EXTERNAL MOMENTUM DEPENDENCE FROM THE HIGGS BOSON MASS

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Outline

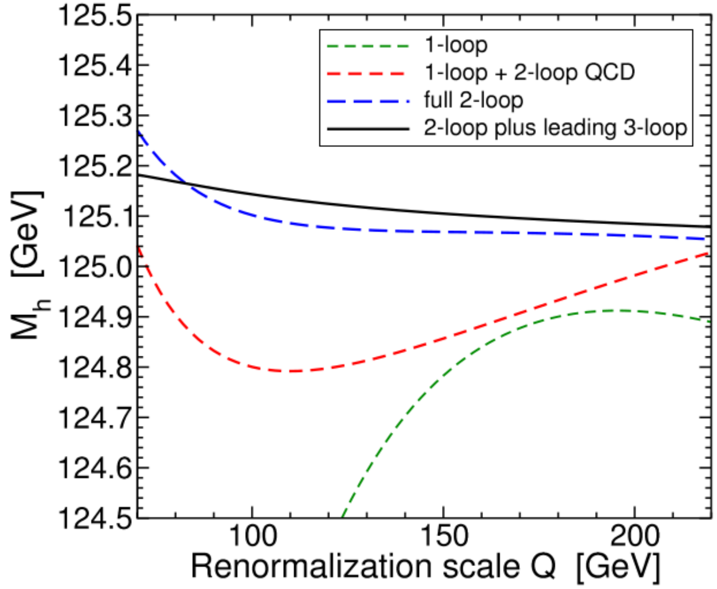
1. STATE OF ART

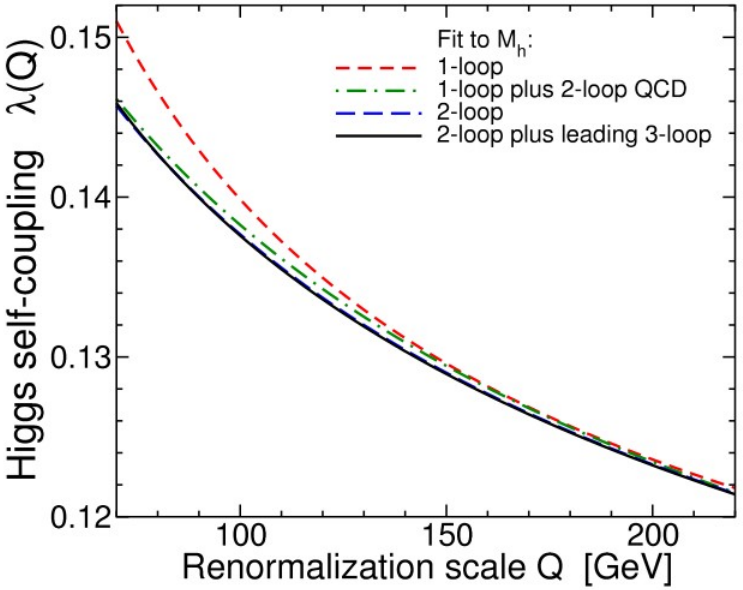
- Three-loop corrections to M_h in the SM
- Three-loop corrections to M_h in the MSSM
- EFT vs Fixed-Order Approaches
- Uncertainties Estimation

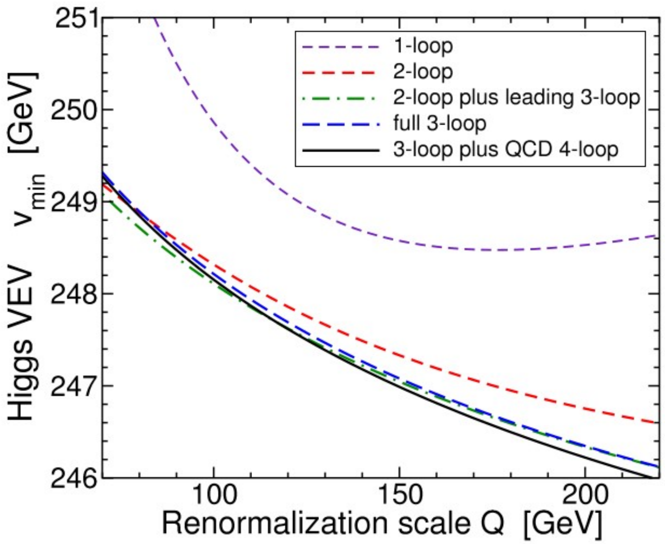
2. EXTERNAL MOMENTUM DEPENDENCE

- Three-Loop Fixed-Order Numerical Results
- Effective-Field-Theory vs Fixed-Order Predictions
- Reduction to a Set of Scalar Integrals
- Master Integrals

Three-Loop Corrections to M_h in the SM







SMDR numerical results:

$$M_h^2 = 2\lambda(Q)\, v^2(Q) \quad \in \boldsymbol{OS}$$

$$\Delta^{(3)} M_h^2 = \frac{1}{(6\pi^2)^3}\left[O(g_s^4\, y_t^4) + O(y_t^8)\right]$$

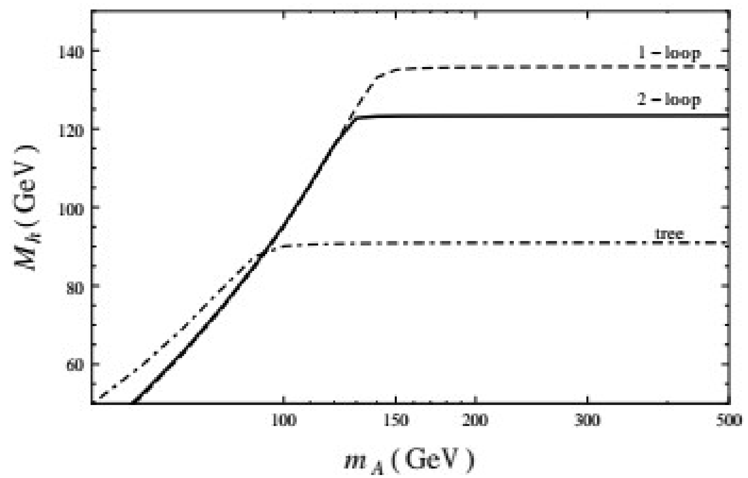
Effective – Potential Approach

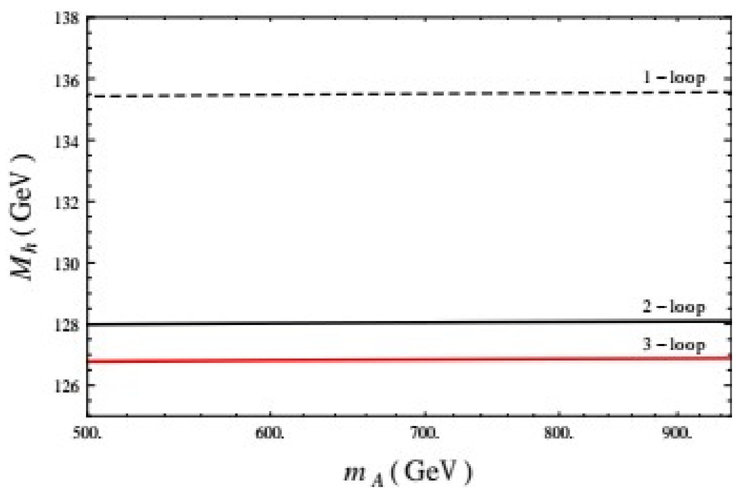
S. P. Martin (2019)

Three-Loop Corrections to M_h in the MSSM

Input parameters for the m_h^{max} and m_h^{mod+} scenarios.

	M_t	M_{SUSY}	X_t	$M_{\tilde{g}}$	μ
m_h^{max}	173.2 GeV	1000 GeV	$2M_{SUSY}$	1500 GeV	200 GeV
m_h^{mod+}	173.2 GeV	1000 GeV	$+1.5M_{SUSY}$	1500 GeV	200 GeV





FeynHiggs 2.14: One- and two- loop M_h -predictions.

Contributions:

$$Tree-level : M_h \lesssim 90\ GeV \quad (60\%)$$
$$One-loop : \Delta M_h \approx 40\ GeV \quad (35\%)$$
$$Two-loop : \Delta M_h \approx 10\ GeV \quad (4\%)$$
$$Three-loop : \Delta M_h \approx 1\ GeV \quad (1\%)$$

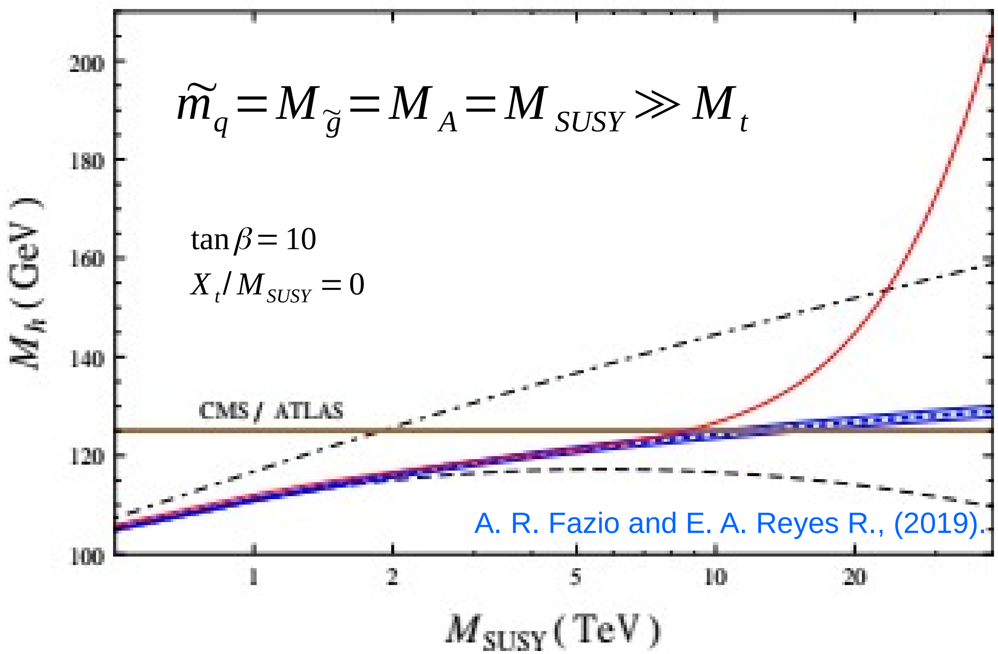
A. R. Fazio and E. A. Reyes R., (2019).

Uncertainty at LHC: **Δ M_h ~ 100 – 200 MeV**, and at **ILC Δ M_h ~ 50 MeV**. But theoretical uncertainty at higher-loop order: **1–5 GeV.**

Hollik '98, G. Degrassi 2003

EFT vs Fixed-Order Predictions

$$M_h^2 \approx M_Z^2 cos^2 2\beta + \frac{3G_F}{\sqrt{2}\pi^2 s_\beta^2} M_t^4 \left[\ln\left(\frac{M_{SUSY}^2}{M_t^2}\right) + \frac{X_t^2}{M_{SUSY}^2} - \frac{X_t^4}{12 M_{SUSY}^4} \right]$$



FEYNHIGGS 2.14:

- Heavy SUSY limit.
- The low-scale effective-field-theory (EFT) is the **SM**.
- SQCD two-loop fixed-order contributions.
- **NNLL** resummation of the large logarithms.
- M_t renormalized in the modified **MS** scheme.
- RUNDEC shifts M_t^{MS} to M_t^{DR} .

RGEs : $\frac{dg_k}{dt} = \beta_k, \quad t = \log(Q), \quad \beta_k = \beta_k^{(1)} + \beta_k^{(2)} + \beta_k^{(3)}, \qquad g_k = \lambda, y_t, g_s, \ldots$

$$\lambda(M_{SUSY}) = \frac{1}{4}[g^2 + g'^2] c_{2\beta}^2 + \underbrace{\Delta^{(1)}\lambda}_{full} + \underbrace{\Delta^{(2)}\lambda^{gaugeless}}_{O(y_{t,b,\tau}^4 g_s^2)} + \underbrace{\Delta^{(2)}\lambda^{EW-QCD}}_{O(y_{t,b}^2 g^2 g_s^2),\ O(y_{t,b}^2 g'^2 g_s^2),\ O(g^4 g_s^2,\ g'^4 g_s^2)}$$

G. Degrassi et al. (2019).

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- Dimensional Regularization Scheme
- Reduction to a Set of Scalar Integrals
- Master Integrals

Evaluation of Unrenormalized Topologies (SM)

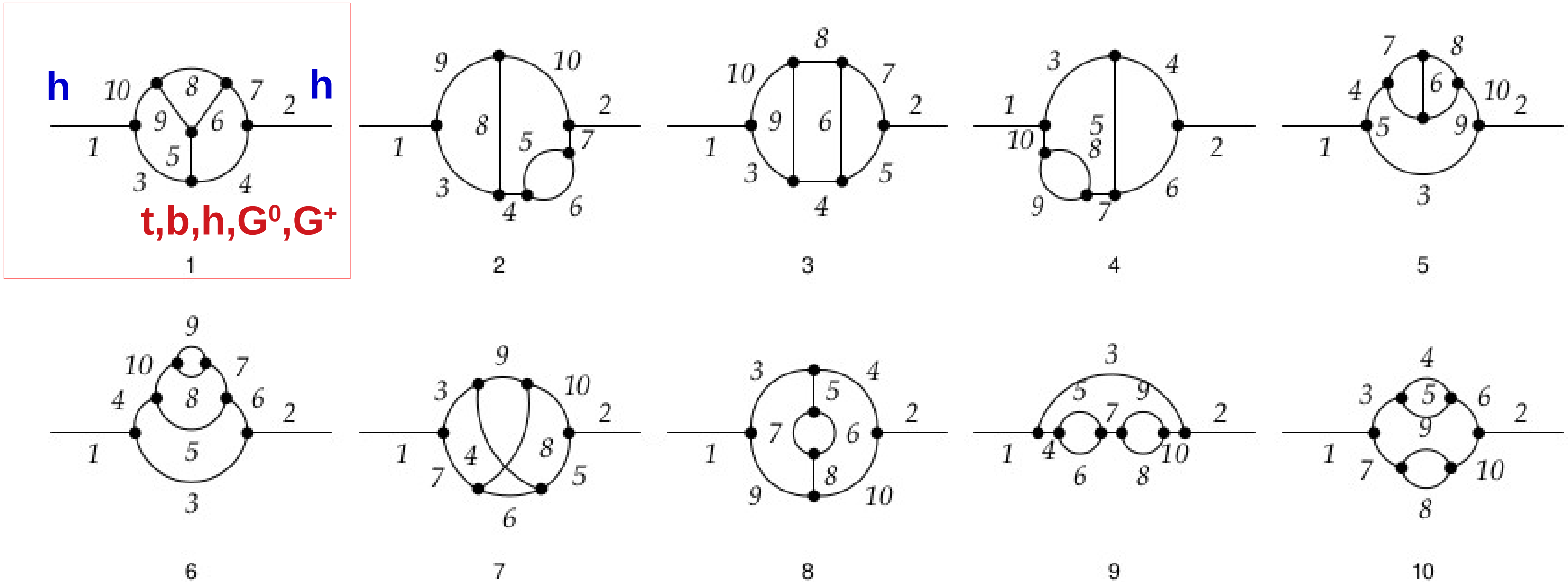
- **Three-loop unrenormalized Higgs self-energies in Mathematica:**

```
Get["~/FeynArts.m"];
Topology = CreateTopologies[ 3, 1 → 1, Options ];
DiagramSelect[ InsertFields[ Topology, h → h, Model → "SM",
Options  ], SelectionRules];
```

← **Topologies, Amplitudes at $O(\alpha_t^3;\ \alpha_t\alpha_s^2)$**

T. Hahn '13,

Generated integrals are in 4-dimensions (they are not regularized)




```
Get["~/FeynCalc.m"];
```

← **Dirac and Color Algebra**

V. Shtabovenko et al. '16

```
Get["~/APART"];
```

← **Irreducible Propagators**

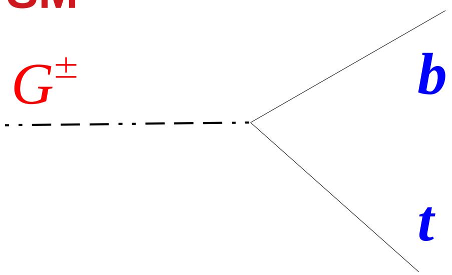
Feng Feng. '18

Dimensional Regularization Scheme (DREG)

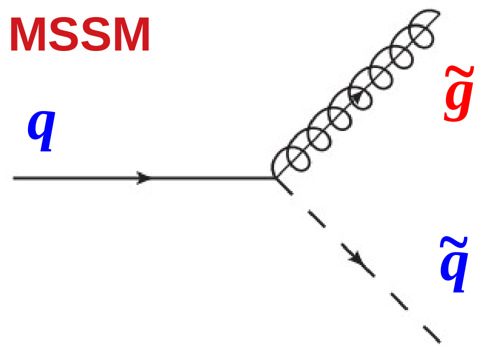
- **Loop momenta, gamma matrices and gauge fields have been continued from 4 to D dimensions.** $q_j^{\mu} \rightarrow \begin{pmatrix} \hat{q}_j^{\mu} \\ 0 \end{pmatrix}.$

- **Be careful with Dirac algebra for diagrams with the cubic vertices:**

SM



MSSM



- **In order to preserve super-symmetry, DRED must be used instead of DREG.**

Get["~/FeynCalc.m"];
DiracOrder[____]; ← Canonical Order

$$\{\gamma^{\mu},\ \gamma^{\nu}\} = 2g^{\mu\nu}\ ;\ \{\gamma_{5,}\ \gamma^{\mu}\} = 0\ ,\quad \mu = 0, 1, , 2, \ldots, D-1$$

$$\gamma_5^2 = 1;\quad Tr(\gamma_5) = 0$$

$$Tr((\gamma^{\mu})\gamma_5) = Tr((\gamma^{\mu}\ \gamma^{\nu})\gamma_5) = Tr((\gamma^{\mu}\ \gamma^{\nu}\ \gamma^{\rho})\gamma_5) = 0$$

$$Tr(\gamma_5(\gamma^{\mu}\ \gamma^{\nu}\ \gamma^{\rho}\ \gamma^{\sigma})) = -4\,i\,\epsilon^{\mu\nu\rho\sigma}$$ ← DiracTrace[___]

J. G. Körner et al. '91

For $p^2 = 0$ non-vanishing traces with a single γ_5 do not occur !

Reduction to a Set of Scalar Integrals

- **Amplitudes can be expressed as a superposition of a set of scalar integrals except for Topology 3 where:**

$$i\left(\frac{\partial}{\partial a_1^\nu}\right)\left(\frac{\partial}{\partial a_2^\rho}\right)\left(\frac{\partial}{\partial a_3^\sigma}\right)\mathrm{Exp}\left[i\left(a_1\cdot q_1+a_2\cdot q_2+a_3\cdot q_3\right)\right]\Big|_{a_j=0}$$

$$\int\int\int d^Dq_1d^Dq_2d^Dq_3\frac{\mathrm{Tr}\left[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma_5\right]p_\mu q_{1\nu}q_{2\rho}q_{3\sigma}}{P_1P_2P_3P_4P_6P_7P_8P_9}, \qquad \text{where}$$

$$P_1=\left(q_1^2-M_t^2\right), \quad P_2=\left(q_2^2-M_b^2\right), \quad P_3=\left(q_3^2-M_t^2\right),$$
$$P_4=\left((q_1-q_2)^2-M_W^2\zeta_W\right), \quad P_5=\left((q_1+q_3)^2-m_5^2\right), \quad P_6=\left((q_2+q_3)^2-M_W^2\zeta_W\right),$$
$$P_7=\left((q_1+p)^2-M_t^2\right), \quad P_8=\left((q_2+p)^2-M_b^2\right), \quad P_9=\left((q_3-p)^2-M_t^2\right).$$

- **Tarasov Method could be useful:**

Schwinger Representation ! — Gaussian integration formula !

$$\prod_{i=1}^{L}\int d^dk_i\prod_{j=1}^{N}P_{\bar{k}_j,m_j}^{\nu_j}\prod_{l=1}^{n_1}\bar{k}_{1\mu_l}\ldots\prod_{s=1}^{n_N}\bar{k}_{N\lambda_s}=i^L\left(\frac{\pi}{i}\right)^{\frac{dL}{2}}\prod_{j=1}^{N}\frac{i^{-\nu_j-n_j}}{\Gamma(\nu_j)}$$
$$\times\prod_{r=1}^{n_1}\frac{\partial}{\partial a_{1\mu_r}}\cdots\prod_{s=1}^{n_N}\frac{\partial}{\partial a_{N\lambda_s}}\int_0^\infty\cdots\int_0^\infty\frac{d\alpha_j\alpha_j^{\nu_j-1}}{[D(\alpha)]^{\frac{d}{2}}}e^{i\left[\frac{Q(\{\bar{s}_i\},\alpha)}{D(\alpha)}-\sum_{l=1}^{N}\alpha_l\left(\bar{m}_l^2-i\epsilon\right)\right]}\Big|_{a_j=0},$$

Differential Operator

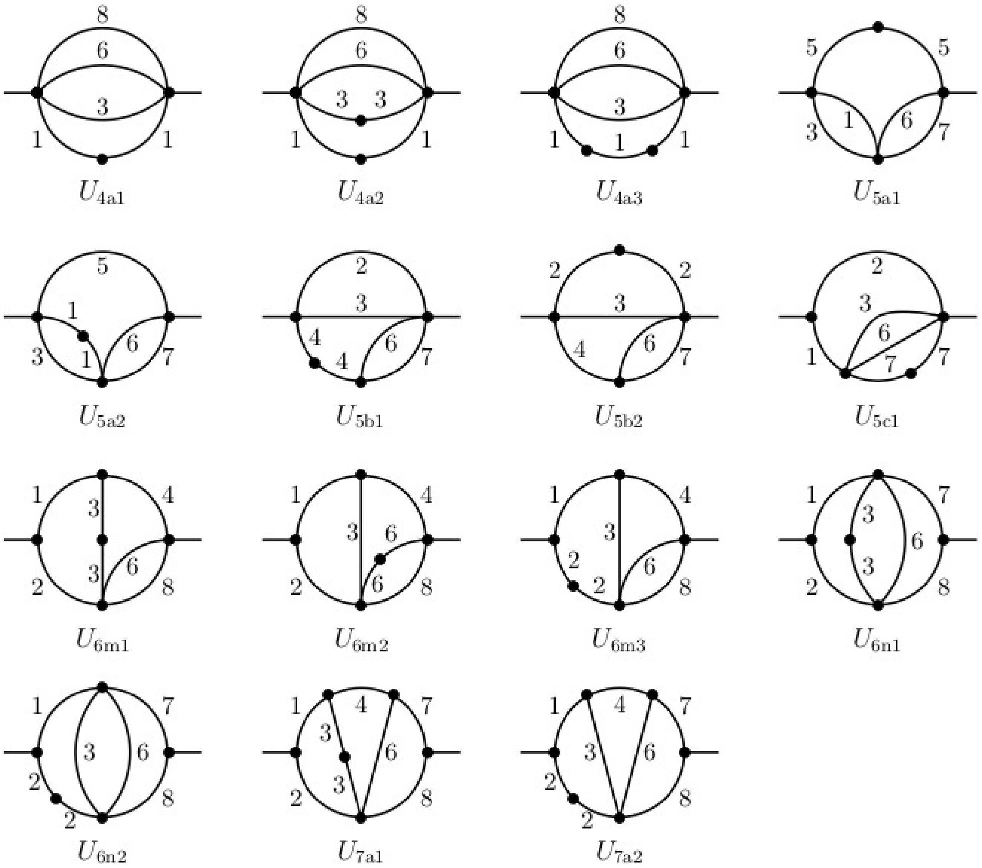
Polynomials in Schwinger parameters !

Master Integrals

- **Reduze 2.1**: C++ implementation of the Laporta algorithm for the IBP indentities. Simplification of the prefactors with GiNaC, Fermat, etc. C. Studerus et al. '10 '12

$$\int d^D q_j \frac{\partial}{\partial q_j^\mu} [k^\mu \boldsymbol{I}(p_1, \ldots, p_m, q_1, \ldots, q_l)] = 0$$

- **Basis of master integrals with doubled propagators for planar-type three-loop self-energies:**



- **Numerical estimation of Master Integrals with TVID2.**

A. Freitas et al. (2020)

- **Sub-loop self-energies are evaluated by using dispersion relations for the sub-loop.**

- **Numerical estimation of non-planar three-loop integrals with pySECDEC.**

Stephan Jahn (2018)

- **AMBRE/MBnumerics project**

T. Riemann et al. (2018)

Computation in progress ...

Thanks for your attention !

Backup Slides

EFT vs Fixed-Order Predictions

- **SM Higgs propagator in the modified MS scheme**:

$$p^2 - 2\lambda(M_t)\, v^2(M_t) + \widetilde{\sum}(p^2) = 0,$$

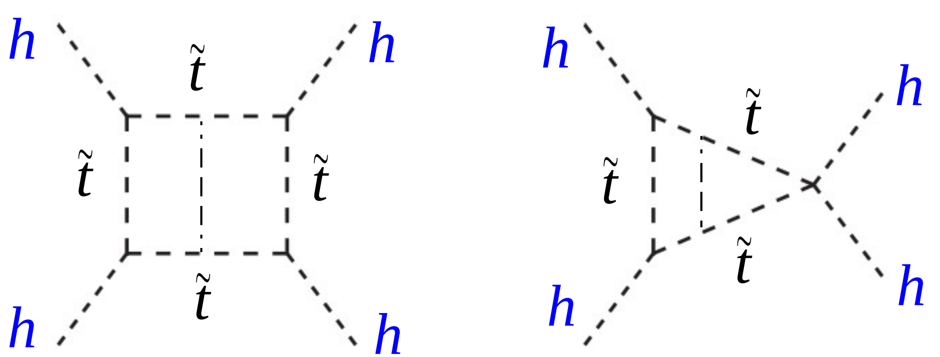
- **Three-loop SM renormalization group equations (RGEs) :**

$$\frac{dg_k}{dt} = \beta_k, \quad t = \log(Q), \quad \beta_k = \beta_k^{(1)} + \beta_k^{(2)} + \beta_k^{(3)}, \qquad g_k = \lambda, y_t, g_s, \ldots$$

- **Two-loop SUSY threshold corrections:**

full $\qquad O(y_{t,b,\tau}^4\, g_s^2)$

$$\lambda(M_{SUSY}) = \frac{1}{4}[g^2(M_{SUSY}) + g'^2(M_{SUSY})]c_{2\beta}^2 + \Delta^{(1)}\lambda + \Delta^{(2)}\lambda^{gaugeless}$$

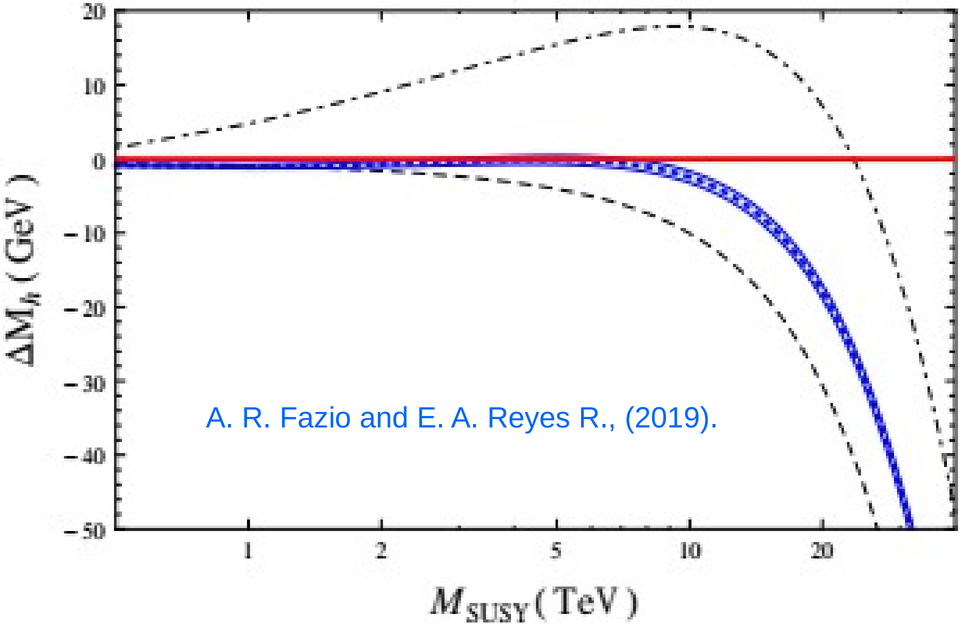


$$\simeq \alpha_j^{m+n-1} \log^m(M_{SUSY}/M_t)$$

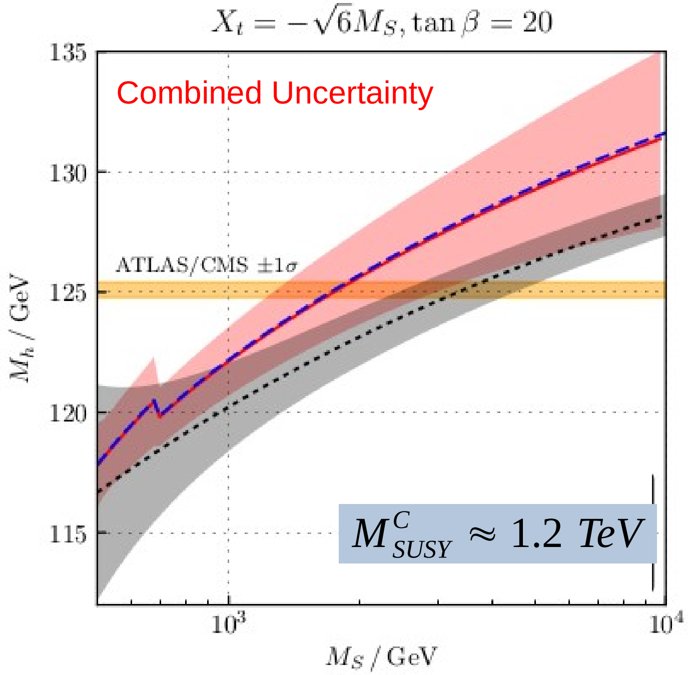
$$j = \lambda, y_t, g_s, \ldots$$

EFT vs Fixed-Order Predictions

- The difference between EFT and fixed-order predictions increases with SUSY scale for energies higher than 10 TeV.



A. R. Fazio and E. A. Reyes R., (2019).



B. C. Allanach and A. Voigt (2018)

FeynHiggs hybrid approach:

$$p^2 - m_h^2 + \widehat{\sum}(p^2) + \Delta_{hh}^{\log} = 0\,, \qquad where$$

$$\Delta_{hh}^{\log} = -[2\lambda(M_t)\,\upsilon^2(M_t)]_{\log} - [\widehat{\sum}(m_h^2)]_{non-\log}$$

log → $\log\left(\frac{M_{SUSY}}{M_t}\right)$

non − log → $\log\left(\frac{\widetilde{m}_q}{M_t}\right)$

$M_t \leq \widetilde{m}_q \leq M_{SUSY}$

Sources of Uncertainty

- Uncertainty at LHC: $\Delta M_h \sim 100 - 200$ MeV, and at ILC $\Delta M_h \sim 50$ MeV. But theoretical uncertainty at higher-loop order: 1–5 GeV (Hollik '98, G. Degrassi 2003).

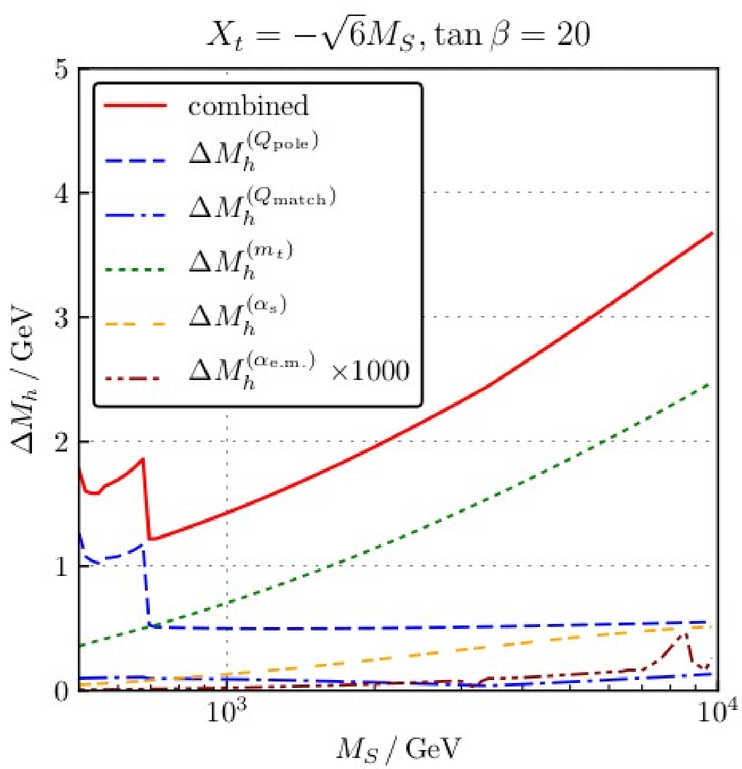


Fig. 2 Individual sources of uncertainty of the three-loop fixed order $\overline{\rm DR}'$ Higgs boson mass prediction of SOFTSUSY.

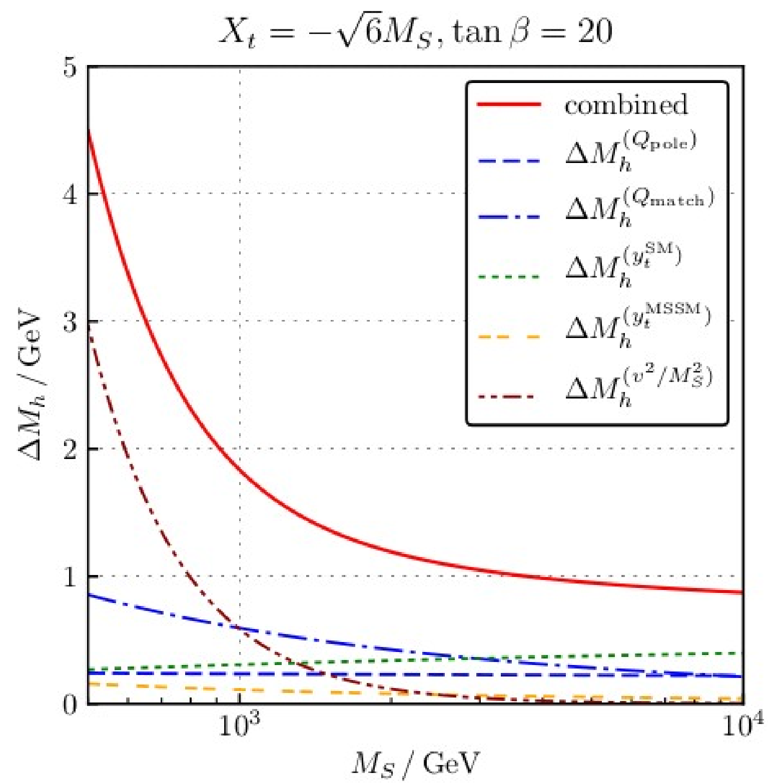


Fig. 3 Individual sources of uncertainty of the two-loop EFT Higgs boson mass prediction of HSSUSY.

B. C. Allanach and A. Voigt (2018)

Scheme Conversion

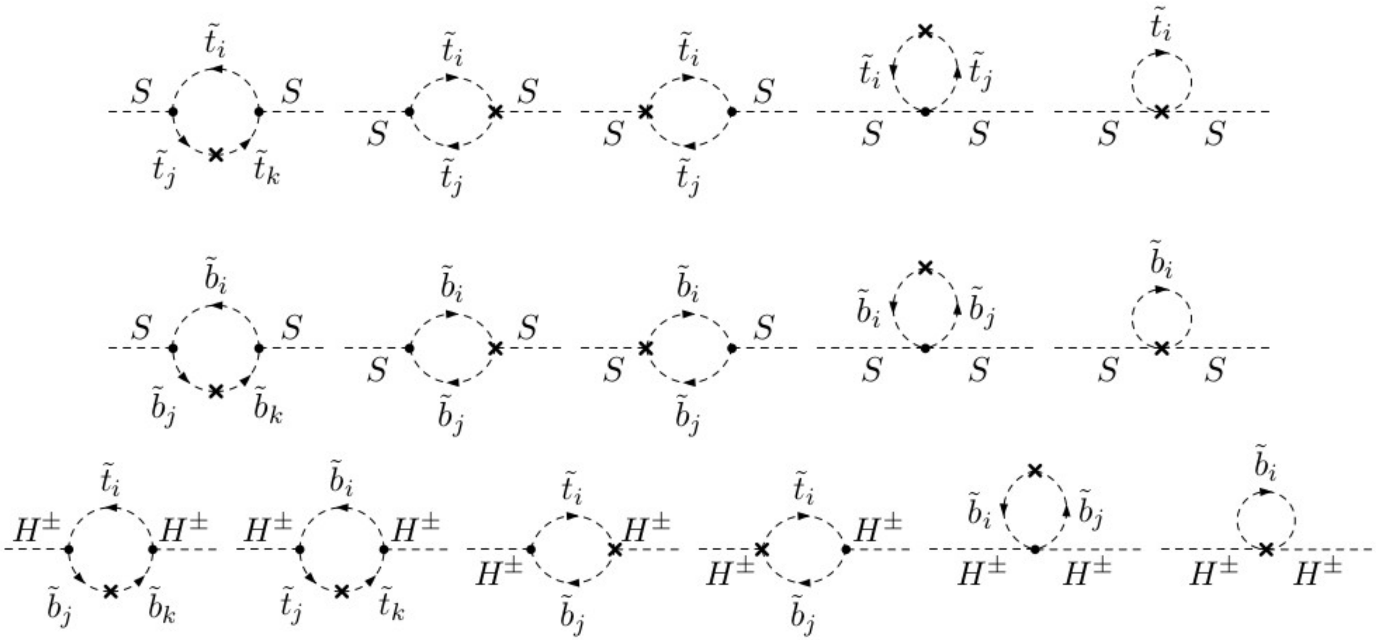


Figure 1: Generic two-loop subloop-renormalization diagrams appearing in the calculation of the $\overline{\text{DR}}$ shifts ($S = h, H, A$ and $i, j, k = 1, 2$). Due to the $SU(2)_L$ symmetry that relates the stop and sbottom sectors, also the diagrams containing only bottom squarks yield contributions involving stop counterterms.

$$\hat{\Sigma}(X_t^{\overline{\text{DR}}}(M_S)) = \hat{\Sigma}(X_t^{\text{OS}}) + \left(\frac{\partial}{\partial X_t}\hat{\Sigma}\right) \cdot \delta^{\text{OS}} X_t(M_S)\Big|_{\text{fin}}$$

Maximal stop-mixing scenario

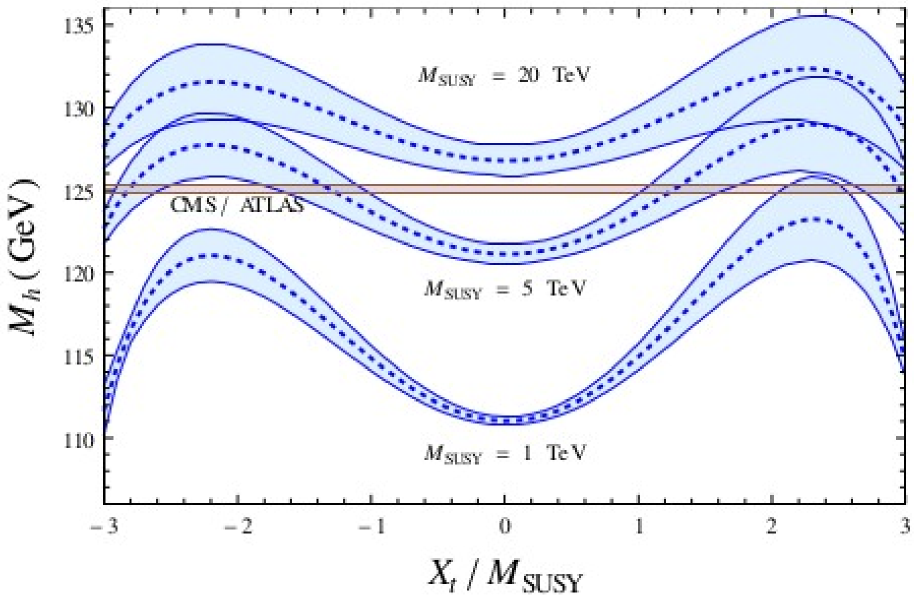


Figure 4-4.: Dependence of M_h on X_t/M_{SUSY} in the heavy SUSY limit evaluated at the same kinematic point considered in Figure **4-3** with $tan\beta = 10$ and $M_{SUSY} = 1$ TeV, 5 TeV and 20 TeV. The blue lines represent the NNLL predictions coming from FeynHiggs and the blue bands are their corresponding theoretical uncertainties.

Dispersion relations

$$\Delta I_{db,fin}(s,m_1^2,m_2^2,m_3^2,m_4^2) = \Delta B_{0,m_1}\left(s,m_1^2,m_2^2\right) \mathrm{Re}\left[B0\left(s,m_3^2,m_4^2\right) - B0\left(s,0,0\right)\right]$$
$$\text{-}\ \Delta B_{0,m_1}\left(s,m_1^2,0\right) \mathrm{Re}\left[B0\left(s,m_3^2,0\right) + B0\left(s,m_4^2,0\right) - 2B0\left(s,0,0\right)\right]$$
$$+\ \mathrm{Re}\left[B_{0,m_1}\left(s,m_1^2,m_2^2\right)\right]\left(\Delta B0\left(s,m_3^2,m_4^2\right) - \Delta B0\left(s,0,0\right)\right)$$
$$\text{-}\ \mathrm{Re}\left[B_{0,m_1}\left(s,m_1^2,0\right)\right]\left(\Delta B0\left(s,m_3^2,0\right) + \Delta B0\left(s,m_4^2,0\right) - 2\Delta B0\left(s,0,0\right)\right). \qquad (3\text{-}31)$$

$\Delta B0$ and $\Delta B_{0,m_j}$ are the discontinuities of the scalar one-loop self-energy function, $B0$, and its mass derivative, $B_{0,m_j} = \frac{\partial}{\partial m_j^2} B0$, given by

$$\Delta B0\left(s,m_a^2,m_b^2\right) = \frac{1}{s}\lambda\left(s,m_a^2,m_b^2\right)\Theta\left(s-(m_a+m_b)^2\right), \qquad (3\text{-}32)$$

$$\Delta B_{0,m_1}\left(s,m_a^2,m_b^2\right) = \frac{m_a^2-m_b^2-s}{s\lambda\left(s,m_a^2,m_b^2\right)}\Theta\left(s-(m_a+m_b)^2\right). \qquad (3\text{-}33)$$

Here $\lambda(x,y,z)$ is the Källen function defined as

$$\lambda(x,y,z) = \sqrt{x^2+y^2+z^2-2(xy+yz+zx)} \qquad (3\text{-}34)$$

Regularization Schemes

- define D–dim. reg. schemes according to different treatment of internal and external vector fields (propagator numerators, polarization sums, ...)

$$Q4S \supset QDS \supset 4S$$

	CDR	HV	DRED	FDH
internal vector fields	$\hat{g}^{\mu\nu}$	$\hat{g}^{\mu\nu}$	$g^{\mu\nu}$	$g^{\mu\nu}$
external vector fields	$\hat{g}^{\mu\nu}$	$\bar{g}^{\mu\nu}$	$g^{\mu\nu}$	$\bar{g}^{\mu\nu}$

- two 'four'-dim. spaces
  - compatible with SUSY: $g^{\mu\nu}$, $\bar{g}^{\mu\nu}$
  - compatible with index counting/helicity methods: $\bar{g}^{\mu\nu}$