Solving the Simplest SUSY RG Flow

w/ L. Fitzpatrick, E. Katz, Y. Xin

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Menu:

1) Motivation (Why)
2) SGNY Model (What)
3) Conformal Truncation (How)
4) Results (Wow)

Big Picture:

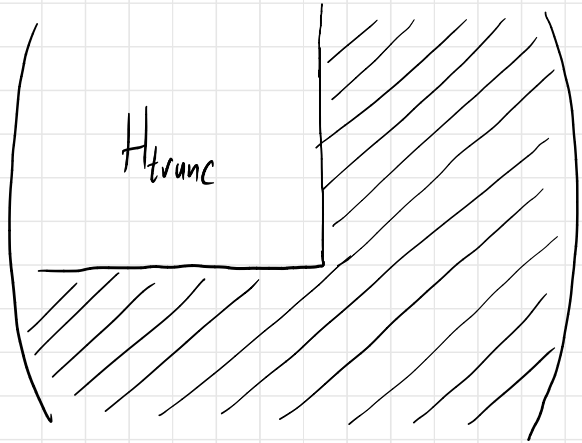
UV CFT $+ \lambda \mathcal{O}_R$

$\downarrow$

IR QFT

1) What does this actually mean?
2) Can I use this to get quantitative results?

Idea: Hamiltonian truncation

$$H = H_0 + \lambda V =$$ 

Know eigenstates

$$\Rightarrow \boxed{\text{Diagonalize } H_{trunc}}$$

Natural basis for UV CFT: $\boxed{\text{Primary Operators}}$

Basis in momentum space: $\quad |\mathcal{O}(p)\rangle \equiv \int d^dx \, e^{-ip\cdot x}\, \mathcal{O}(x)|0\rangle$

Organize basis by scaling dimension + truncate $\quad \boxed{\Delta \leq \Delta_{max}} \quad$ (all spin)

As Δ_{max} increases, approximation improves

$$S = S_{CFT} + \lambda \int d^dx \, \mathcal{O}_R(x) \quad \Rightarrow \quad H = \underbrace{H_{CFT}}_{H_0} + \underbrace{\lambda \int d^{d-1}x \, \mathcal{O}_R(x)}_{V}$$

$$\langle \mathcal{O}(p)|V|\mathcal{O}'(p')\rangle = C_{\mathcal{O}\mathcal{O}'\mathcal{O}_R}\, M^{\mathcal{O}_R}_{\mathcal{O}\mathcal{O}'}(p,p')$$

OPE coefficient

Kinematic function of Δ's + spins

Back to Big Picture:

$$\bullet\ \text{UV CFT} + \lambda \mathcal{O}_R \Longleftarrow \Delta_i, C_{ijk}$$

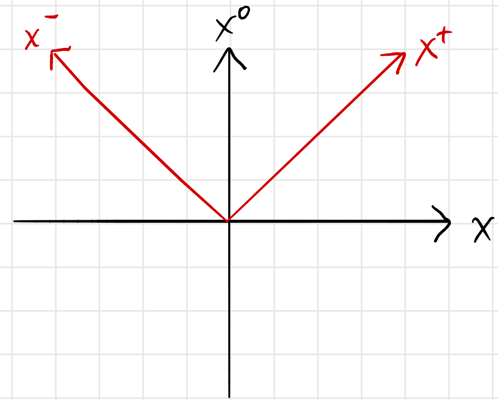
$$\downarrow$$

$$\text{IR QFT} \Longrightarrow m_i^2,\ |\psi_i\rangle,\ \rho_{\mathcal{O}}(\mu),\ S(s,t,u),\ \ldots$$

Tool for turning CFT data into quantitative calculations of real-time, infinite-volume dynamics in strongly-coupled QFT

Important but Technical Point:

Need to choose quantization scheme



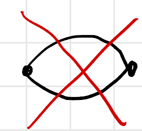
LC quantization: $x^\pm \equiv x^0 \pm x^1$

$$p_\pm = p_0 \pm p_1$$

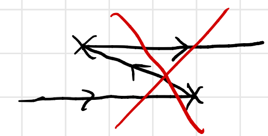
$$p^2 = p_+ p_- - |\vec{p}_\perp|^2$$

$$\boxed{p_+ : \text{ Hamiltonian} \qquad p_-, \vec{p}_\perp : \text{ ``Spatial'' Momentum}}$$

Why? 1) No vacuum renormalization $\Rightarrow$ Infinite volume



2) Selection rules simplify Hamiltonian



Ex: $\mathcal{O}_R = m^2 \phi^2$ $\quad \langle 0 | V | \phi^2 \rangle = 0, \quad \langle \phi | V | \phi^3 \rangle = 0$

SUSY Gross-Neveu-Yukawa (SGNY) Model: $\quad (d=1+1)$

$\mathcal{N}=(1,1)$ SUSY: Q_+, Q_-

Single real superfield: $\underbrace{\Phi = \phi}_{\text{Real scalar}} + \theta\psi + \bar{\theta}\chi + \theta\bar{\theta}F \qquad (\theta \equiv \theta^-, \ \bar{\theta} \equiv \theta^+)$

Left- + right-moving fermions

$$\boxed{W = h\,\Phi + \frac{g}{3!}\,\Phi^3} \quad \Rightarrow \quad V = \frac{1}{2}\left(W'(\phi)\right)^2 + W''(\phi)\,\psi\chi$$

$$= \boxed{\frac{g^2}{2}\left(\frac{h}{g} + \frac{1}{2}\phi^2\right)^2 + g\phi\psi\chi}$$

$\mathbb{Z}_2$ symmetry: $\phi \to -\phi, \quad \psi \to -\psi, \quad \chi \to +\chi$

Two Phases
$$\begin{cases} \dfrac{h}{g} > 0: \ \langle W'(\phi)\rangle \neq 0 \ \Rightarrow \ \cancel{\text{SUSY}}, \ \mathbb{Z}_2 \\[2ex] \dfrac{h}{g} < 0: \ \langle \phi \rangle \neq 0 \ \Rightarrow \ \text{SUSY}, \ \cancel{\mathbb{Z}_2} \end{cases}$$

Can shift $\Phi \to \Phi + c$ to eliminate tadpole:

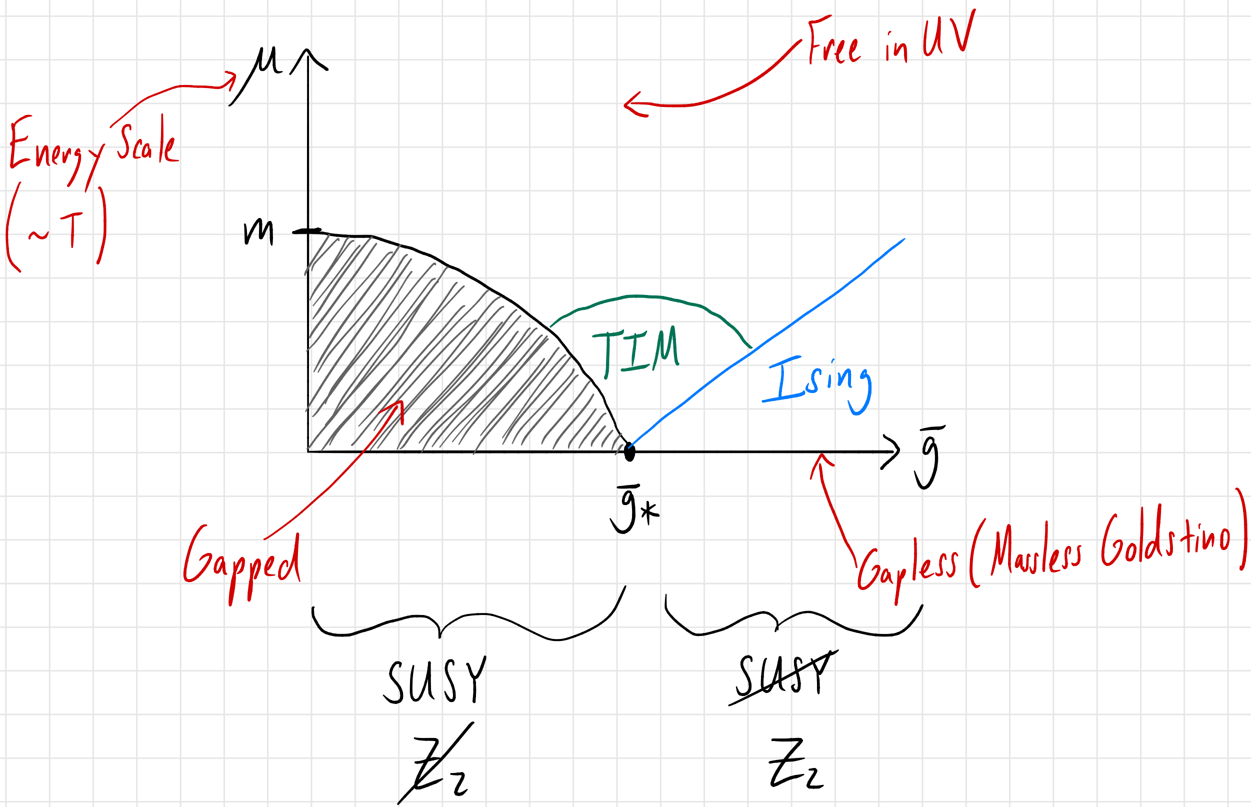
$$\boxed{W(\Phi) = \frac{1}{2} m \Phi + \frac{g}{3!} \Phi^3} \Rightarrow \boxed{V = \frac{1}{2}(m\phi + g\phi^2)^2 + (m + g\phi)\psi\chi}$$

$$\bar{g} \equiv \frac{g}{m}$$

$\bar{g} < \bar{g}_*$: SUSY, $\cancel{\mathbb{Z}_2}$

$\bar{g} > \bar{g}_*$: $\cancel{\text{SUSY}}$, $\mathbb{Z}_2$

Phase Diagram:



Near $\bar{g}_*$: $\boxed{\text{Tricritical Ising Model (TIM)}}$

Minimal model w/ $c = \frac{7}{10}$

6 Virasoro primaries:

$\mathbb{1}, \varepsilon, \boxed{\varepsilon'}, \varepsilon'', \sigma, \sigma'$

Vacancy deformation, $\Delta_{\varepsilon'} = \frac{6}{5}$

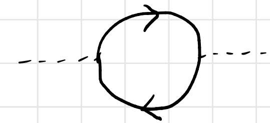
$$\Phi_{TIM} = \varepsilon + \theta\psi + \bar{\theta}\chi + \theta\bar{\theta}\varepsilon' \quad (\neq \Phi_{SGNY})$$

$$\boxed{SGNY\Big|_{g_*} + (g_* - g)\,\Phi^3 \approx TIM + m_{gap}^{4/5}\,\varepsilon'} \qquad \begin{pmatrix} g \to g_* \\ \mu \to 0 \end{pmatrix}$$

IR theory is integrable! (full SGNY isn't)

UV		IR
ϕ	$\Longrightarrow$	ε
$W'(\phi)$	$\Longrightarrow$	ε'
$(\partial\phi)^2\psi\chi$	$\Longrightarrow$	ε''

Naively: Construct P_+, Truncate in Δ, + Diagonalize

BUT: UV Divergences  + 

- Should cancel, but do not @ finite truncation
- Must add counterterm + tune for each value of coupling + Δ_{max}
- SUSY not manifest in matrix elements or results

Alternative: $P_\pm = Q_\pm^2 \implies$ Truncate Q_+ + Diagonalize Q_+^2 !

$$Q_- = \int dx^- \, \psi \partial_- \phi \qquad\qquad Q_+ = \int dx^- \, \psi W'(\phi) = \int dx^- \, \psi (m\phi + g\phi^2)$$

Schematically:

$$Q_+ = \begin{array}{cc|l} \text{Bosons} & \text{Fermions} & \\ \left(\begin{array}{c|c} 0 & Q_{BF} \\ \hline Q_{FB} & 0 \end{array}\right. & \left.\right) & \begin{array}{l} \text{Bosons} \\ \text{Fermions} \end{array} \end{array}$$

$$\Rightarrow \quad P_+ = Q_+^2 = \left(\begin{array}{c|c} Q_{BF}Q_{FB} & 0 \\ \hline 0 & Q_{FB}Q_{BF} \end{array}\right)$$

$\Rightarrow$ Exact degeneracy between bosons + fermions!

- Divergences perfectly cancel, even at finite truncation

Basis: Operators of the form $\boxed{\mathcal{O}(x) = \sum_{\{k,l\}} C_{k,l}\, \partial_-^{k_1}\phi \cdots \partial_-^{k_m}\phi\, \partial_-^{l_1}\psi \cdots \partial_-^{l_n}\psi}$

$$\partial_+\psi = 0\,, \quad \partial_+\phi = 0\,, \quad \chi = 0$$

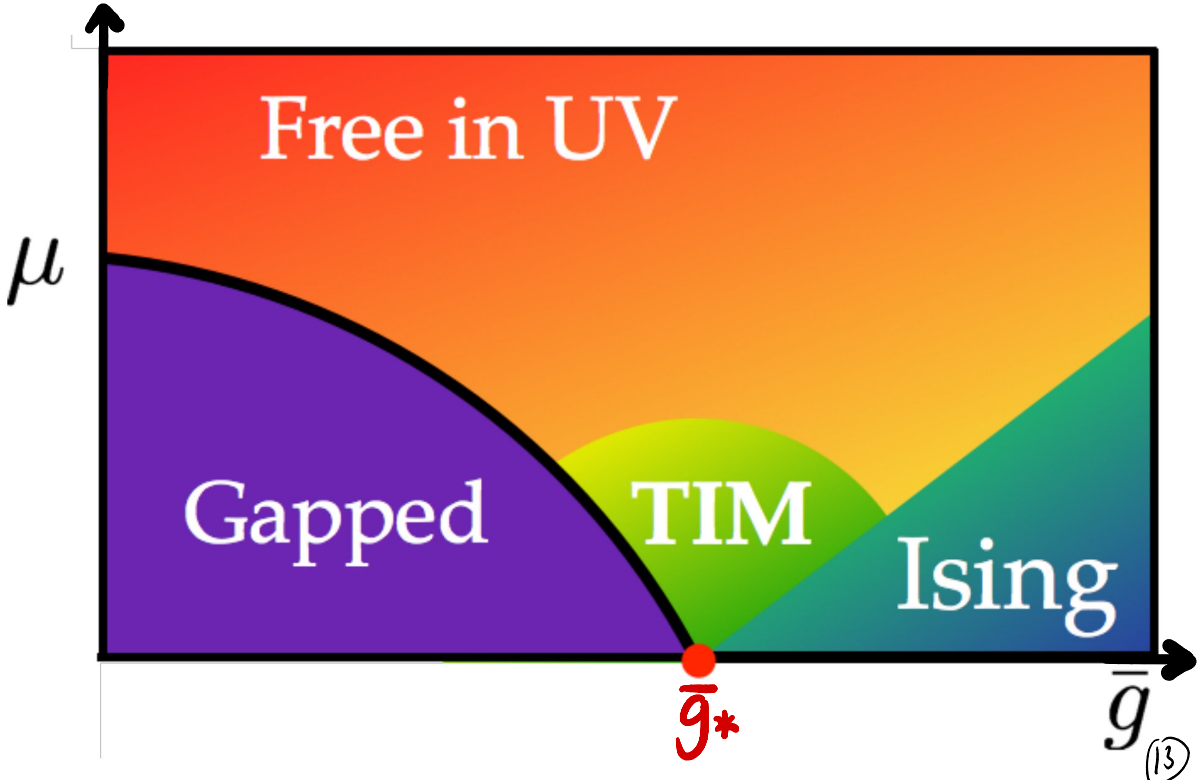
Left-moving

LC Quantization (Right-moving modes non-dynamical)

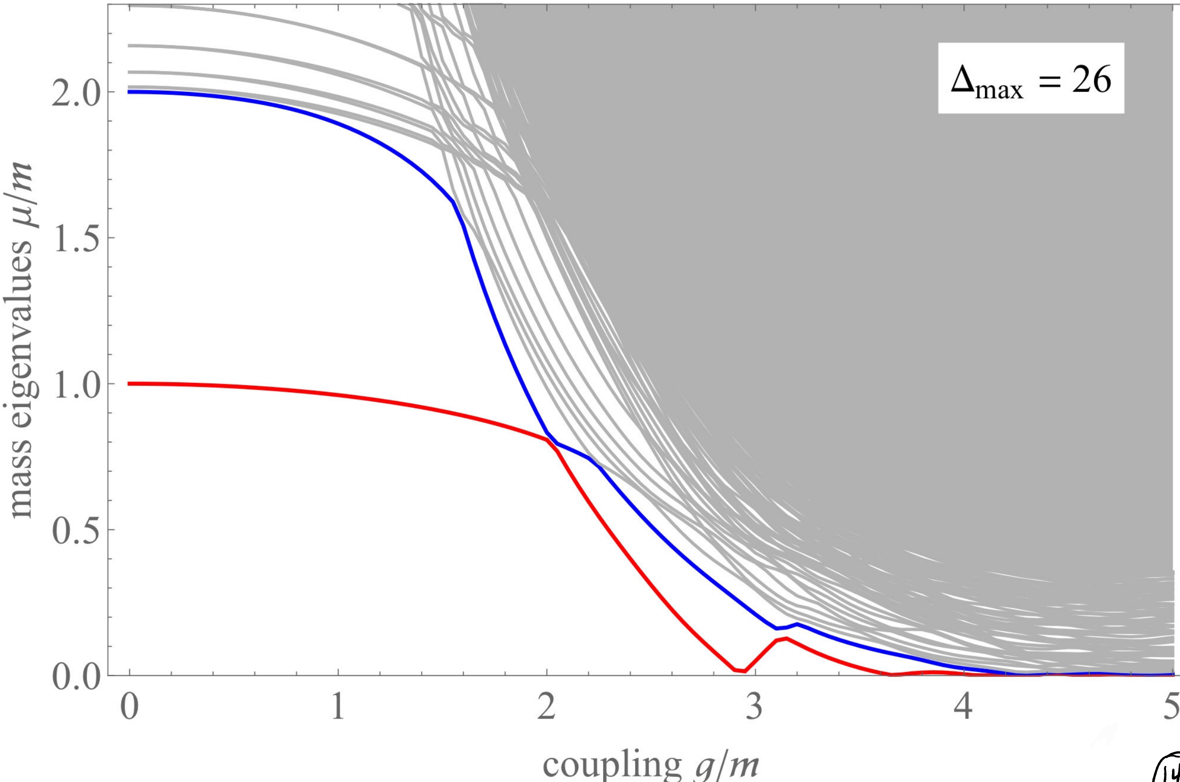
Matrix Elements: for Q_+, three kinds of terms

$m\phi\psi$ $\qquad$ $g\phi^2\psi$

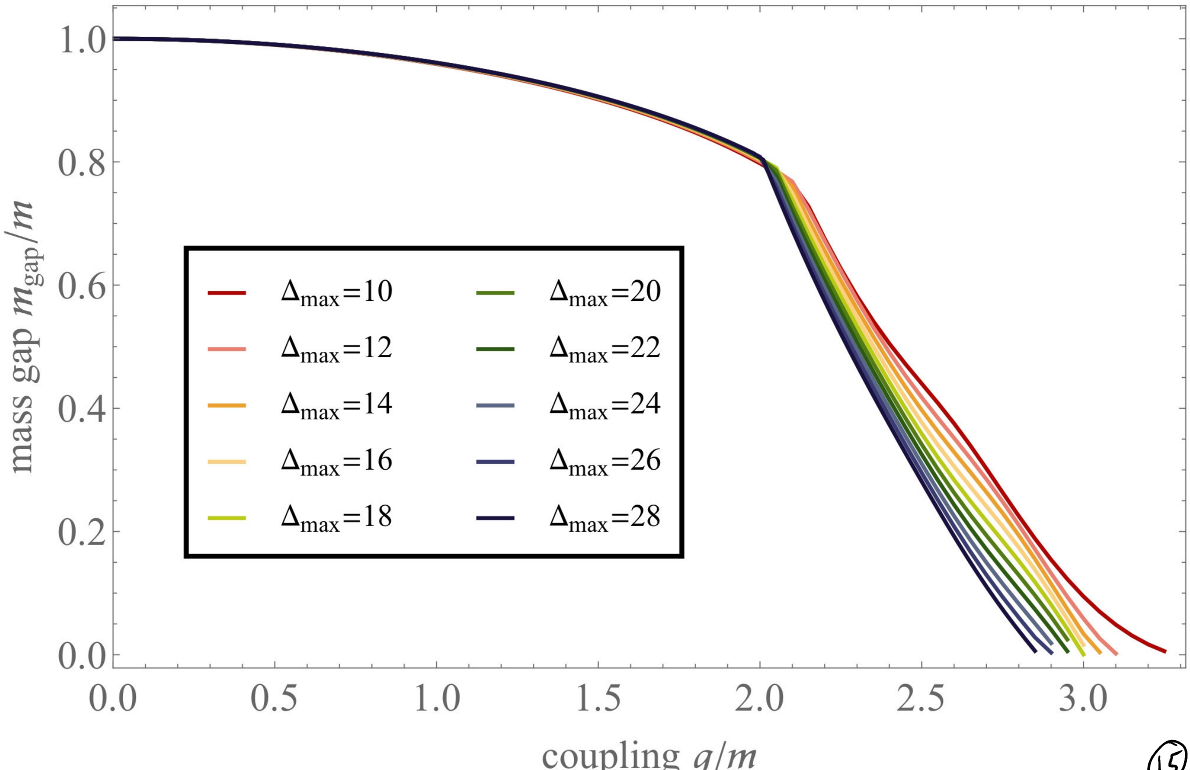
Truncate @ Δ_{max}, Diagonalize Q_+^2 to obtain mass eigenstates $|\mu_i\rangle$



Free in UV
Gapped
TIM
Ising
μ
$\bar{g}$
$\bar{g}_*$
13



$\Delta_{\max} = 26$
mass eigenvalues μ/m
coupling g/m
0.0
0.5
1.0
1.5
2.0
0
1
2
3
4
5

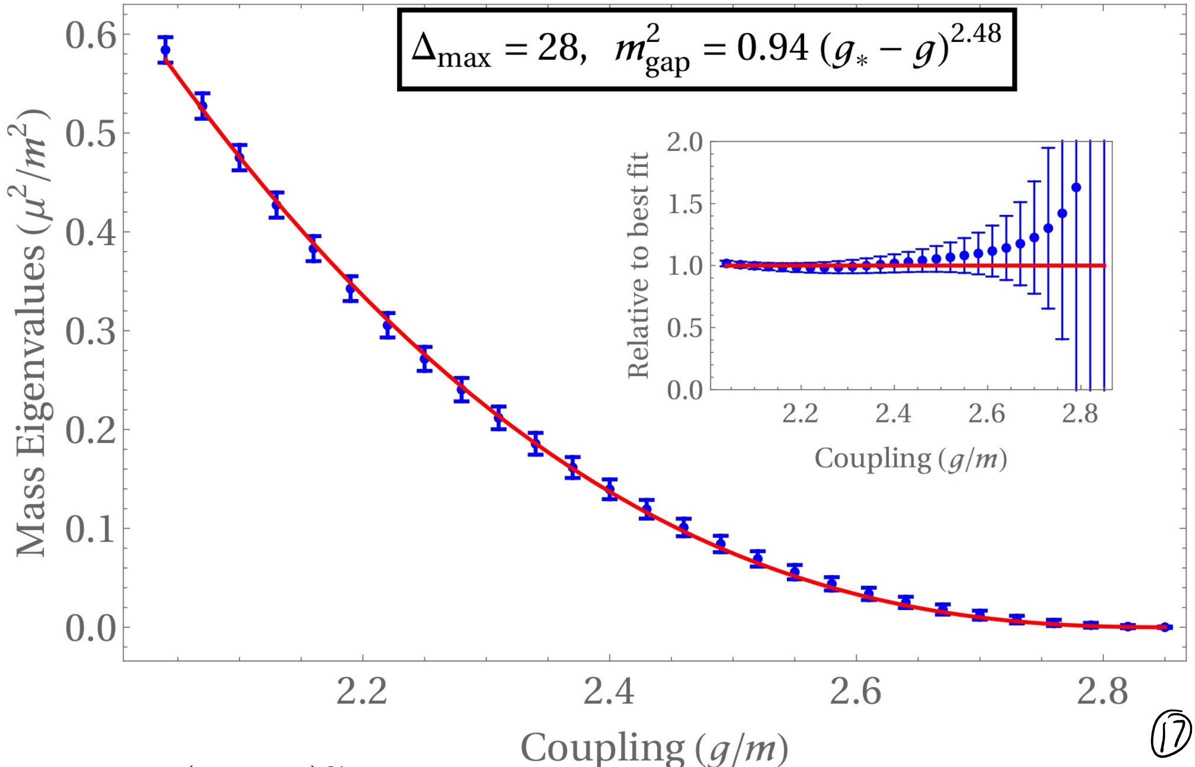


mass gap m_{gap}/m
coupling g/m
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\Delta_{\mathrm{max}}=10$
$\Delta_{\mathrm{max}}=12$
$\Delta_{\mathrm{max}}=14$
$\Delta_{\mathrm{max}}=16$
$\Delta_{\mathrm{max}}=18$
$\Delta_{\mathrm{max}}=20$
$\Delta_{\mathrm{max}}=22$
$\Delta_{\mathrm{max}}=24$
$\Delta_{\mathrm{max}}=26$
$\Delta_{\mathrm{max}}=28$

Critical Exponent:

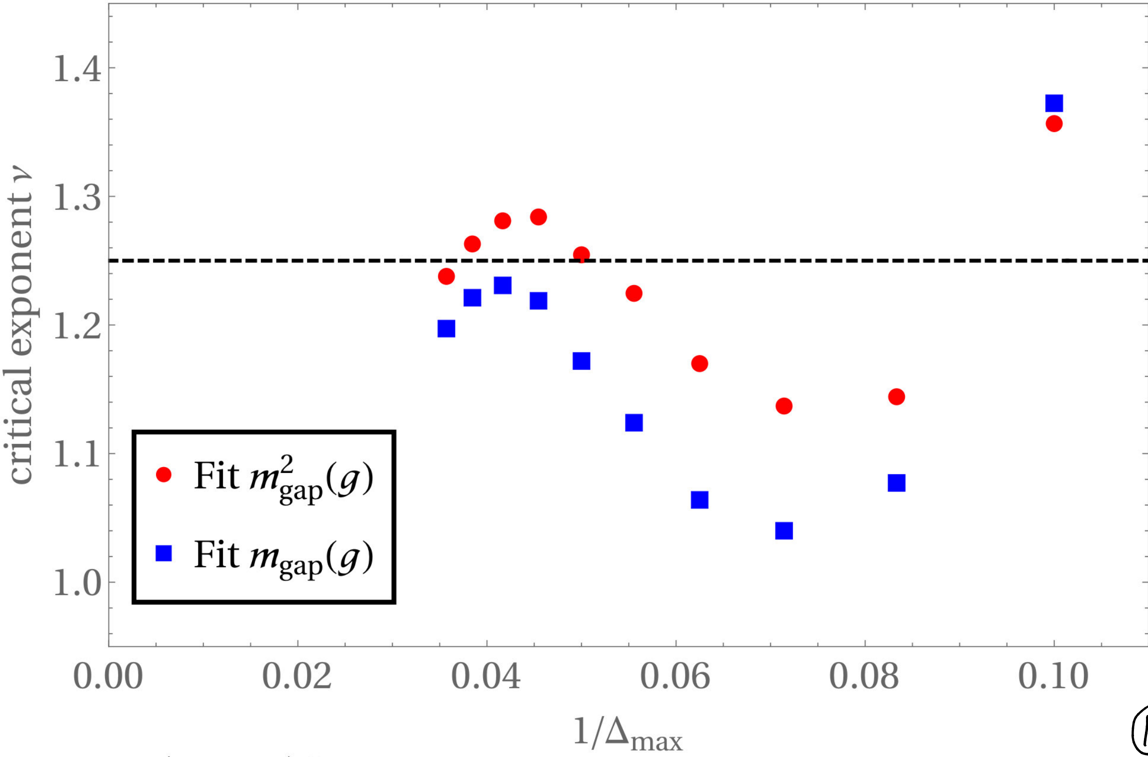
$$(g_* - g)\,\Phi^3 \sim m_{gap}^{2-\Delta_{\varepsilon'}}\,\varepsilon' \quad \Longrightarrow \quad m_{gap} \sim (g_* - g)^{\nu}$$

$$\Delta_{\varepsilon'} = \frac{6}{5} \quad \Longrightarrow \quad \nu = \frac{1}{2-\Delta_{\varepsilon'}} = \boxed{\frac{5}{4}}$$



$m_{\rm gap} \sim (g_* - g)^{\nu}$

$\nu_{\rm TIM} = 1.25$



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$m_{\mathrm{gap}} \sim (g_* - g)^\nu$ $\nu_{\mathrm{TIM}} = 1.25$

Spectral Densities:

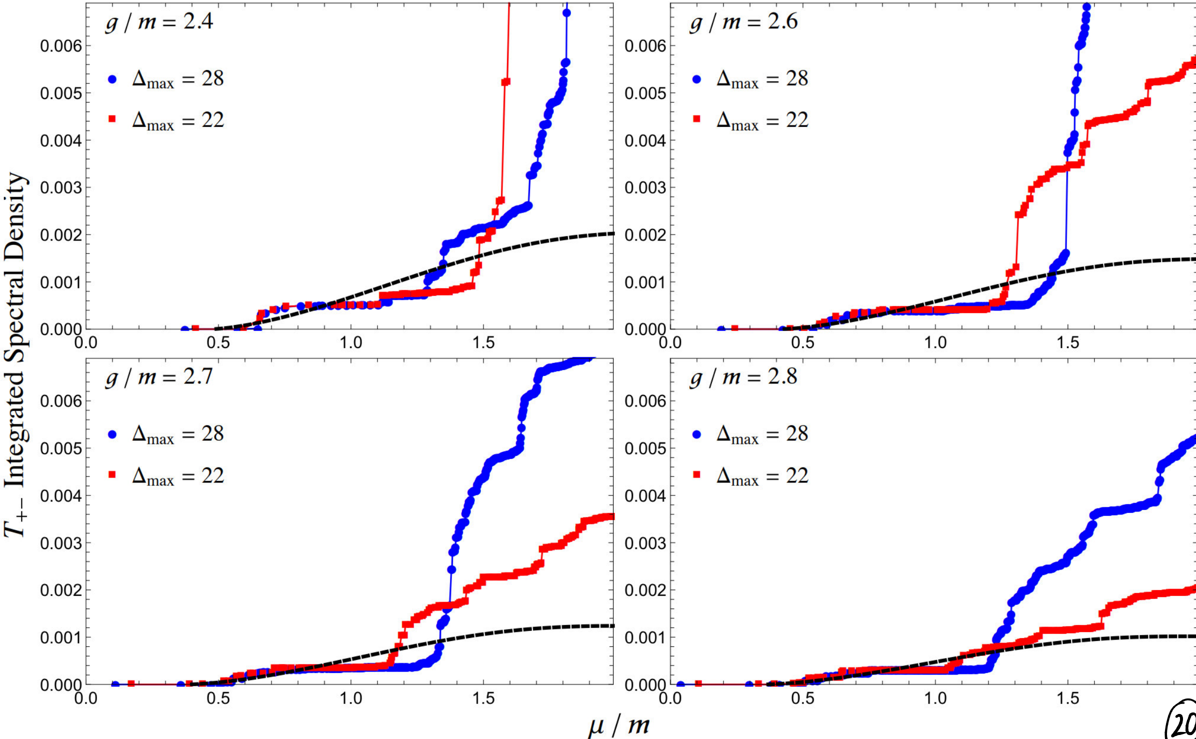
$$\rho_{\mathcal{O}}(\mu) = \sum_i \left|\langle \mu_i | \mathcal{O}(0) \rangle\right|^2 \delta(\mu^2 - \mu_i^2)$$

Simpler to study integrated spectral densities:

$$I_{\mathcal{O}}(\mu) \equiv \int_0^{\mu^2} d\mu'^2 \, \rho_{\mathcal{O}}(\mu') = \sum_{\mu_i \leq \mu} \left|\langle \mu_i | \mathcal{O}(0) \rangle\right|^2$$

or even double-integrated spectral densities:

$$I^{(2)}_{\mathcal{O}}(\mu) \equiv \int_0^{\mu^2} d\mu'^2 \, I_{\mathcal{O}}(\mu')$$



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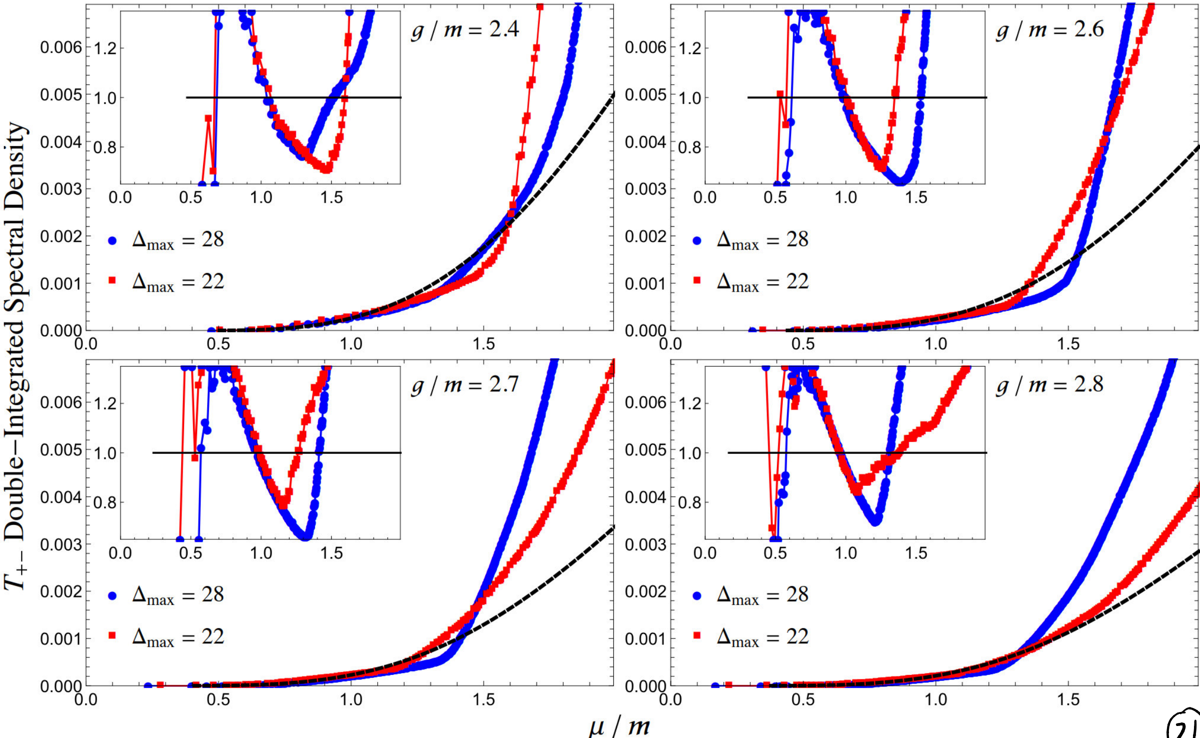
$$T_{+-}^{\mathrm{TIM}} \approx m_{\mathrm{gap}}^{4/5}\varepsilon' + \frac{1}{\Lambda^{6/5}}\partial^2\varepsilon'$$

$$\frac{\Lambda}{m} \approx 10$$



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$$\frac{\Lambda}{m} \approx 10$$

Zamolodchikov C-function:

$$C(\mu) = \frac{12\pi}{p_-^4}\int_0^{\mu^2} d\mu'^2\, \rho_{T_{--}}(\mu')$$

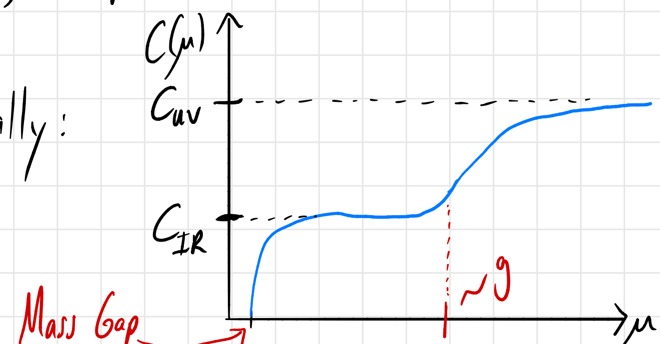
Central charge $(\langle TT\rangle \sim c)$

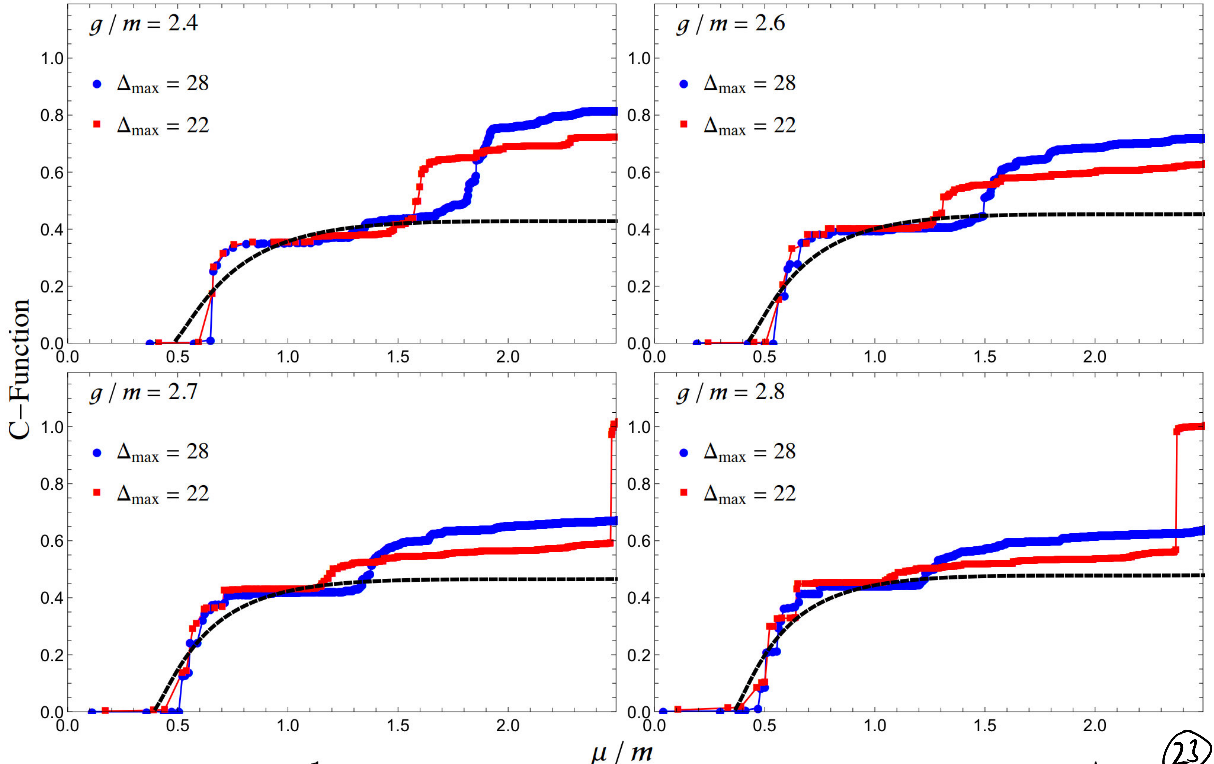
At CFT: $\rho_{T_{--}}(\mu) \sim c\,\delta(\mu^2)$

$C(\mu) \to c_{IR} \quad (\mu \to 0)$ $\qquad$ $C(\mu) \to c_{UV} \quad (\mu \to \infty)$

μ parametrizes RG flow

Schematically:





$$T_{+-}^{\mathrm{TIM}} \approx m_{\mathrm{gap}}^{4/5}\varepsilon' + \frac{1}{\Lambda^{6/5}}\partial^2\varepsilon'$$

$$\frac{\Lambda}{m} \approx 10$$

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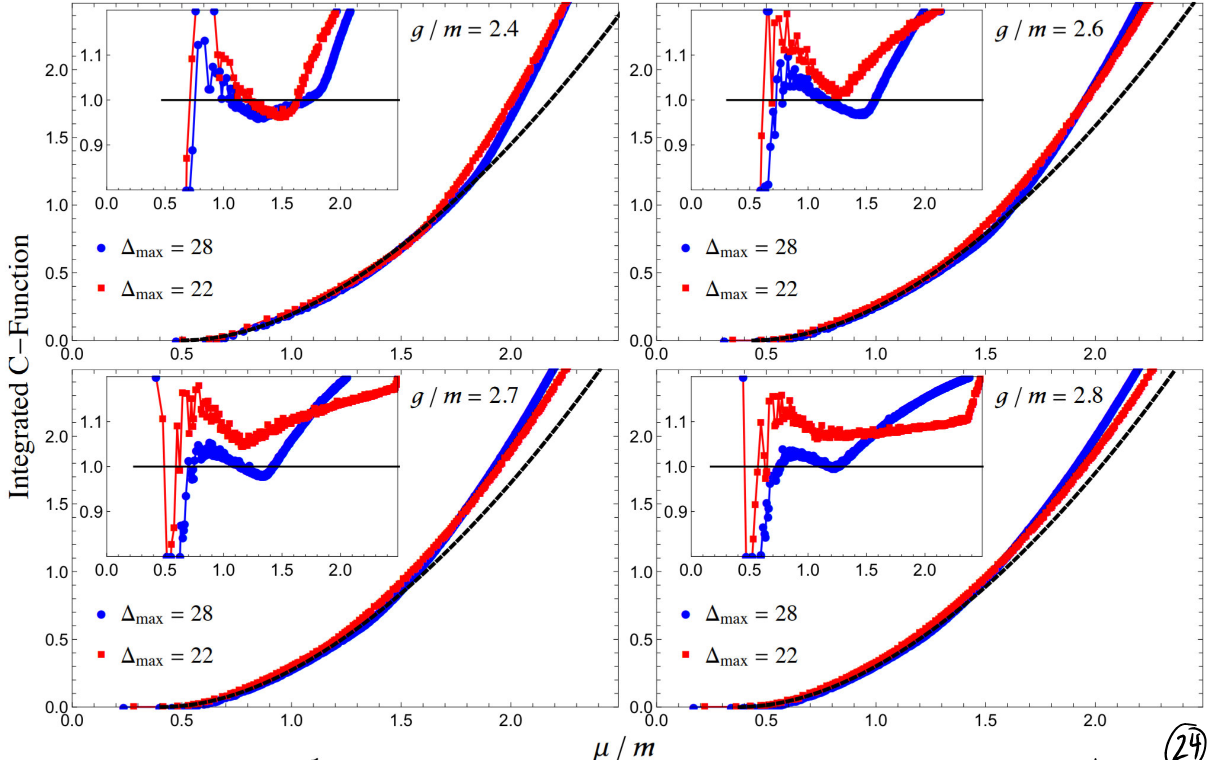

$$T_{+-}^{\text{TIM}} \approx m_{\text{gap}}^{4/5}\varepsilon' + \frac{1}{\Lambda^{6/5}}\partial^2\varepsilon'$$

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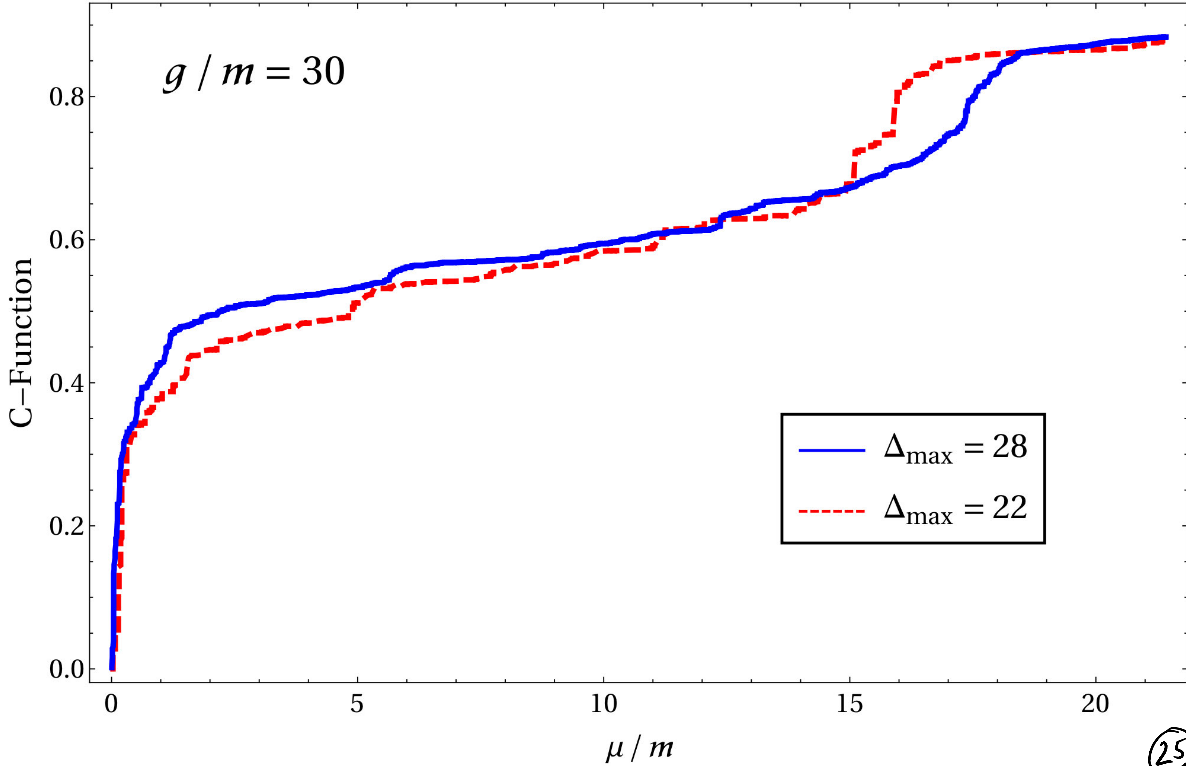
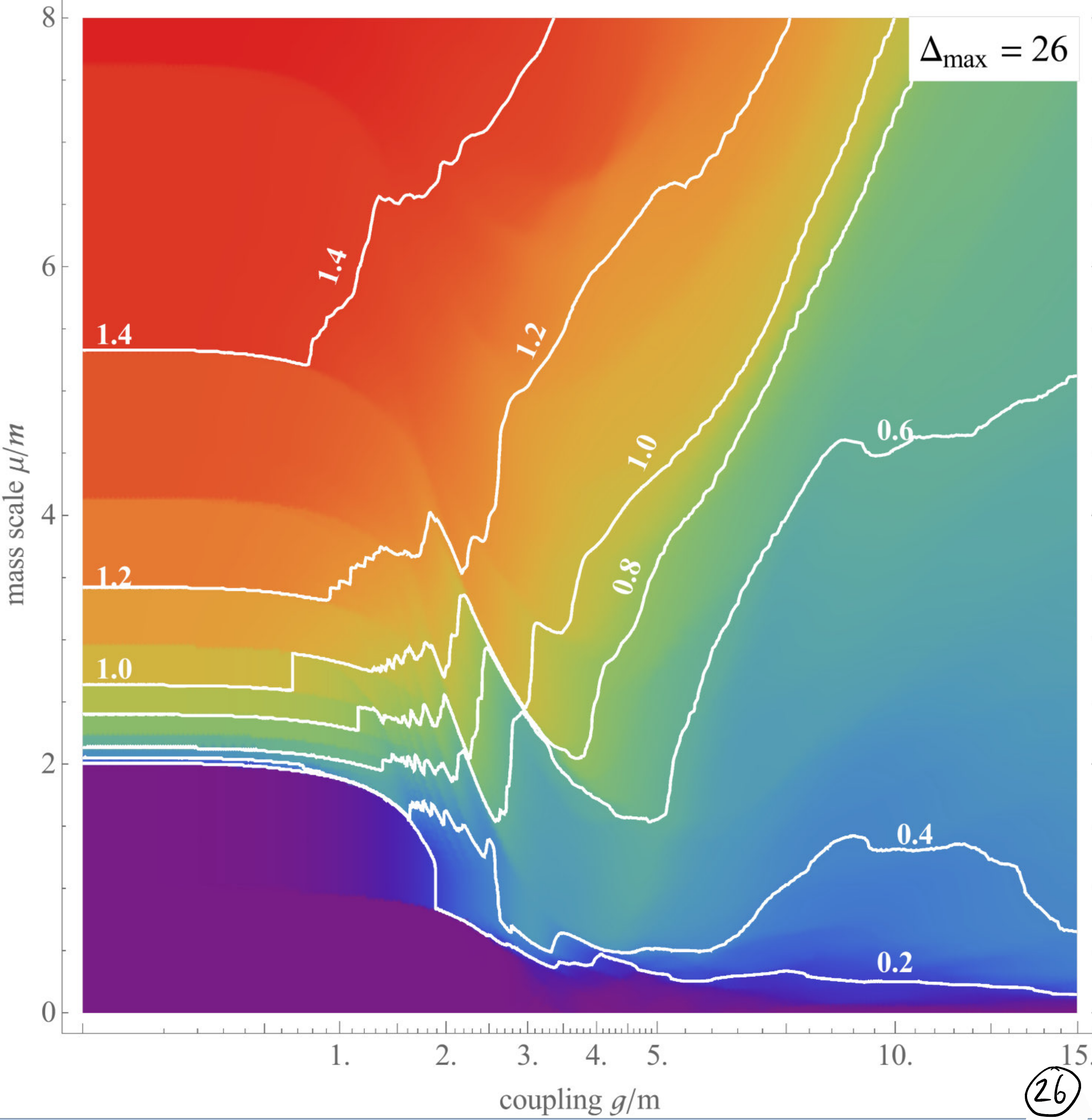

$g / m = 30$
C–Function
0.0
0.2
0.4
0.6
0.8
0
5
10
15
20
μ / m
$\Delta_{\max} = 28$
$\Delta_{\max} = 22$



Δmax = 26
1.4
1.4
1.2
1.2
1.0
1.0
0.8
0.6
0.4
0.2
mass scale μ/m
coupling g/m
8
6
4
2
0
1.
2.
3.
4.
5.
10.
15.

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Moving Forward

- Apply to theories in higher dimensions

  1) SGNY in $d = 2+1$: conceptually very similar, flows to $\mathcal{N}=1$ SCFT at critical coupling; behavior away from critical pt. not known

  2) $\mathcal{N}=4$ SYM $\Rightarrow$ $\mathcal{N}=2^*$ or $\mathcal{N}=1^*$ in $d=3+1$: study gauge theories w/o need for dim-reg or alternative regulator, useful bridge to pure YM, QCD, etc.

- Generalize "SUSY tricks" to non-SUSY theories

  Ex: write P_+ as "Q_+^2" w/ fictitious fields running in middle, allows us to control/remove UV divergences