$	4230 ± 8	5.37
2	4383.1	2	$\psi(4360)$	4368 ± 13	0.35
3	4705.1	2	$\psi(4360)$	4643 ± 9	1.34

Bonus: Tetraquarks as Multi-quark states

Holographic spectrum		Non-$q\,\bar{q}$ states			
$\Delta = 6$ and $\bar{m}_{\text{hadronic molecule}}$		Multiquark state			
$\alpha = 0.863$ and $\kappa = 11649$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_B mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	10410.9	1	$Z_B(10610)$	10607.2 ± 2	1.85
2	10669.3	2	$Z_B(10650)$	10652.2 ± 1.5	0.16

Table: Summary of results for the set of non-$q\,\bar{q}$ states considered in this work. Experimental results are read from PDG.

Where we have used:

$$\begin{aligned}
\bar{m}_{\text{diquark-Antidiquark}} &= \bar{m}_c \\
\bar{m}_{\text{Hadrocharmonium}} &= \frac{1}{2} m_{J/\psi} + \frac{1}{4}\left(\bar{m}_u + \bar{m}_d\right) \\
\bar{m}_{\text{hadronic molecule}} &= \frac{1}{3} m_{J/\psi} + \frac{2}{3} m_\rho \text{ for } \psi \\
\bar{m}_{\text{hadronic molecule}} &= 0.283\, m_{J/\psi} + 0.717\, m_\rho \text{ for } Z_C \\
\bar{m}_{\text{hadronic molecule}} &= 0.458\, m_{\Upsilon(1S)} + 0.542\, m_\rho \text{ for } Z_B \\
m_{J/\psi} = 3077.9\,\text{MeV}, &\qquad m_{\Upsilon(1S)} = 9460.3 \text{ MeV, and } m_\rho = 770\,\text{MeV}.
\end{aligned}$$

Bonus: Tetraquarks as Gluonic Excitations States

Holographic spectrum		Non-$q\,\bar{q}$ states			
$\Delta = 5$ and $\bar{m}_{\text{Hybrid Meson}}$		Gluonic excitation state			
$\alpha = 0.0367$ and $\kappa = 488$ MeV		$I^G(J^{CP}) = 0^-(1^{+-})$ π_1 mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	1351.7	1	$\pi_1(1400)$	1354 ± 25	0.16
2	1646.6	2	$\pi_1(1600)$	1660^{+15}_{-11}	0.8
3	1901.7	3	$\pi_1(2015)$	$2014 \pm 20 \pm 16$	5.58
$\Delta = 5$ and $\bar{m}_{\text{Hybrid meson}}$		Gluonic Excitation			
$\alpha = 0.539$ and $\kappa = 2151$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_c mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	3721.9	1	$Z_c(3900)$	3887.2 ± 2.3	4.24
2	4156.4	2	$Z_c(4200)$	4196^{+35}_{-32}	0.94
3	4513.2	3	$Z_c(4430)$	4478^{+15}_{-18}	0.78
$\Delta = 7$ and $\bar{m}_{\text{Hybrid Meson}}$		Gluonic excitation state			
$\alpha = 0.863$ and $\kappa = 11649$ MeV		$I^G(J^{CP}) = 1^+(1^{+-})$ Z_B mesons			
n	M_{Th} **(MeV)**	**n**	**State**	M_{Exp} **(MeV)**	ΔM **(%)**
1	10346.7	1	$Z_B(10610)$	10607.2 ± 2	2.52
2	10696.6	2	$Z_B(10650)$	10652.2 ± 1.5	0.42

Table: Summary of results for the set of non-$q\,\bar{q}$ states considered in this work. Experimental results are read from PDG.

Bonus: Tetraquarks as Gluonic Excitations States

Where we have used:

$$\bar{m}_{\text{hybrid meson}} = P_q\, m_q + P_{\bar{q}}\, m_{\bar{q}} + P_G\, m_G\,, \tag{3}$$

with the following probabilities:

Vector hybrid meson	P_q	$P_{\bar{q}}$	P_G
π_1	0.497	0.497	6×10^{-3}
Z_c	0.49	0.49	0.02
Z_b	0.495	0.495	0.01

Table: Summary of coefficients fixed for each hybrid meson candidate.

Important Remark:

In the case of Z_b we are considering two flux tubes instead one.

Our results and conclusions

By looking the tables, and based on the small RMS criterion, we can conclude that:

- Constituent gluons are not so revelevant in order to define non-$q\,st\bar{a}tes$.
- Z_C, Z_B and ψ states are better described as hadronic molecules.
- The hybrid meson descriptions fits well the π_1 spectrum (RMS less than 5%).

General Conclusions

Conclusions

- The family of isovector mesons were fitted as non-linear Regge Trajectories.
- This approach allows us to extend the model by extrapolation to other hadronic vector species.
- Holographic non-linear trajectories provide a good tool to describe mesonic systems. The RMS error for fitting 27 mesonic states was near 13%,with the with 15 parameters organized as:
  - Two parameters, κ and α, for each isovector meson family, i.e., ω, ϕ, J/Ψ and Υ, implying eight in total.
  - One $\bar{m}$ for the vector kaon K^* system.
  - Six $\bar{m}$ for each heavy-light vector meson considered, i.e., D^{*0}, D^{+0}, D_s^{*0}, B^*, B^{*0} and B_s^{0*}.
- Therefore an RMS error around 13% is reasonable for this model, considering the simplicity of the proposal done and the complexity of the QCD physics at a strong regime.
- Things to do next: to extend this model to other hadronic properties as wave functions, decay constants, form factors, melting temperatures and density effects.

Thank you!

Appendix: WKB method for a General Dilaton Field

Consider the general trajectory

$$M_n^2 = a(n+b)^\nu .$$

Obtained as a eigenvalue spectrum from the holographic potential

$$V(z) = V_{\text{AdS}}(z) + \frac{1}{4}\Phi'(z)^2 - \frac{1}{2\,z}\Phi''(z) - \frac{\beta}{2}\Phi'(z), \tag{4}$$

where

$$V_{\text{AdS}}(z) = \frac{\beta(\beta-2) + 4\,M_{d+2}^2\,R^2}{4\,z^2}. \tag{5}$$

At large z, only positive powers of z will contribute to the potential. These powers can be added into a single function $H(z)$, defined as

$$H(z) = \frac{1}{4}\Phi'(z)^2 - \frac{1}{2}\Phi''(z) - \frac{\beta}{2\,z}\Phi'(z). \tag{6}$$

Appendix: WKB method for a General Dilaton Field

The $H(z)$ function is entirely determined by the WKB method since we can always reconstruct the potential from a given spectrum. Thus applying WKB we have

$$z(V) = 2\int_0^V \frac{d\,M^2}{\frac{d\,M^2}{d\,n}\left(V - M_n^2\right)^{1/2}}. \tag{7}$$

Performing the spectrum derivative we obtain:

$$\frac{d\,M_n^2}{d\,n} = a\,\nu\,(n+b)^{\nu-1} = a\,\nu\left(\frac{M_n^2}{a}\right)^{\frac{\nu-1}{\nu}}. \tag{8}$$

Solving this equation we obtain the approximation for the holographic potential

$$V(z) = \left[\frac{a^{1/\nu}}{2\,\pi^{1/2}}\,\frac{\nu\,\Gamma\left(\frac{\nu+2}{2\,\nu}\right)}{\Gamma\left(\frac{1}{\nu}\right)}\,z\right]^{\frac{2\,\nu}{2-\nu}} \equiv C(\nu,a)\,z^{\frac{2\,\nu}{2-\nu}}. \tag{9}$$

Where $0 < \nu < 2$. Notice that if we fix $\nu = 1$ we recover the soft wall model case. Now, we can solve the dilaton for the nonlinear trajectory case.

Appendix: WKB method for a General Dilaton Field

From the solution for the potential we can infer the dilaton by solving:

$$C(\nu,a)\,z^{\frac{2\,\nu}{2-\nu}} = \frac{1}{4}\Phi'(z)^2 - \frac{1}{2}\Phi''(z) - \frac{\beta}{2}\Phi'(z), \tag{10}$$

where the boundary conditions should be fixed as

$$\Phi'(z\to 0) = 0 \text{ and } \Phi'(z\to\infty) = \frac{2\,\nu}{2-\nu}\,C(\nu,a)\,z_\infty^{\frac{2\,\nu}{2-\nu}-1},$$

where z_∞ is large z value used to fit the numerical infinity.

Appendix: WKB method for a General Dilaton Field

Quadratic Dilaton

We will fix $\nu = 1$, implying that the potential will have the harmonic form. Therefore, the equation for the dilaton would be

$$\frac{a^2}{16} z^2 = \frac{1}{4}\Phi'(z)^2 - \frac{1}{2}\Phi''(z) - \frac{\beta}{2\,z}\Phi'(z) \tag{11}$$

This equation has an analytical solution

$$\Phi(z) = c_1 - 2\log\left[\cosh\left(\frac{a}{8}z^2 - 2\,c_2\right)\right]. \tag{12}$$

Now we can fix $c_2 \to \infty$ implying that $\cosh 2\,a\,c_2 = \sinh 2\,a\,c_1 = 1$. Therefore we obtain

$$\cosh\frac{a^2}{8}z^2 - \sinh\frac{a}{8}z^2 = e^{-\frac{a}{8}z^2}. \tag{13}$$

Now fix $c_1 = 0$ and finally, we will obtain to the standard quadratic dilaton

$$\Phi(z) = -2\log\left(e^{-\frac{a}{8}z^2}\right) = \frac{a}{4}z^2 \equiv k^2\,z^2 \tag{14}$$

Appendix: WKB method for a General Dilaton Field

Other cases

The dominant terms in the holographic potential at large z come from the $\Phi'(z)^2$ term in the potential. Thus, we can approximately fulfill the following condition

$$V_{\mathrm{WKB}}(z) = \frac{1}{4}\Phi'(z)^2 \tag{15}$$

at large z. Therefore, we can suppose an ansatz for the dilaton as

$$\Phi(z) = (\kappa\, z)^{\gamma}, \tag{16}$$

where κ is an energy scale. Calculating the derivative, we can construct the contribution to the potential at large z:

$$\Phi(z)^2 = \gamma^2\, \kappa^{2\gamma}\, z^{2(\gamma-1)}. \tag{17}$$

Thus, for the WKB potential condition (15) we have

$$C(\nu, a)\, z^{\frac{2\,\nu}{2-\nu}} = \frac{1}{4}\, \gamma^2\, \kappa^{2\gamma}\, z^{2(\gamma-1)}, \tag{18}$$

Appendix: WKB method for a General Dilaton Field

Other cases

If we require that both parts of the equation match, we will obtain the following relations for the exponent γ and the energy scale κ as:

$$\gamma = \frac{2}{2-\nu} \tag{19}$$

$$\kappa = \left[(2-\nu)^2 \; C(\nu, a)\right]^{\frac{2-\nu}{4}}. \tag{20}$$

If we do the definition $\gamma = 2 - \alpha$, we obtain the non-quadratic dilaton proposed here written in terms of the trajectory parameters.