331 MODEL WITH RIGHT-HANDED NEUTRINOS, IMPLEMENTED IN SARAH AND SPHENO

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INTRODUCTION

- The standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ with all its successes, fails to explain several fundamental issues such as hierarchical charged fermion masses, fermion mixing, strong CP violation, replication of families, neutrino masses and oscillations, etc. All this make us think that we must call for extensions of the model.
- The simplest way is extending the SM in its gauge, scalar, and fermion sectors.
- Popular and well-motivated extensions of the SM are based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ (called hereafter 3-3-1 for short).
- Some outstanding features of 3-3-1 models are:

1. The simple models are free of gauge anomalies, if and only if the number of families is a multiple of three (becoming just three by imposing QCD asymptotic freedom).
2. One quark family has different quantum numbers than the other two, a fact that may be used to explain the heavy top quark mass.
3. The scalar sector includes several good candidates for dark matter.
4. The lepton content is suitable for explaining some neutrino properties.
5. The hierarchy in the Yukawa coupling constants can be avoided by implementing several universal seesaw mechanisms.

- In this analysis we are going to study one of 3-3-1 models without exotic electric charges called 3-3-1 with right-handed neutrinos. In this model, we will use a scalar sector with three triplets of Higgs.
- The goal of this work is the implementation of the 3-3-1 model with right-handed neutrinos in computer tools like SARAH, which is a package of MATHEMATICA, and Spheno program. This is a great help to reduce the time of calculation and obtain analytical and numerical results efficiently.
- As proof of this, we present the mass matrices for the quark sectors, where a Higgs sector with three scalar triplets has been used. Then, using the Spheno program, a numerical analysis of the analytical outputs obtained with SARAH is performed.

3-3-1 MODEL WITH RIGHT-HANDED NEUTRINOS

Let us review briefly the so-called 3-3-1 model with right-handed neutrinos:

- The gauge sector:

The gauge bosons sector is the same for all 3-3-1 models without exotic electric charges.

As was stated, the model we are interested in is based on the local gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ which has 17 gauge bosons: one gauge field B^μ associated with $U(1)_X$, eight gluon fields G^μ associated with $SU(3)_c$ which remain massless after spontaneous breaking of the electroweak symmetry, and another eight gauge fields associated with $SU(3)_L$ that we write for convenience as

$$\sum_{\alpha=1}^{8} \lambda^\alpha A^\mu_\alpha = \sqrt{2}\begin{pmatrix} D^\mu_1 & W^{+\mu} & K^{+\mu} \\ W^{-\mu} & D^\mu_2 & K^{0\mu} \\ K^{-\mu} & \bar{K}^{0\mu} & D^\mu_3 \end{pmatrix}$$

where $D^\mu_1 = A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, $D^\mu_2 = -A^\mu_3/\sqrt{2} + A^\mu_8/\sqrt{6}$, and $D^\mu_3 = -2A^\mu_8/\sqrt{6}$. λ_α, $\alpha = 1, 2, \ldots, 8$ are the eight Gell-Mann matrices normalized as $\mathrm{Tr}(\lambda^\alpha \lambda^\beta) = 2\delta_{\alpha\beta}$.

- The fermion sector:

$$
\begin{aligned}
L_{lL}^{T} &= (l^{-}, \nu_l^{0}, \nu_l^{0c})_L \sim (1, 3^*, -1/3), \quad l_L^{+} \sim (1,1,1), \\
Q_{iL}^{T} &= (u_i, d_i, D_i)_L \sim (3, 3, 0), \\
Q_{3L}^{T} &= (d_3, u_3, U)_L \sim (3, 3^*, 1/3), \\
u_{aL}^{c} &\sim (3^*, 1, -2/3), \quad d_{aL}^{c} \sim (3^*, 1, 1/3), \\
D_{iL}^{c} &\sim (3^*, 1, 1/3), \quad U_L^{c} \sim (3^*, 1, -2/3),
\end{aligned}
$$

where $i = 1, 2$ for two families, D_{iL} are two extra quarks of electric charge -1/3, U_L is an extra quark of electric charge 2/3. The right-handed quarks which belong to $SU(3)_L$ singlets are u_{aL} *and* d_{aL} with a=1,2,3 a family index. The lepton content is given by the three $SU(3)_L$ triplets L_{lL} *for* $l = e, \mu, \tau$ a lepton family index, and three singlets l_L, where ν_l^0 is the neutrino field associated with the lepton l^-, and ν_l^{0c} plays the role of the right-handed neutrino field associated with the same flavor.

- The scalar sector:

$$\Phi_1(1,3,1/3) = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \\ \phi_1'^0 \end{pmatrix}, \text{ con VEV: } \langle \Phi_1 \rangle = \begin{pmatrix} 0 \\ 0 \\ V \end{pmatrix}$$

$$\Phi_2(1,3,1/3) = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \\ \phi_2'^0 \end{pmatrix}, \text{ con VEV: } \langle \Phi_2 \rangle = \begin{pmatrix} 0 \\ v_1 \\ 0 \end{pmatrix}$$

$$\Phi_3(1,3,-2/3) = \begin{pmatrix} \phi_3^0 \\ \phi_3^- \\ \phi_3'^- \end{pmatrix}, \text{ con VEV: } \langle \Phi_3 \rangle = \begin{pmatrix} v_2 \\ 0 \\ 0 \end{pmatrix}$$

with the hierarchy $v_1 \sim v_2 \sim 10^2$ GeV $<< V \sim$ TeV.

The analysis shows that this set of VEVs breaks the 3-3-1 symmetry in two steps:

$$3-3-1 \xrightarrow{V} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$
$$\xrightarrow{v_i} SU(3)_c \otimes U(1)_{\mathrm{EM}},$$

for i=1,2, and $U(1)_{EM}$ is the Abelian gauge group of the electromagnetism.

SARAH MODEL IMPLEMENTATION: ANALYTICAL RESULTS

- What is SARAH?

SARAH is a Mathematica package for building and analyzing particle physics models. Although originally SARAH was designed to work only with supersymmetric models, after version 3 non-supersymmetric ones can be implemented as well. Once a model is defined in SARAH, the user can get all sorts of details about it: all vertices, mass matrices, tadpoles equations, 1-loop corrections for tadpoles and self-energies, and 2-loop renormalization group equations (RGEs). All this information about the model is derived by SARAH analytically and the user can simply handle it in Mathematica and use it for his own purposes.

Technical details:

Author: Florian Staub (florian.staub@cern.ch)

Type of code: Mathematica package

Website: http://sarah.hepforge.org/

- Defining 3-3-1 model in SARAH:

We must create a new folder in $PATH/SARAH-X.Y.Z/Models with the name of your model, por example, 331v3. Then, we must define four files:

1. 331-v3.m → This file contains the basic definitions of the model: multiplets, Lagrangian, gauge symmetries and their breaking.
2. parameters.m → In this file we provide additional information about the parameters of the model.
3. particles.m → This one is devoted to the particles in the model, with some details not present in 331-v3.m
4. SPheno.m → This file is only required if we want to create a SPheno module for our model to perform numerical calculation.

This four files are shown in detail on the website: https://github.com/amtapia/SARAH-331v3

331-v3.m file:

```
(* ::Package:: *)

Off[General::spell]

Model`Name = "331v3";
Model`NameLaTeX ="3-3-1 Model with Right-Handed Neutrinos";
Model`Authors = "R. Benavides & A. Tapia";
Model`Date = "2019-02-15";

(*-------------------------------------------*)
(*   Particle Content*)
(*-------------------------------------------*)

(* Gauge     fields *)

Gauge[[1]]={B,  U[1],  xcharge,      g1,False, 0, 1};
Gauge[[2]]={WB, SU[3], left,         g2,True,  0, 1};
Gauge[[3]]={G,  SU[3], color,        g3,False, 0, 1};
```

```
(* Chiral     fields *)

FermionFields[[1]] = {Q1, 1, {dt1L, d1L, u1L}, 0, 3, 3 };
ScalarFields[[1]] =  {phi1,  1, {phi10p, phi10, phi1m},  1/3, 3, 1};

(* Gauge Sector *)


DEFINITION[EWSB][GaugeSector] =
{
  {{VB,VWB[7],VWB[8]},{VP,VZ,VZp},ZZ},
  {{VWB[2],VWB[5],VWB[6],VWB[3]},{VWp,conj[VWp],VXp,conj[VXp]},ZW}
};
```

331-v3.m file:

```
(* ::Package:: *)

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(*-------------------------------------------*)
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(*-------------------------------------------*)

(* Gauge      fields *)

Gauge[[1]]={B,  U[1],  xcharge,     g1,False, 0, 1};
Gauge[[2]]={WB, SU[3], left,        g2,True,  0, 1};
Gauge[[3]]={G,  SU[3], color,       g3,False, 0, 1};
```

```
(* Chiral     fields *)
FermionFields[[1]] = 
```
$Q_{iL}^T = (u_i, d_i, D_i)_L \sim (3, 3, 0)$

```
ScalarFields[[1]] = 
```
$\langle \phi_1^T \rangle = \langle (\phi_1^+, \phi_1^0, \phi_1'^0) \rangle = \langle (0, 0, V) \rangle \sim (1, 3, 1/3)$

```
(* Gauge Sector *)

DEFINITION[EWSB][GaugeSector] =
{
```
$\{B, W_7, W_8\} \rightarrow \{\gamma, Z, Z'\}$

$\{W_2, W_5, W_6, W_3\} \rightarrow \{W^+, W^-, W'^+, W'^-\}$

```
};
```

Next, we define the decomposition of the scalar fields into their CP-even and CP-odd components, including also the possibility of having VEVs. We must also define the mass eigenstates in terms of the gauge eigenstates. Finally, we define Dirac spinors. This is because so far all the fermions we have considered are 2-components Weyl spinors, since this is the way they are internally handled by SARAH. Therefore, we must tell SARAH how to combine them to form 4-component Dirac fermions, more common in particle physics calculations.

parameters.m file

```
ParameterDefinitions = {

 {Gf,{
      Description -> "Fermi's constant",
      Dependence -> None,
      DependenceNum -> None,
      DependenceOptional -> None,
      DependenceSPheno -> None,
      Real -> True,
      Value -> 0.0000116639,
      LesHouches -> {SMINPUTS, 2},
      LaTeX -> "G_f",
      OutputName -> Gf}},
```

particles.m

```
ParticleDefinitions[EWSB] = {

  {Fu,{
       Description -> "Up-Quarks",
       FeynArtsNr -> 3,
       LaTeX -> "u",
       Mass -> {0.0015, 1.27, 171.2, LesHouches},
       OutputName -> "Fu",
       PDG -> {2, 4, 6, 1006},
       ElectricCharge -> 2/3,

ParticleDefinitions[GaugeES] = {

  {VWB,{
        Description -> "W-Bosons",
        FeynArtsNr -> 2,
        LaTeX -> "W",
        Mass -> 0,
        OutputName -> "W",
        PDG -> {0, 79, 80, 81, 82, 83, 84, 85}
```

SPheno.m file:

```
OnlyLowEnergySPheno = True;

MINPAR={
  {1,Lambda1IN},
  {2,Lambda2IN},
  {3,Lambda3IN},
  {4,Lambda12IN},
  {5,Lambda13IN},
  {6,Lambda23IN},
  {7,Lambda12TIN},
  {8,Lambda13TIN},
  {9,Lambda23TIN},
  {10,fInput},
  {11,VnIN},
  {12,v1IN}
};

ParametersToSolveTadpoles = {mu12,mu22,mu32};
```

```
DEFINITION[MatchingConditions]= {
  {v2,vSM},
  {g1, g2SM/0.57},
  {g2, g2SM},
  {g3, g3SM},
  {hll1, YeSM*vSM/v2}
 };

BoundaryLowScaleInput={

  {l1,Lambda1IN},
  {l2,Lambda2IN},
  {l3,Lambda3IN}, ...

 ListDecayParticles = {Fu,Fe,Fd,hh,Ah,Hpm,Dpm,VZ,VZp,VXp};


 DefaultInputValues = {
   Lambda1IN -> 0.1,
   Lambda2IN -> 0.1,
   Lambda3IN -> 0.1,  ...
```

- Analytical results:

First, we must load SARAH and 331v3 model. We can do that with these Mathematica commads:

```
<< /home/.Mathematica/Applications/SARAH-X.Y
    .Z/SARAH.m
```

```
Start["331/v3"]
```

After a few seconds all the initial SARAH computations will be finished and we will be ready to execute all kinds of commands to get analytical information about the model.

We can run the command **MatrixForm**[MassMatrix[Fu]] and SARAH return the 4 x 4 quark mass matrix for the up-quark sector:

$$\begin{pmatrix} v2\,hup11[1] & v2\,hup11[2] & v2\,hup11[3] & hUp1\,v2 \\ v2\,hup21[1] & v2\,hup21[2] & v2\,hup21[3] & hUp2\,v2 \\ v1\,hu12[1] & v1\,hu12[2] & v1\,hu12[3] & hU2\,v1 \\ V\,hu11[1] & V\,hu11[2] & V\,hu11[3] & hU1\,V \end{pmatrix} \Rightarrow M_u = \begin{pmatrix} v_2 h^{u'}_{11} & v_2 h^{u'}_{12} & v_2 h^{u'}_{13} & v_2 h^{U'}_{1} \\ v_2 h^{u'}_{21} & v_2 h^{u'}_{22} & v_2 h^{u'}_{23} & v_2 h^{U'}_{2} \\ v_1 h^{u}_{12} & v_1 h^{u}_{22} & v_1 h^{u}_{32} & v_1 h^{U}_{2} \\ V h^{u}_{11} & V h^{u}_{21} & V h^{u}_{31} & V h^{U}_{1} \end{pmatrix}$$

PRD 80, 113009 (2009)

We can run the command `MatrixForm[MassMatrix[Fd]]` and SARAH return the 5 x 5 quark mass matrix for the down-quark sector:

$$
\begin{pmatrix}
\mathrm{v1\,hd1[1]} & \mathrm{v1\,hd1[2]} & \mathrm{v1\,hd1[3]} & \mathrm{v1\,hdt1[1]} & \mathrm{v1\,hdt1[2]} \\
\mathrm{v1\,hd2[1]} & \mathrm{v1\,hd2[2]} & \mathrm{v1\,hd2[3]} & \mathrm{v1\,hdt2[1]} & \mathrm{v1\,hdt2[2]} \\
\mathrm{v2\,hd3[1]} & \mathrm{v2\,hd3[2]} & \mathrm{v2\,hd3[3]} & \mathrm{v2\,hdt3[1]} & \mathrm{v2\,hdt3[2]} \\
\mathrm{V\,hdp11[1]} & \mathrm{V\,hdp11[2]} & \mathrm{V\,hdp11[3]} & \mathrm{V\,hdtp11[1]} & \mathrm{V\,hdtp11[2]} \\
\mathrm{V\,hdp1[1]} & \mathrm{V\,hdp1[2]} & \mathrm{V\,hdp1[3]} & \mathrm{V\,hdtp1[1]} & \mathrm{V\,hdtp1[2]}
\end{pmatrix}
$$

$$
M_d = \begin{pmatrix}
v_1 h_{11}^{d} & v_1 h_{12}^{d} & v_1 h_{13}^{d} & v_1 h_{11}^{D} & v_1 h_{12}^{D} \\
v_1 h_{21}^{d} & v_1 h_{22}^{d} & v_1 h_{23}^{d} & v_1 h_{21}^{D} & v_1 h_{22}^{D} \\
v_2 h_{31}^{d} & v_2 h_{32}^{d} & v_2 h_{33}^{d} & v_2 h_{31}^{D} & v_2 h_{32}^{D} \\
V h_{11}^{d'} & V h_{12}^{d'} & V h_{13}^{d'} & V h_{11}^{D'} & v_2 h_{12}^{D'} \\
V h_{1}^{d'} & V h_{2}^{d'} & V h_{3}^{d'} & V h_{1}^{D'} & V h_{2}^{D'}
\end{pmatrix}
$$

PRD 80, 113009 (2009)

SPHENO MODEL IMPLEMENTATION: NUMERICAL RESULTS

- What is SPheno?

SPheno is a spectrum calculator: it takes the values of the input parameters and computes, numerically, all masses and mixing matrices in the model. Besides, with this information it also computes the vertices, decay rates and many flavor observables. Although it was originally designed to cover just a few specific supersymmetric models, now it has become available for many other models (including non-supersymmetric ones) thanks to SARAH. The code is written in Fortran.

Technical details:

Author: Werner Porod (porod@physik.uni-wuerzburg.de) and Florian Staub (florian.staub@cern.ch)

Type of code: Fortran

Website: https://spheno.hepforge.org

Defining 3-3-1 model in SARAH:

We must run, after loading SARAH and initializing our model, the following command in Mathematica `MakeSPheno[]`. After some minutes, the SPheno module will be created.

Then, we copy the SPheno module to SPheno program directory and we must compile the Fortran code running:

```
$make Model=331v3
```

Finally, we can execute the code. This is done with:

```
$bin/SPheno331v3 LesHouches.in.331v3_low
```

Where LesHouches is the file with the input values in the corresponding physic entries of the model. For example, the VEVs values used for our model are:

$$v_1 = 100,\ v_2 = 245,\ \text{y}\ V = 5000\ \text{GeV.}$$

```
Block MODSEL        #
 1 0                #  1/0: High/low scale
     input
 2 1                # Boundary Condition
 6 1                # Generation Mixing
 12 173.5           # Renormalization scale
Block SMINPUTS      # Standard Model inputs
 2 1.166370E-05     # G_F,Fermi constant
 3 1.187000E-01     # alpha_s(MZ) SM MSbar
 4 9.118870E+01     # Z-boson pole mass
 5 4.180000E+00     # m_b(mb) SM MSbar
 6 1.735000E+02     # m_top(pole)
 7 1.776690E+00     # m_tau(pole)
Block MINPAR        # Input parameters
 11 5.00E+03        # VnIN
 12 1.00E+02        # v1IN
 13 2.45E+02        # v2IN
```

LesHouches.in.331v3_low

Numerical results:

With the mentioned VEVs values, we obtain the simplest solution to up-quark and down-quark sector:

$$M_{up} = h^{u'}_{11} v_2,\ M_c = h^{u'}_{22} v_2,\ M_t = h^{u}_{32} v_1 \text{ y } M_U = h^{U}_{1} V$$

With Yukawa values:

$$h^{u'}_{11} = 9.35 \times 10^{-6},\ h^{u'}_{22} = 0.7053,\ h^{u}_{32} = 0.01275,\ h^{U}_{1} = 1$$

Numerical up-quark sector masses according with PDG mass values!

$$M_d = h^{d}_{11} v_1,\quad M_s = h^{d}_{22} v_1,\quad M_b = h^{d}_{33} v_2,$$
$$M_{D1} = h^{D'}_{11} V \text{ y } M_{D2} = h^{D'}_{2} V;$$

With Yukawa values:

$$h^{d}_{11} = 0.000048,\ h^{d}_{22} = 0.00095,\ h^{d}_{33} = 0.016992,$$
$$h^{D'}_{11} = 0.3 \text{ y } h^{D'}_{2} = 0.6,$$

Numerical down-quark sector masses according with PDG mass values!

Block MASS # Mass spectrum
PDG code mass particle

PDG code	mass	particle	
36	4.50435683E+03	# Ah_3	
41	4.46524104E+03	# Dpm_2	
1	4.80000000E-03	# Fd_1	d
3	9.50000000E-02	# Fd_2	s
5	4.18377991E+00	# Fd_3	b
1001	1.50000000E+03	# Fd_4	
1003	3.00000000E+03	# Fd_5	
11	5.10998930E-04	# Fe_1	
13	1.05658372E-01	# Fe_2	
15	1.77669000E+00	# Fe_3	
2	2.30216232E-03	# Fu_1	u
4	1.27500000E+00	# Fu_2	c
6	1.73659367E+02	# Fu_3	t
1006	5.00000000E+03	# Fu_4	
25	1.03211827E+02	# hh_1	
35	2.23656366E+03	# hh_2	
1035	4.50404752E+03	# hh_3	
37	2.30145102E+03	# Hpm_3	
39	5.10386526E+03	# Hpm_4	
24	8.03497269E+01	# VWp	
224	2.34618648E+03	# VXp	
23	9.11887000E+01	# VZ	
2023	3.85889765E+03	# VZp	

Down sector Exotic quarks (Fd_4, Fd_5)

Up sector Exotic quark (Fu_4)

CONCLUSIONS

- We have shown that SARAH package of Mathematica generate correctly the analytical quark mass matrices for the 3-3-1 model with right-handed neutrinos, and with three scalar triplets.
- SARAH calculate not only fermionic masses, but also for de Gauge bosons, analysis of the Higgs sector, currents etc., Reducing the performance time of analytical calculations in tree level and in a higher perturbative order.
- We have shown that using SPheno code it is possible obtain the numerical values of the quark masses from our model.
- With three scalar triplets, we showed that all the quark masses are generated correctly to tree level in our model, this allows us to conclude that it is a realistic model, at least generating the quark masses.
- This work will be published the next year in the Revista Mexicana de Física E.

THANK YOU!